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**CS 5154**

**Logic Coverage**

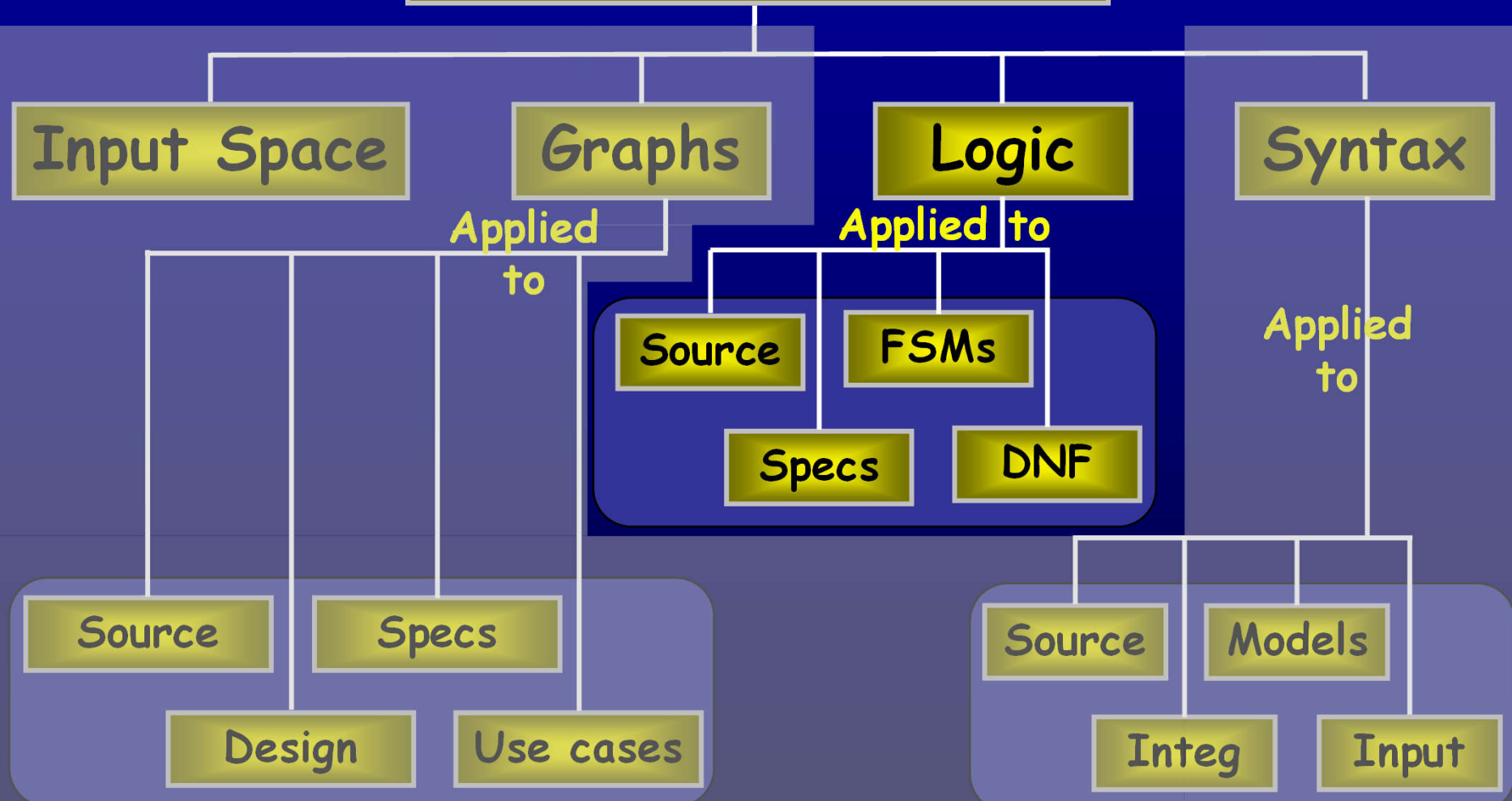
Owolabi Legunsen

**The following are modified versions of the publicly-available slides for Chapter 8  
in the Ammann and Offutt Book, “Introduction to Software Testing”**

(<http://www.cs.gmu.edu/~offutt/softwaretest>)

# Logic Coverage

## Four Structures for Modeling Software



# Semantic Logic Criteria

- Logic expressions show up in many situations
- Covering logic expressions is required by the US Federal Aviation Administration for safety critical software
- Logical expressions can come from many sources
  - Decisions in programs
  - FSMs and statecharts
  - Requirements
- Tests are intended to choose some subset of the total number of truth assignments to the expressions

# Logic Predicates and Clauses

- **Predicate** : an expression that evaluates to a **boolean** value
- Predicates can contain
  - **boolean variables**
  - non-boolean variables that are related by  $>$ ,  $<$ ,  $==$ ,  $>=$ ,  $<=$ ,  $!=$
  - **function** calls that return booleans
- Internal structure is created by logical operators
  - $\neg$  – the *negation* operator
  - $\wedge$  – the *and* operator
  - $\vee$  – the *or* operator
  - $\rightarrow$  – the *implication* operator
  - $\oplus$  – the *exclusive or* operator
  - $\leftrightarrow$  – the *equivalence* operator
- A **clause** is a predicate with no logical operators

# Example and Facts

- $(a < b) \vee f(z) \wedge D \wedge (m \geq n * o)$  has four clauses:
  - $(a < b)$  – relational expression
  - $f(z)$  – boolean-valued function
  - $D$  – boolean variable
  - $(m \geq n * o)$  – relational expression
- Most predicates have **few clauses**
  - 88.5% have 1 clause
  - 9.5% have 2 clauses
  - 1.35% have 3 clauses
  - Only 0.65% have 4 or more !
- **Sources** of predicates
  - Decisions in **programs**
  - Guards in **finite state machines**
  - Decisions in **UML** activity graphs
  - **Requirements**, both formal and informal
  - **SQL** queries

*from a study of non-FAA, 63 open-source programs with >400,000 predicates*

# Translating from English

- “I am interested in CS 5154 and CS 5150”
- $course == cs5154 \text{ OR } course == cs5150$

Humans have  
trouble translating  
from English to logic

- “If you leave before 6:30 AM, take Braddock to 495, if you leave after 7:00 AM, take Prosperity to 50, then 50 to 495”
- $(time < 6:30 \rightarrow path = Braddock) \wedge (time > 7:00 \rightarrow path = Prosperity)$
- Hmm ... this is incomplete !
- $(time < 6:30 \rightarrow path = Braddock) \wedge (time \geq 6:30 \rightarrow path = Prosperity)$

# Logic Coverage Criteria

- We use predicates in testing as follows :
  - Developing a model of the software as a set of predicates
  - Requiring tests to satisfy some combination of clauses
- Abbreviations that we will use in later slides:
  - $P$  is the set of predicates
  - $p$  is a single predicate in  $P$
  - $C$  is the set of clauses in  $P$
  - $C_p$  is the set of clauses in predicate  $p$
  - $c$  is a single clause in  $C$

# Predicate and Clause Coverage

- The first (and simplest) two criteria require that each predicate and each clause evaluate to both true and false

**Predicate Coverage (PC)** : For each  $p$  in  $P$ ,  $TR$  contains two requirements:  $p$  evaluates to true, and  $p$  evaluates to false.

- If predicates are conditions on edges, PC is equivalent to edge coverage
- PC does not evaluate all the clauses, so ...

**Clause Coverage (CC)** : For each  $c$  in  $C$ ,  $TR$  contains two requirements:  $c$  evaluates to true, and  $c$  evaluates to false.



# Predicate Coverage Example

$$((a < b) \vee D) \wedge (m \geq n * o)$$

predicate coverage

Predicate = true

$$a = 5, b = 10, D = \text{true}, m = 1, n = 1, o = 1$$

$$= (5 < 10) \vee \text{true} \wedge (1 \geq 1 * 1)$$

$$= \text{true} \vee \text{true} \wedge \text{TRUE}$$

$$= \text{true}$$

Predicate = false

$$a = 5, b = 10, D = \text{true}, m = 0, n = 1, o = 1$$

$$= (5 < 10) \vee \text{true} \wedge (0 \geq 1 * 1)$$

$$= \text{true} \vee \text{true} \wedge \text{FALSE}$$

$$= \text{false}$$

# Clause Coverage Example

$$((a < b) \vee D) \wedge (m \geq n * o)$$

Clause coverage

<u><math>(a &lt; b) = \text{true}</math></u> a = 5, b = 10	<u><math>(a &lt; b) = \text{false}</math></u> a = 10, b = 5	<u>D = true</u> D = true	<u>D = false</u> D = false
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<u><math>m \geq n * o = \text{true}</math></u> m = 1, n = 1, o = 1	<u><math>m \geq n * o = \text{false}</math></u> m = 1, n = 2, o = 2
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true cases

- Two tests
- 1) a = 5, b = 10, D = true, m = 1, n = 1, o = 1
  - 2) a = 10, b = 5, D = false, m = 1, n = 2, o = 2

false cases

# Problems with PC and CC

- PC does not **fully exercise** all the clauses, especially in the presence of short circuit evaluation
- CC does not always **ensure PC**
  - That is, we can satisfy CC without causing the predicate to be both true and false
  - Example:  $a \vee b$
  - This is definitely not what we want !
- The simplest solution is to test **all combinations** ...

# Combinatorial Coverage

- CoC requires every possible combination
- Sometimes called Multiple Condition Coverage

**Combinatorial Coverage (CoC) :** For each  $p$  in  $\underline{P}$ , TR has test requirements for the clauses in  $\underline{C}_p$  to evaluate to each possible combination of truth values.

	$a < b$	D	$m \geq n * o$	$((a < b) \vee D) \wedge (m \geq n * o)$
1	T	T	T	T
2	T	T	F	F
3	T	F	T	T
4	T	F	F	F
5	F	T	T	T
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

# Combinatorial Coverage

- CoC is simple, neat, clean, and comprehensive ...
- But quite **expensive!**
- $2^N$  tests, where  $N$  is the number of clauses
  - Impractical for predicates with more than 3 or 4 clauses
- The literature has lots of suggestions – some confusing
- The general idea is simple:

**Test each clause independently from the other clauses**

- Getting the details right is hard
- What exactly does “independently” mean ?
- The book presents this idea as “*making clauses active*” ...

# Active Clauses

- Clause coverage has a **weakness** : The values do not always make a difference
- Consider the first test for **clause coverage**, which caused each clause to be true:
  - $((5 < 10) \vee true) \wedge (1 \geq 1 * 1)$
  - Only the last clause **counts** !
- To really test the results of a clause, the clause should be the **determining factor** in the value of the predicate

## **Determination :**

A clause  $C_i$  in predicate  $p$ , called the *major clause*, **determines**  $p$  if and only if the values of the remaining *minor clauses*  $C_j$  are such that changing  $C_i$  changes the value of  $p$

- This is considered to **make the clause active**

# Determining Predicates

$$\underline{P = A \vee B}$$

if  $B = \text{true}$ ,  $p$  is always true.  
so if  $B = \text{false}$ ,  $A$  determines  $p$ .  
if  $A = \text{false}$ ,  $B$  determines  $p$ .

$$\underline{P = A \wedge B}$$

if  $B = \text{false}$ ,  $p$  is always false.  
so if  $B = \text{true}$ ,  $A$  determines  $p$ .  
if  $A = \text{true}$ ,  $B$  determines  $p$ .

- **Goal** : Find tests for each clause when the clause determines the value of the predicate
- This goal is formalized in a **family of criteria** that have subtle, but very important, differences

# Active Clause Coverage

**Active Clause Coverage (ACC)** : For each  $p$  in  $P$  and each major clause  $c_i$  in  $C_p$ , choose minor clauses  $c_j, j \neq i$ , so that  $c_i$  determines  $p$ . TR has two requirements for each  $c_i$  :  $c_i$  evaluates to true and  $c_i$  evaluates to false.

$p = a \vee b$	
1)	$a = \text{true}, b = \text{false}$
2)	$a = \text{false}, b = \text{false}$
3)	$a = \text{false}, b = \text{true}$
4)	$a = \text{false}, b = \text{false}$

a is major clause

b is major clause

Duplicate

- This is a form of **MCDC**, which is required by the FAA for safety critical software
- **Ambiguity** : Do the minor clauses have to have the **same values** when the major clause is true and when it is false?



# Resolving the Ambiguity

$$p = a \vee (b \wedge c)$$

Major clause : **a**

**a** = true, **b** = false, **c** = true

**a** = false, **b** = false, **c** = false

Is this allowed ?

- This question caused **confusion** among testers for years
- Considering this carefully leads to **three** separate criteria :
  - Minor clauses **do not** need to be the same
  - Minor clauses **do** need to be the same
  - Minor clauses **force the predicate** to become both true and false

# General Active Clause Coverage

**General Active Clause Coverage (GACC)** : For each  $p$  in  $P$  and each major clause  $c_i$  in  $C_p$ , choose minor clauses  $c_j, j \neq i$ , so that  $c_i$  determines  $p$ . TR has two requirements for each  $c_i$  :  $c_i$  evaluates to true and  $c_i$  evaluates to false. The values chosen for the minor clauses  $c_j$  do not need to be the same when  $c_i$  is true as when  $c_i$  is false, that is,  $c_j(c_i = \text{true}) = c_j(c_i = \text{false})$  for all  $c_j$  **OR**  $c_j(c_i = \text{true}) \neq c_j(c_i = \text{false})$  for all  $c_j$ .

- This is **complicated** !
- We can satisfy GACC **without satisfying** predicate coverage
- We **want** to cause predicates to be both true and false

# Restricted Active Clause Coverage

Restricted Active Clause Coverage (RACC) : For each  $p$  in  $P$  and each major clause  $c_i$  in  $C_p$ , choose minor clauses  $c_j, j \neq i$ , so that  $c_i$  determines  $p$ . TR has two requirements for each  $c_i$ :  $c_i$  evaluates to true and  $c_i$  evaluates to false. The values chosen for the minor clauses  $c_j$  must be the same when  $c_i$  is true as when  $c_i$  is false, that is, it is required that  $c_j(c_i = \text{true}) = c_j(c_i = \text{false})$  for all  $c_j$ .

- This was a **common interpretation** by aviation developers
- RACC often leads to **infeasible test requirements**
- There is **no logical reason** for such a restriction

# Correlated Active Clause Coverage

**Correlated Active Clause Coverage (CACCC)** : For each  $p$  in  $P$  and each major clause  $c_i$  in  $C_p$ , choose minor clauses  $c_j, j \neq i$ , so that  $c_i$  determines  $p$ . TR has two requirements for each  $c_i$  :  $c_i$  evaluates to true and  $c_i$  evaluates to false. The values chosen for the minor clauses  $c_j$  must cause  $p$  to be true for one value of the major clause  $c_i$  and false for the other, that is, it is required that  $p(c_i = \text{true}) \neq p(c_i = \text{false})$ .

- A more recent interpretation
- **Implicitly** allows minor clauses to have different values
- Explicitly satisfies (**subsumes**) predicate coverage

# CACC and RACC

	a	b	c	$a \wedge (b \vee c)$
1	T	T	T	T
2	T	T	F	T
3	T	F	T	T
4	T	F	F	F
5	F	T	T	F
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

	a	b	c	$a \wedge (b \vee c)$
1	T	T	T	T
2	T	T	F	T
3	T	F	T	T
4	T	F	F	F
5	F	T	T	F
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

major clause

$P_a : b = \text{true} \text{ or } c = \text{true}$

CACC can be satisfied by choosing any of rows 1, 2, 3 AND any of rows 5, 6, 7 – a total of nine pairs

RACC can only be satisfied by row pairs (1, 5), (2, 6), or (3, 7)  
Only three pairs

# Inactive Clause Coverage

- The active clause coverage criteria ensure that “major” clauses **do affect** the predicates
- Inactive clause coverage takes the opposite approach – major clauses **do not affect** the predicates

**Inactive Clause Coverage (ICC)** : For each  $p$  in  $P$  and each major clause  $c_i$  in  $C_p$ , choose minor clauses  $c_j, j \neq i$ , so that  $c_i$  **does not determine**  $p$ . TR has **four** requirements for each  $c_i$  : (1)  $c_i$  evaluates to true with  $p$  true, (2)  $c_i$  evaluates to false with  $p$  true, (3)  $c_i$  evaluates to true with  $p$  false, and (4)  $c_i$  evaluates to false with  $p$  false.

# General and Restricted ICC

- Unlike ACC, the notion of correlation is not relevant
  - $c_i$  does not determine  $p$ , so cannot correlate with  $p$
- Predicate coverage is always guaranteed

**General Inactive Clause Coverage (GICC)** : For each  $p$  in  $P$  and each major clause  $c_i$  in  $C_p$ , choose minor clauses  $c_j, j \neq i$ , so that  $c_i$  does not determine  $p$ . The values chosen for the minor clauses  $c_j$  do not need to be the same when  $c_i$  is true as when  $c_i$  is false, that is,  $c_j(c_i = \text{true}) = c_j(c_i = \text{false})$  for all  $c_j$  **OR**  $c_j(c_i = \text{true}) \neq c_j(c_i = \text{false})$  for all  $c_j$ .

**Restricted Inactive Clause Coverage (RICC)** : For each  $p$  in  $P$  and each major clause  $c_i$  in  $C_p$ , choose minor clauses  $c_j, j \neq i$ , so that  $c_i$  does not determine  $p$ . The values chosen for the minor clauses  $c_j$  must be the same when  $c_i$  is true as when  $c_i$  is false, that is, it is required that  $c_j(c_i = \text{true}) = c_j(c_i = \text{false})$  for all  $c_j$ .

# Infeasibility & Subsumption

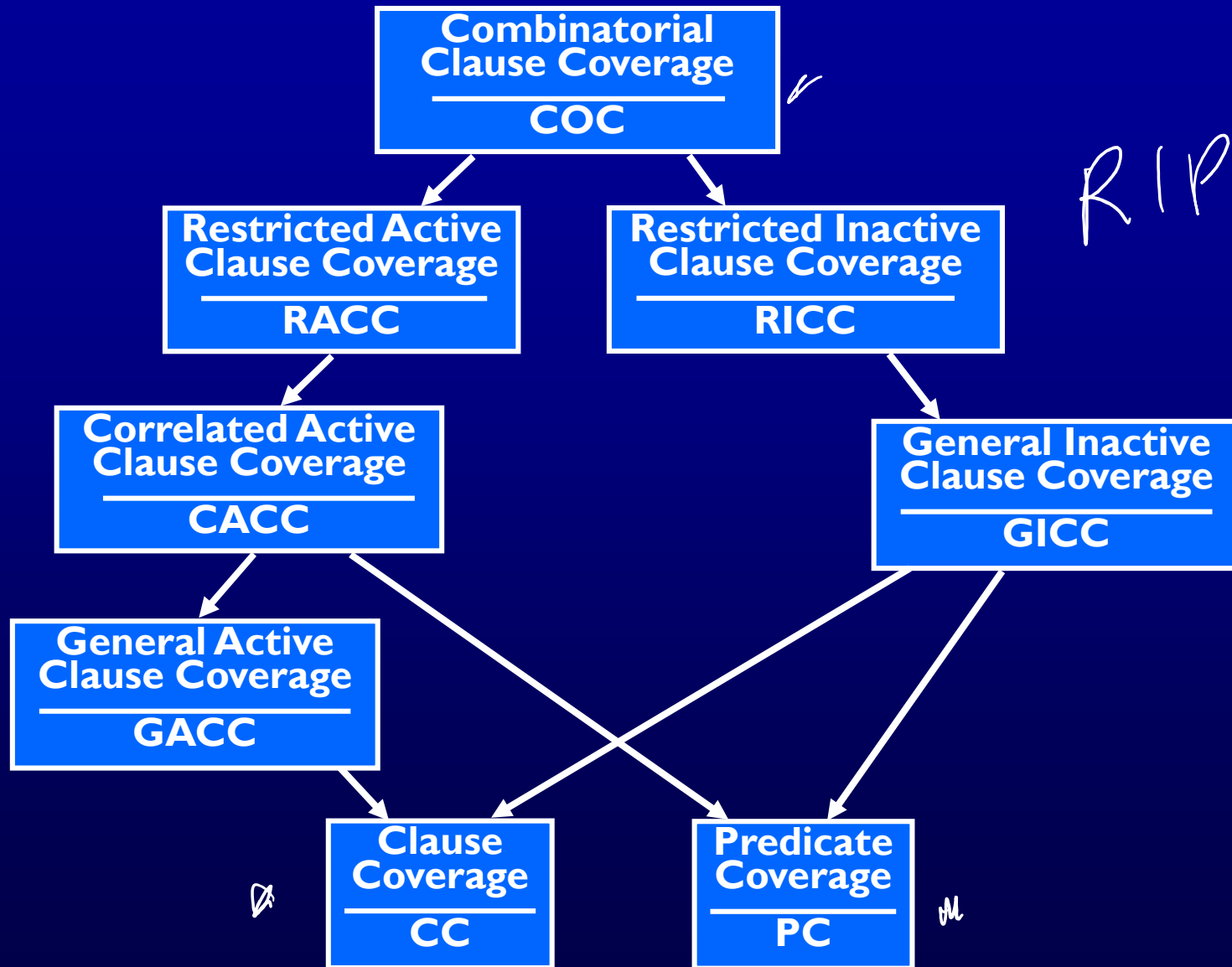
- Consider the predicate:

$$(a > b \wedge b > c) \vee c > a$$

- $(a > b) = true, (b > c) = true, (c > a) = true$  is infeasible
- As with graph-based criteria, infeasible test requirements have to be **recognized** and **ignored**
- Recognizing infeasible test requirements is hard, and in general, **undecidable**



# Logic Criteria Subsumption



# Making Clauses Determine a Predicate

- Finding values for minor clauses  $c_j$  is easy for simple predicates
  - But how to find values for more complicated predicates ?
  - Definitional approach:
    - $p_{c=true}$  is predicate  $p$  with every occurrence of  $c$  replaced by *true*
    - $p_{c=false}$  is predicate  $p$  with every occurrence of  $c$  replaced by *false*
  - To find values for the minor clauses, connect  $p_{c=true}$  and  $p_{c=false}$  with exclusive OR
- $$\underline{p}_c = p_{c=true} \oplus p_{c=false}$$
- After solving,  $\underline{p}_c$  describes exactly the values needed for  $c$  to determine  $p$

# Examples

$$\underline{p = a \vee b}$$

$$\begin{aligned} P_a &= P_{a=\text{true}} \oplus P_{a=\text{false}} \\ &= (\text{true} \vee b) \text{ XOR } (\text{false} \vee b) \\ &= \text{true} \text{ XOR } b \\ &= \neg b \end{aligned}$$

$$\underline{p = a \wedge b}$$

$$\begin{aligned} P_a &= P_{a=\text{true}} \oplus P_{a=\text{false}} \\ &= (\text{true} \wedge b) \oplus (\text{false} \wedge b) \\ &= b \oplus \text{false} \\ &= b \end{aligned}$$

$$\underline{p = a \vee (b \wedge c)}$$

$$\begin{aligned} P_a &= P_{a=\text{true}} \oplus P_{a=\text{false}} \\ &= (\text{true} \vee (b \wedge c)) \oplus (\text{false} \vee (b \wedge c)) \\ &= \text{true} \oplus (b \wedge c) \\ &= \neg (b \wedge c) \\ &= \neg b \vee \neg c \end{aligned}$$

- “**NOT  $b \vee \text{NOT } c$** ” means either  $b$  or  $c$  can be false
- **RACC** requires the same choice for both values of  $a$ , **CACC** does not

# XOR Identity Rules

Exclusive-OR (*xor*,  $\oplus$ ) means both cannot be true

That is,  $A \text{ xor } B$  means

*“A or B is true, but not both”*

$$\begin{aligned} p &= A \oplus A \wedge b \\ &= A \wedge \neg b \end{aligned}$$

$$\begin{aligned} p &= A \oplus A \vee b \\ &= \neg A \wedge b \end{aligned}$$

with fewer symbols ...

$$\begin{aligned} p &= A \text{ xor } (A \text{ and } b) \\ &= A \text{ and } !b \end{aligned}$$

$$\begin{aligned} p &= A \text{ xor } (A \text{ or } b) \\ &= !A \text{ and } b \end{aligned}$$

# A More Subtle Example

$$\underline{p = (a \wedge b) \vee (a \wedge \neg b)}$$

$$\begin{aligned} p_a &= p_{a=\text{true}} \oplus p_{a=\text{false}} \\ &= ((\text{true} \wedge b) \vee (\text{true} \wedge \neg b)) \oplus ((\text{false} \wedge b) \vee (\text{false} \wedge \neg b)) \\ &= (b \vee \neg b) \oplus \text{false} \\ &= \text{true} \oplus \text{false} \\ &= \text{true} \end{aligned}$$

$$\underline{p = (a \wedge b) \vee (a \wedge \neg b)}$$

$$\begin{aligned} p_b &= p_{b=\text{true}} \oplus p_{b=\text{false}} \\ &= ((a \wedge \text{true}) \vee (a \wedge \neg \text{true})) \oplus ((a \wedge \text{false}) \vee (a \wedge \neg \text{false})) \\ &= (a \vee \text{false}) \oplus (\text{false} \vee a) \\ &= a \oplus a \\ &= \text{false} \end{aligned}$$

- **a** always determines the value of this predicate
- **b** never determines the value – **b** is **irrelevant** !

# Tabular Method for Determination

- The math sometimes gets complicated
- A truth table can sometimes be simpler

• Example

**b & a differ to determine value of p**     **For a and b, only one pair, TFT and TFF, cause c to determine the value of p**     **Likewise, for clause c, only one pair, TFT and TFF, cause c to determine the value of p**

	a	b	c	$a \wedge (b \vee c)$	$p_a$	$p_b$	$p_c$
1	T	T	T	T			
2	T	T	F	T			
3	T	F	T	T			
4	T	F	F	F			
5	F	T	T	F			
6	F	T	F	F			
7	F	F	T	F			
8	F	F	F	F			

**In sum, three separate pairs of rows can cause a to determine the value of p, and only one pair each for b and c**

# Logic Coverage Summary

- Predicates are often **very simple**—in practice, most have less than 3 clauses
  - In fact, most predicates only have one clause !
  - With only one clause, PC is enough
  - With 2 or 3 clauses, CoC is practical
  - Advantages of ACC & ICC criteria significant for large predicates
    - CoC is impractical for predicates with many clauses
- **Control software** often has many complicated predicates, with lots of clauses

# Next

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- Applying Logic Coverage to source code
- Group assignments, start working on your projects
- Reminder: HW2 is due on Monday 3/29 at 9:30am EST