Translate the following expressions into English.

Let $\mathbb{P}$ equal ***Prop***, the type of propositions. Let $P : Form \rightarrow \mathbb{P}$ be the propositional functions on Formulas. Recall that SAT(S) for an infinite set of formulas S means that they are all simultaneously satisfiable using an assignment (valuation) of Var.

1. $\forall X : Form.\ \forall v : Var(X) \rightarrow \mathbb{B}.\ Boolean(bval(X), SubForm(X)).$

2. $\forall X : Form.\ (\ TAUT(X) \Rightarrow\ \exists T : Tableau(FX).$
   $(Completed(T) \Rightarrow Closed(T))\ ).$

3. $\forall e : \mathbb{N}\ \rightarrow Form.\ \forall n : \mathbb{N}.\ SAT(\{e(1), ..., e(n)\})\ \Rightarrow\ \exists v : Var \rightarrow \mathbb{B}.$
   $\forall i : \mathbb{N}.\ bval(e(i), v) = t\ in\ \mathbb{B}.$

4. $\forall P : Form \rightarrow \mathbb{P}.\ (\ \forall X : Form.$
   $(\ (\forall Y : SubForm(X).\ P(Y))\ )\ \Rightarrow P(X)\ )\ \Rightarrow\ \forall X : Form.\ P(X).$

5. $\forall X : Form.\ PROV(X) \Leftrightarrow (\ \exists T : Tableau(FX).Closed(T)).$

6. $\exists Decide : Form \rightarrow \mathbf{B}.\ \forall X : Form.(Decide(X) = t\ in\ \mathbb{B})\ \Leftrightarrow\ PROV(X).$

Here are acceptable answers.

1. For every formula X and assignment v to the variables of X, the (recursive) function bval, having inputs X and v, as a function of v is a Boolean valuation of the subformulas of X.

   Note, you need to recall that the recursive function we defined in class, bval, is a function of X and v, i.e. bval(X,v) produces a Boolean. So bval(X) is a function of one argument as Smullyan requires.

   $\forall X : Form.\ \forall v : Var(X) \rightarrow \mathbb{B}.\ Boolean(bval(X), SubForm(X)).$

2. For every formula X, if it is a tautology, then we can find a tableau T for FX which is completed and closed (or if completed is closed).

   $\forall X : Form.\ (\ TAUT(X) \Rightarrow\ \exists T : Tableau(FX).$
   $(Completed(T)\ \&\ Closed(T))\ ).$

3. Given an enumeration e of propositional formulas such that every enumerated initial segment up to n is satisfiable, there is a valuation v of all the propositional variables that satisfies every formula enumerated.

   $\forall e : \mathbb{N}\ \rightarrow Form.\ \forall n : \mathbb{N}.\ SAT(\{e(1), ..., e(n)\})\ \Rightarrow\ \exists v : Var \rightarrow \mathbb{B}.$
   $\forall i : \mathbb{N}.\ bval(e(i), v) = t\ in\ \mathbb{B}.$

4. Given any propositional function P on formulas, if for any formula X assuming that P(Y) holds for every subformula Y of X implies that P holds for X, then we know that P is true for all formulas.

   $\forall P : Form \rightarrow \mathbb{P}.\ (\ \forall X : Form.$
   $(\ (\forall Y : SubForm(X).\ P(Y))\ )\ \Rightarrow P(X)\ )\ \Rightarrow\ \forall X : Form.\ P(X).$

5. A propositional formula X is provable iff there is a closed tableau T for FX.

   $\forall X : Form.\ PROV(X) \Leftrightarrow (\ \exists T : Tableau(FX).Closed(T)).$

6. There is a function Decide from Formulas to Booleans that has the value true on any formula X if and only if X is provable.

   $\exists Decide : Form \rightarrow \mathbf{B}.\ \forall X : Form.(Decide(X) = t\ in\ \mathbb{B})\ \Leftrightarrow\ PROV(X).$