Problem Set 5 Due Thursday October 28, 2010 CS4860 Fall 2010

We discussed in class on Tuesday October 19 the apparent property of First-OrderTableau proof attempts (searches) that they cannot find a simple two element *counter model* for the following proposition:

(All x)(Exits y). Axy → (Exists y)(All x). Axy

The counter model M2 we found in class by *model checking* consists of two elements, a1, a2 where Aa1a1 and Aa2a2 are both true, the two other combinations Aa1a2 and Aa2a1 are both false. Call the counter model with the opposite assignments M2'.

It seems impossible to find either simple model M2 or M2' using Tableau because the restrictions on Rule D, the rule for δ formulas, prevent us from introducing the parameter a1 to witness T Exists y.Aa1y. Even the Liberal rule D does not allow a1 as a parameter in this subgoal, see Smullyan page 54. We can however find an *infinite* counter model Mω by applying the rules, and the systematic procedure generates such a model (very slowly indeed).

Consider these questions and do your best to contribute in writing ideas for solving them.

Question 1: Is there a legal Tableau search (proof attempt) to find a finite counter model M2 or M2'? Try to prove or disprove the conjecture that *there is no such search (proof attempt).*

Question 2: Is there a further liberalization of Rule D on page 54 that is sound and allows generating M2? Try to find such a rule that is in the spirit of Tableau. Recall that Liberal Rule D applied to a δ-formula has these *three restrictions* from page 54 (my labeling of them): D1 parameter *a* has not been previously introduced by a D rule, and D2: *a* does not already occur in the δ-formula as a parameter, and D3: no parameter of the δ-formula has been previously introduced by a D rule.

Can we safely remove condition D2? If not, why not? Recall that in class we found a model if we violated the D2 restriction. Does this example illustrate why each of D1, D2, and D3 is necessary?

Question 3: Is there another example of a formula with a finite counter model for which you can prove that Tableau search will not find it.

Question 4: The example at the bottom of page 62 finds a finite counter model for a simple formula. Does this example help us understand our problem? Does it reveal general conditions under which proof attempts find finite counter models?