

Encoding Graph Coloring

Consider the following encoding of graph coloring into CNF formulas.

The task is to color planar graphs so that no adjacent countries have the same color. Here is an example from the map of Europe.



Let the countries with the color i be B_i, G_i, F_i, L_i . Consider only three colors, red(r), blue(b), green(g). Then the possible coloring of France is $F_r _ F_b _ F_g$.

The constraints on pairs of neighbors derived from the graph include pairs such as $_ (B_b \wedge L_b)$, $_ (B_r \wedge L_r)$, etc. These are equivalent to $(_ B_b _ L_b) \wedge (_ B_r _ L_r) \wedge \dots$

Write down the CNF claim that red, green, and blue suffice to color this map and prove it is unsatisfiable.

