Homework Assignment 1 Due Tue Sept 14 CS4860 Fall 2010

Reading: Smullyan Chapter I and II.1

Problem 1

(a) Give types for the following objects using B for Booleans, Form for Formulas, Var for Variables, Ops for the type {not, or, and, imp} of logical operators, N for natural numbers.
  - (i) functions from formulas to their outer operators
  - (ii) functions from formulas to Booleans
  - (iii) ordered pairs of Formulas and their depth
  - (iv) valuations of the type of all formulas (see pages 9 and 10)
  - (v) interpretations of a formula X (see page 10)

(b) Using these types write a definition of tautologies as a subtype of Form, use the set type notation from lecture, {x:T | P(x)}.

Problem 2

(a) Solve Exercise 3 page 13 of textbook

(b) Solve Exercise 3 for Conjunctive Normal Form (hint use part (a)).

Problem 3

Solve Exercise 5 page 14.

Problem 4

Construct tableau proofs or falsifications for these formulas (→ is implication)

- (i) ~((p and q) or (~p and r)) → ((~p or ~q) and (p or ~r))
- (ii) (p → (q → r)) → ((p → q) → (p → r))
- (iii) (p →(q or r)) → ((q → r) → (p → r))

Extra Credit Problems

Problem 5

Write a more detailed definition of Smullyan's trees following the outline from class and prove the theorem that for every node x in S, there is a *unique* path p(x) such that end(p(x)) = x. Try to do the proof so that by following it, a reader can actually construct the path. Note in your proof where you use the fact that you can decided whether two points of S are equal. This proof will require you to know that every interior point (a point not equal to the origin) has a unique predecessor.

Problem 6

Write a recursive data type or define a recursive set that corresponds a close as possible to Smullyan's simple definition of formulas on page 7. Based on this data type or set, define a recursive function which is a valuation for any formula given an interpretation for its variables.

Extra Reading: Look at the paper Expressing and Implementing the Computational Content Implicit in Smullyan's Account of Boolean Valuations. It is posted as part of Lecture 3.