Reading

Please read the notes on Refinement Logic for next Thursday,
the notes on second order propositional logic for Thursday, March 5
and pp. 43–52 in Smullyan for Tuesday, March 10.

Questions

(1) Let S be a set of formulas such that for any interpretation v_0 there is a formula $X \in S$ with $value(X, v_0) = t$. Show, using the compactness theorem, that there is a finite subset $\{X_1, .., X_n\}$ of S such that $X_1 \vee .. \vee X_n$ is a tautology.

(2) Translate the following arguments into propositional formulas and prove them right or wrong using (multi-conclusioned) Gentzen Systems:

    (a) If there is a snowstorm then roads will be closed. The roads are open. Thus there can't be a snowstorm.

    (b) If there is a snowstorm then roads will be closed. There is no snowstorm. Hence the roads must be open.

    (c) If there can't be no snowstorm then there is one.

    (d) If either wages or prices are raised, there will be inflation. If there is inflation, then either Congress must regulate it or the people will suffer. But if the people suffer, then congressmen will be unpopular. However, Congress does not regulate inflation and congressmen are not unpopular. Therefore, wages do not rise.

    (e) Jones has filed a lawsuit against Rogers because of a contract. Jones will win the lawsuit if and only if Rogers has entered into the contract, the contract is legal, and Rogers has not performed the contract. Rogers has entered into the contract only if he has accepted Jones' offer. But, in fact, Rogers has not accepted Jones' offer. Therefore Jones will lose the lawsuit.

(3) Write down the rules for a Gentzen system based on the *choice operator* $|$ (c.f. homework 1) and the *joint denial operator* $\downarrow$ (see p. 14 of Smullyan).