Note on integrity: You may discuss problems with fellow students, but all written work must be entirely your own, and should not be from any other course, present or past. If you use a solution from another source you must cite it, including from other people who help you.

## Reading

Read pp. 1-24 in Smullyan for next Tuesday.

## Questions

(1) Do Exercise 1 in Smullyan (p.13):

Let $F$ be a formula containing a formula $X$ as a part and $F[Y / X]$ denote the formula that results from $F$ by replacing one (or more) occurrences of $X$ by the formula $Y$.
Prove for any formula $F$ that $F[Y / X]$ is truth functionally equivalent to $F$ (written as $F[Y / X] \equiv F)$ if $Y \equiv X$
(2) Do Exercise 5 in Smullyan (p.14) for the Sheffer stroke operator $\mid$.
(Read Exercise 4 for a definition of "definable".)
Bonus: Do the same for $\downarrow$, the joint denial operator.
(3) Use the DPLL procedure to check whether the following formulas are satisfiable and provide satisfying valuations if they are.
(a) $(\neg p \vee \neg q \vee r) \wedge(p \vee \neg q \vee r) \wedge(p \vee q \vee \neg r) \wedge(p \vee q \vee r) \wedge(\neg p \vee q \vee \neg r) \wedge(p \vee \neg q \vee \neg r) \wedge(\neg p \vee q \vee r)$
(b) $(x \vee y \vee \neg z \vee t \vee u) \wedge(\neg x \vee y \vee z \vee \neg t \vee u) \wedge(x \vee \neg y \vee z \vee t \vee \neg u) \wedge(y \vee z \vee t \vee \neg u)$
$\wedge(y \vee z \vee \neg t \vee u) \wedge(y \vee \neg z \vee \neg u) \wedge(\neg y \vee \neg z \vee u) \wedge(x \vee y \vee z \vee u) \wedge(\neg x \vee \neg y \vee \neg u)$
$\wedge(x \vee y \vee u) \wedge(\neg x \vee \neg y \vee u) \wedge(x \vee y \vee t) \wedge(x \vee \neg z \vee t) \wedge(\neg x \vee z \vee \neg t \vee u)$

