Applied Logic	**Homework 8**
CS 486 Spring 2005	**Due Date: Thursday, May 5, 2005**

Questions

(I use $\supset$ for implication.)

(1) Prove that $\Box\varphi \wedge \Box(\varphi \supset \psi) \supset \Box\psi$ is valid in all frames.

(2) In class, we showed that if F is a reflexive frame, then for all formulas φ, $\Box\varphi \supset \varphi$ is valid in F. Prove the converse: if for all formulas φ, $\Box\varphi \supset \varphi$ is valid in a frame F, then F is a reflexive frame. **Hint**: show that if F is not reflexive, then $\Box\varphi \supset \varphi$ is not valid in F.

This result says that the formula schema $\Box\varphi \supset \varphi$ *corresponds to* reflexive frames, since F is reflexive if and only if for all φ, $\Box\varphi \supset \varphi$ is valid in F.

(3) An equivalence relation on W is a relation R that is:

- reflexive: for all $x \in W$, $(x, x) \in R$;
- symmetric: for all $x, y \in W$, $(x, y) \in R$ implies that $(y, x) \in R$;
- transitive: for all $x, y, z \in W$, if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$.

A frame $F = \langle W, R\rangle$ is an equivalence frame if R is an equivalence relation on W. Show that the following formula schemas correspond to equivalence frames:

$$\Box\varphi \supset \varphi$$
$$\Box\varphi \supset \Box\Box\varphi$$
$$\neg\Box\varphi \supset \Box\neg\Box\varphi$$