Applied Logic	**Homework 7**
CS 486 Spring 2005	**Due Date: Tuesday, April 26, 2005**

Questions

(1) **(Gentzen System for First-Order Logic)**

(a) Write down a Gentzen system for first-order logic, following the Gentzen system we initially gave for propositional logic.

(b) Give an argument showing that the system you define is correct and complete. (I don't want a formal proof of this, just a reasonable argument, of the kind we gave in class for the Gentzen system for propositional logic.)

(2) **(Functions and Equality)**

(a) Extend the tableau proof system for first-order logic given in class to deal with equality.

(b) Extend the Gentzen system in Question (1) with appropriate rules to deal with equality.

(c) Do we need to extend the tableau proof system to deal with function symbols? If so, how; if not, why not?

(3) Prove, either using a tableau proof or a sequent proof, the validity of the following formulas:

$$AR \Rightarrow (\forall x)[(\forall y)(y + x = y) \Rightarrow x = 0]$$
$$AR \Rightarrow (\forall x)(\forall y)(\forall z)(x + y = x + z \Rightarrow y = z)$$

Recall that AR is the conjunction of all the axioms for arithmetic I gave in Lecture 19. Note that you will have to use either the tableau proof system you extended in Question (2) for equality and functions, or the Gentzen system developed in Question (1) and extended in Question (2) for equality and functions.