Applied Logic	**Homework 7**
CS 486 Spring 2005	**Partial solutions**

Solutions

(1) (a)

$G, A \wedge B \vdash H$ $\quad G, A, B \vdash H$	$G \vdash H, A \wedge B$ $\quad G \vdash H, A$ $\quad G \vdash H, B$
$G, A \vee B \vdash H$ $\quad G, A \vdash H$ $\quad G, B \vdash H$	$G \vdash H, A \vee B$ $\quad G \vdash H, A, B$
$G, A \Rightarrow B \vdash H$ $\quad G \vdash H, A$ $\quad G, B \vdash H$	$G \vdash H, A \Rightarrow B$ $\quad G, A \vdash H, B$
$G, \neg A \vdash H$ $\quad G \vdash H, A$	$G \vdash H, \neg A$ $\quad G, A \vdash H$
$G, A \vdash H, A$	
$G, (\forall x)A \vdash H$ $\quad G, A[t/x], (\forall x)A \vdash H$	$G \vdash H, (\forall x)A$ $\quad G \vdash H, A[y/x]$
$G, (\exists x)A \vdash H$ $\quad G, A[y/x] \vdash H$	$G \vdash H, A[t/x], (\exists x)A$ $\quad G \vdash H, (\exists x)A$

I use $A[y/x]$ to mean replacing every free occurence of x in formula A by y. In the rules above, t is an arbitrary term (more precisely, there is an instance of the rule for every term t), and y is taken to be a new variable not appearing in G,H, or A. See the corresponding tableaux rules given in class for more details on this restriction.

(b) *See lecture notes for February 24, on the relationship between Gentzen systems and block tableaux.*

(2) (a) *Let me give only the Gentzen rules in (b).*

(b) *There are a number of equivalent ways of doing this; this is one of the simplest.*

For every choice of t, we have a rule:

$$\begin{array}{l} G \vdash H \\ \quad G, t = t \vdash H \end{array}$$

For every choices of n-ary predicate P and terms $t_1, \ldots, t_n$ and $t'_1, \ldots, t'_n$, we have a rule:

$$\frac{G \vdash H}{G, (t_1 = t'_1 \wedge \cdots \wedge t_n = t'_n \wedge P(t_1, \ldots, t_n)) \Rightarrow P(t'_1, \ldots, t'_n) \vdash H}$$

This rule of course captures the idea that we can substitute equals for equals.

It's one of the simplest approaches because the only thing that these rule do is introduce a new formula in the goal sequents. The others rules of the Gentzen system are used to take these new formulas apart. Among other things, we don't need to fuss with substitution. It makes the resulting Gentzen system nicely uniform.

(c) *The answer to this question depends of course on what you did for (a) and (b). Again, let me talk about Gentzen systems only. Given what I gave for (b), I need the following rules to deal with functions, again capturing the fact that I can substitute equals for equals:*

For every choices of n-ary function symbol f and terms $t_1, \ldots, t_n$ and $t'_1, \ldots, t'_n$, we have a rule:

$$\frac{G \vdash H}{G, (t_1 = t'_1 \wedge \cdots \wedge t_n = t'_n) \Rightarrow f(t_1, \ldots, t_n) = f(t'_1, \ldots, t'_n) \vdash H}$$

(3) *Given the rules above, this is just a straightforward application of the rules.*