

Reading

Read pp. 57–65 in Smullyan for Tuesday, April 5

Questions

- (1) Show that validity and satisfiability are “monotone” wrt. the universe
 - (a) If a formula A is satisfiable in \mathcal{U} (i.e true under some interpretation with universe \mathcal{U}) then it is satisfiable in any superset of \mathcal{U} .
 - (b) If a formula A is valid in \mathcal{U} then it is valid in any subset of \mathcal{U} .
 - (c) Give an example of a formula that is satisfiable in an infinite universe but not in any finite one.
- (2) Show that validity and satisfiability are “complementary” to each other
 - (a) A formula A is valid iff $\sim A$ is unsatisfiable
 - (b) A formula A is satisfiable iff $\sim A$ is invalid
- (3) Show that boolean and first-order valuations are “the same” in a quantifier-free logic
 - (a) Every quantifier-free sentence that is truth-functionally satisfiable is also first-order satisfiable.
 - (b) Every (finite or infinite) set S of quantifier-free sentences that is (simultaneously) truth-functionally satisfiable is also (simultaneously) first-order satisfiable.
 - (c) Every valid quantifier-free sentence is also a tautology
- (4) Construct a first-order tableaux proof for the formula
$$(\forall x)Sx \wedge (\forall y)(\sim(Ty \supset Ry) \supset Py) \supset \sim((\exists z)((Pz \supset Qz) \wedge (Tz \supset Rz))) \supset \sim\sim Pa \wedge Sa \wedge Sb$$