Applied Logic	**Homework 4**
CS 486 Spring 2005	**Due Date: Tuesday, March 15, 2005**

Reading

Please read the notes on second order propositional logic for Thursday, March 10 and pp. 43–52 in Smullyan for Thursday, March 17.

Project Work

During the second half of this course you should work on a self-chosen project related to the topic of applied logic. This could, for instance, be a literature study about an interesting or the implementation (and documentation) of a proof environment. We will discuss a few possibilities in class. Please prepare a project proposal (about half a page) for Tuesday, March 29.

Questions

(1) Prove the following formulas in the multi-conclusioned sequent calculus

(a) $(P \Rightarrow Q) \Rightarrow \neg Q \Rightarrow \neg P$

(b) $\neg(P \wedge Q) \Rightarrow \neg P \vee \neg Q$

(c) $\neg\neg P \Rightarrow P$

(2) Prove the above formulas in refinement logic

(3) Write down the rules for a Gentzen system based on the *choice operator* $\mid$ (c.f. homework 1) and the *joint denial operator* $\downarrow$ (see p. 14 of Smullyan).

(4) Prove that the `magic` rule is equivalent to the following rule

$H \vdash G$ by `contradiction`
$H, \neg G \vdash$ `f`

Show how to derive `contradiction` in refinement logic and how to derive `magic` in a refinement logic with `contradiction` (and without `magic`).

Note that using the `cut` *rule is not necessary but simplifies the derivation.*

(5) **Bonus**: Show that refinement logic becomes incomplete if we change the `impL` rule from

$$\frac{H, A \Rightarrow B, H' \vdash G \qquad H, A \Rightarrow B, H' \vdash A \qquad H, H', B \vdash G}{H, A \Rightarrow B, H' \vdash G} \quad \text{to} \quad \frac{H, A \Rightarrow B, H' \vdash G \qquad H, H', B \vdash G}{H, A \Rightarrow B, H' \vdash G}$$