Applied Logic	Homework
CS 486 Spring 2005	Due Date: Thursday, March 3, 2005

Reading

Read pp. 101–108 in Smullyan for Thursday, February 24.

Questions

(1) Prove that any closed tableau can be further extended into an atomically closed tableau.

Hint: Show by induction on the degree of a formula X that any set S that contains both X and $\bar{X}$ has an atomically closed tableau.

(2) Let S be a set of formulas such that for any interpretation v_0 there is a formula $X \in S$ with $value(X, v_0) = t$. Show, using the compactness theorem, that there is a finite subset $\{X_1, .., X_n\}$ of S such that $X_1 \vee .. \vee X_n$ is a tautology.

(3) A set of formulas S is *complete* if every formula or its negation belongs to S. A (deductively closed) *theory* is a consistent set of formulas T such that every formula deducible from T belongs to T.

Prove that every theory is the intersection of all its complete consistent extensions.