Note on integrity: You may discuss problems with fellow students, but all written work must be entirely your own, and should not be from any other course, present or past. If you use a solution from another source you must cite it, including from other people who help you.

## Reading

Read pp.1-24 in Smullyan for next tuesday.

## Questions

(1) Explain, in your own words, why we need a uniqueness decomposition lemma (p. 6 in Smullyan).

Bonus: Prove the unique decomposition lemma.
(2) Prove that if $v$ and $v^{\prime}$ are Boolean valuations that extend the same interpretation $v_{0}$, then $v$ and $v^{\prime}$ agree on all formulas.
(3) Let $v_{0}$ be an interpretation, and define $v(\phi)=\operatorname{value}\left(\phi, v_{0}\right)$. Prove that $v$ is a Boolean valuation that extends $v_{0}$.
(4) Do Exercise 5 in Smullyan (p.14) for the choice operator |. (Read Exercise 4 for a definition of "definable".)
Bonus: Do the same for $\downarrow$, the joint denial operator.

