Introduction to Analysis of Algorithms	Problem Set 1
CS 4820, Summer 2013	Due at 11pm, Monday, July 8

Hand in your solutions electronically using CMS and submit a paper copy in class. Write your name and NetID on the first page.

Remember that when a problem asks you to design an algorithm, you must also prove the algorithm's correctness and analyze its running time. The running time must be bounded by a polynomial function of the input size, unless specified otherwise.

1. (a) [5 points] Prove or disprove: for each $n > 2$ there exists an instance of the stable matching problem such that, when one runs the Gale-Shapley algorithm on this instance to produce a stable matching, no man is matched to his highest-ranked woman.

(b) [10 points] In an instance of the stable matching problem, suppose that each woman assigns a *score* to each man, by numbering the men from 1 to n in order from best to worst according to her ordering. Let $score(w, m)$ denote the score that w assigns to m. Similarly, each man m assigns a score to each woman w, denoted by $score(m, w)$, by numbering the women from 1 to n (best to worst) according to his ordering. Prove that if S is any stable perfect matching, then

$$\sum_{(m,w)\in S} score(m, w) + score(w, m) \quad \leq \quad n^2 + n.$$

2. [5 points] Exercise 4.5 from the text.

3. [5 points] In this problem, you are in charge of maintaining a chemistry lab (this is a much safer job than doing the problem set, after all). Next month, you are expecting a delivery of new chemicals that need to be stored at specified low temperatures, and in preparation for that, you are going to order a few refrigerators. These refrigerators come with a highly accurate thermostats, but this makes them extremely expensive. Given the safe storage temperature ranges $[s_i, f_i]$ $(1 \leq i \leq n)$ of these n chemicals, design an efficient algorithm to find the smallest number of refrigerators needed to store all these n chemicals. When a refrigerator is set to maintain the temperature of t, any number of chemicals can be simultaneously stored in it, as long as $s_i \leq t \leq f_i$ for each chemical i to be stored in this refrigerator.

4. [10 points] Exercise 4.21 from the text.