I was asked in office hours today to provide a sample formalization of a word problem as an example. Here is a real one from *today*.

A certain course at Cornell has an enrollment of 281 students and a course staff of 22 members. The first homework set consists of three problems. It is due soon and will have to be graded. All members of the course staff will participate equally (or as equally as possible) in the grading. As one of the senior TAs for the course, you have been asked by the instructor to allocate the students' submitted solutions to the staff members to grade. The requirements are as follows.

(i) Each problem of each student is to be graded by exactly one staff member.

(ii) Except for the instructor, each staff member grades only instances of one of the three problems.

(iii) The allocation is as equitable as possible, in the sense that all staff members grade roughly the same number of problems.

Solution. Let E be a set representing the enrolled students. Let P be a set representing the different problems on the homework set. Let M be a set representing the staff members. One of the members of M, say i, is designated as the *instructor*. The problem is to produce a function

$$f : E \times P \to M$$

with the following properties:

(a) If $f(e,p) = f(e',p')$ and $f(e,p) \neq i$, then $p = p'$.

(b) The value
$$\max_{m,m' \in M} |\#f^{-1}(m) - \#f^{-1}(m')| \tag{1}$$
is minimum among all $f : E \times P \to M$ satisfying (a), where

- $f^{-1}(m) = \{(e,p) \mid f(e,p) = m\}$, the set of pairs (e,p) assigned to m,
- $\#A$ denotes the size of the set A, and
- $|\cdot|$ denotes absolute value.

The fact that f is a function $E \times P \to M$ captures requirement (i): the value $f(e,p)$ is the staff member assigned to grade problem p of student e. Property (a) captures requirement (ii), that no staff member other than the instructor grades instances of two different problems. Finally, property (b) captures the equitability requirement (iii). Specifically, the set $f^{-1}(m)$ is the set of $(\text{student}, \text{problem})$ pairs assigned to staff member m, and the number $\#f^{-1}(m)$ is the size of this set. Minimizing the expression (1) says that the maximum difference in the number of problems assigned to any two staff members is as small as possible.

For the purposes of computation, the function f can be represented as a bipartite graph with edges between two sets of nodes $E \times P$ and M, with an edge between (e,p) and m if $f(e,p) = m$.