The following table gives the number of respondents who obtained each score.

score	14	13	12	11	10
number	3	10	9	3	4

The numbers in parentheses below show the number of people who missed each question.

Tell whether the following sets are regular or nonregular.

1. $\{a^n b^m \mid n = 2m\}$ **nonregular (0)**
2. $\{a^n b^{2m} \mid n \geq 0 \text{ and } m \geq 0\}$ **regular (4)**
3. $\{a^n b^m c^n \mid n \geq 0 \text{ and } m \geq 0\}$ **nonregular (7)**
4. $\{x \in \{0,1\}^* \mid x \text{ contains more 0's than 1's}\}$ **nonregular (1)**
5. $\{a^n b^m \mid n \neq m\}$ **nonregular (1)**
6. $\{a^n b^{n+4810} \mid n \geq 0\}$ **nonregular (4)**

True or false? Every equivalence relation is

7. reflexive **true (0)**
8. symmetric **true (0)**
9. antisymmetric **false (2)**
10. transitive **true (0)**
11. a partial order **false (4)**
12. a homomorphism **false (7)**
13. refined by the identity relation **true (8)**

Here is a tricky one!

14. One of the following subsets of $\{a, b, c\}^*$ is regular and the other is nonregular. Which is which? **(15)**
    (i) $\{xy \mid x, y \in \{a, b\}^*,\ \#a(x) = \#b(y)\}$
    (ii) $\{xcy \mid x, y \in \{a, b\}^*,\ \#a(x) = \#b(y)\}$

The set (ii) is nonregular: if you intersect with a^*cb^*, then delete the c's with a homomorphism $h(a) = a$, $h(b) = b$, $h(c) = \varepsilon$, you get your favorite nonregular set $\{a^n b^n \mid n \geq 0\}$.

The set (i) is regular, and in fact is just $\{a, b\}^*$. That is, *every* string $z \in \{a, b\}^*$ can be expressed as xy with $\#a(x) = \#b(y)$ for some x, y. Suppose $|z| = n$. Let $f(i) = \#a(x_i) - \#b(y_i)$, where $z = x_i y_i$ with $|x_i| = i$ and $|y_i| = n - i$, $0 \leq i \leq n$. Then $f(0) = \#a(\varepsilon) - \#b(z) \leq 0$, $f(n) = \#a(z) - \#b(\varepsilon) \geq 0$, and $f(i + 1) = f(i) + 1$, $0 \leq i \leq n - 1$. There must be an i such that $f(i) = 0$, i.e. $\#a(x_i) = \#b(y_i)$.