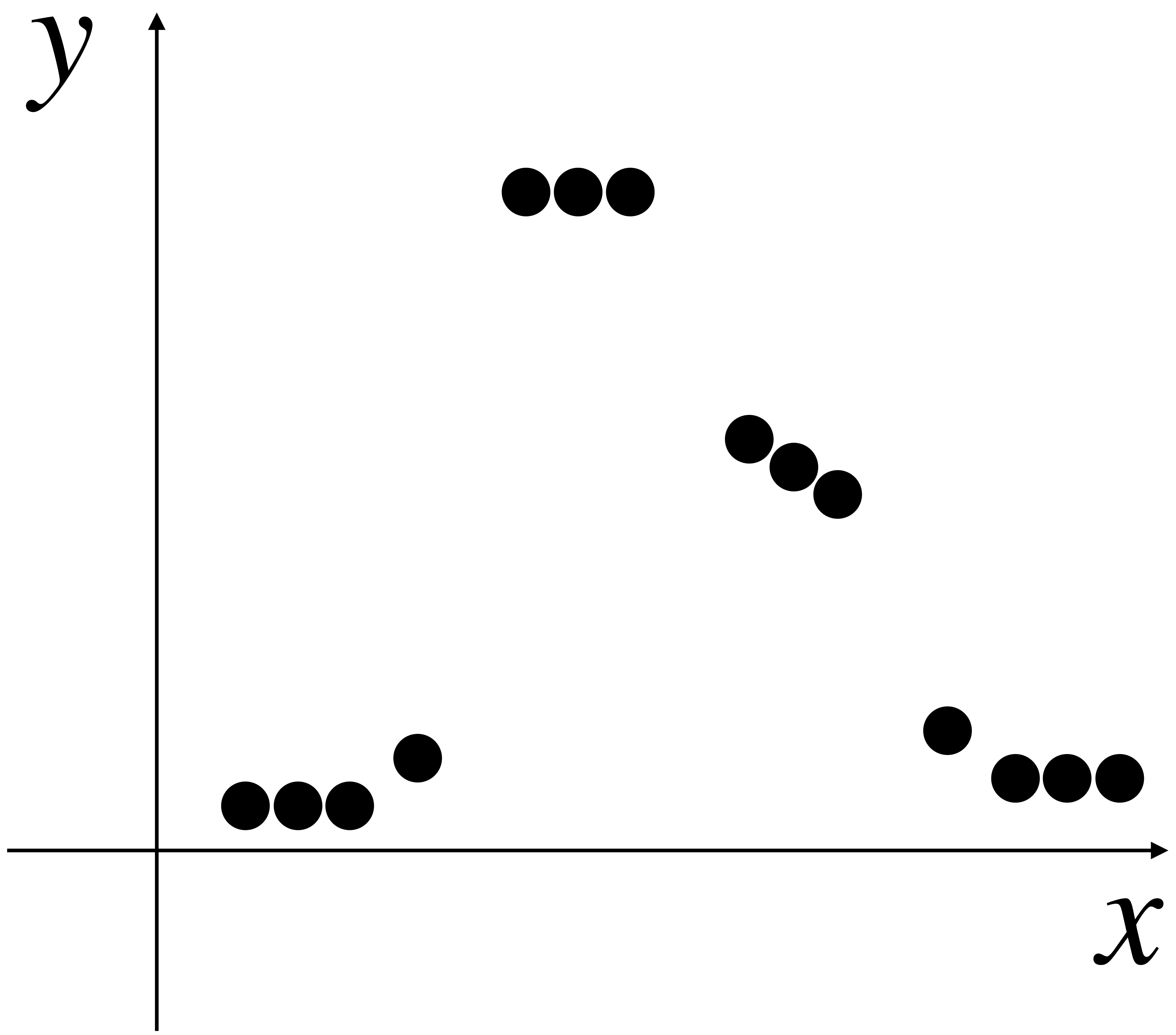
Ensemble Methods:
Bagging & Random Forest

Announcements

Recap on Decision (Regression) Tree

Regression dataset $\mathcal{D} = \{x_i, y_i\}_{i=1}^{n}, \quad (x_i, y_i) \sim P$



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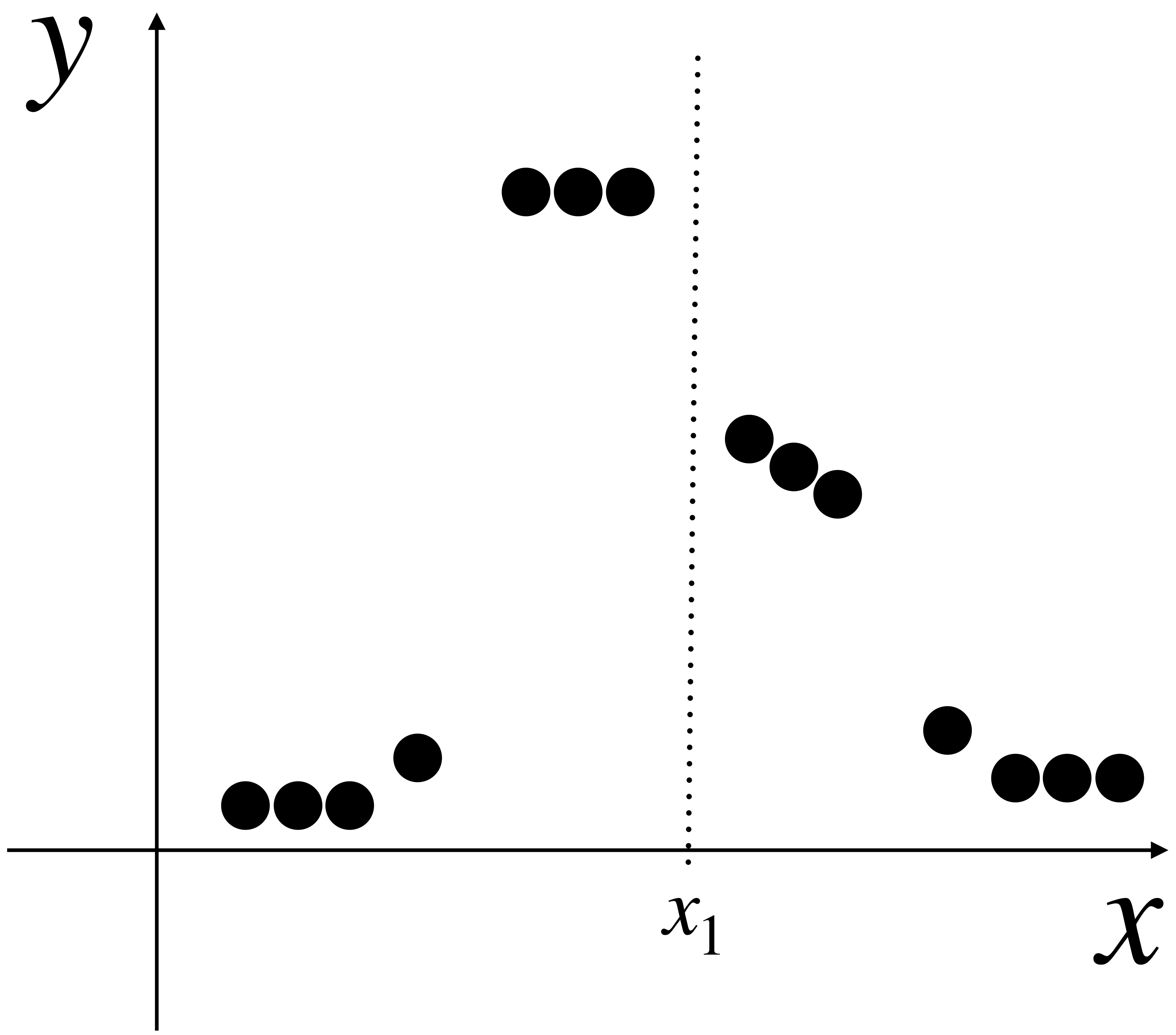

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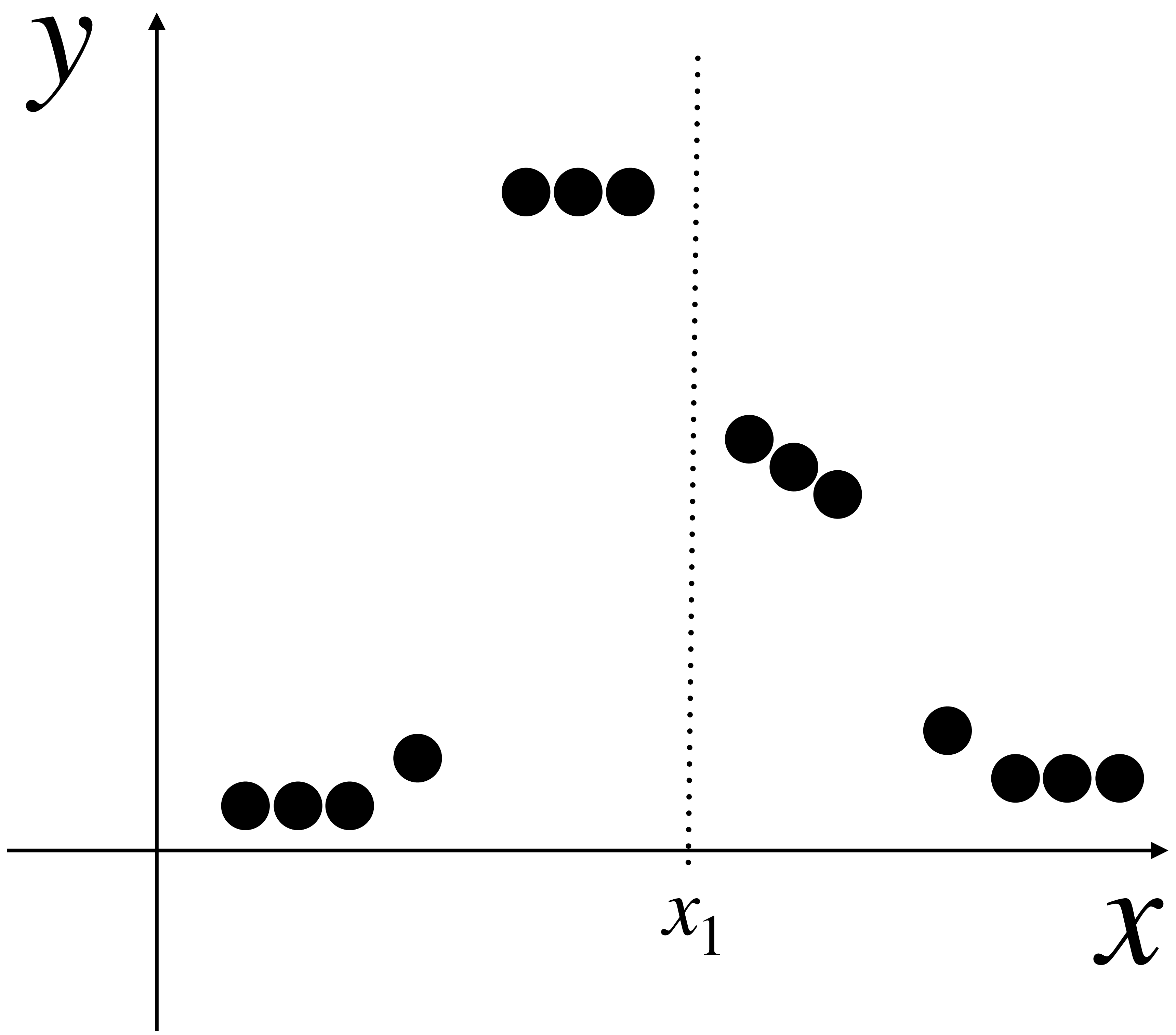

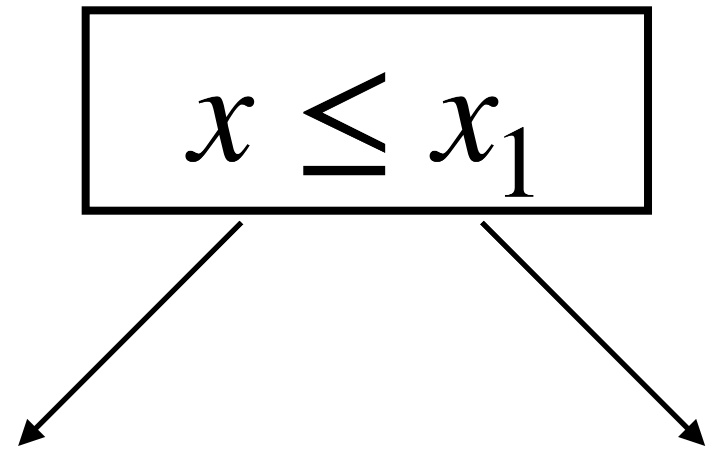

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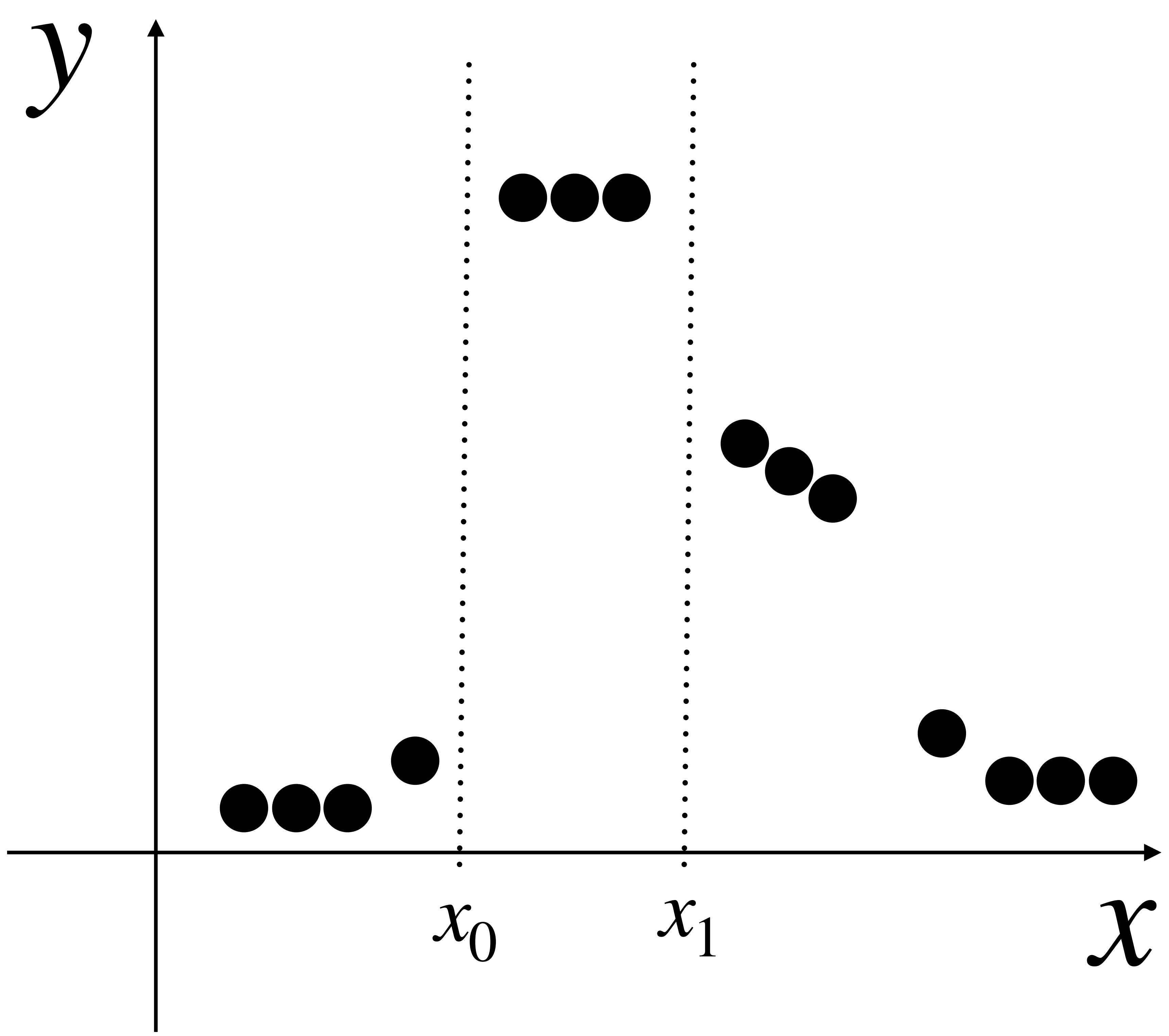

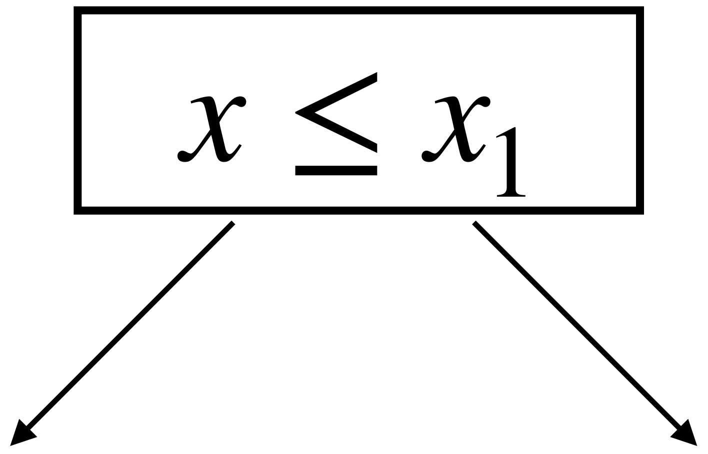

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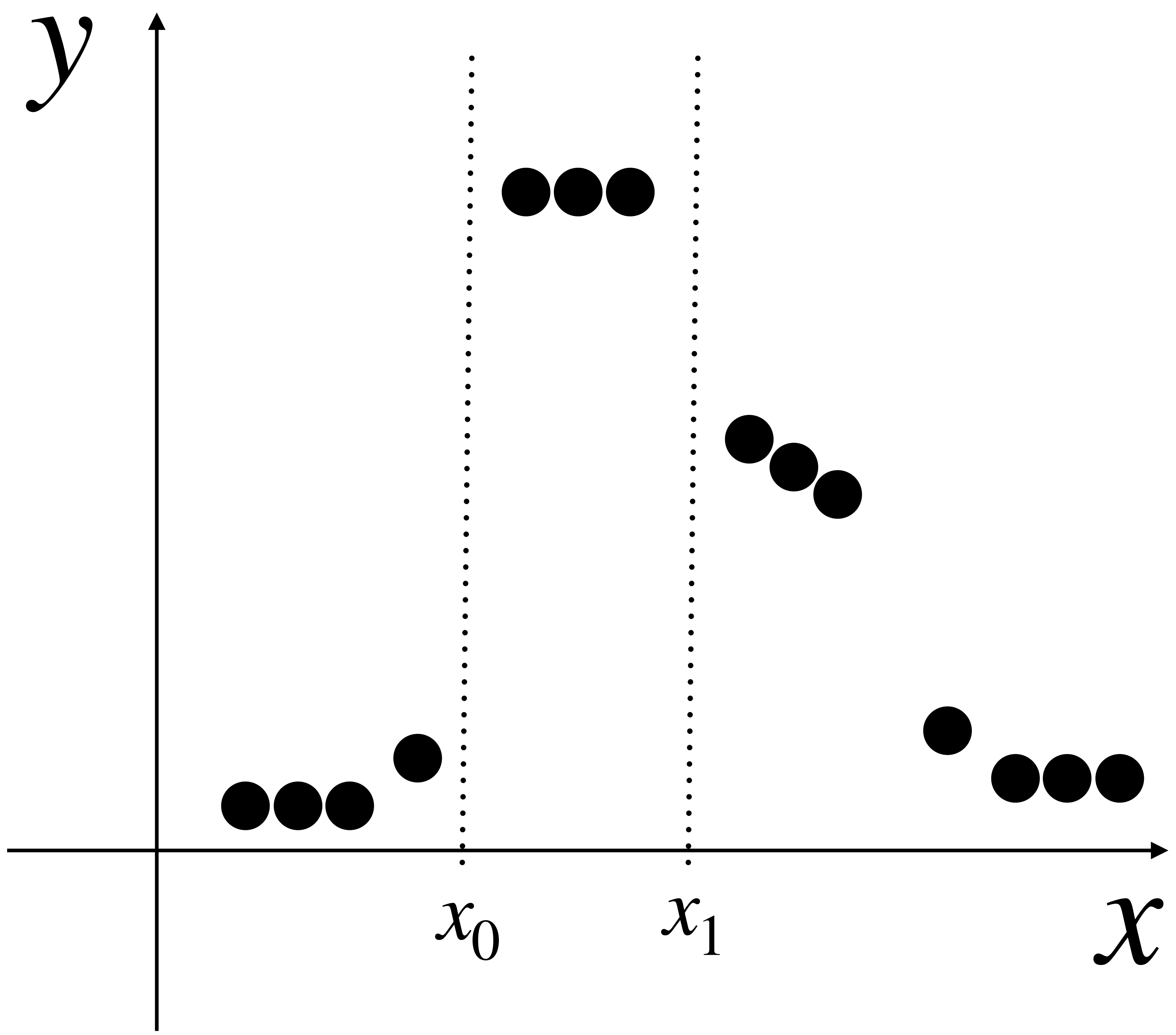

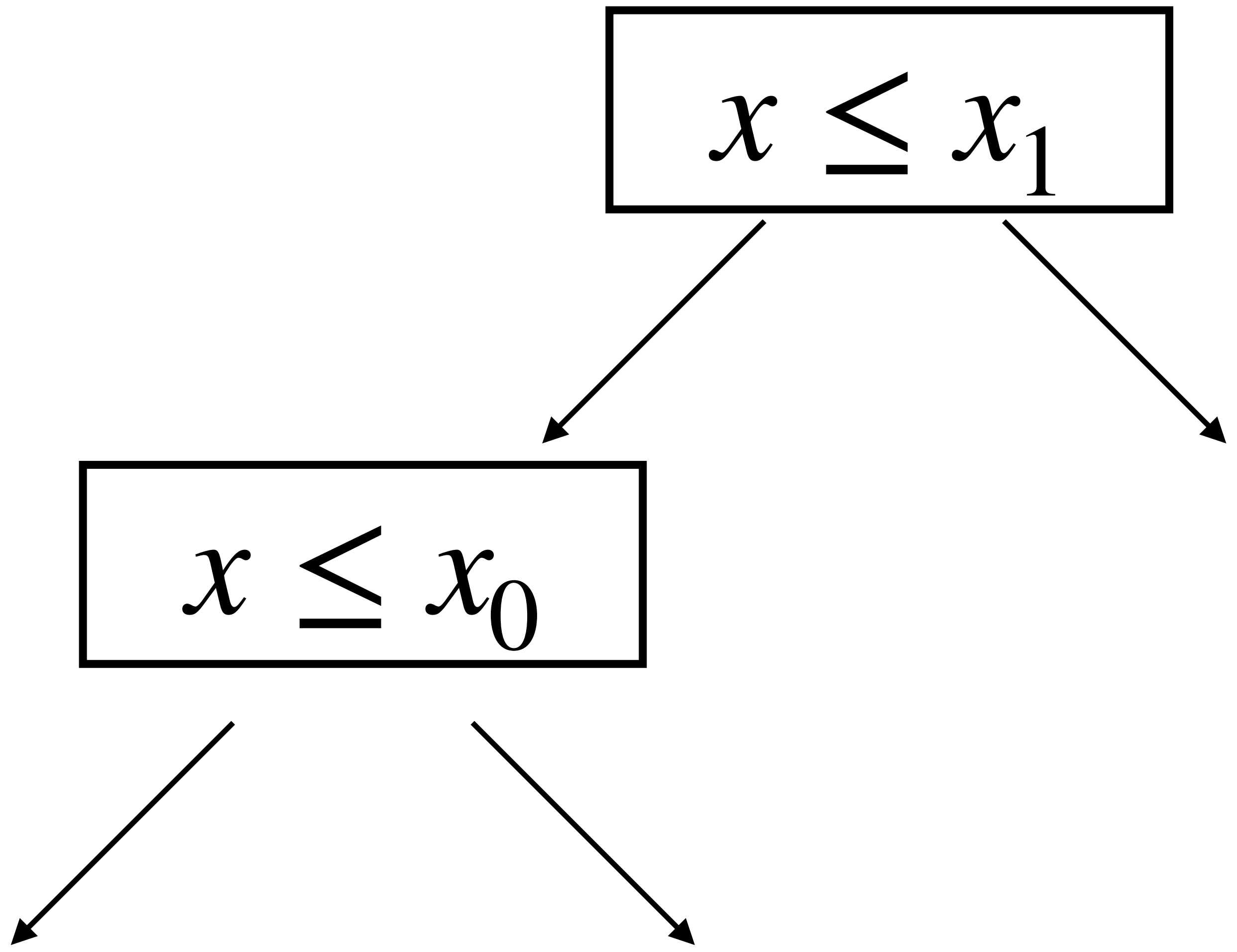

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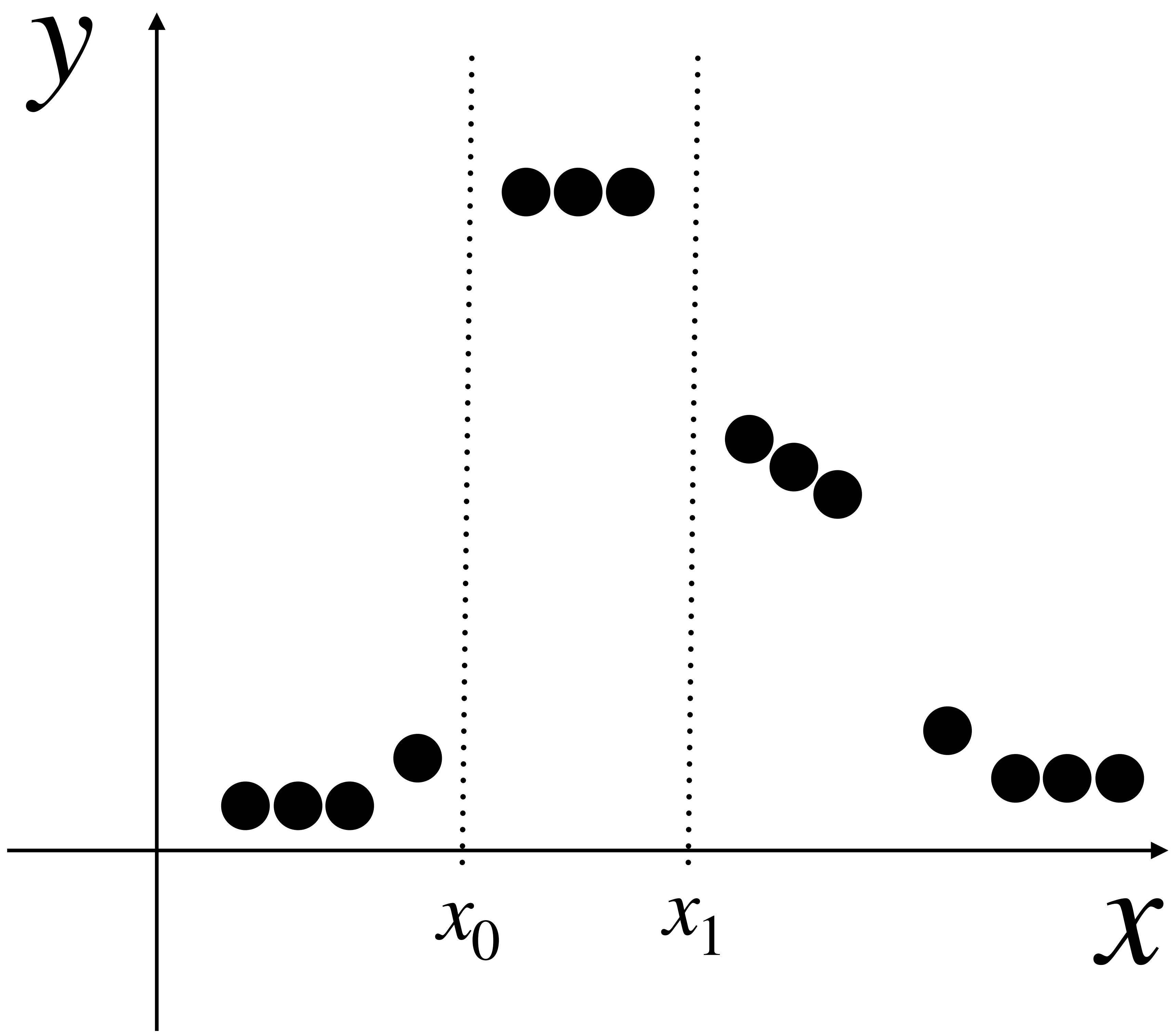

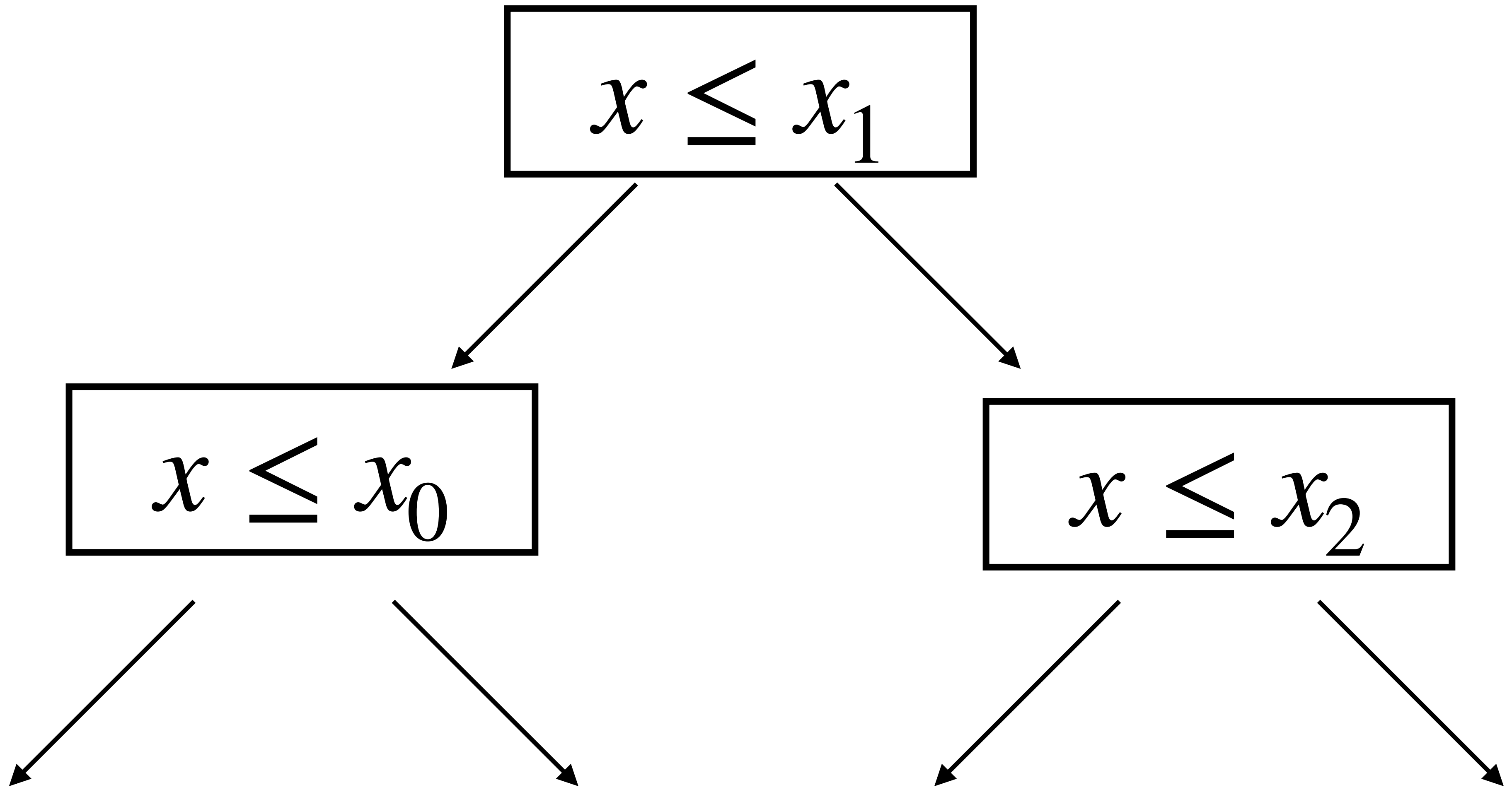

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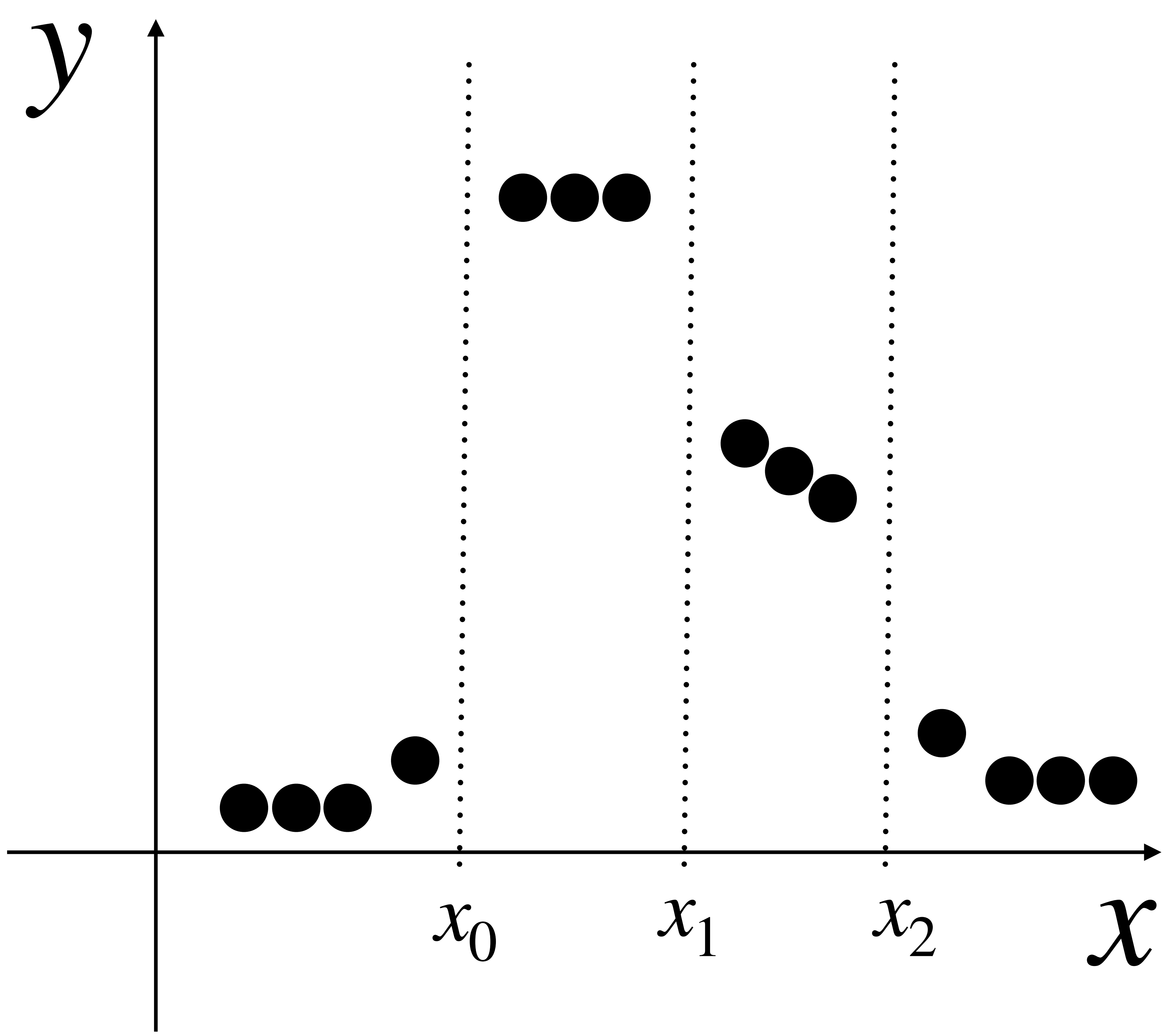

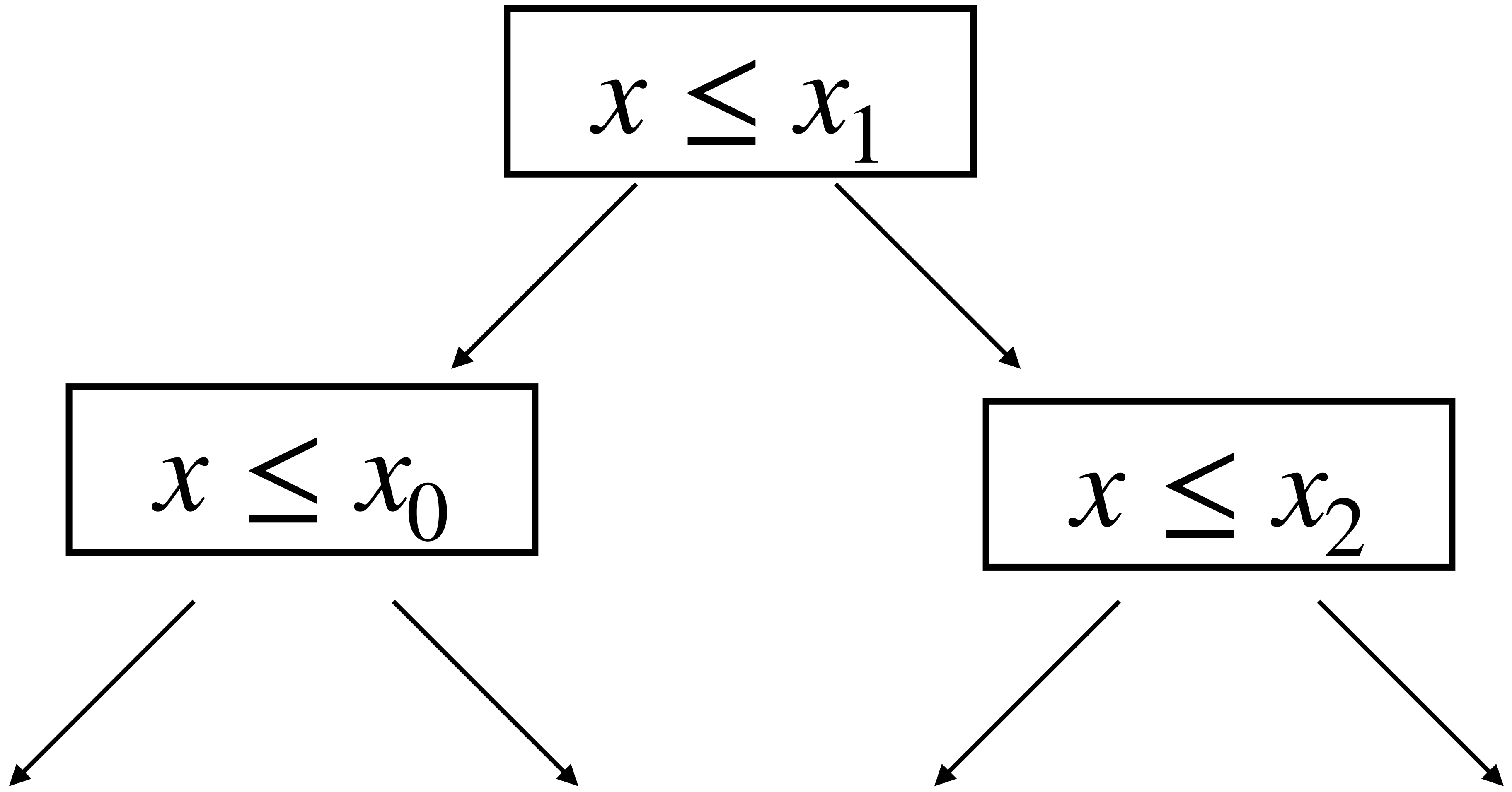

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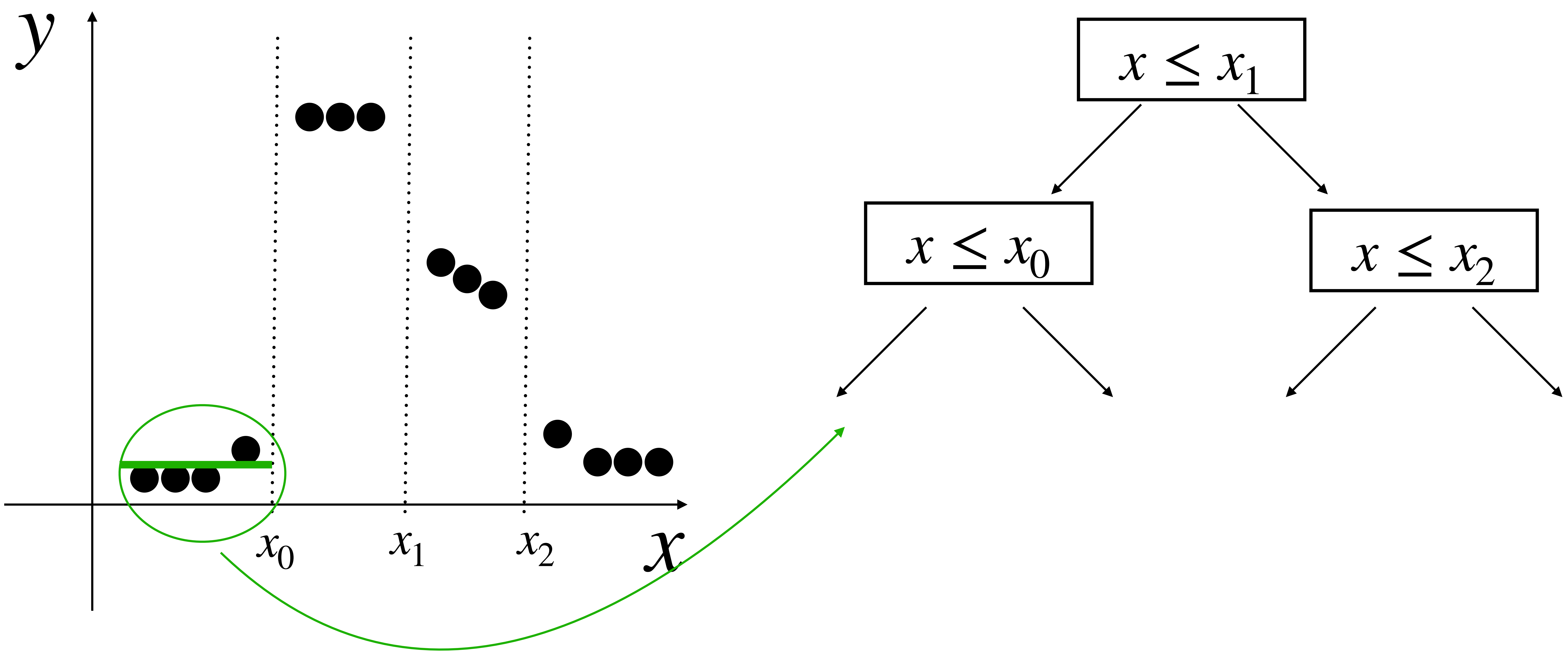

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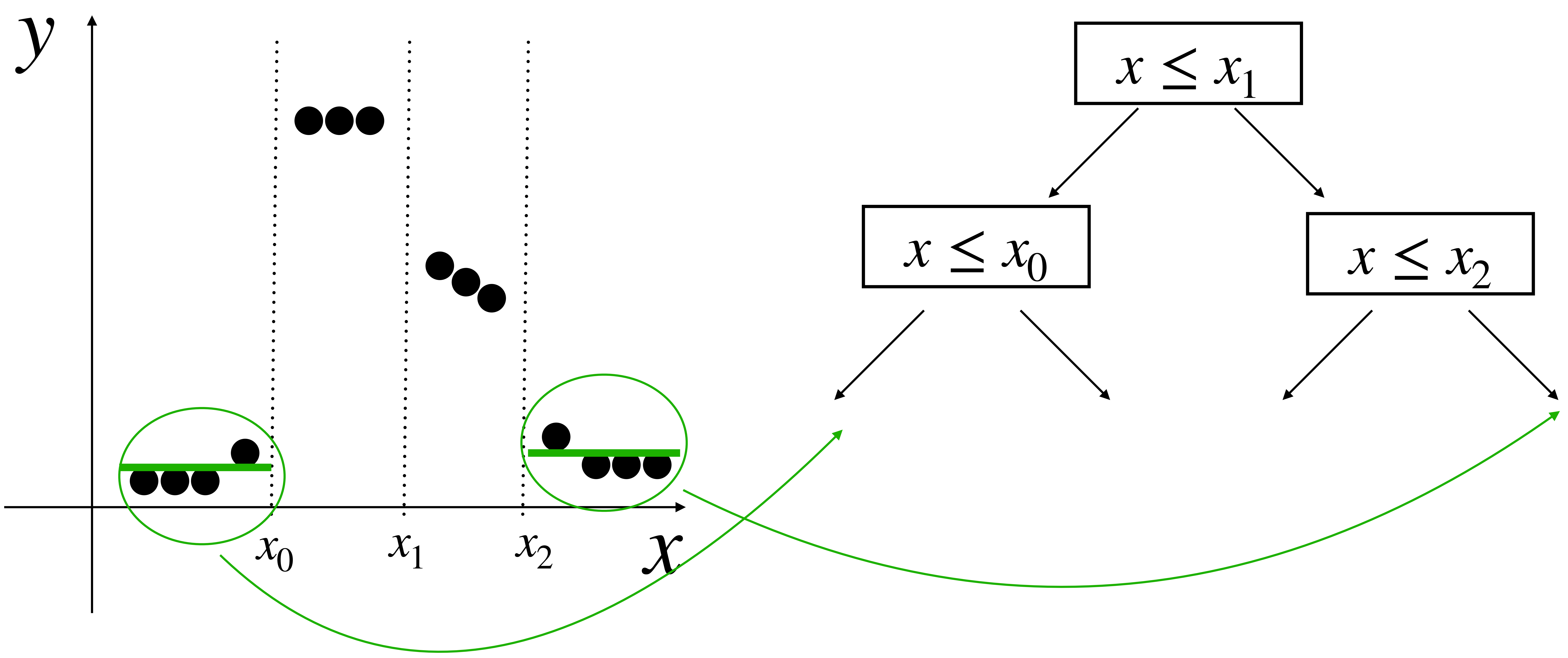

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Define the sample mean $\hat{y}_S = \sum_{i=1}^{m} y_i / m$

Impurity: sample variance $\widehat{Var}(S) = \sum_{i=1}^{m} (y_i - \bar{y}_S)^2 / m$

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The regression Tree algorithm

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  Call Regression_Tree(S_L) & Regression_Tree(S_R)

Issues of Decision Trees

Decision Tree can have high variance, i.e., overfilling!

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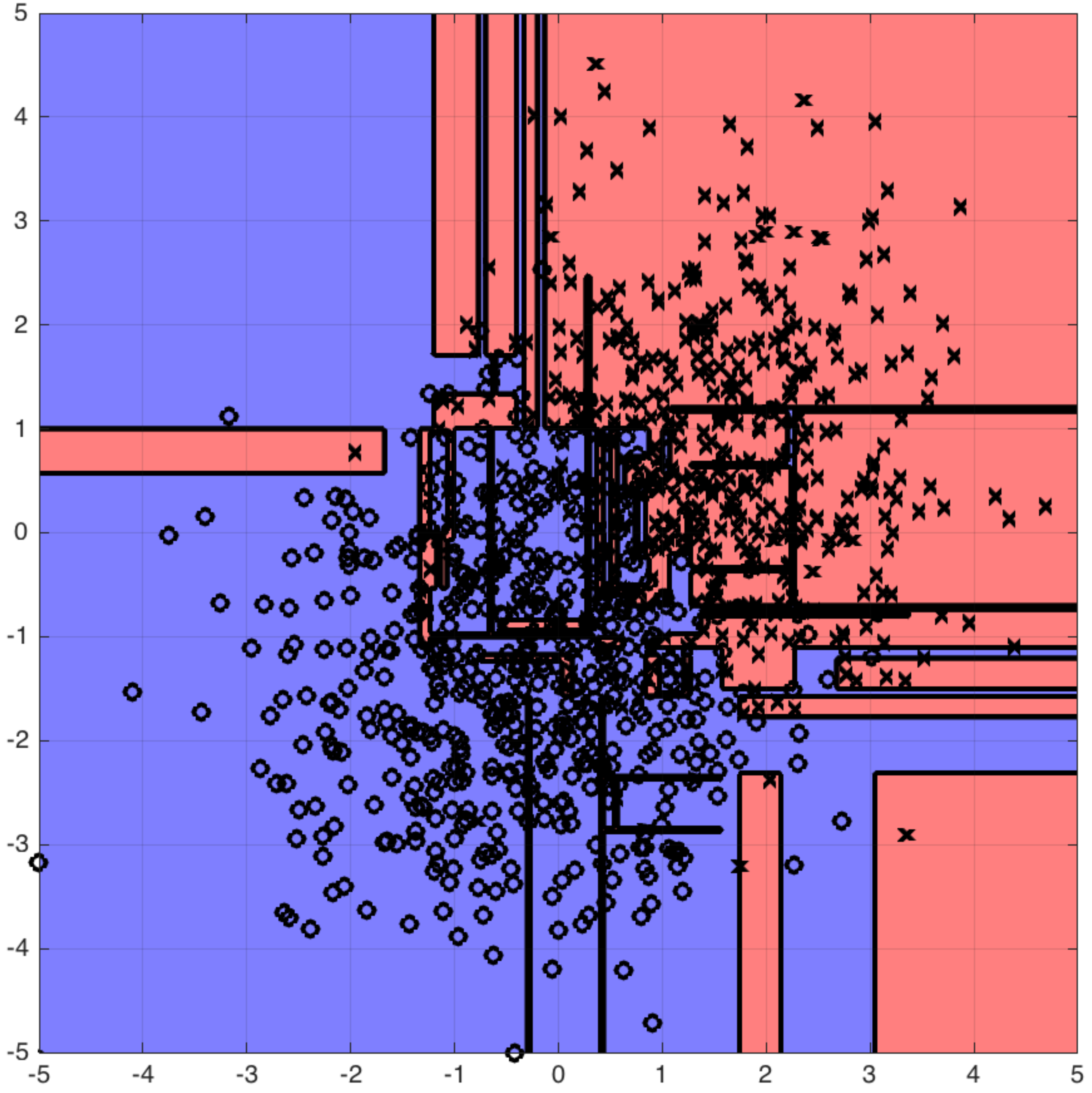

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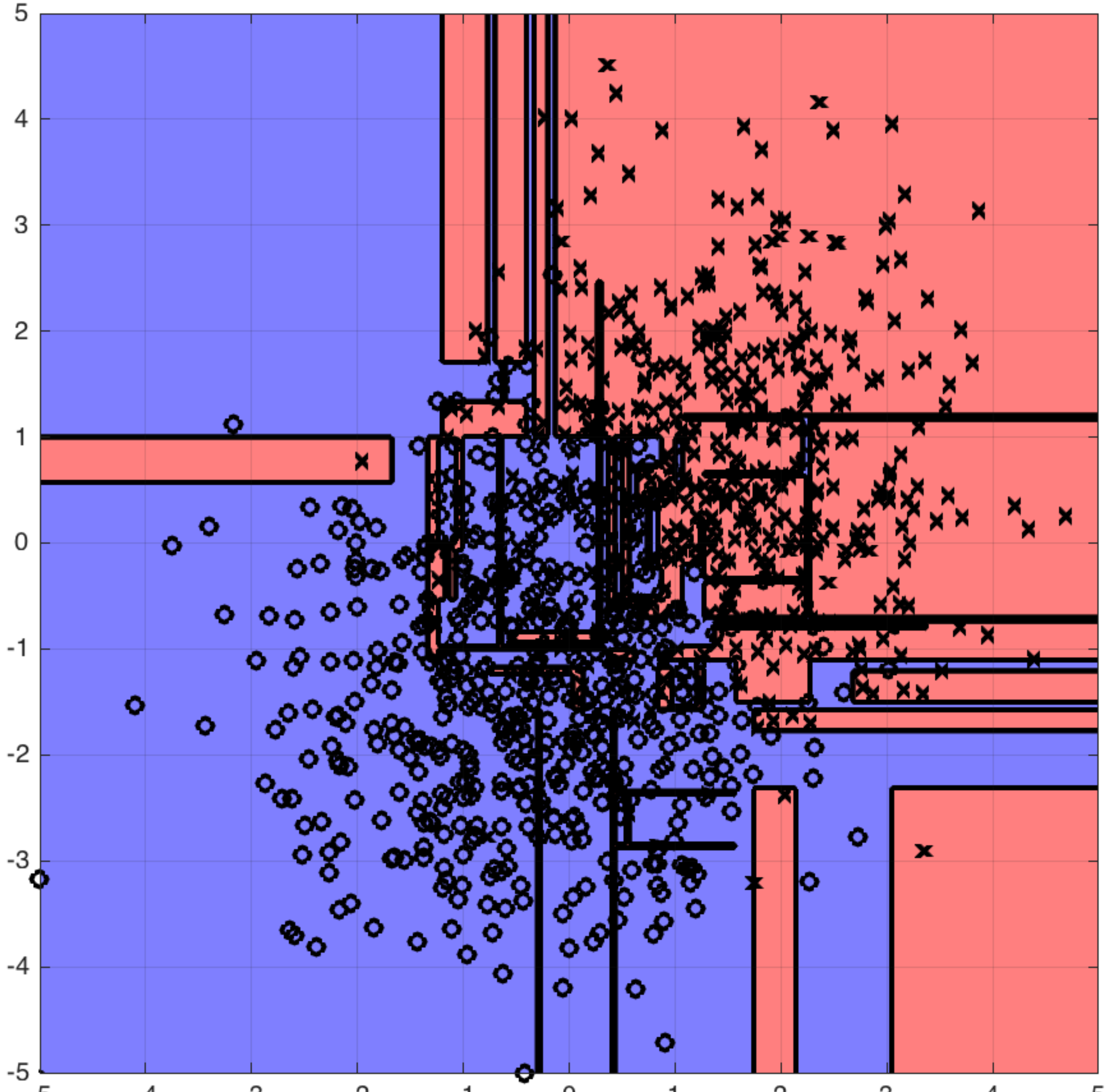

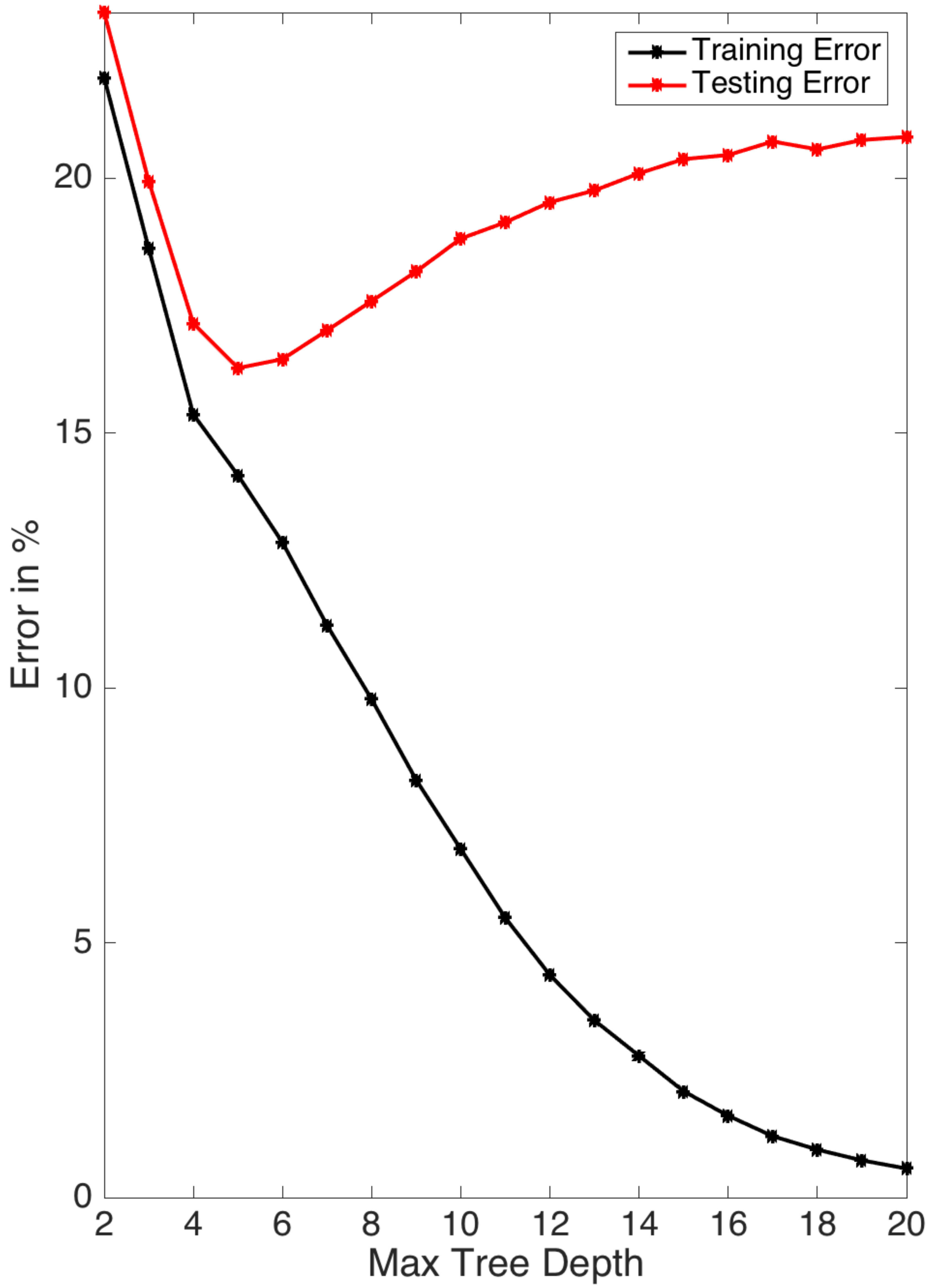

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3. Maximum number of nodes

Stop the tree if it hits max # of nodes

Outline of Today

1. Variance Reduction using averaging

2. Bagging: Bootstrap Aggregation

3. Random Forest

Variance Reduction via Averaging

Consider i.i.d random variables $\{x_i\}_{i=1}^n, \quad x_i \sim \mathcal{N}(0, \sigma^2)$

$$\text{Var}(x_i) = \sigma^2$$

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Avg significantly reduced variance!

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Worst case: when these RVs are positively correlated, averaging may not reduce variance

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Yes, we do this via Bootstrap

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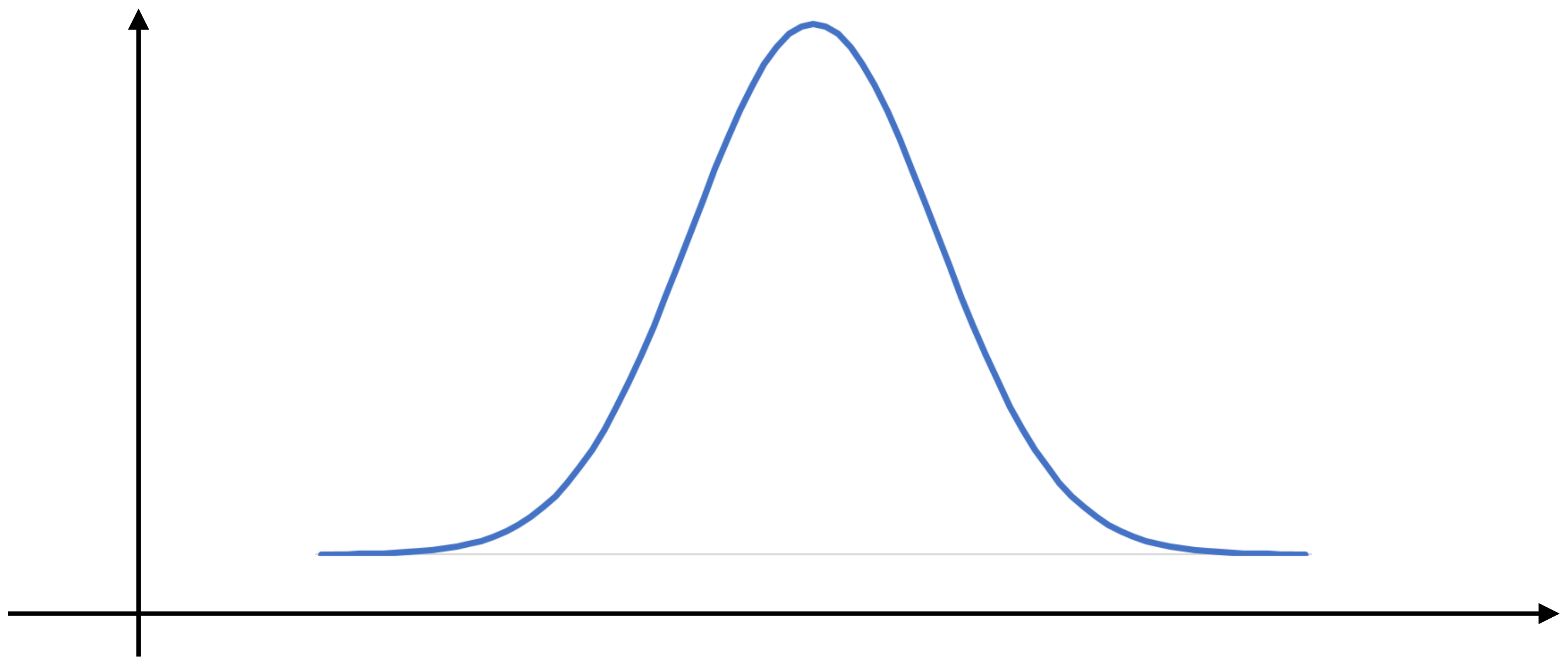

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2. In fact for any $f : Z \rightarrow \mathbb{R}$

$$\mathbb{E}_{z\sim\hat{P}}[f(z)] = \sum_{i=1}^{n} \frac{f(z_i)}{n} \rightarrow \mathbb{E}_{z\sim P}[f(z)]$$

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A: $(1 - 1/n)^n \rightarrow 1/e, n \rightarrow \infty$

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The step that reduces Var!

Bagging in Test Time

Given a test example x_{test}

We can use $\{\hat{h}_i\}_{i=1}^{k}$ to form a distribution over labels:

$$\hat{y} = \begin{bmatrix} p \\ 1 - p \end{bmatrix}$$

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where:

$$p = \frac{\text{\# of trees predicting -1}}{k}$$

Bagging reduces variance

$$\bar{h} = \sum_{i=1}^{k} \hat{h}_i / k$$

What happens when $k \to \infty$?

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The expected decision tree (under true P)

Deterministic, i.e., zero variance

Outline of Today

1. Variance Reduction using averaging

2. Bagging: Bootstrap Aggregation

3. Random Forest

Motivation of Random Forest

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Recall that: $\text{Var}(\bar{x}) = \sigma^2/3 + \sum_{i \neq j} \sigma_{i,j}/9$

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Recall that: $\text{Var}(\bar{x}) = \sigma^2/3 + \sum_{i \neq j} \sigma_{i,j}/9$

To avoid positive correlation, we want to make $\hat{h}_i, \hat{h}_j$ as independent as possible

Random Forest

Key idea:

In ID3, for every split, **randomly select** k $(k < d)$ many features, find the split **only using these k features**

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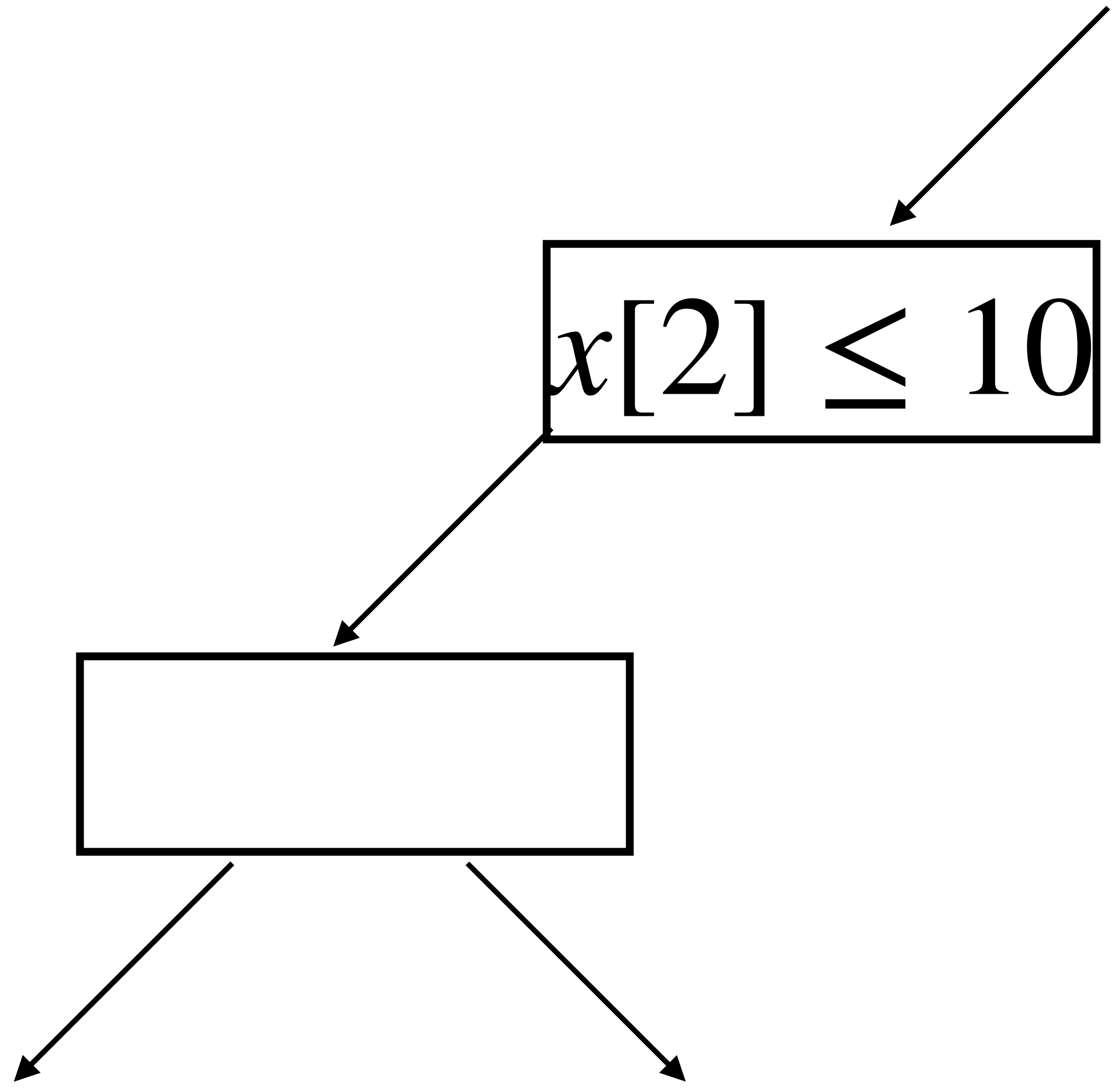

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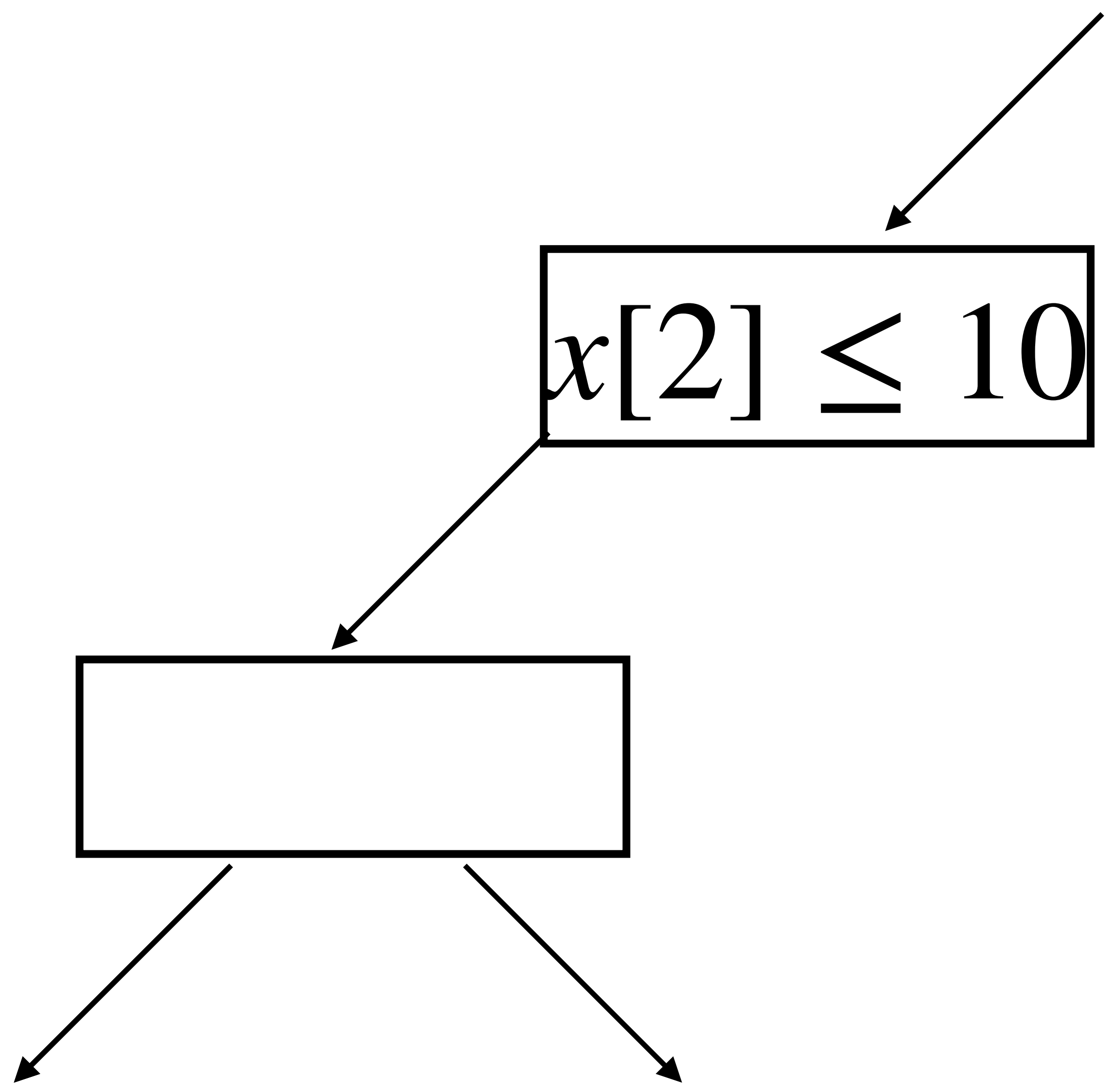

Regular ID3: looking for split in all d dimensions

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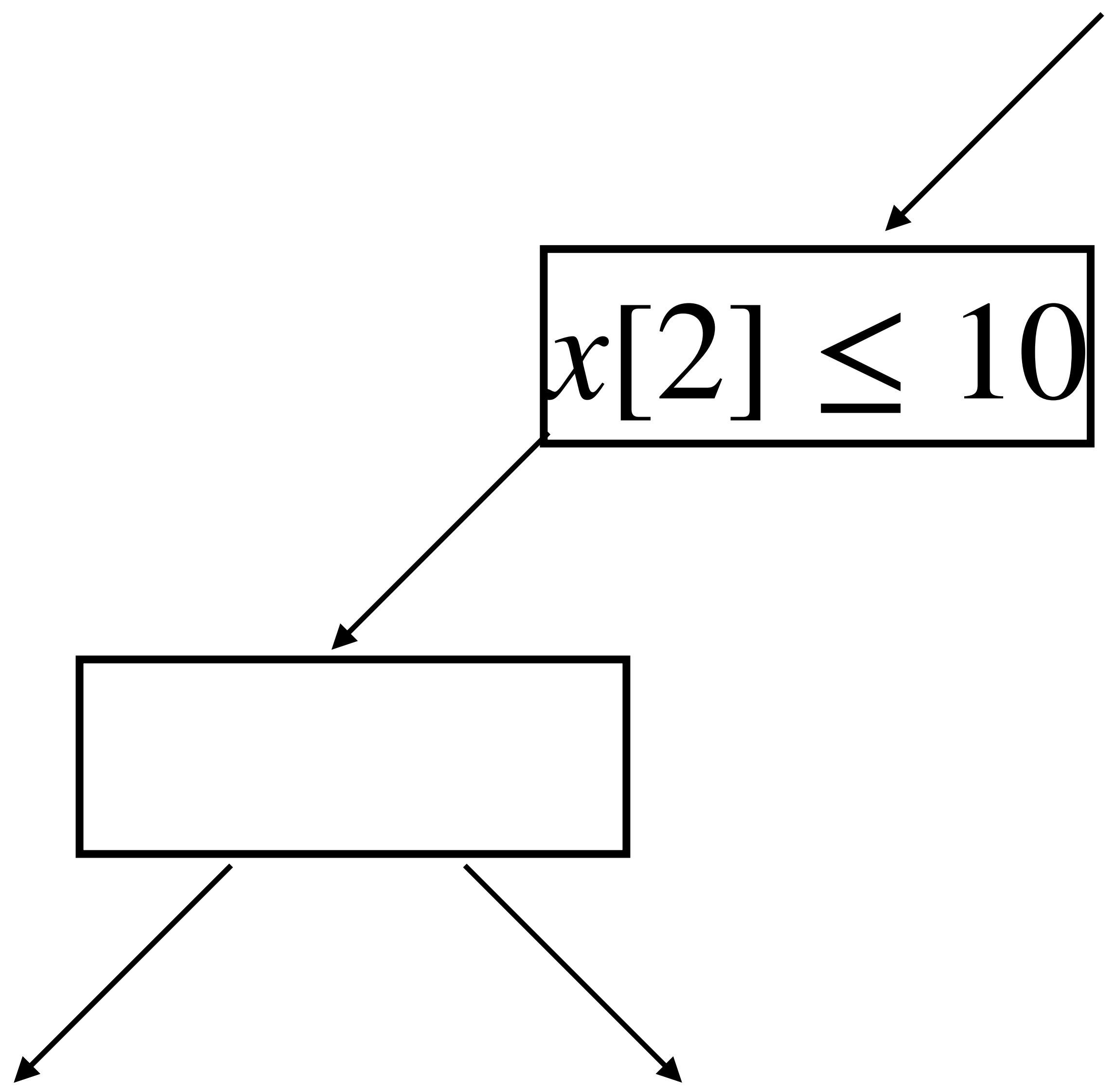


Regular ID3: looking for split in all d dimensions

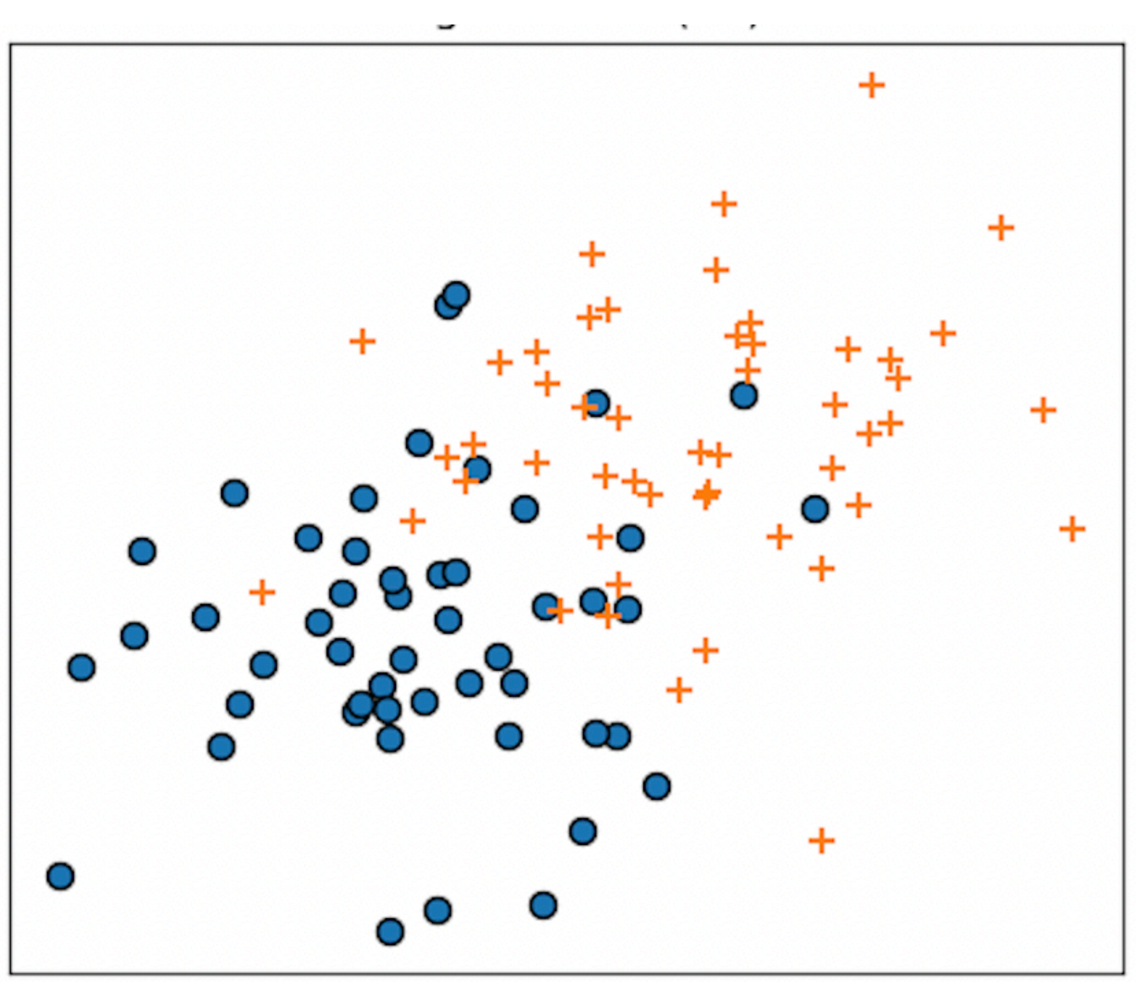
ID3 in RF: looking for split in k randomly picked dimensions

Benefit of Random Forest

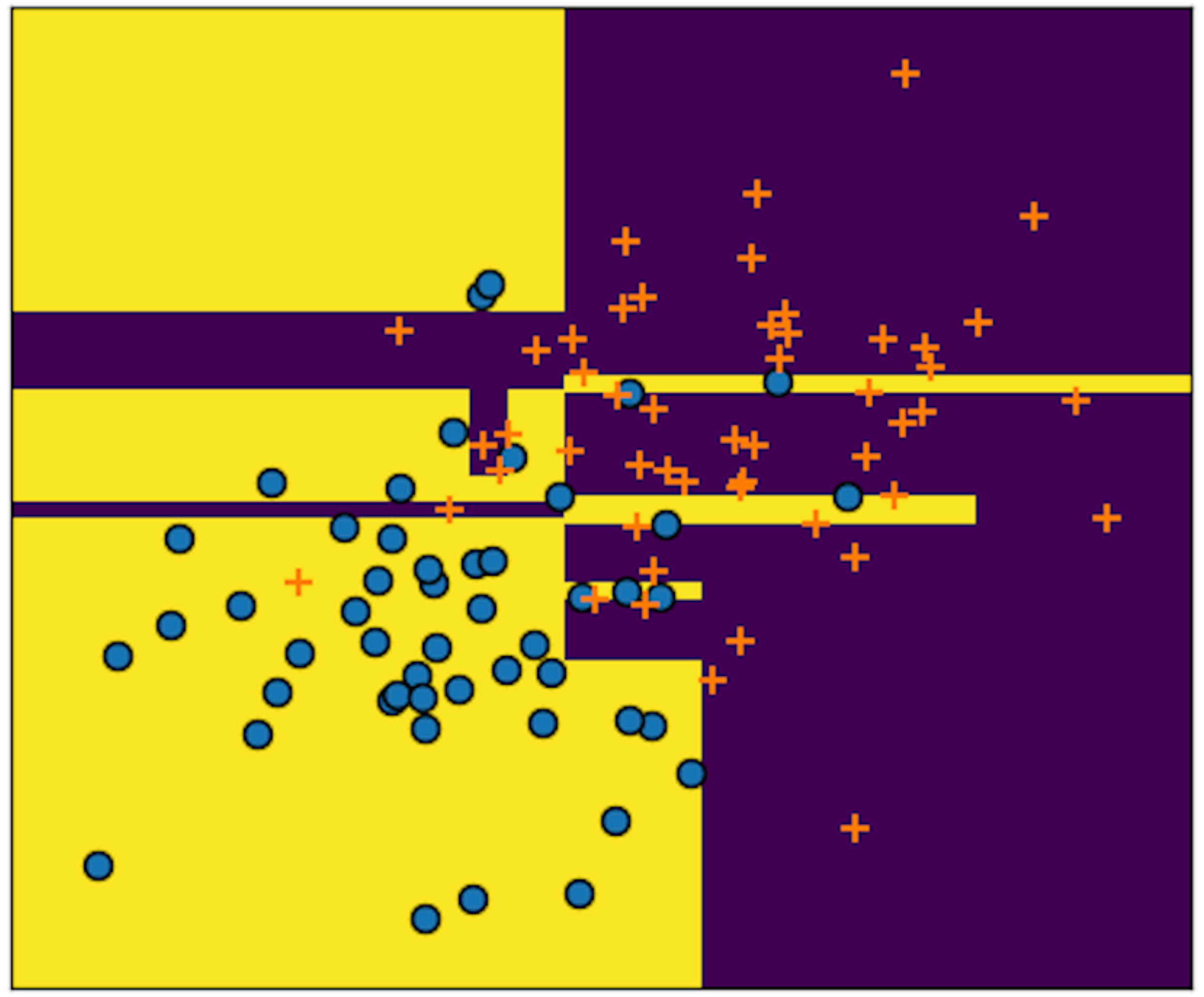
By always randomly selecting subset of features for **every tree, and every split**:

We further reduce the correlation between $\hat{h}_i$ & $\hat{h}_j$

Demo of Random Forest



DT w/ Depth 10



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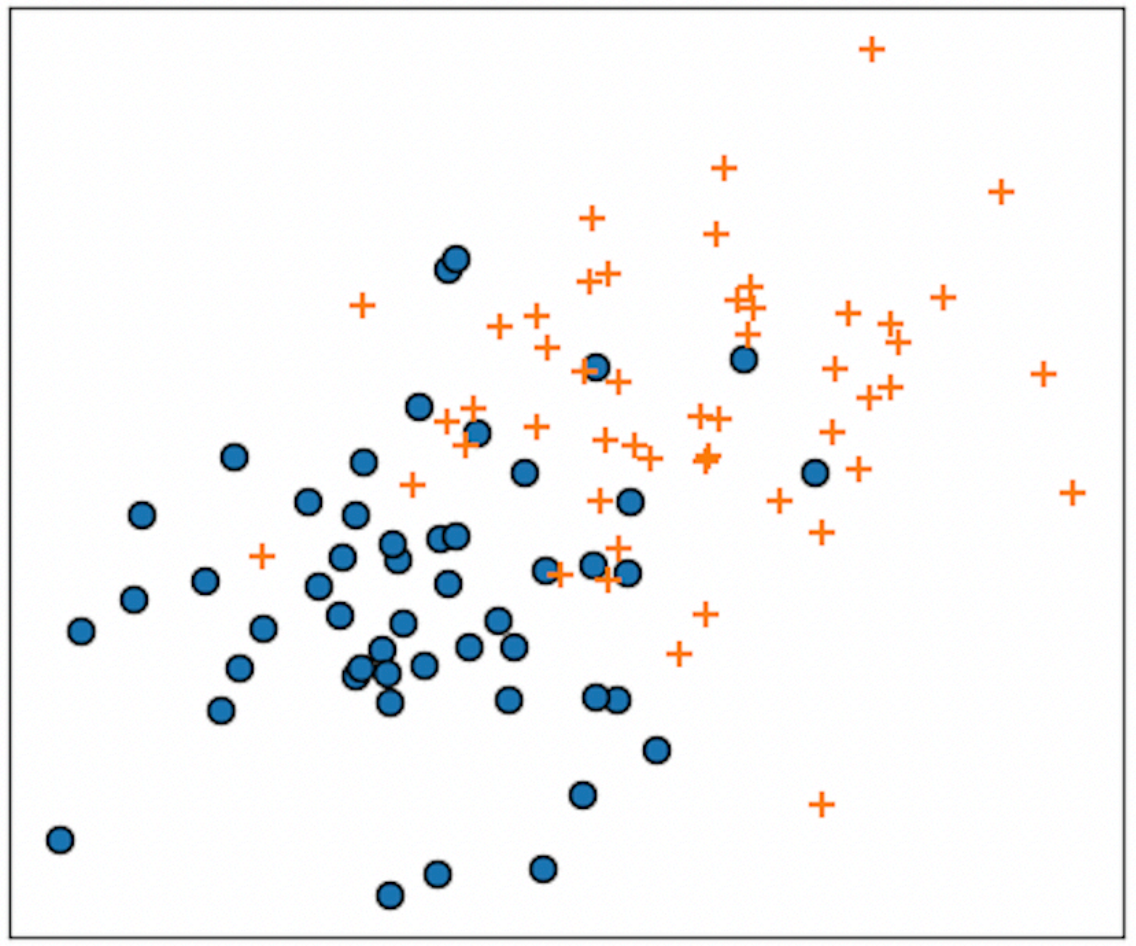

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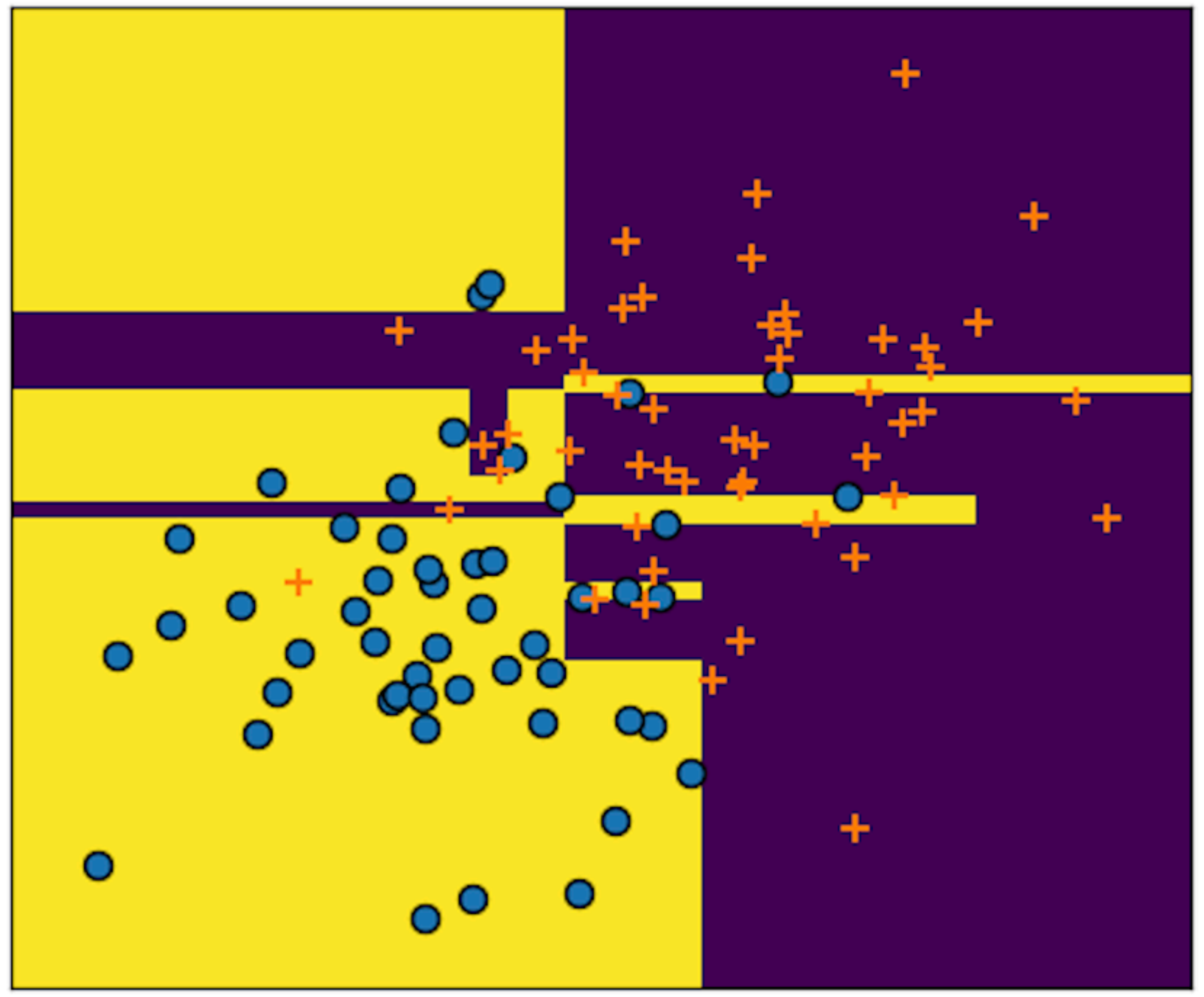
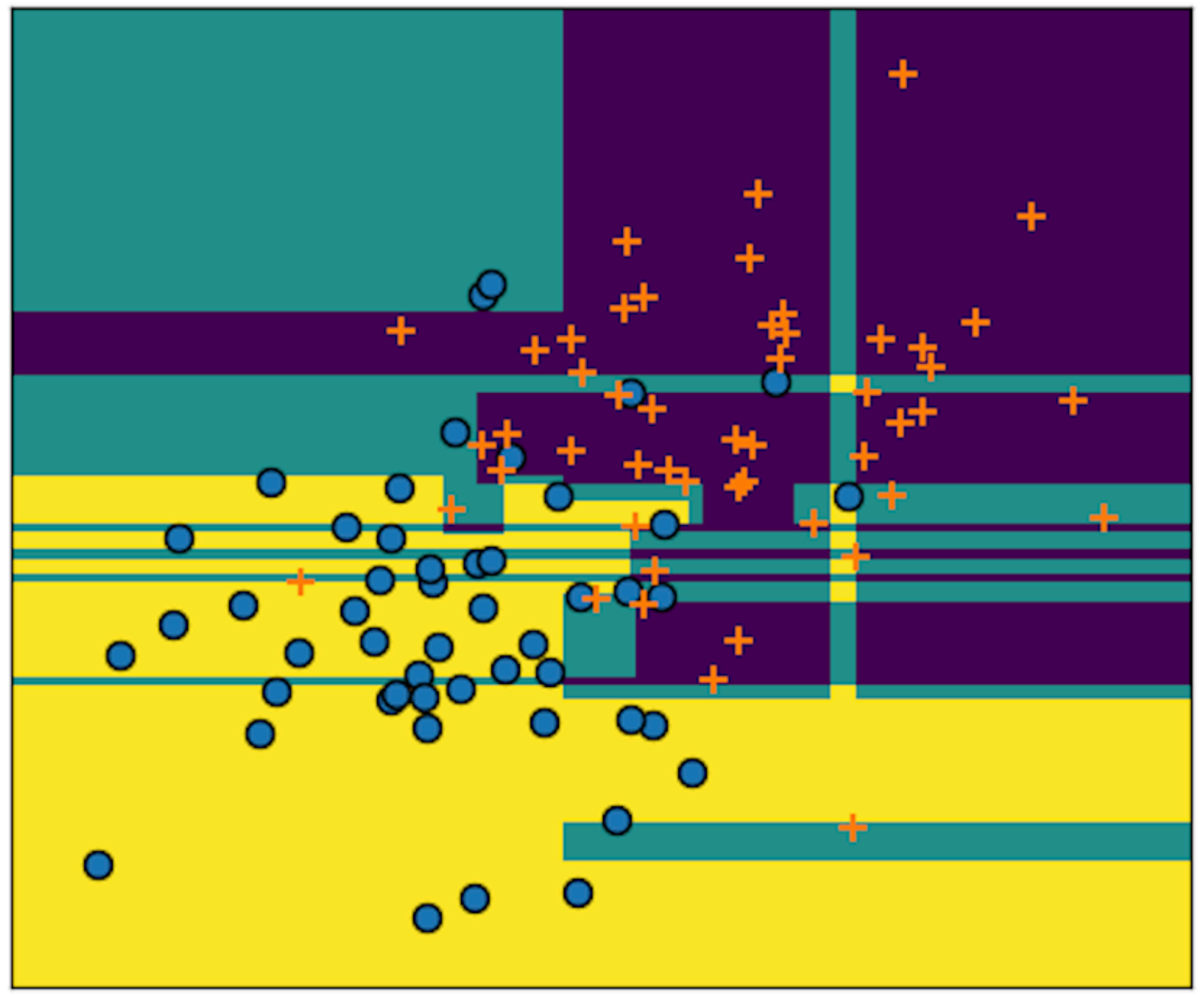


RF w/ 2 trees



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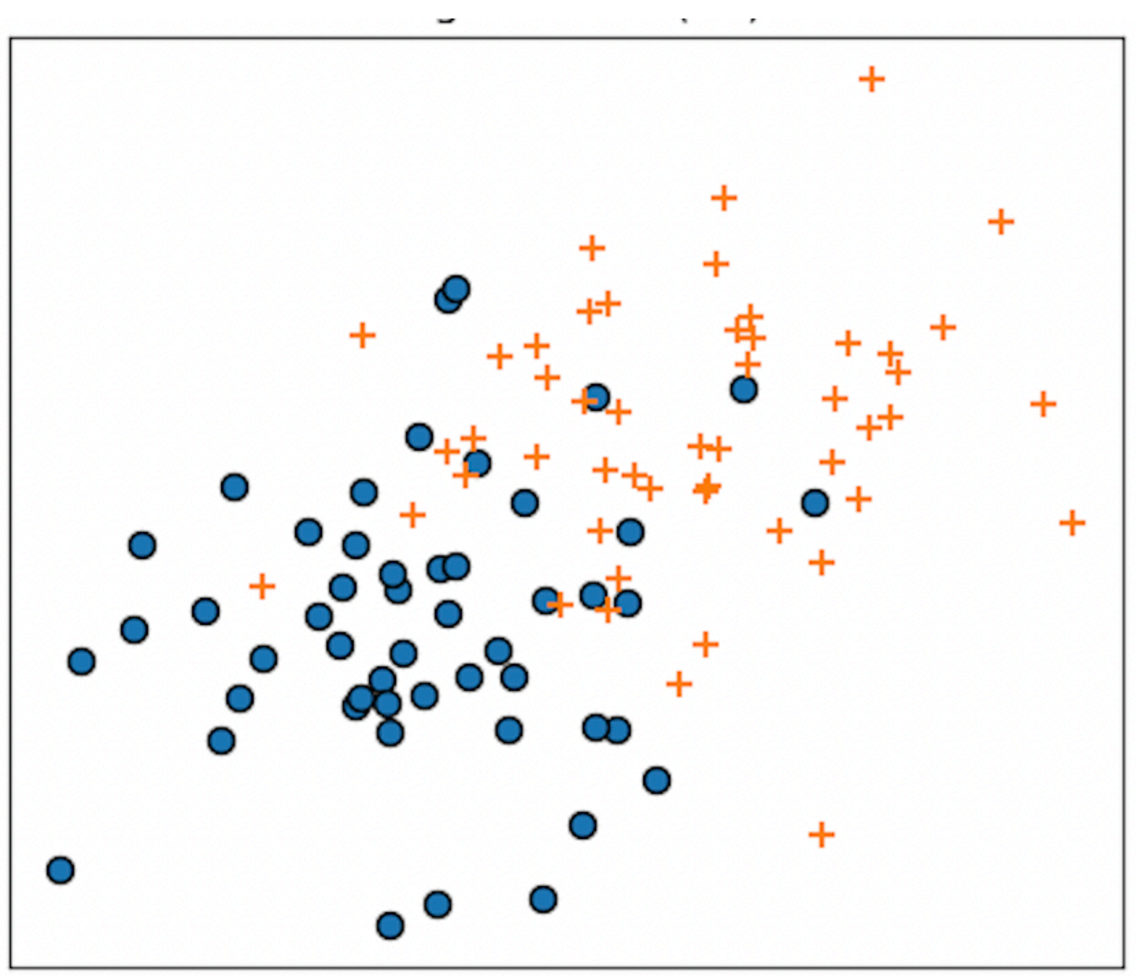

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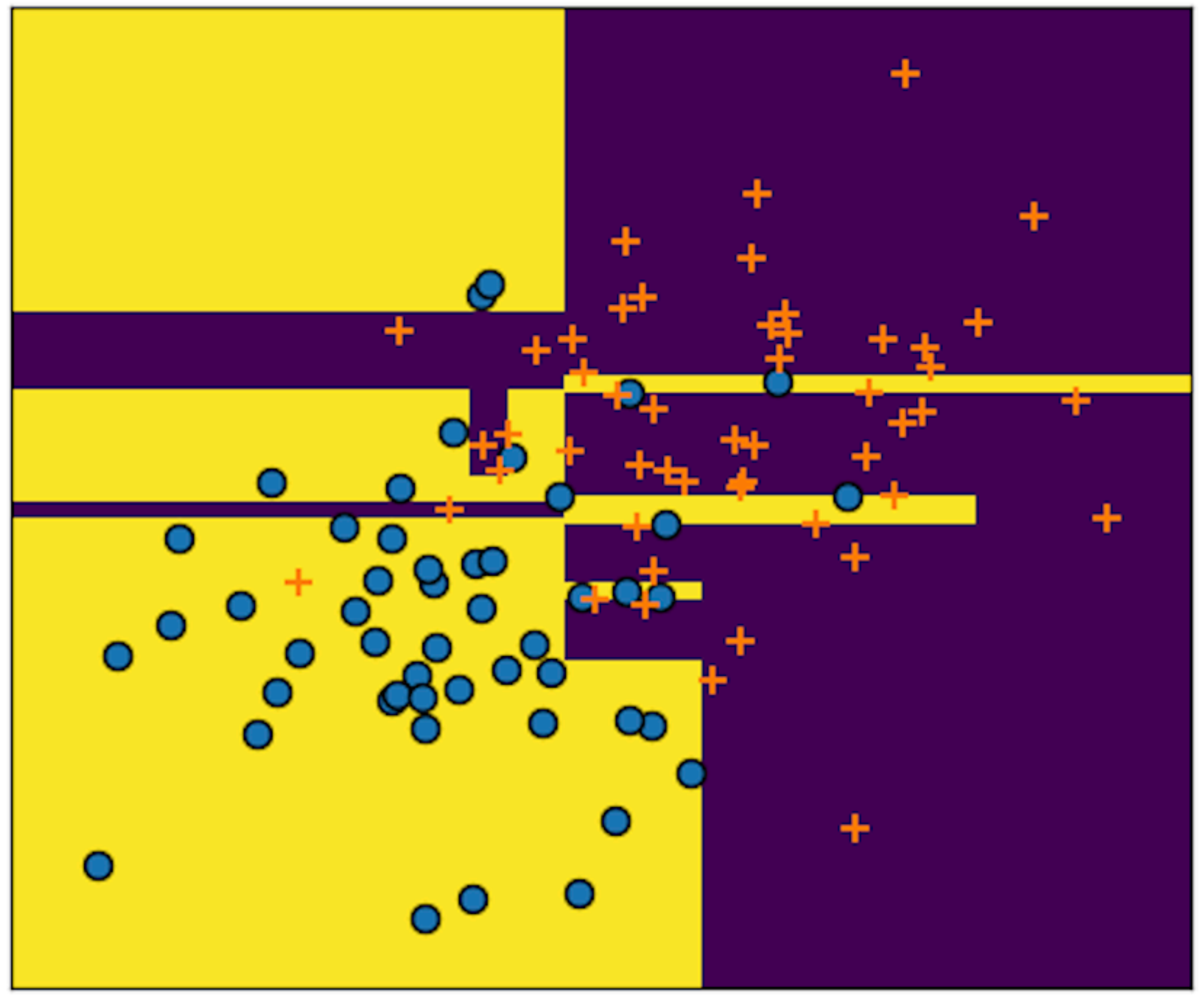

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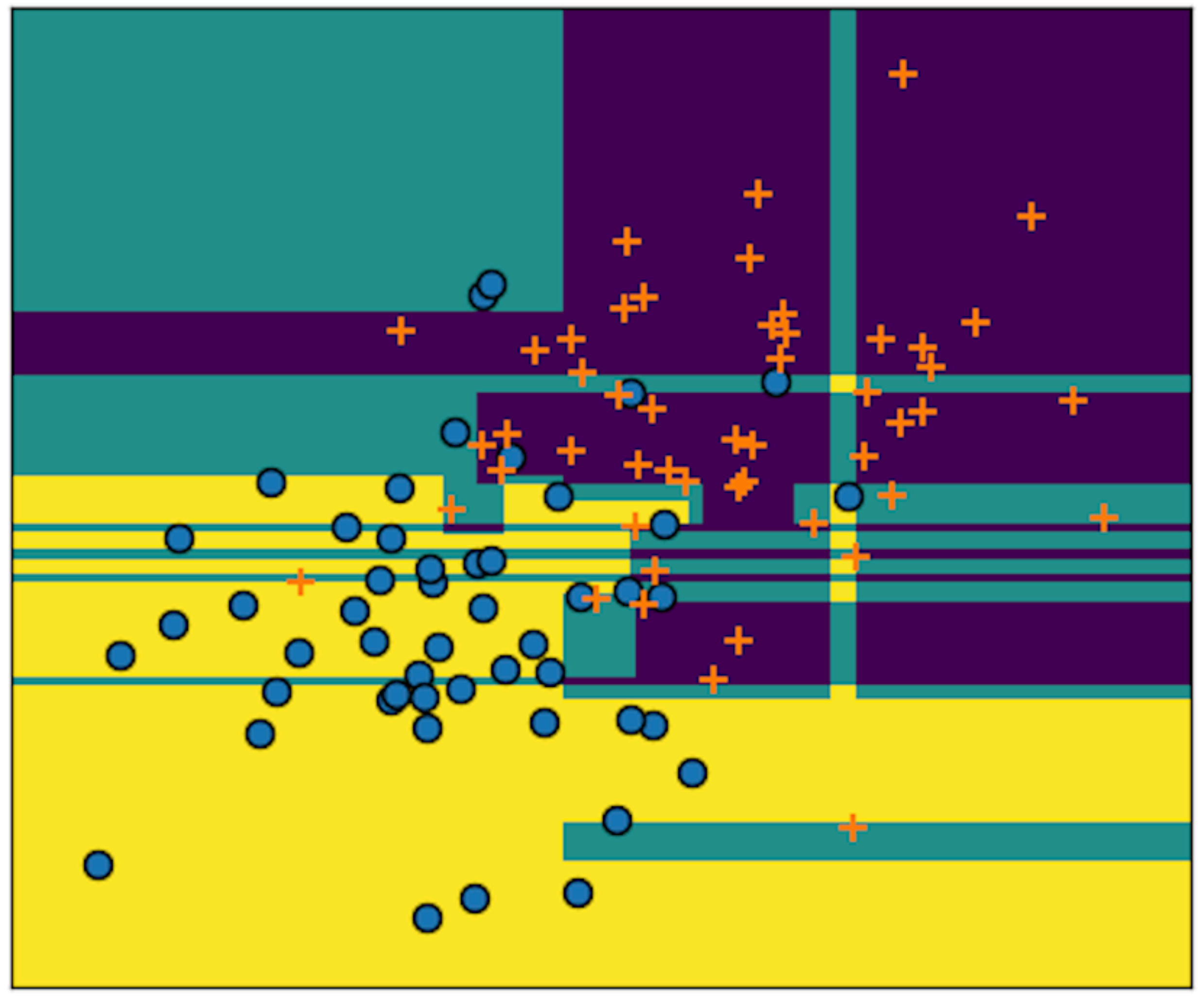
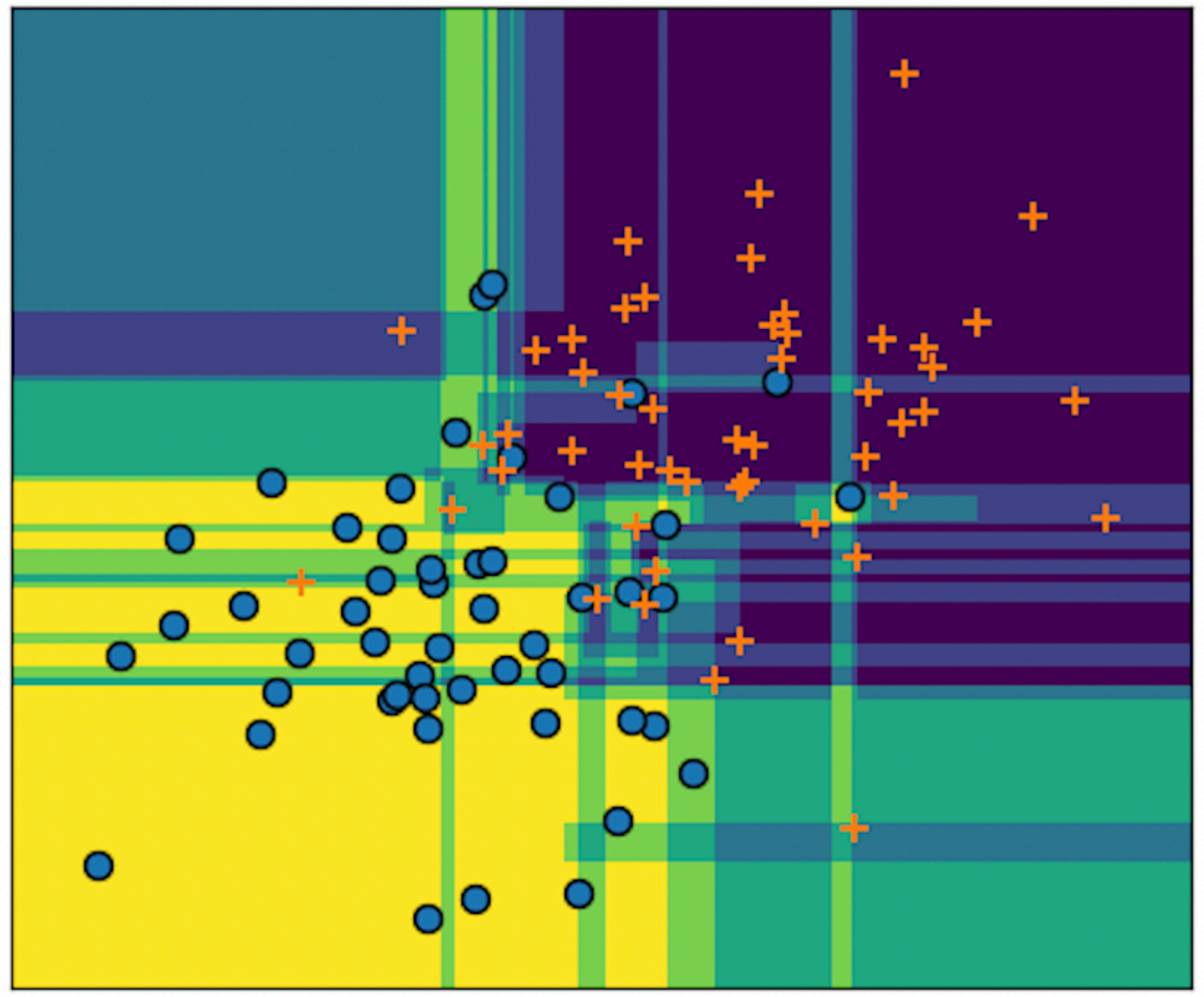


RF w/ 5 trees



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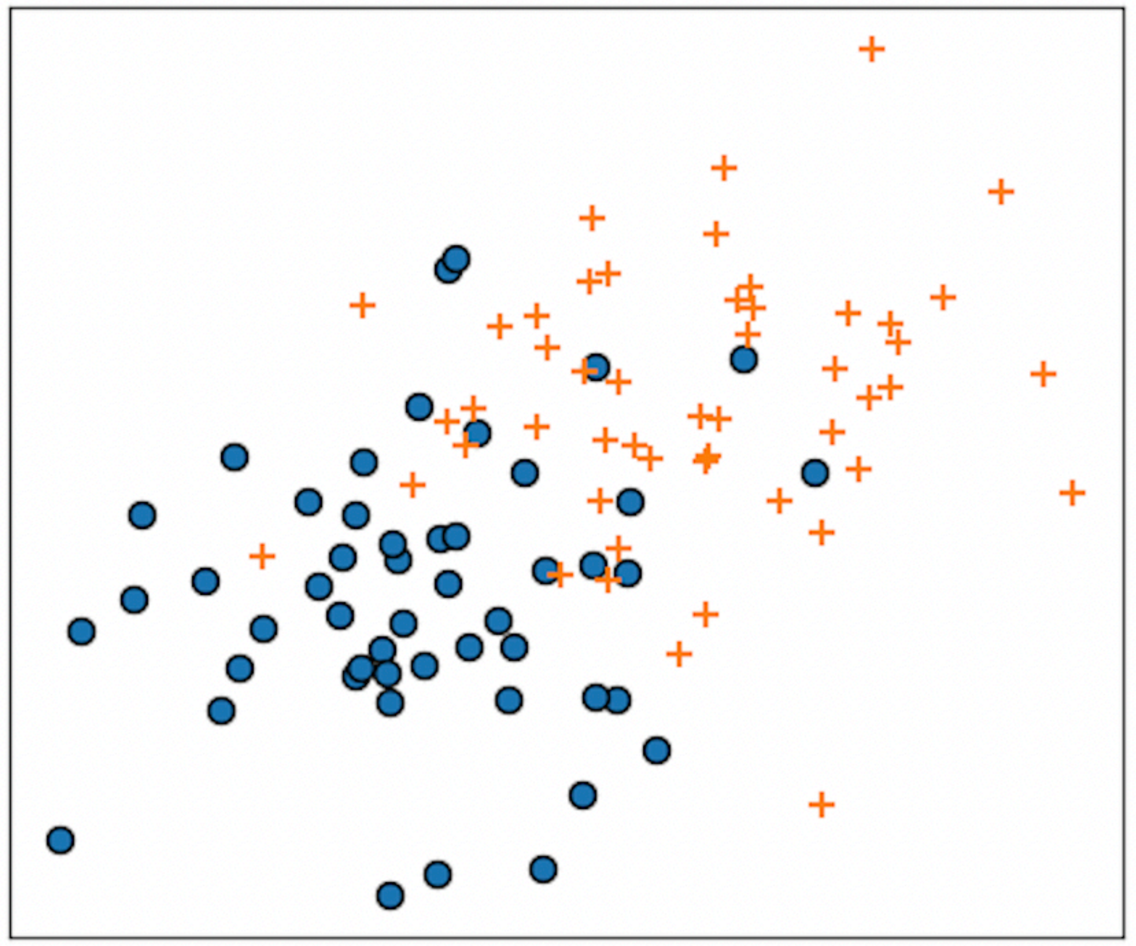

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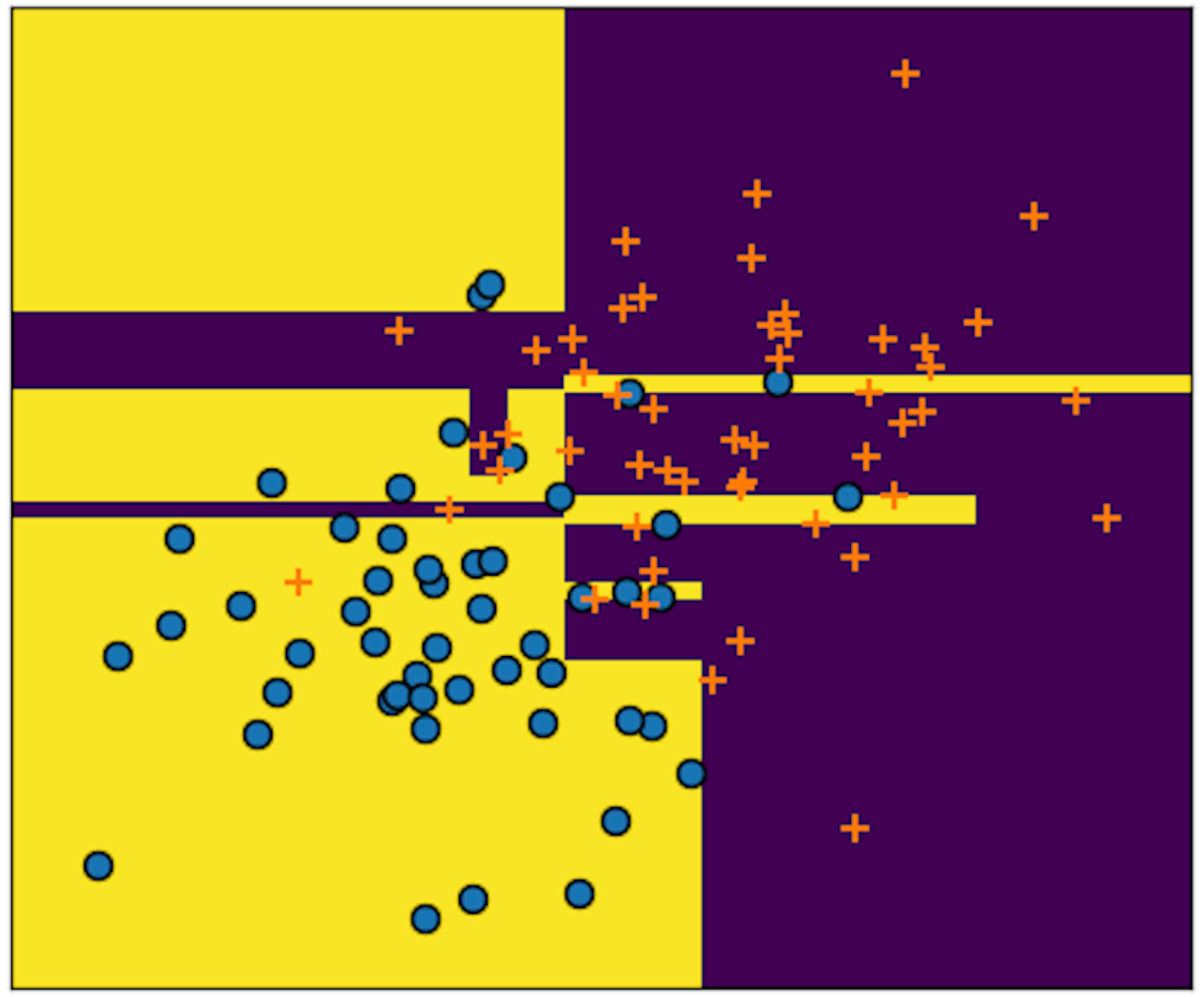

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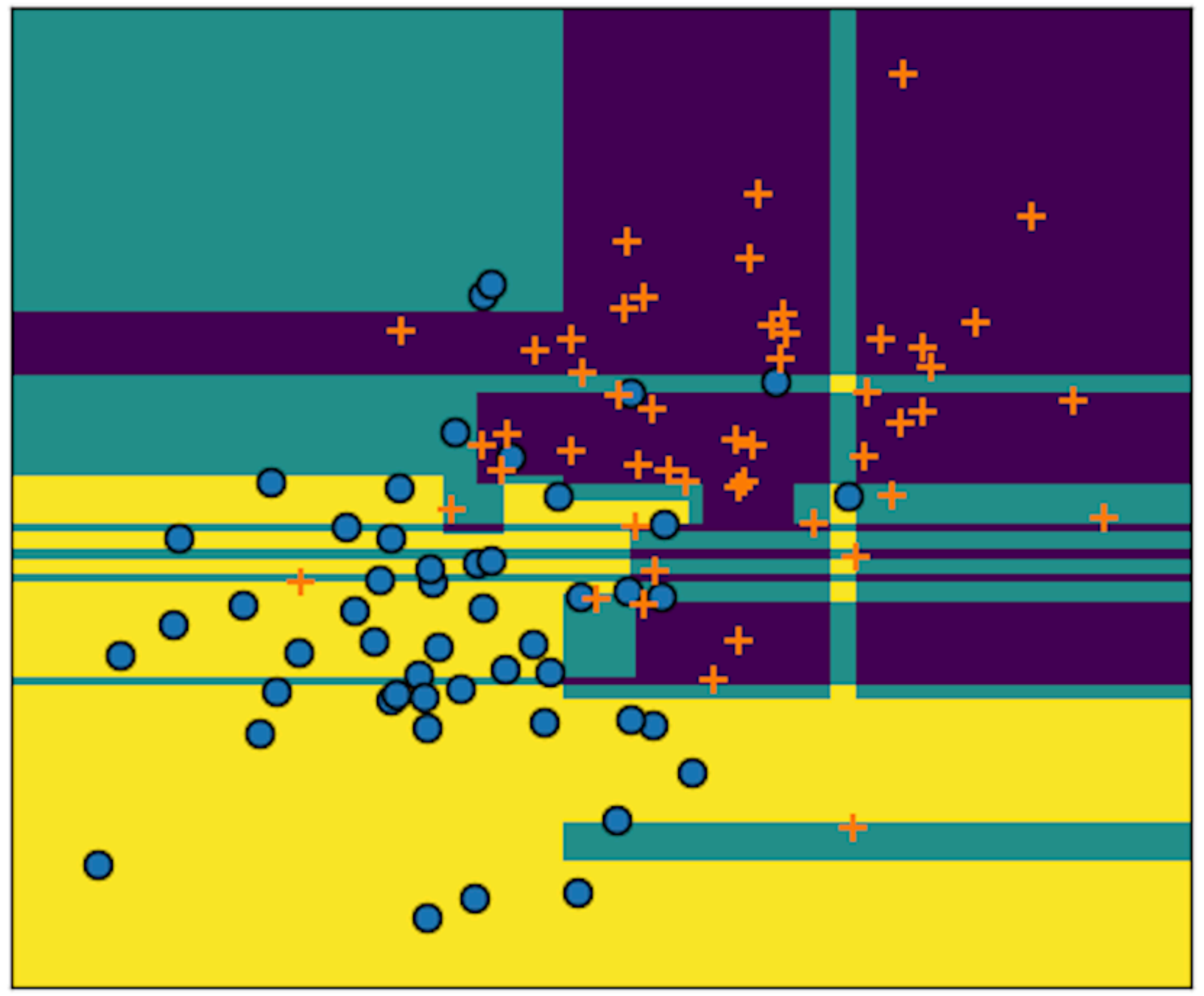

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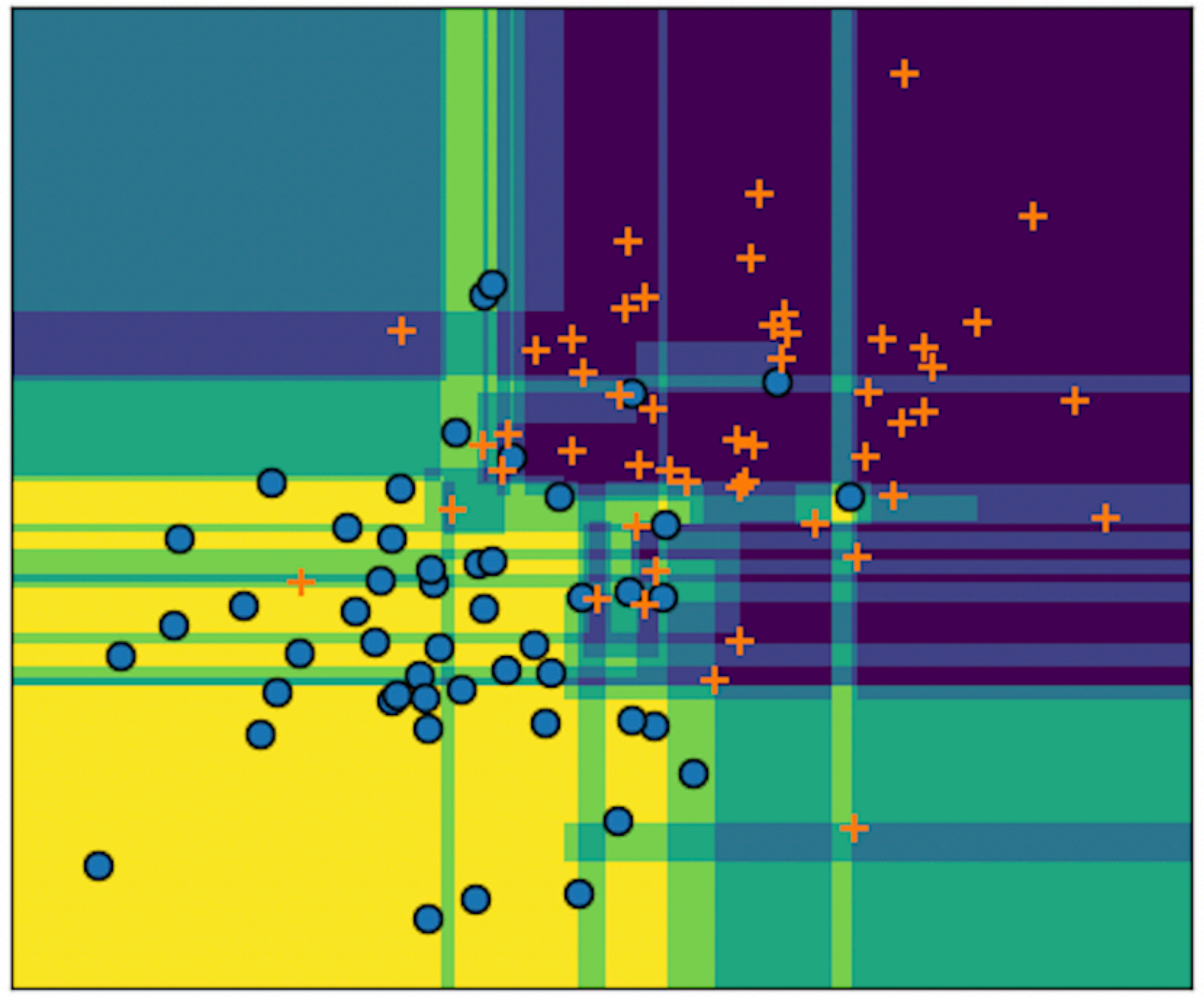
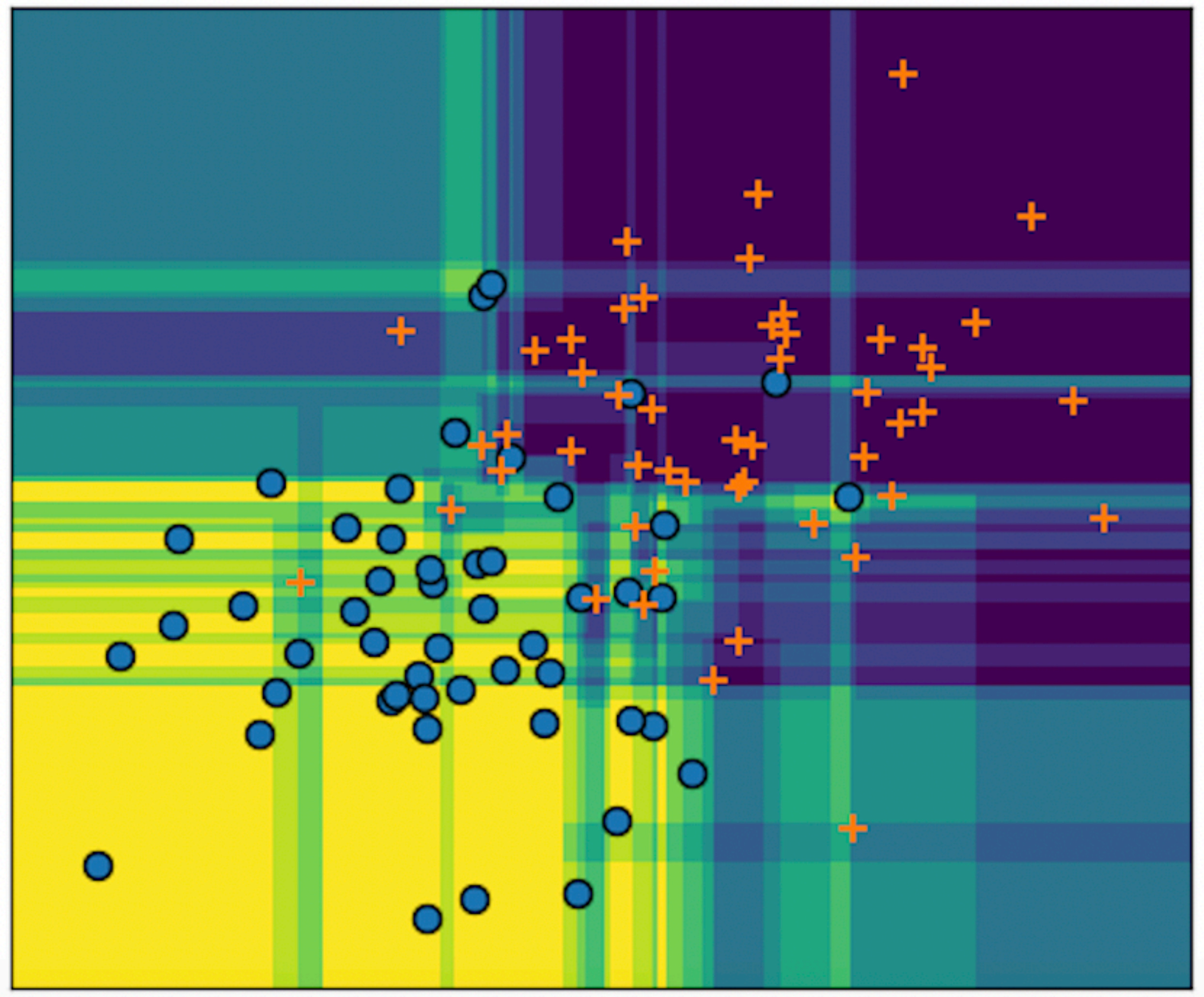


RF w/ 10 trees



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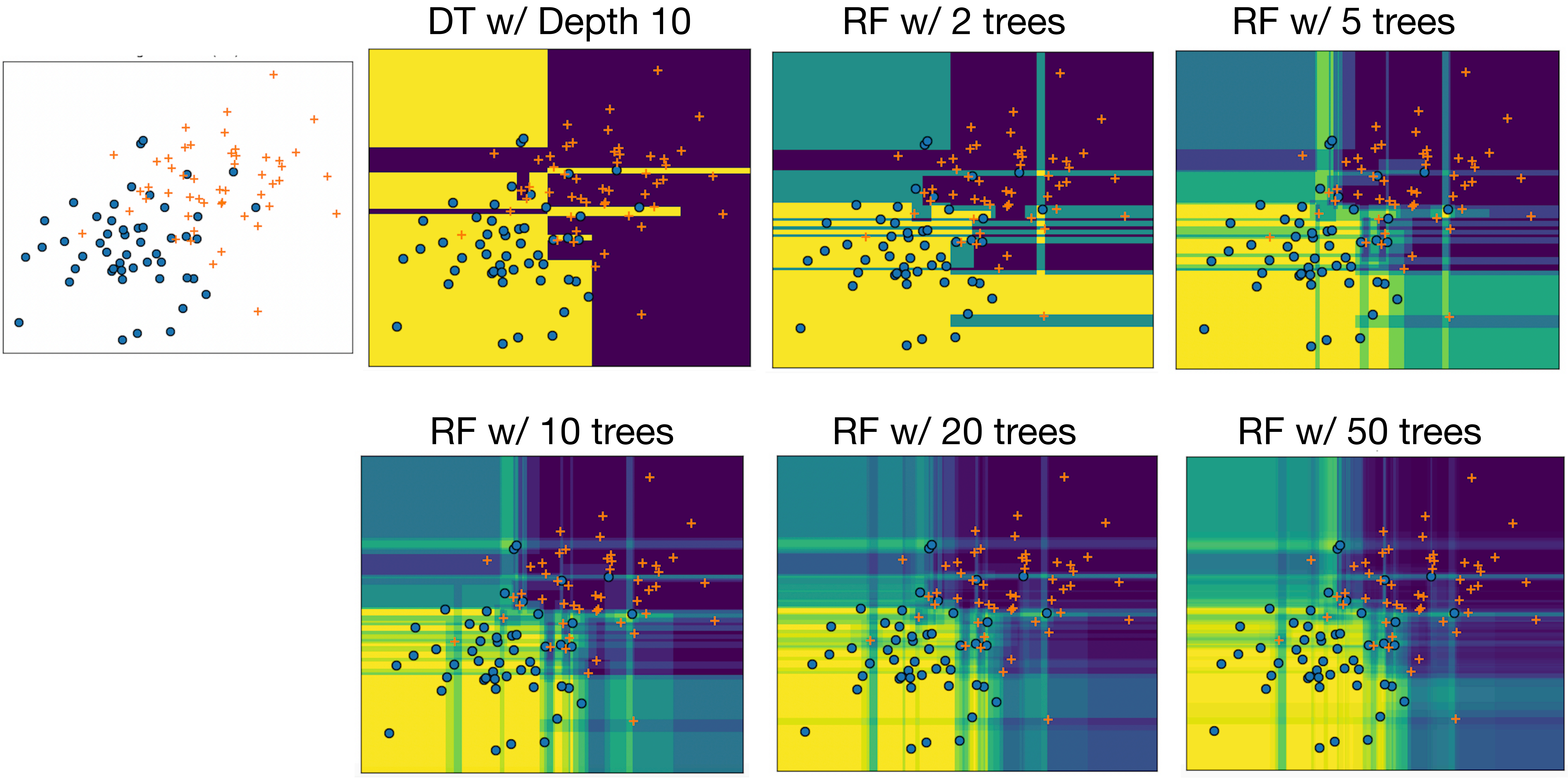

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1. Create datasets via bootstrapping + train classifiers on them + averaging

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1. Create datasets via bootstrapping + train classifiers on them + averaging

2. To further reduce correlation between classifiers, RF randomly selects subset of dimensions for every split.