Support Vector Machine

Announcements

1. Prelim Conflict form is going out soon

2. Prelim practice: we will release previous semesters' prelims w/ solutions

Outline for Today

1. Functional Margin & Geometric Margin

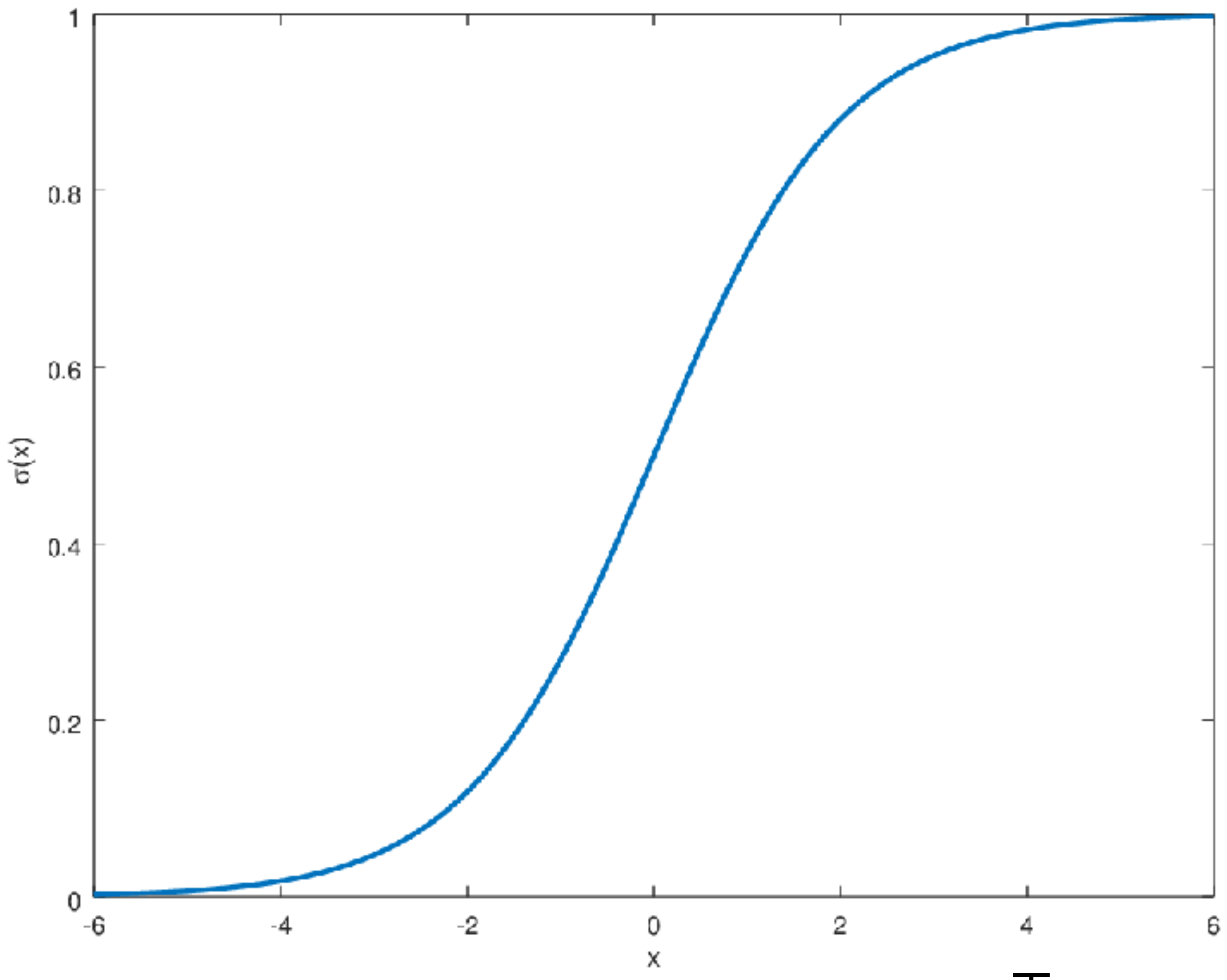
2. Support Vector Machine for separable data

3. SVM for non-separable data

Recall Logistic Regression

Binary classification with $\mathscr{D} = \{x_i, y_i\}_{i=1}^{n}, x_i \in \mathbb{R}^d, y_i \in \{-1,1\}$

Logistic Regression asumes $P(y \,|\, x; w, b) = \dfrac{1}{1 + \exp(-y(w^\top x + b))}$



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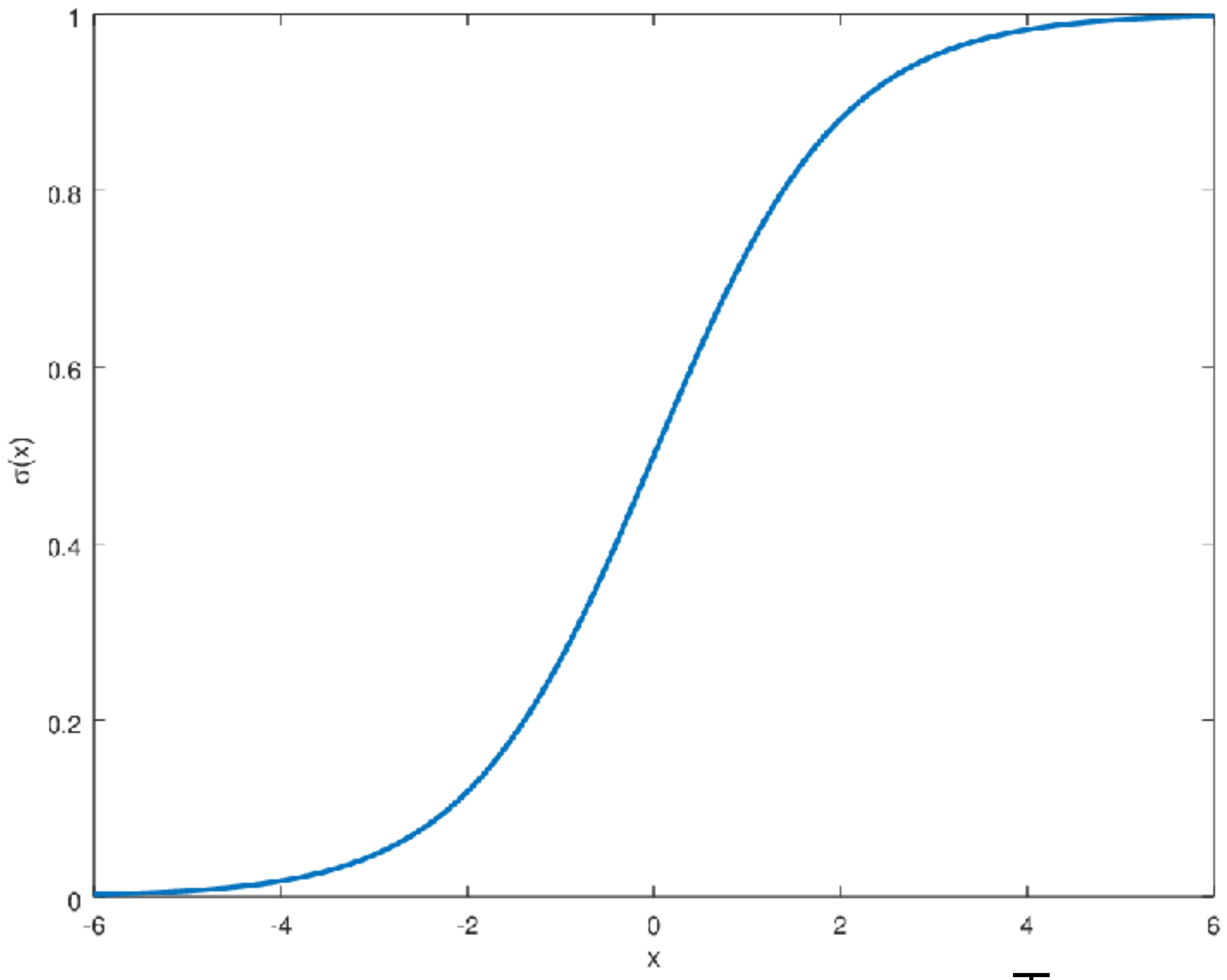

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Given (x, y), our model predict label y, if

$P(y \mid x; w, b) > 0.5$, or equivalently $y(w^\top x + b) > 0$

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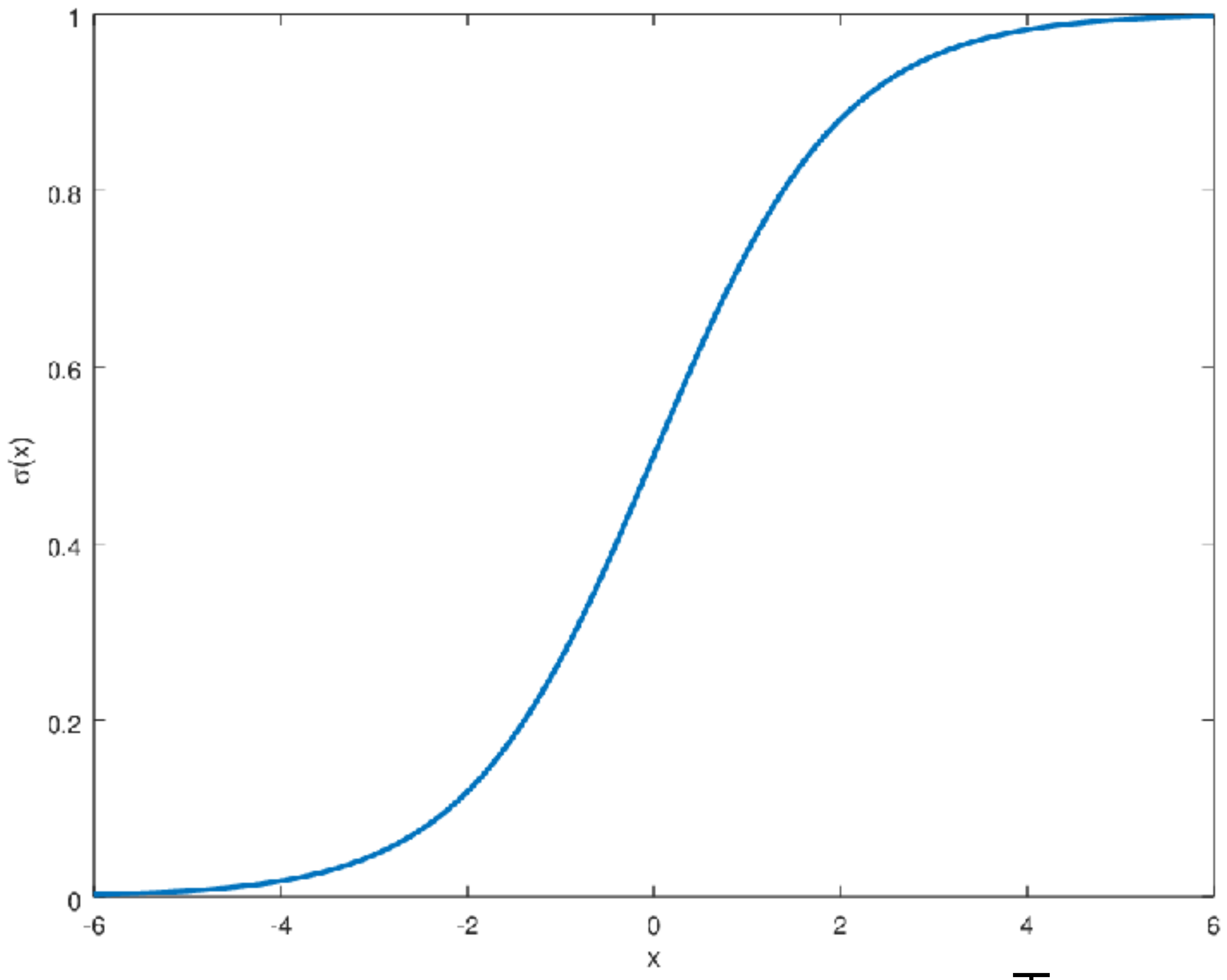

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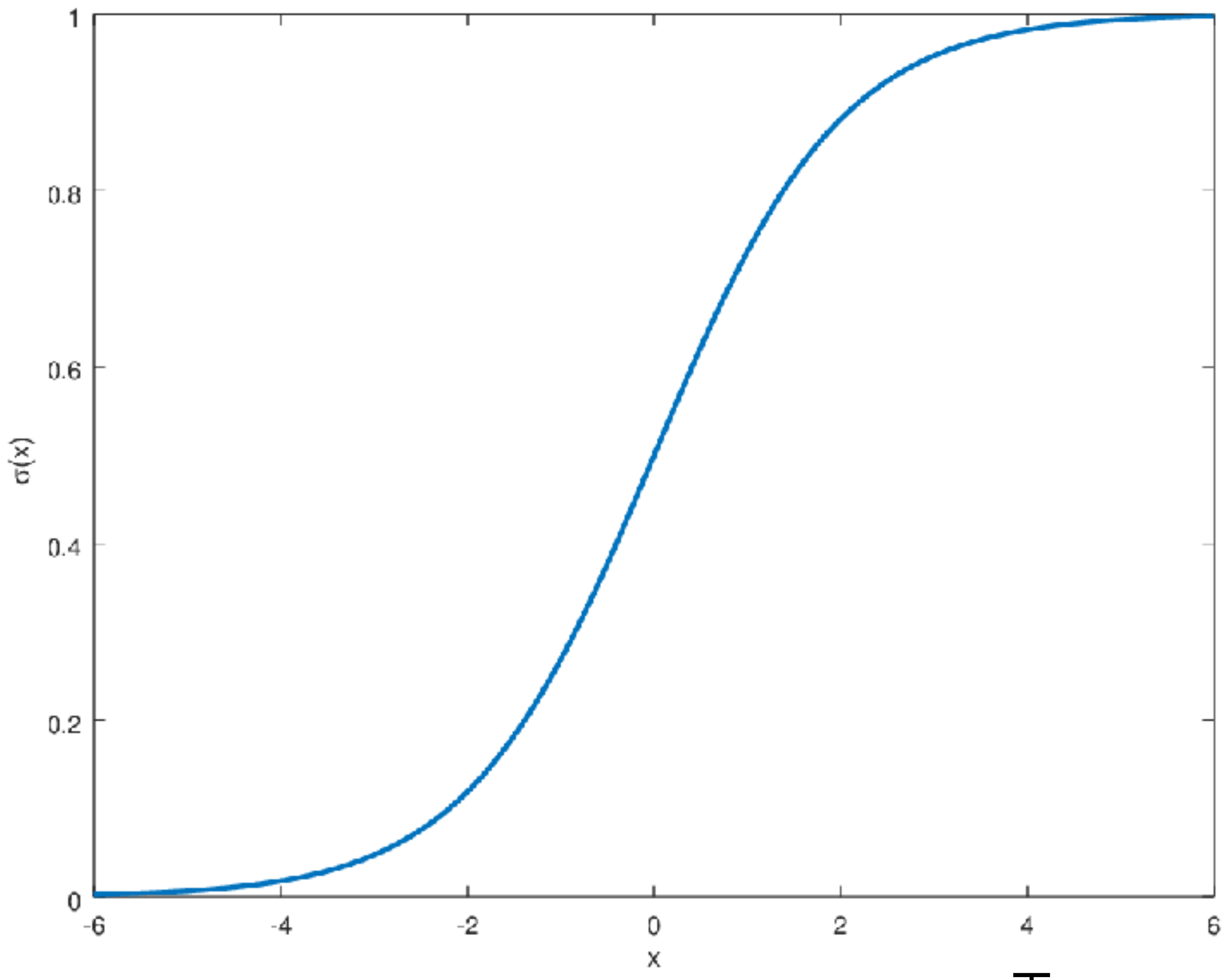

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Functional margin

"confidence"

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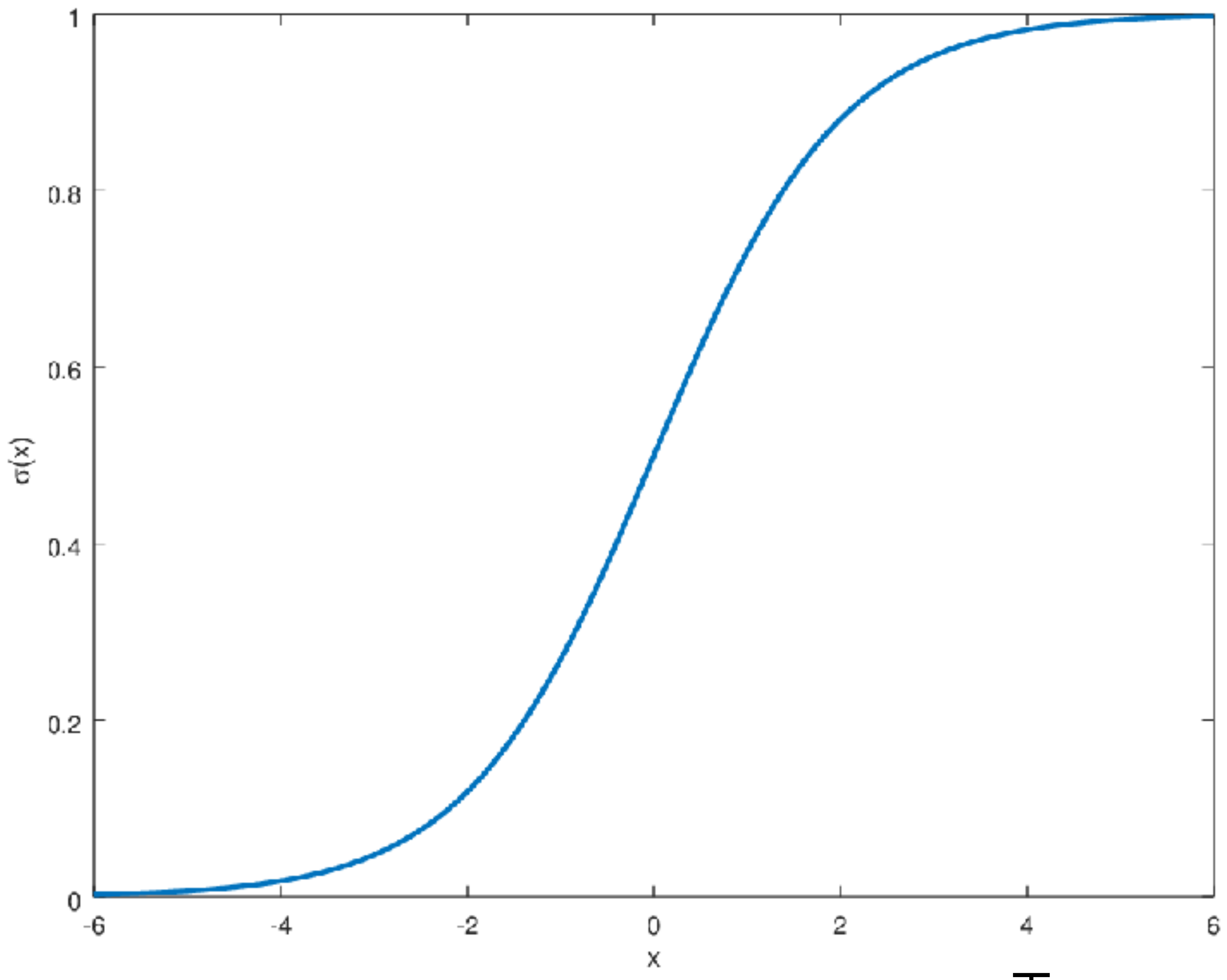

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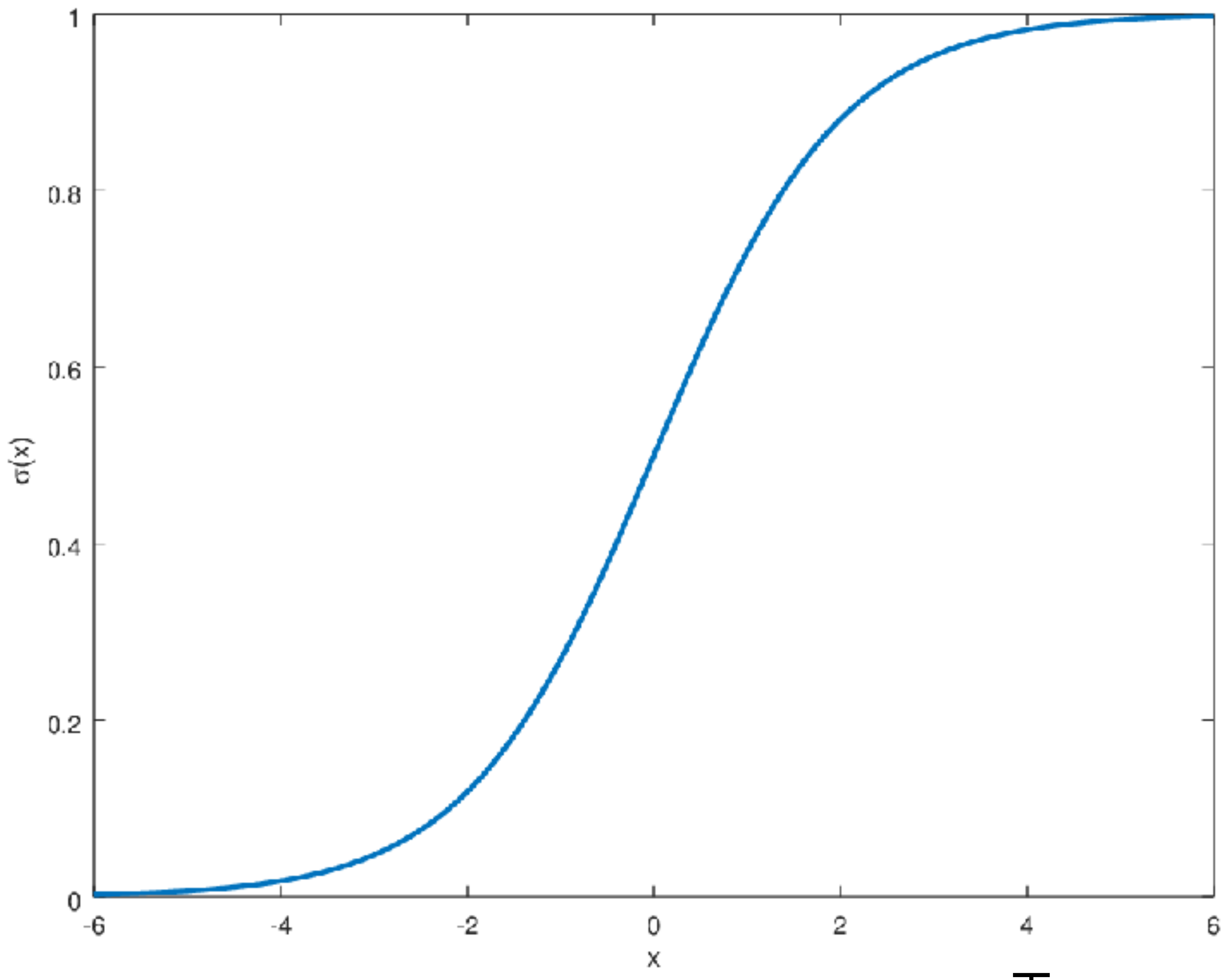

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A good classifier should have large functional margin on training examples:

For all (x_i, y_i), $y_i(w^\top x_i + b) \gg 0$

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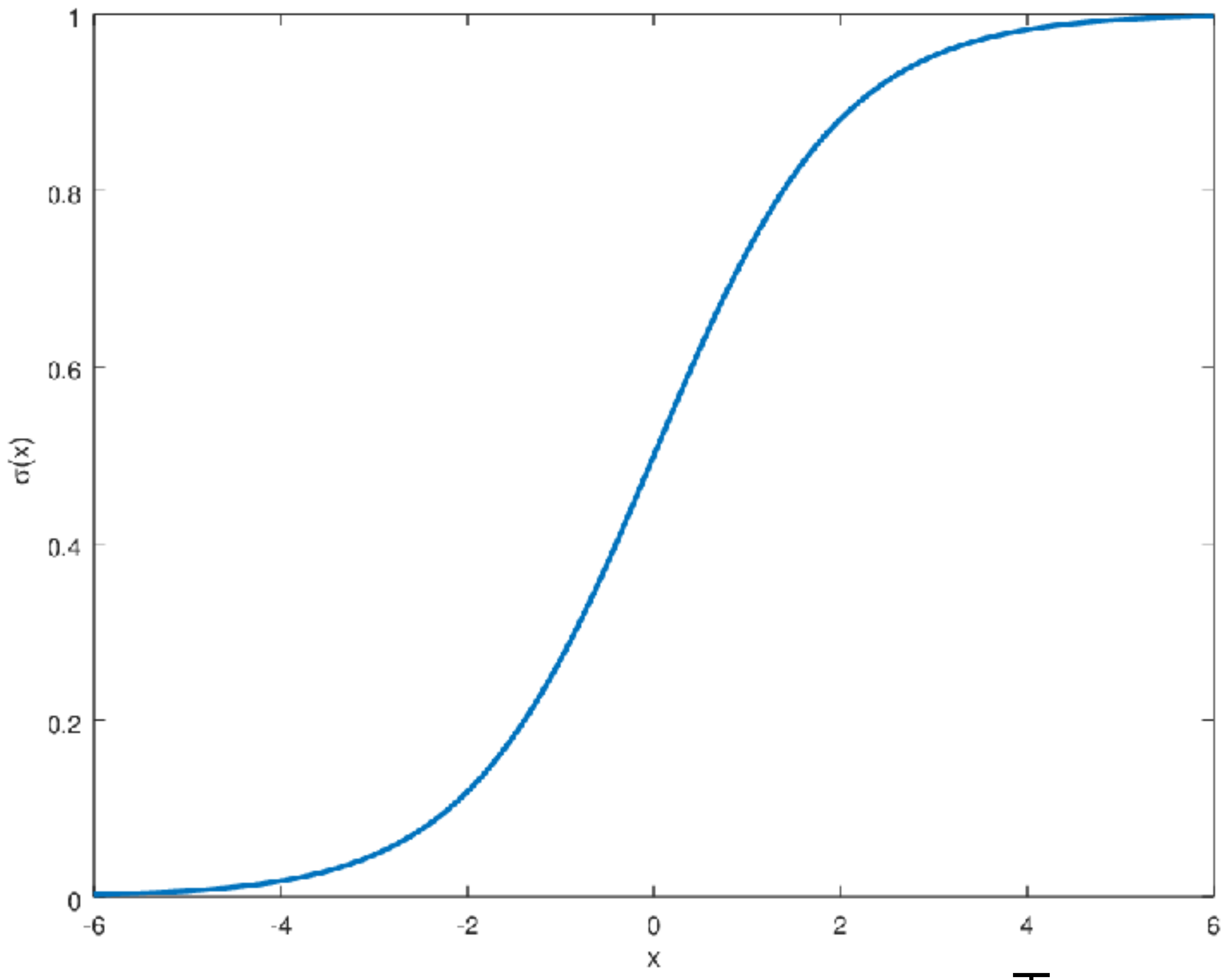

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A good classifier should have large functional margin on training examples:

For all (x_i, y_i), $y_i(w^\top x_i + b) \gg 0$

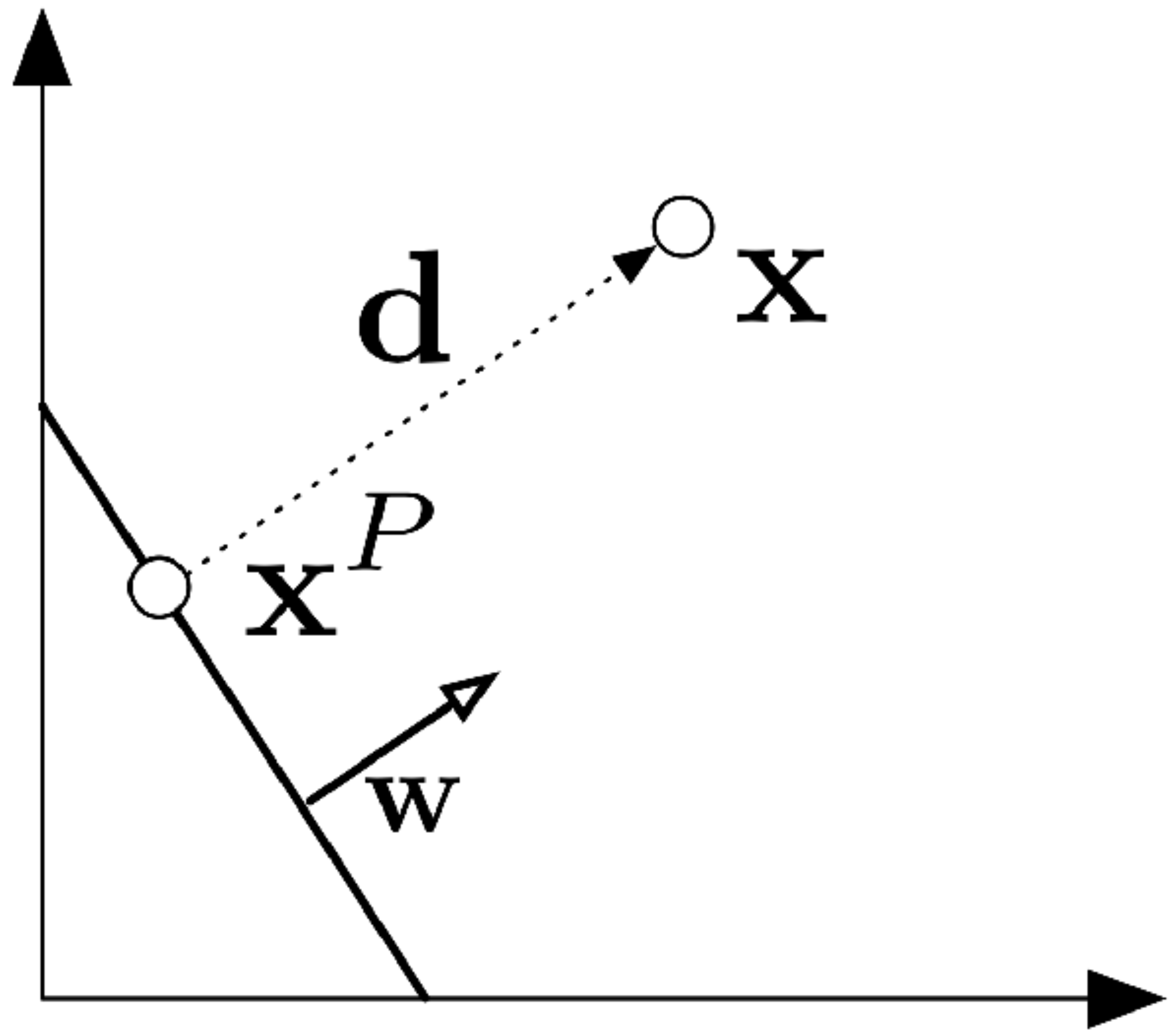
However, functional margin is NOT scale-invariant:

Consider $(2w, 2b)$: functional margin is doubled

Geometric Margin

Hyperplane defined by (w, b), i.e.,

$\{x : w^\top x + b = 0\}$



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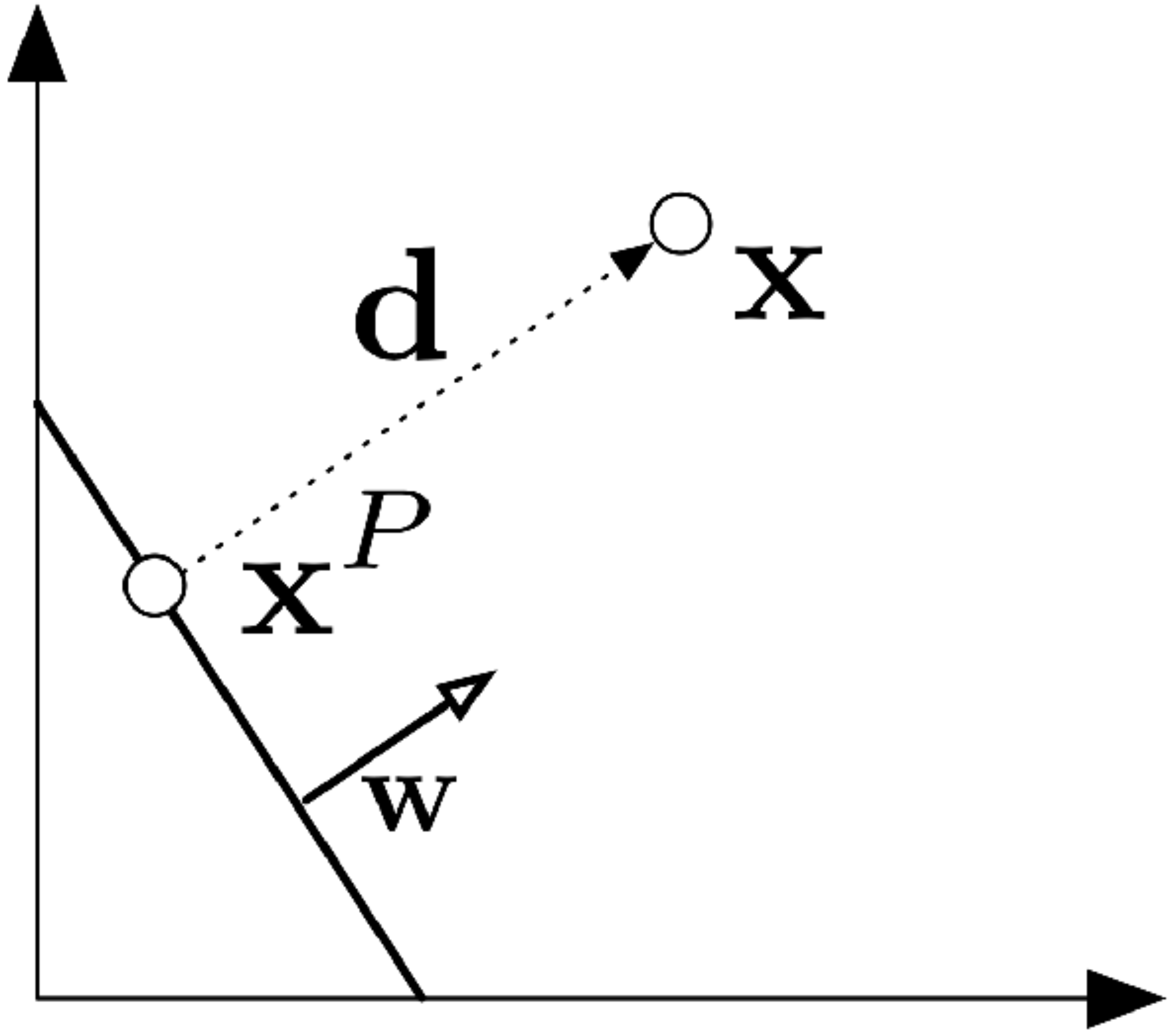

Fact 1. $x - x^P$ is parallel to w:

$$x - x^p = \alpha w$$

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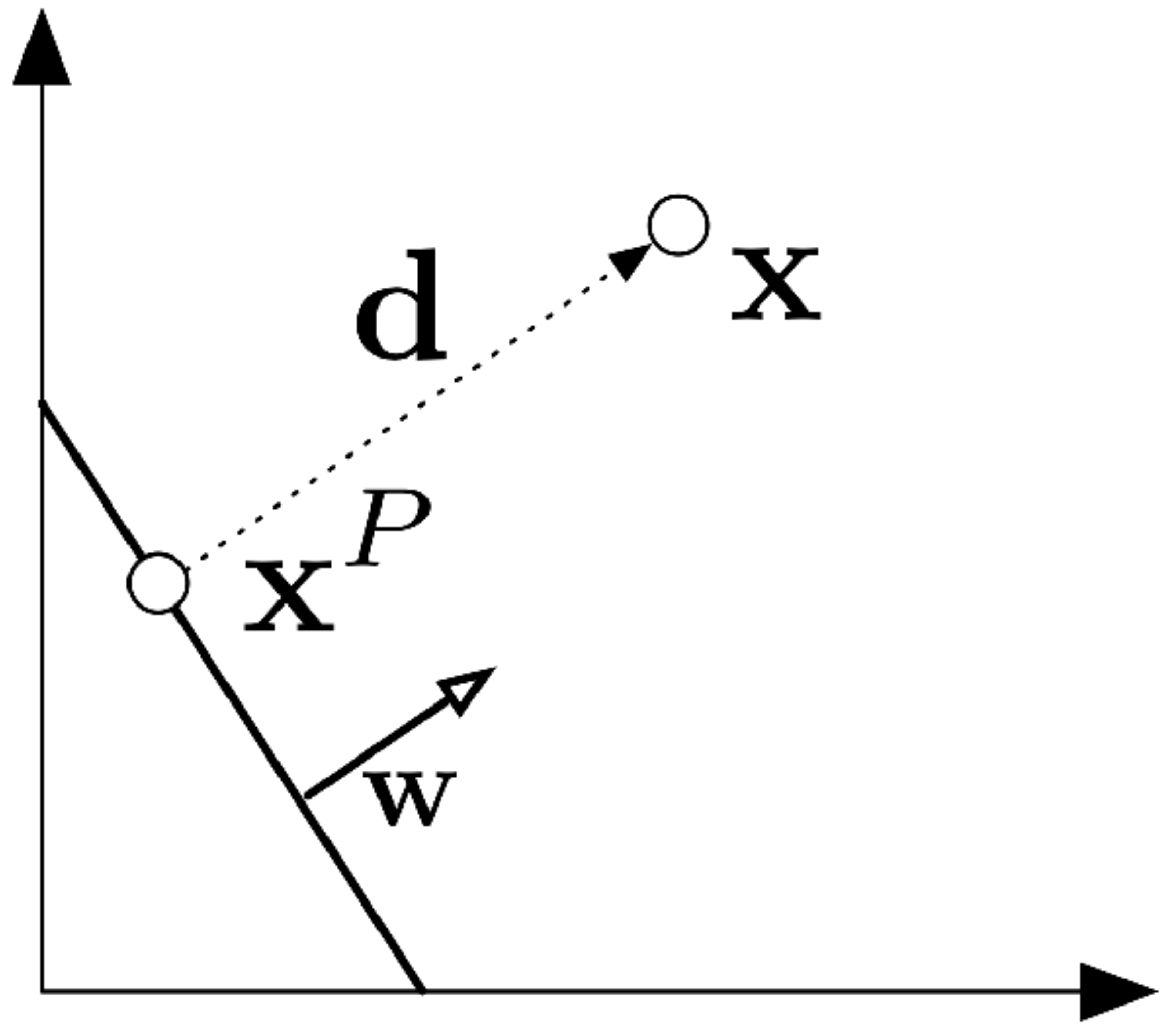

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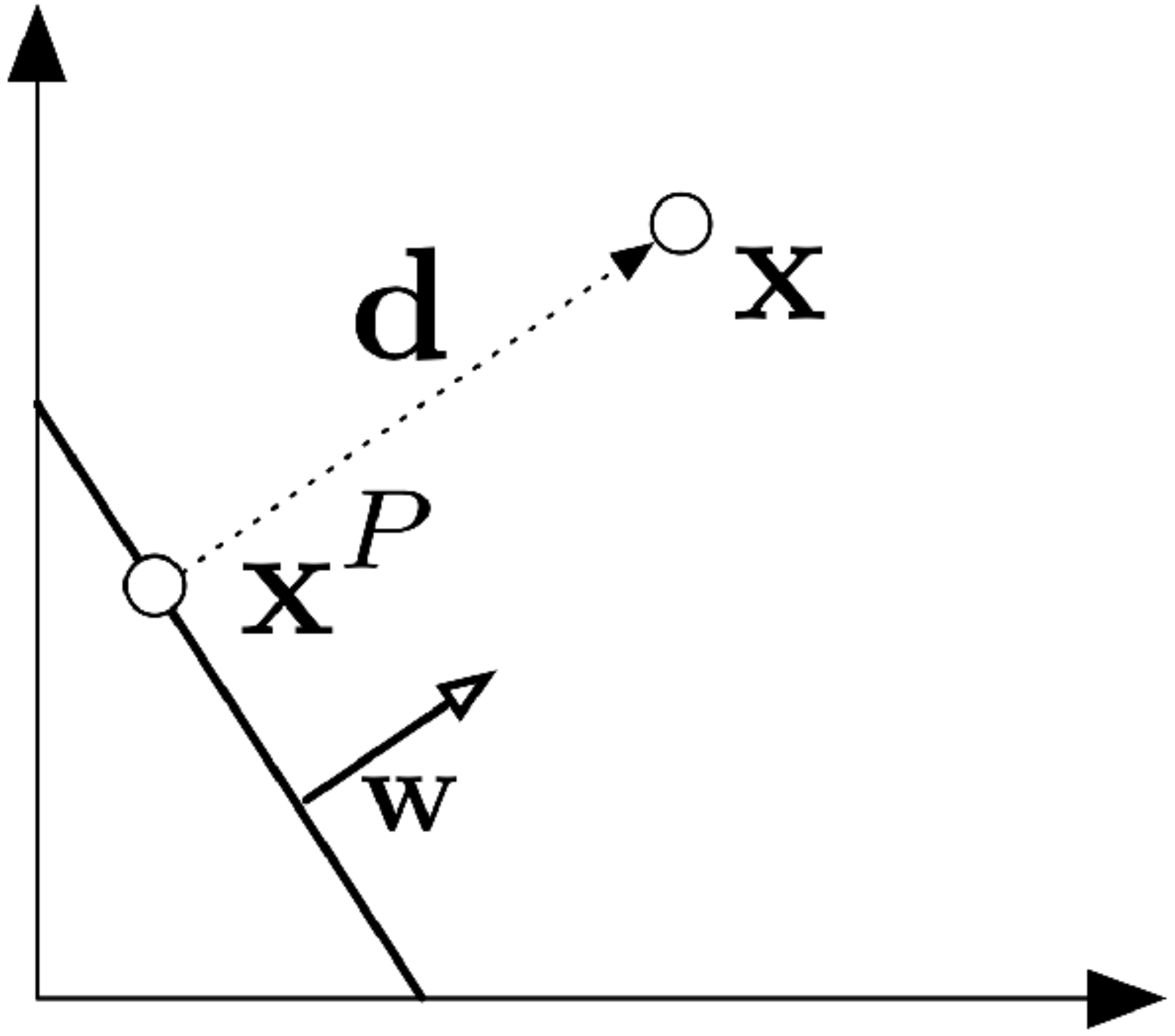

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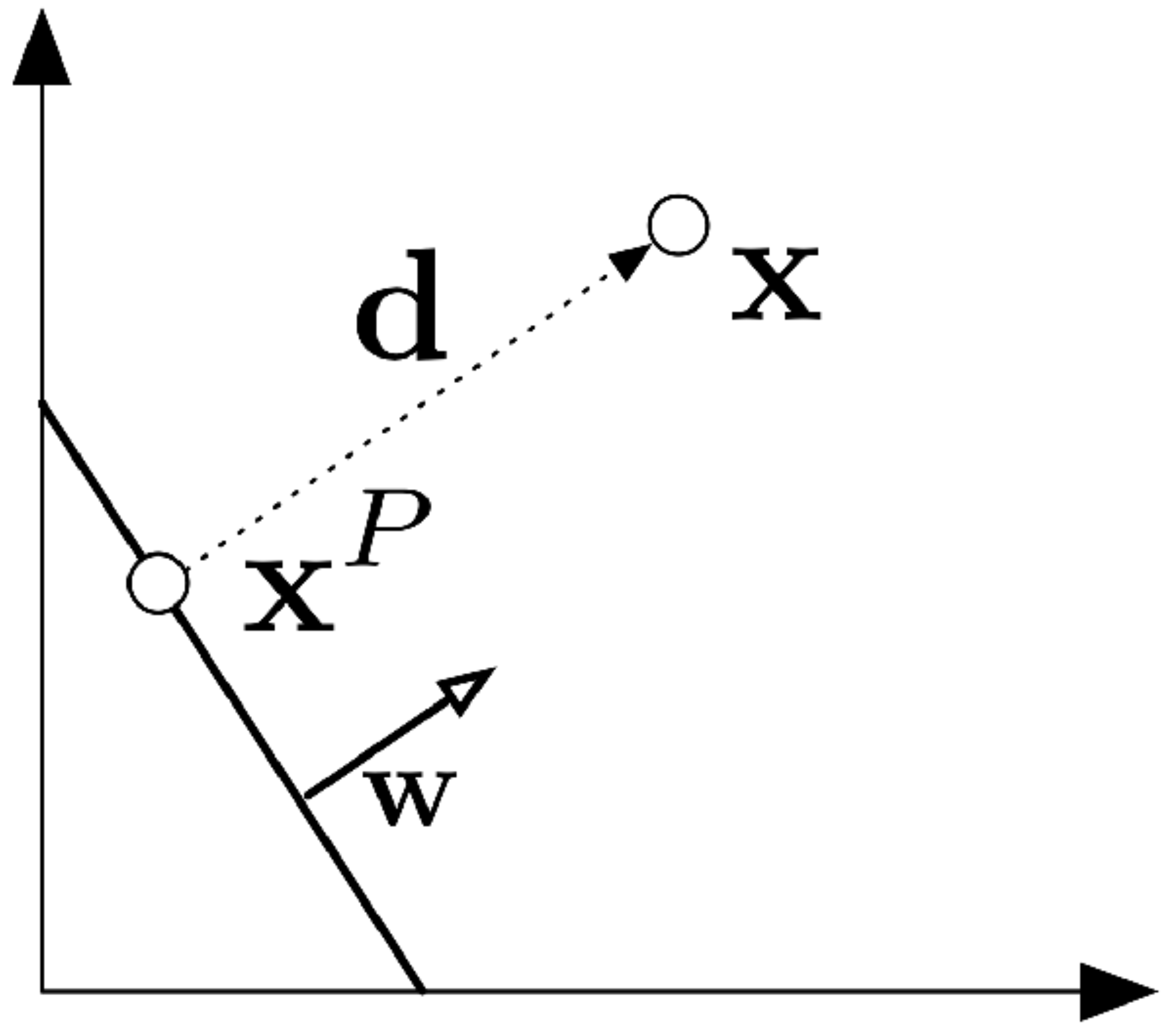

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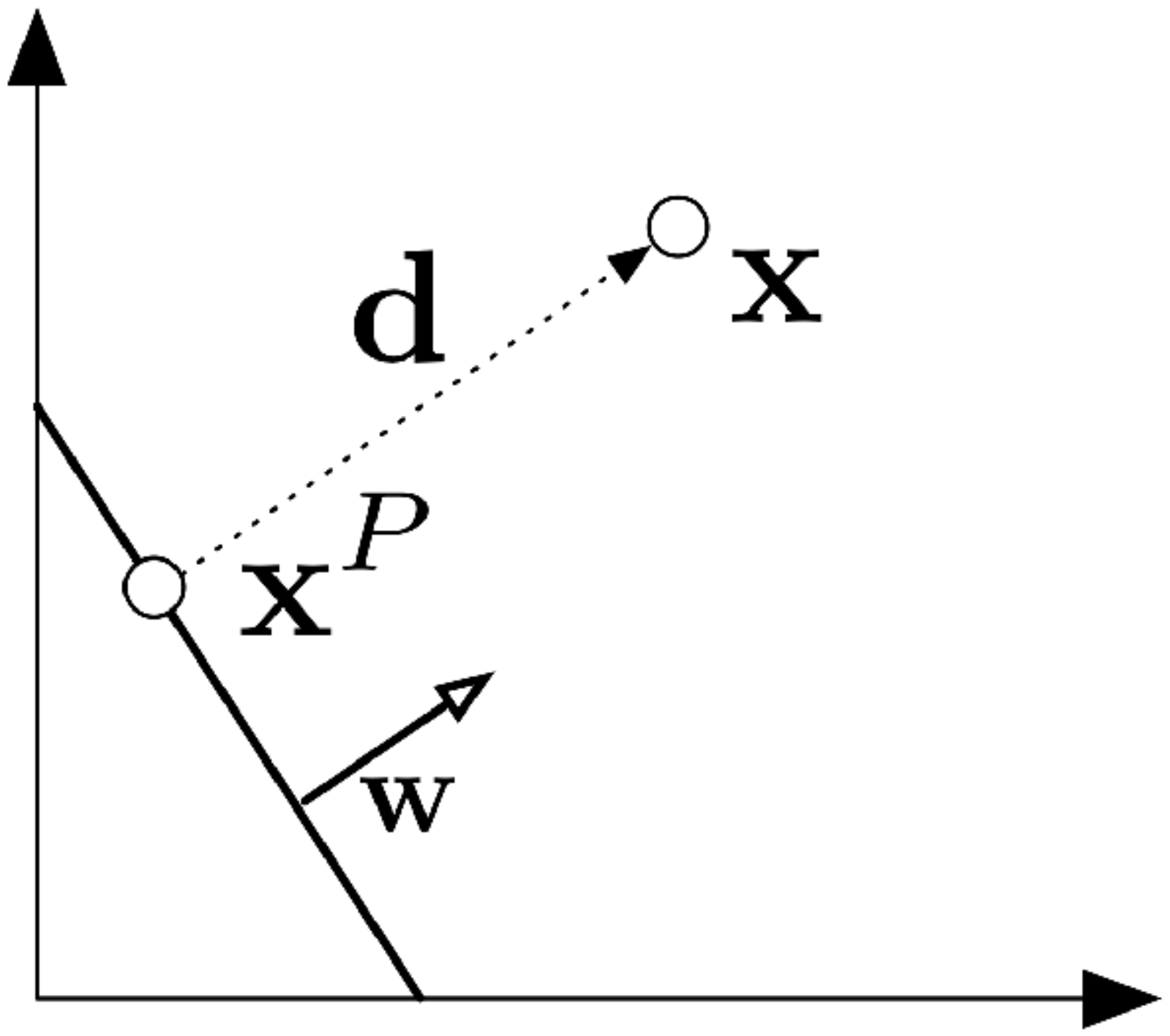

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Final step:

$$\|d\|_2 = \|x - x^p\|_2 = \|\alpha w\|_2$$

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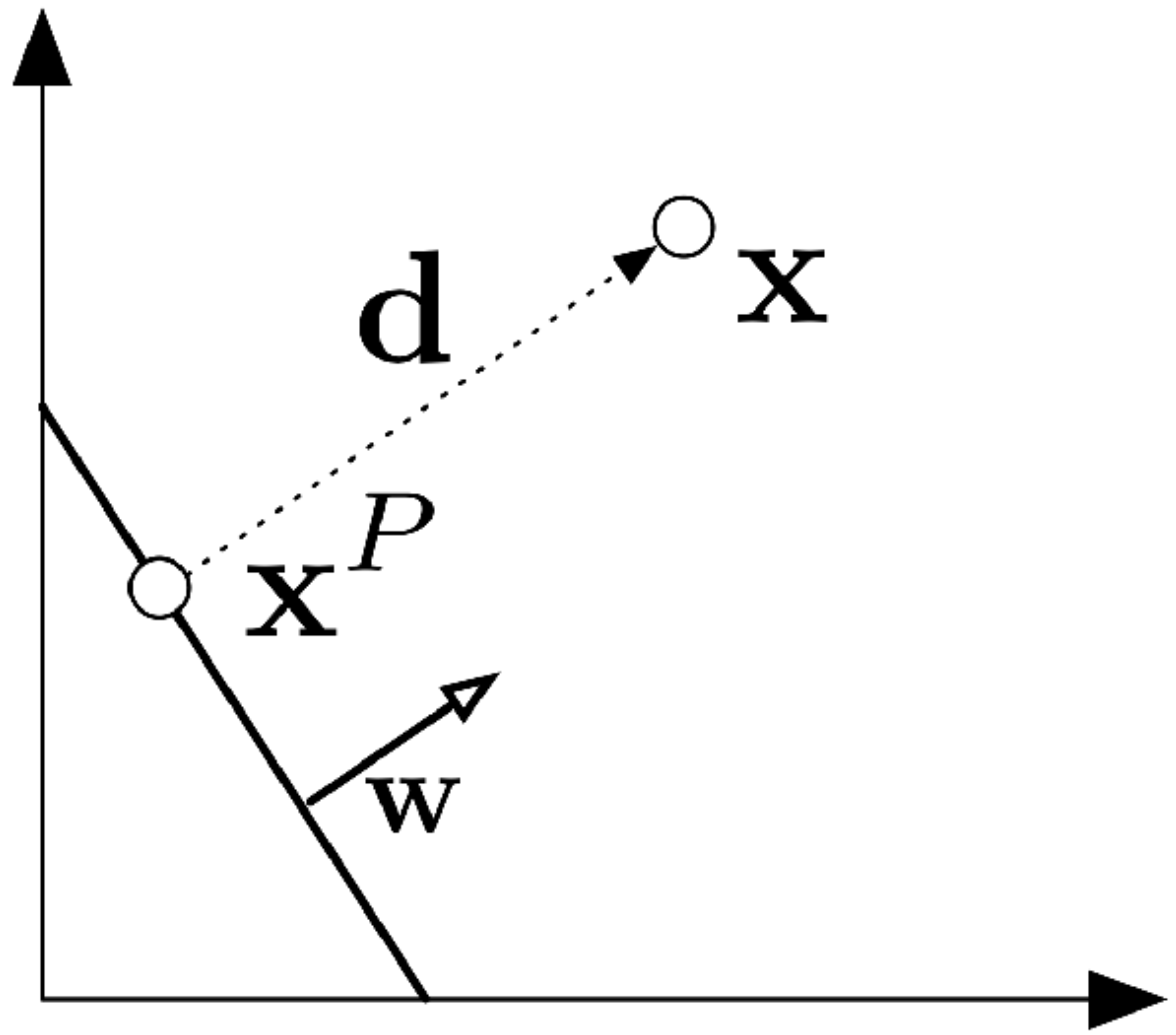

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Geometric Margin is Scale Invariant

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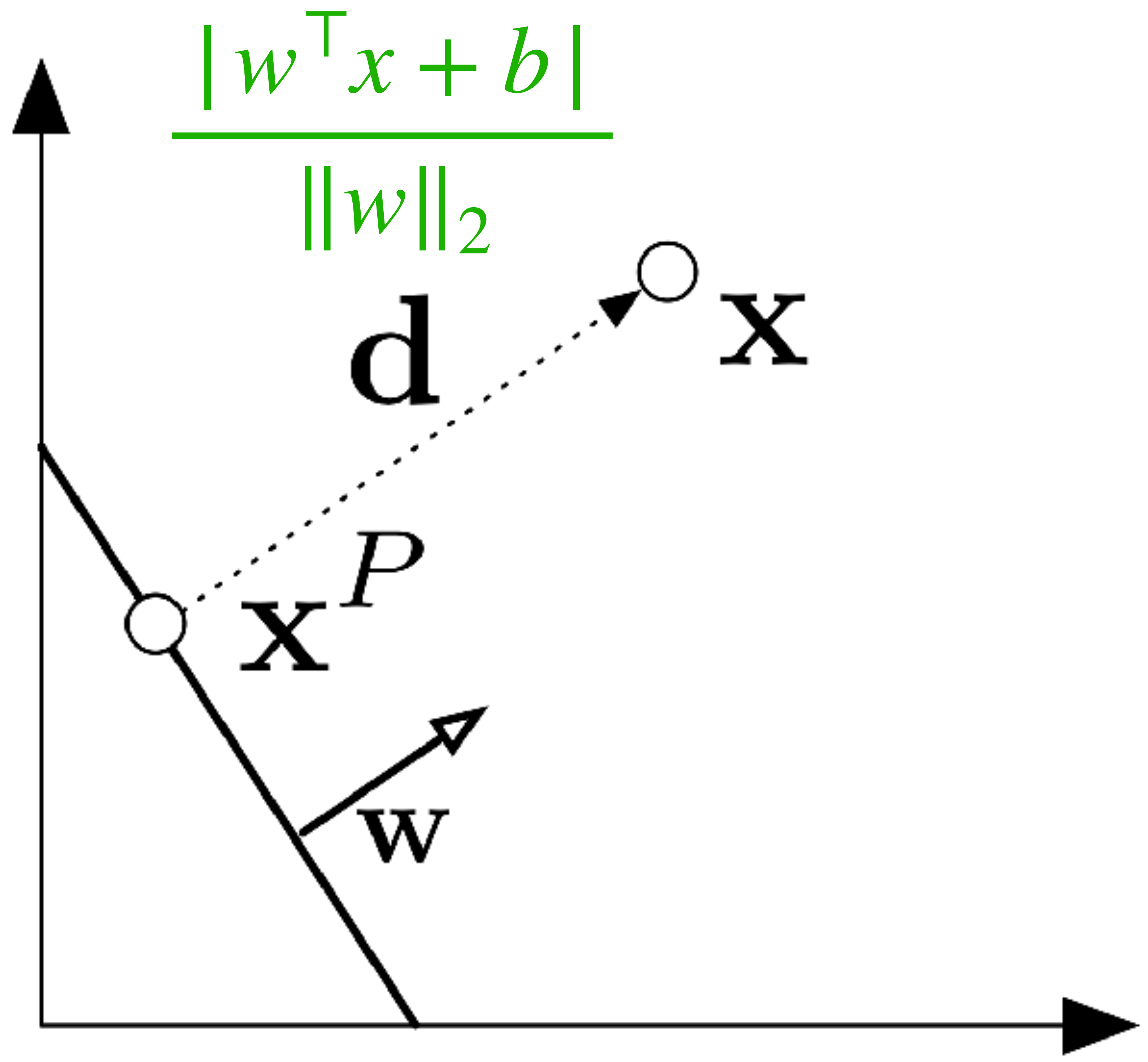

We scale (w, b) by a constant $\gamma \in \mathbb{R}^+$

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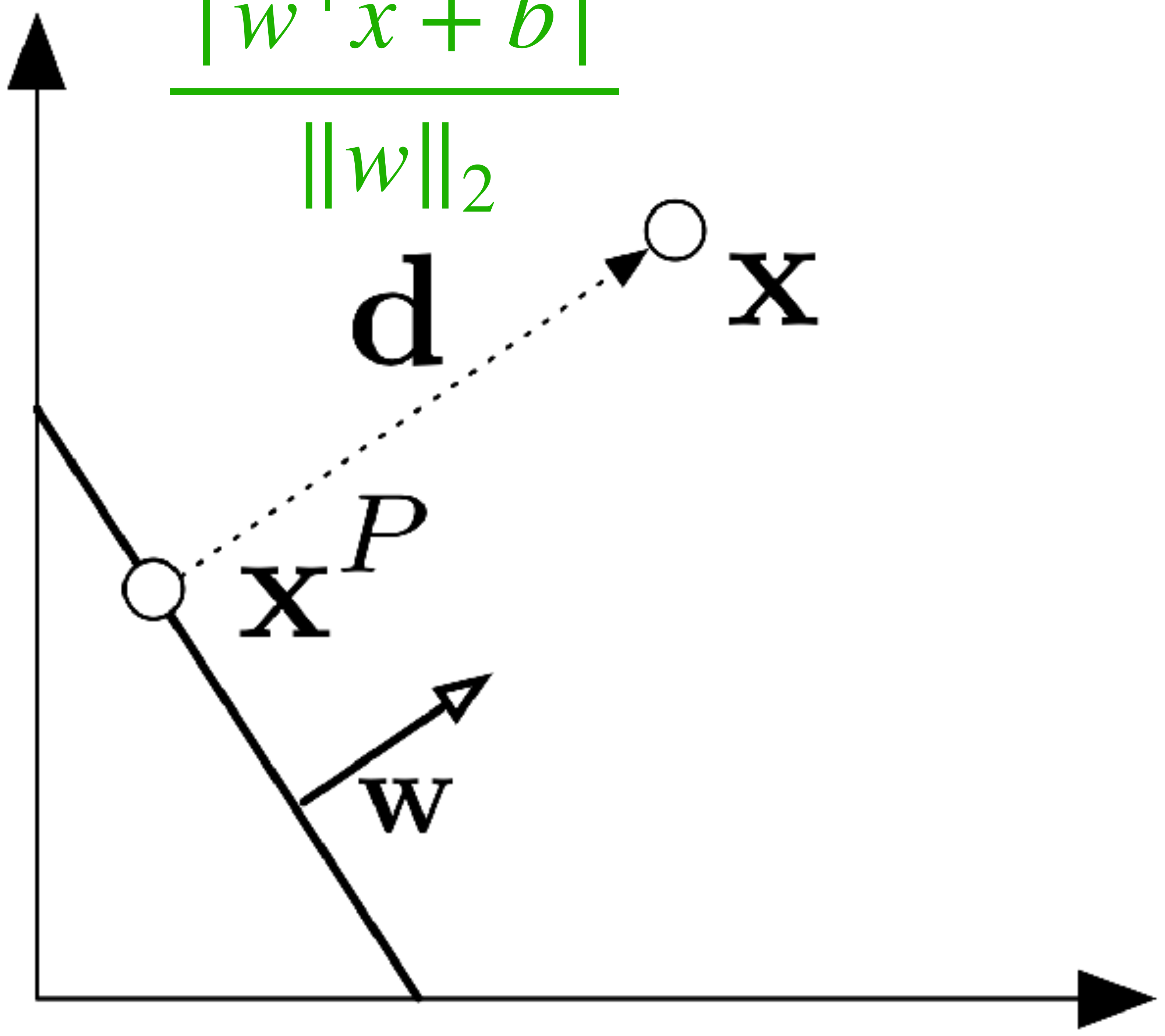

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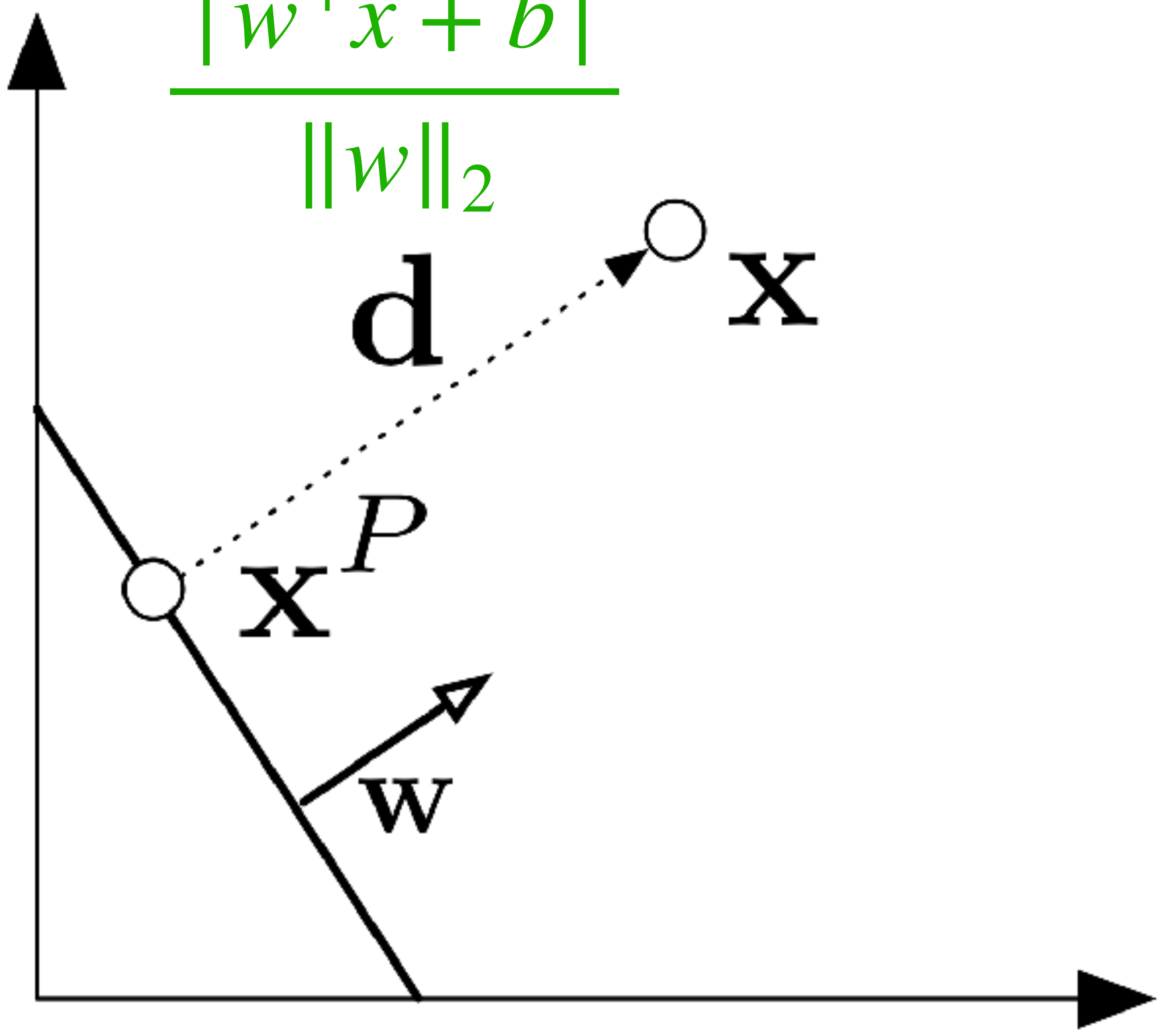

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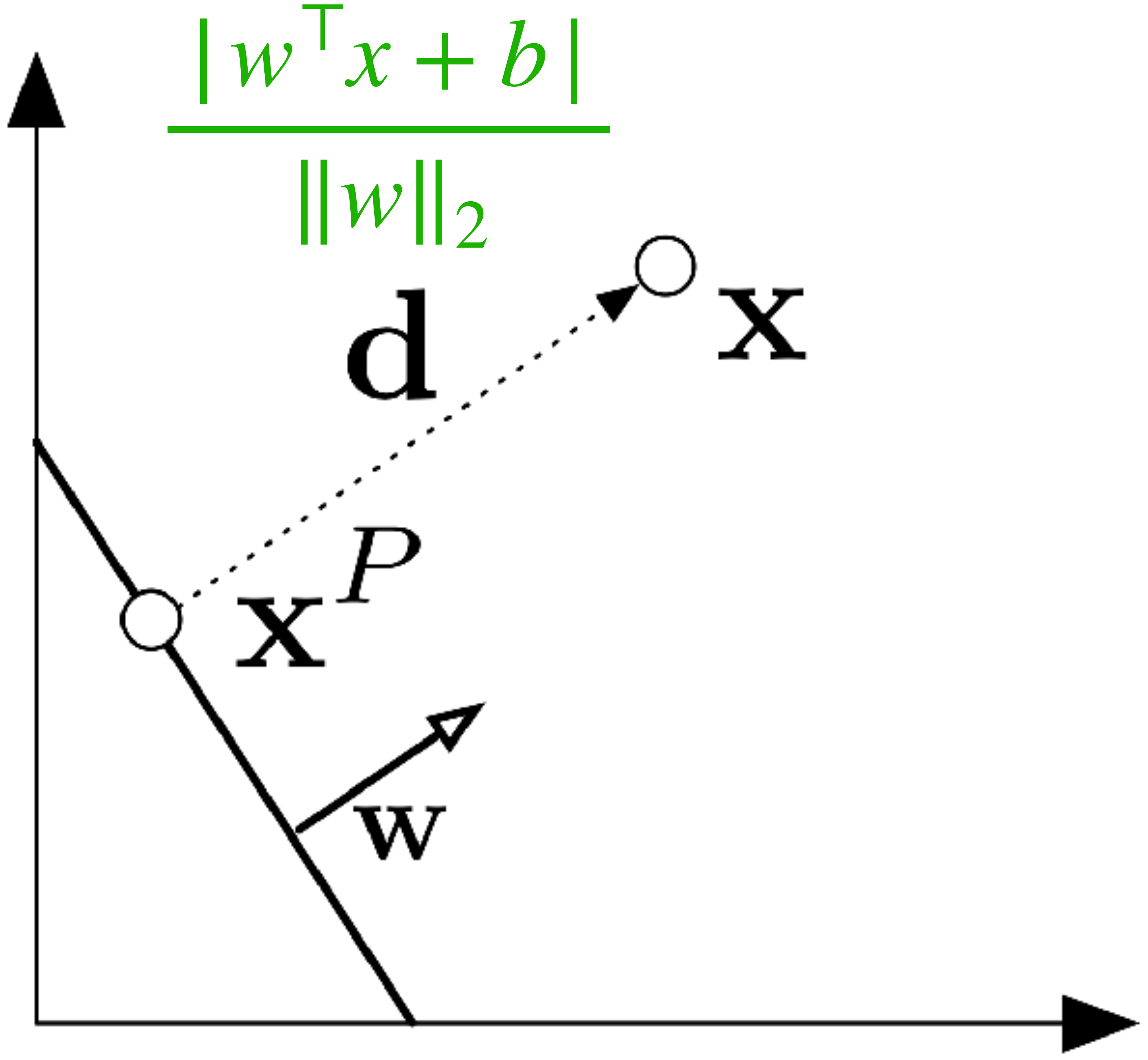

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Hyperplane & Geometric margin are **scale invariant**!

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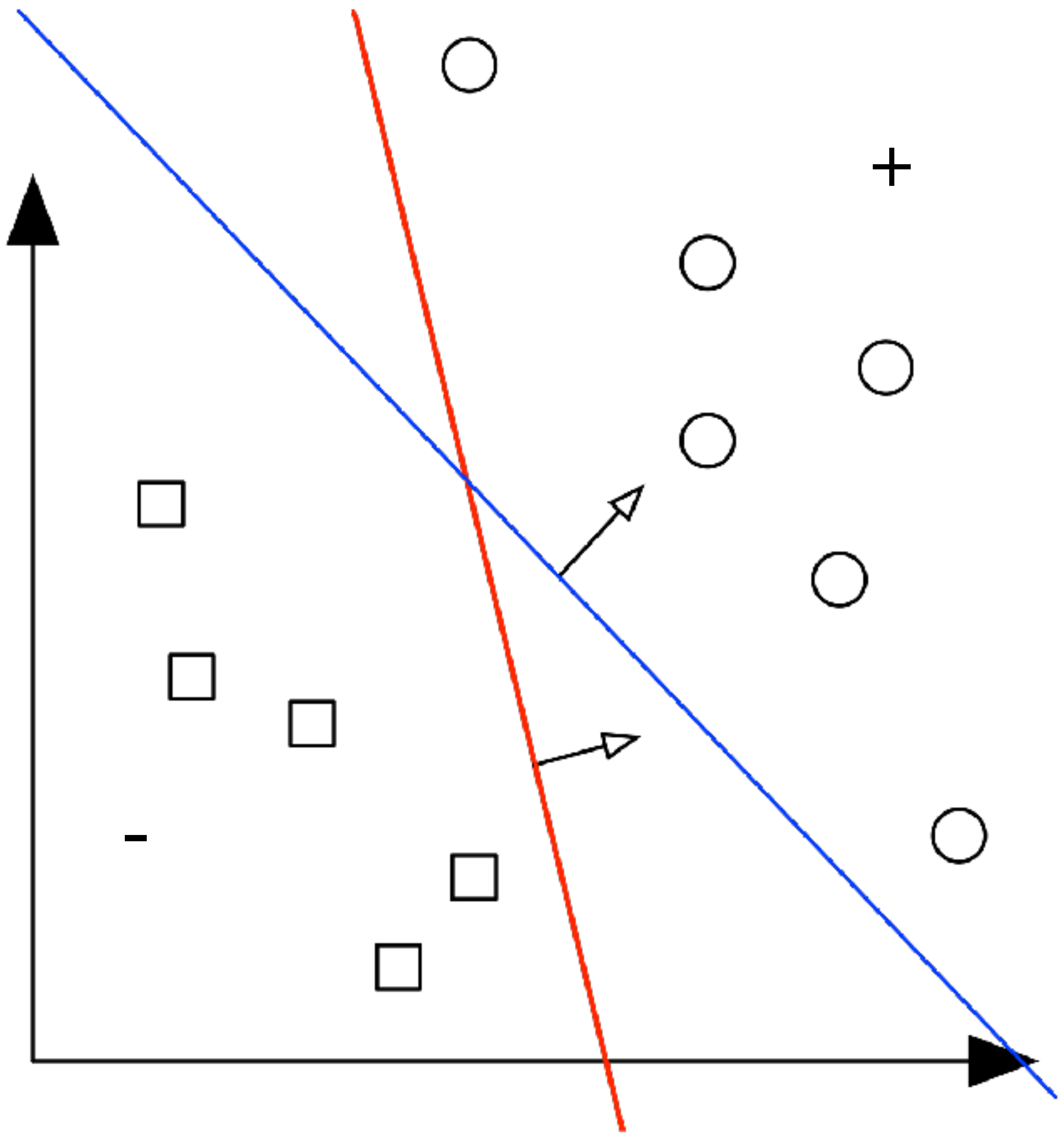
2. Support Vector Machine for separable data

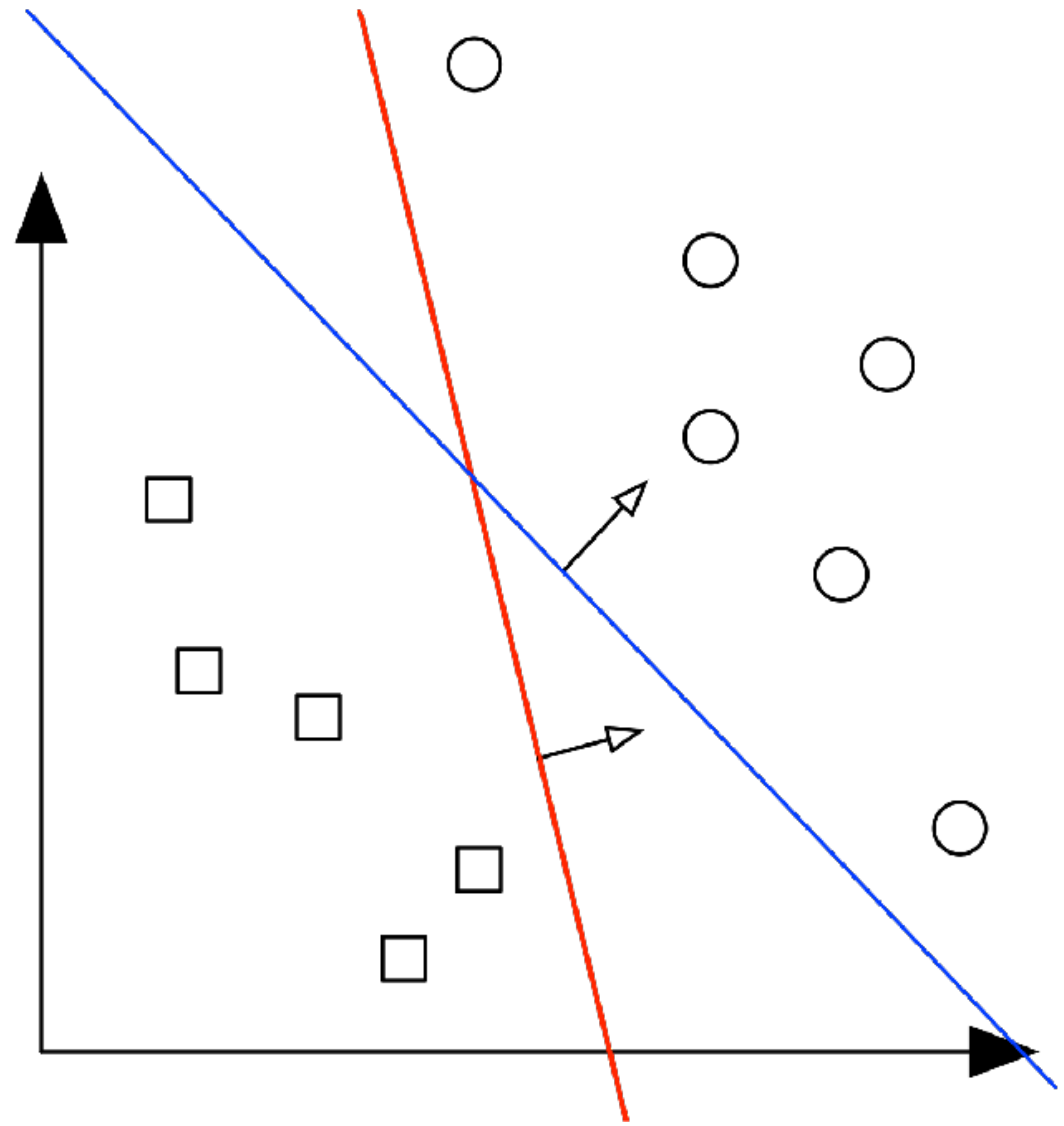
3. SVM for non-separable data

Which linear classifier is Better?



Both hyperplanes correctly separate the data

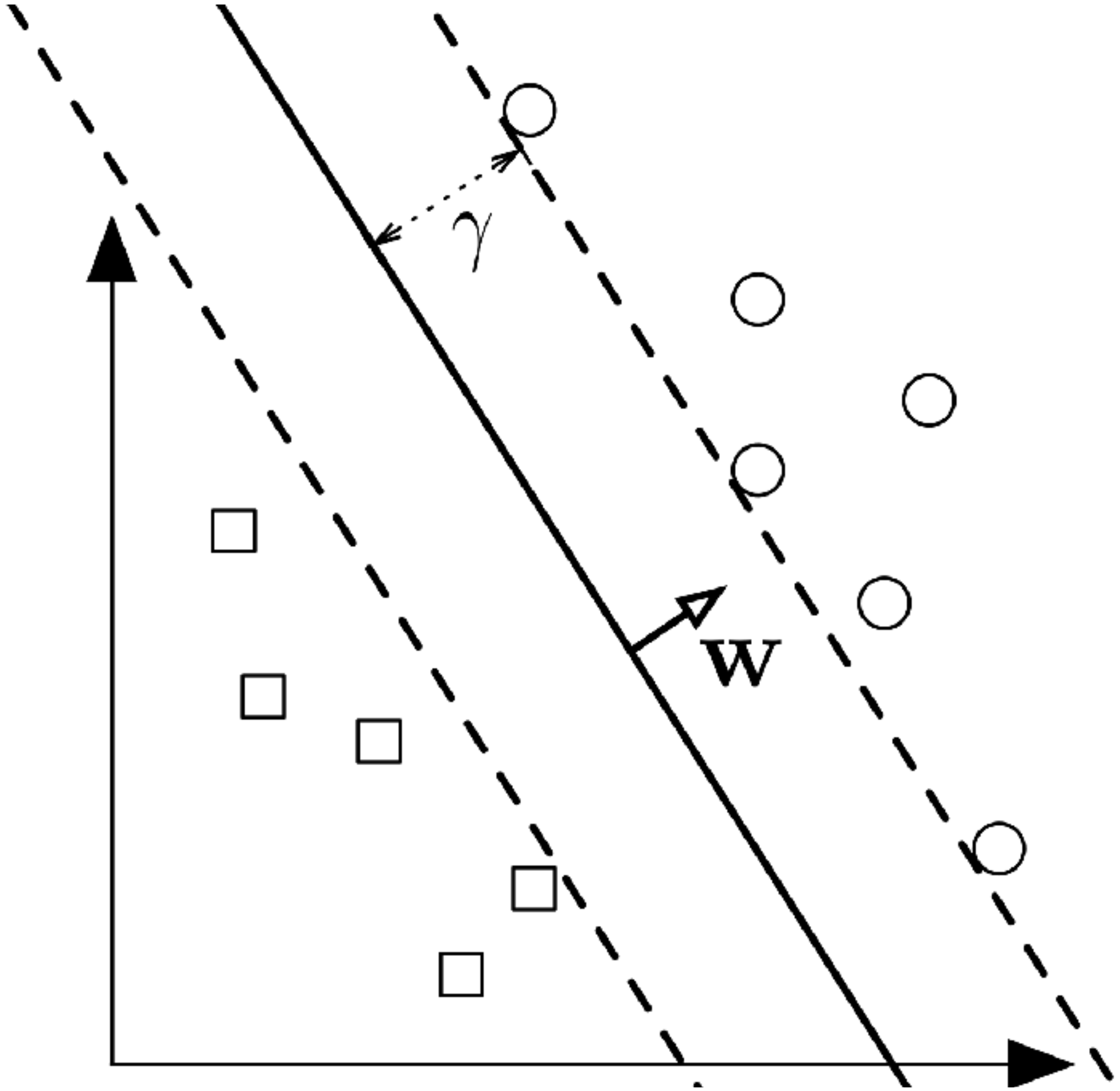
Max Margin Classifier



The Goal of SVM:

Find a hyperplane that has the largest Geometric margin

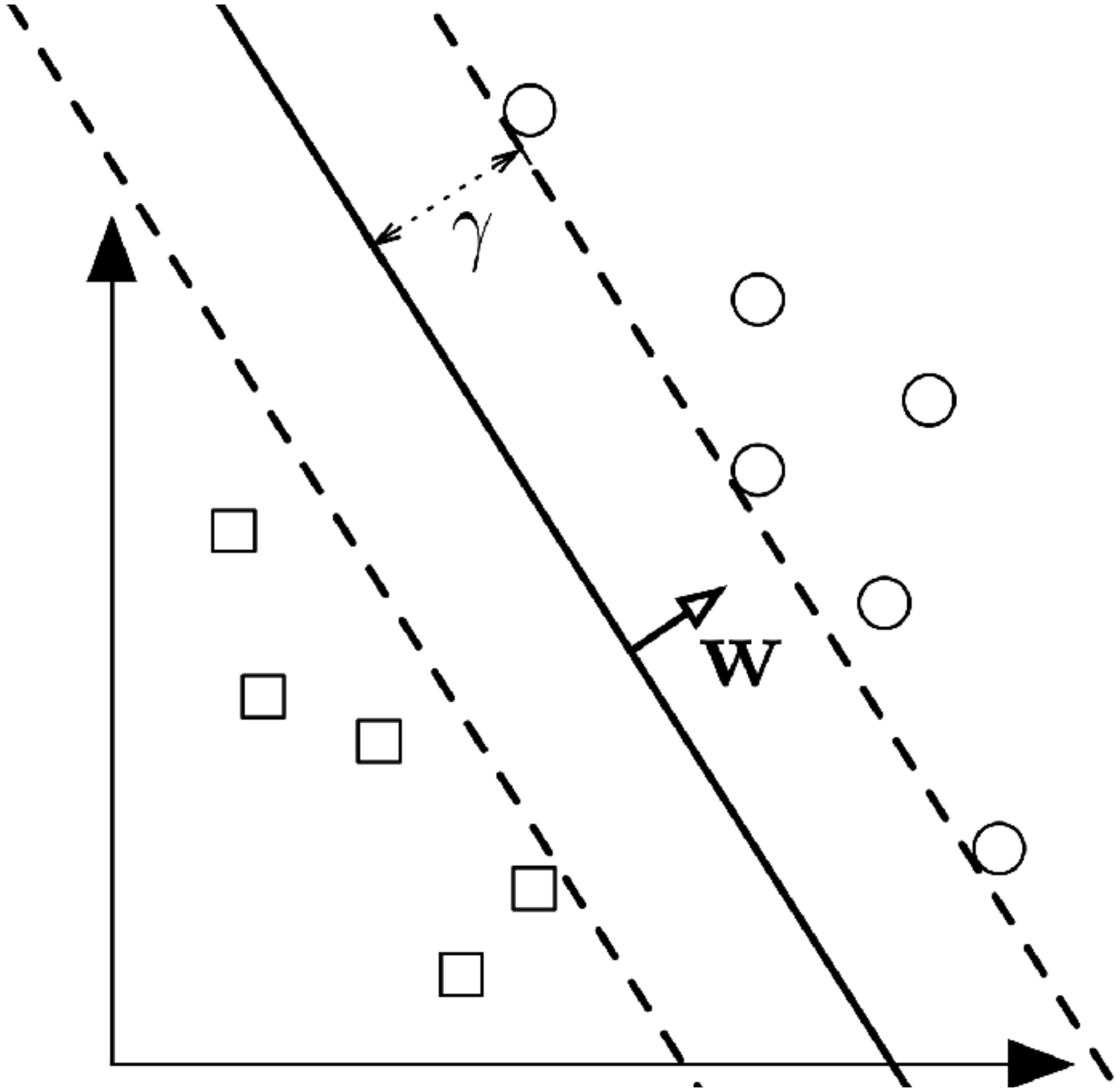
Max Margin Classifier



Given a linearly separable dataset $\{x_i, y_i\}_{i=1}^n$, the minimum geometric margin is defined as

$$\gamma(w, b) := \min_{x_i \in \mathcal{D}} \frac{|x_i^\top w + b|}{\|w\|_2}$$

Max Margin Classifier



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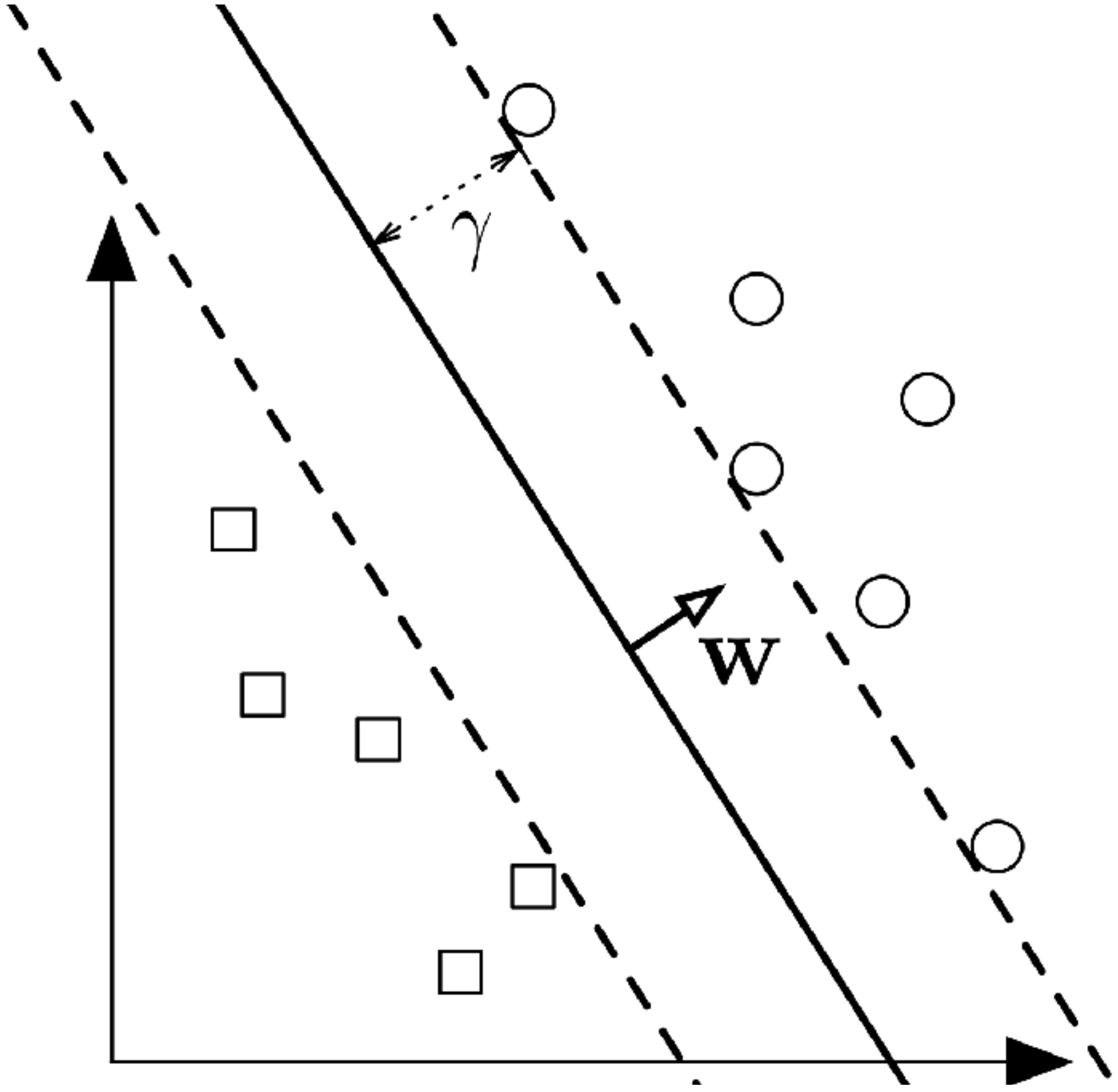
Max Margin Classifier



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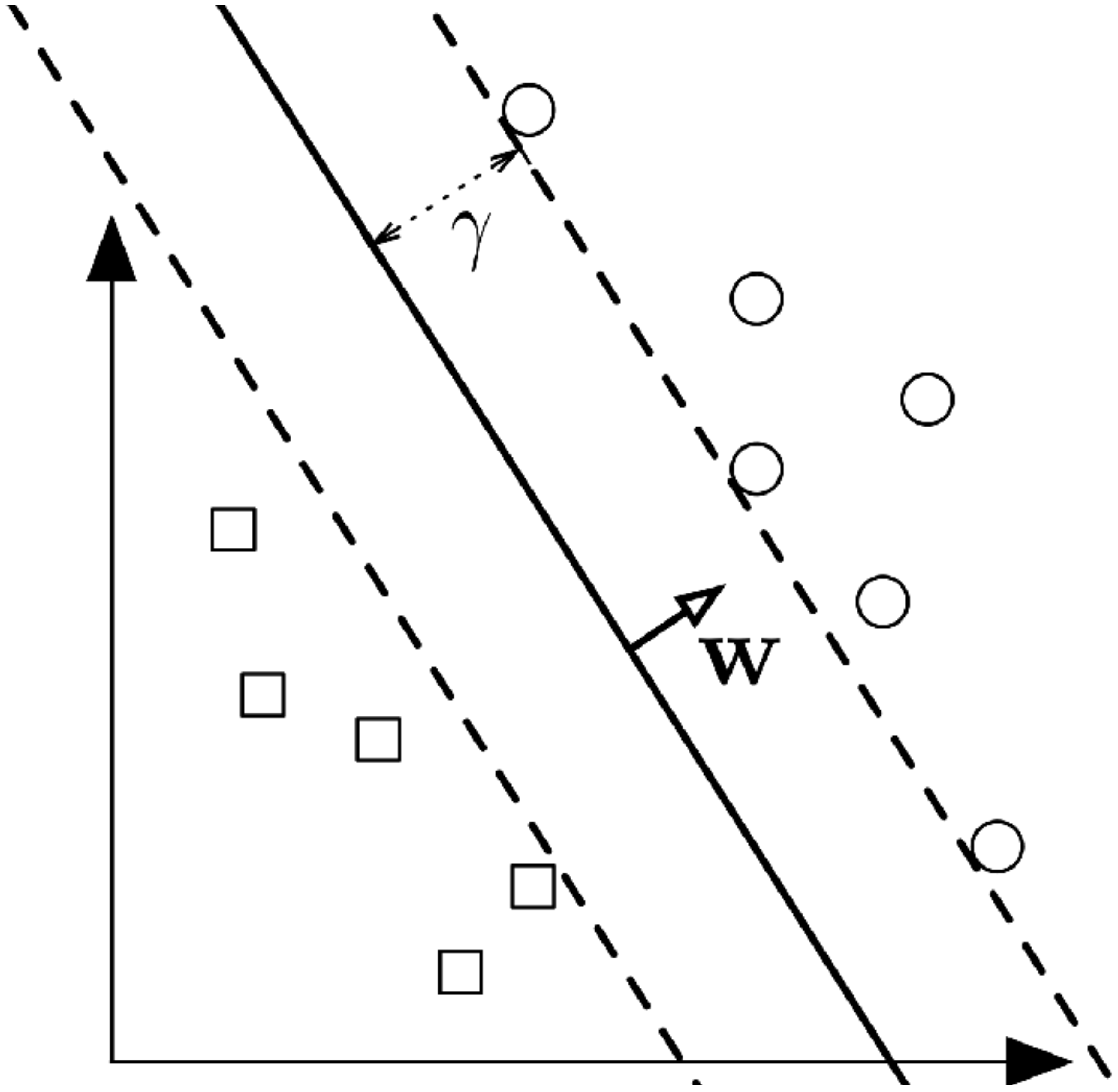
Max Margin Classifier



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Plug in the def of $\gamma(w, b)$:

$$\max_{w,b} \frac{1}{\|w\|_2} \min_{x_i} |w^\top x_i + b|$$

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SVM for separable data: Max Margin Classifier

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Without loss of generality, let's just focus on such (w, b) pairs with $\min_{x_i} |w^\top x_i + b| = 1$

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You will prove that in HW4!

Summary for Max Margin Classifier

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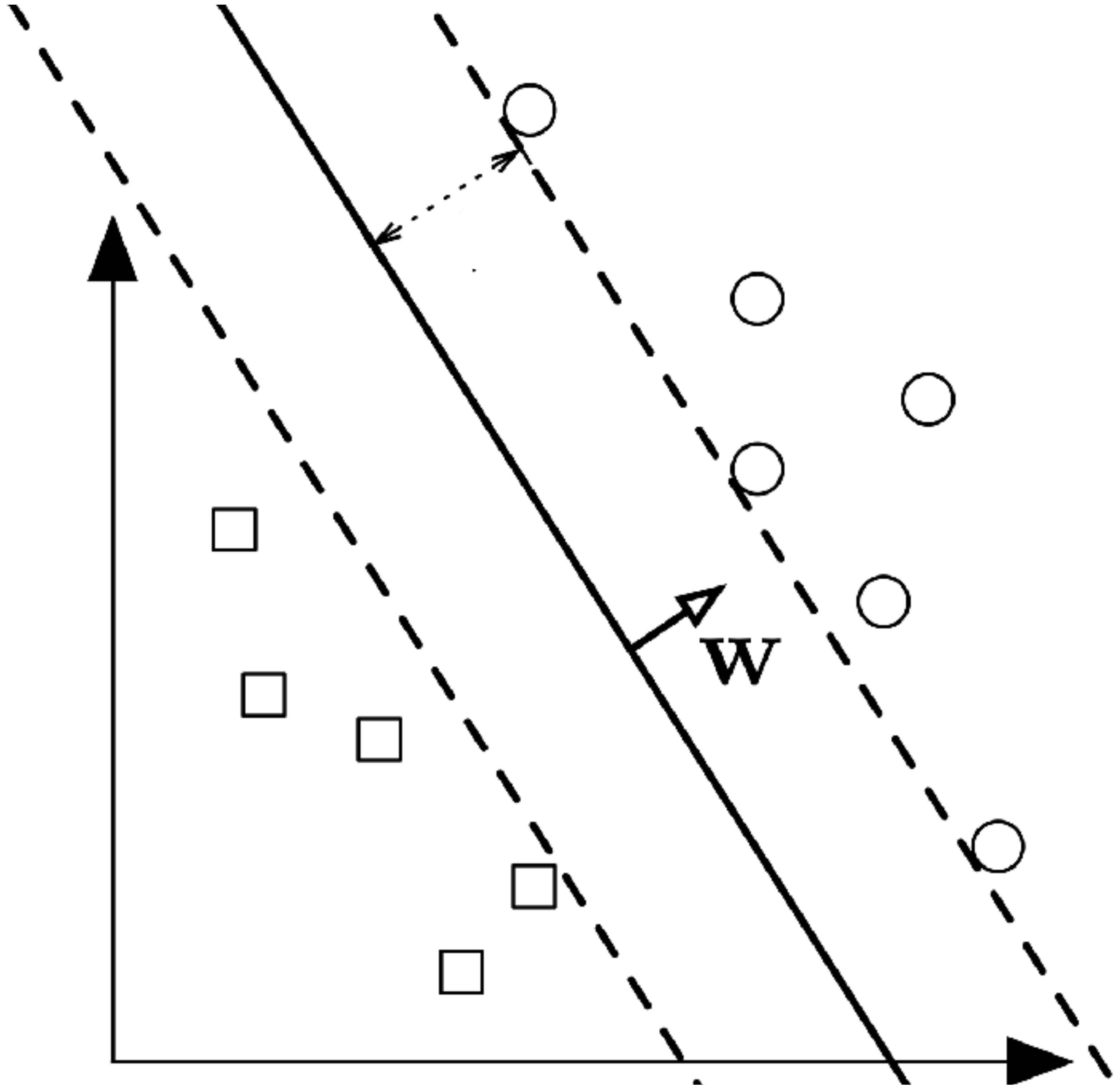
Not only linearly separable, but also has functional margin no less than 1

Always remember **where we started**:

We want to find (w, b) s.t. it separates the data, and maximizes $\gamma(w, b)$

Support Vectors

$w^\top x + b = -1$ $\quad$ $w^\top x + b = 1$



for the optimal (w, b) pair, points x_i such that $y_i(w^\top x_i + b) = 1$ are called **support vectors**

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$$\forall i: \; y_i(w^\top x_i + b) \geq 1 - \xi_i,$$

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SVM for non-separable data

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This always has feasible solutions (e.g., take $\xi_i = +\infty$)

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A: set $\xi_i = \max\{0, \, 1 - y_i(w^\top x_i + b)\}$

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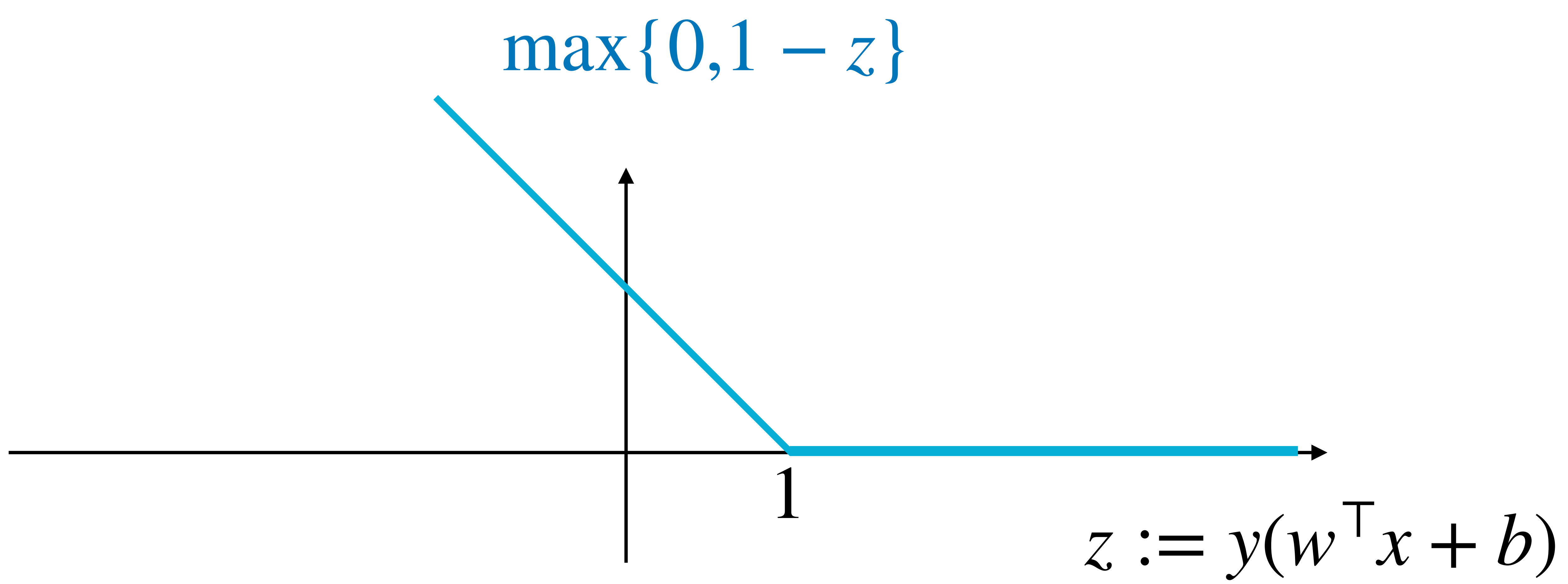

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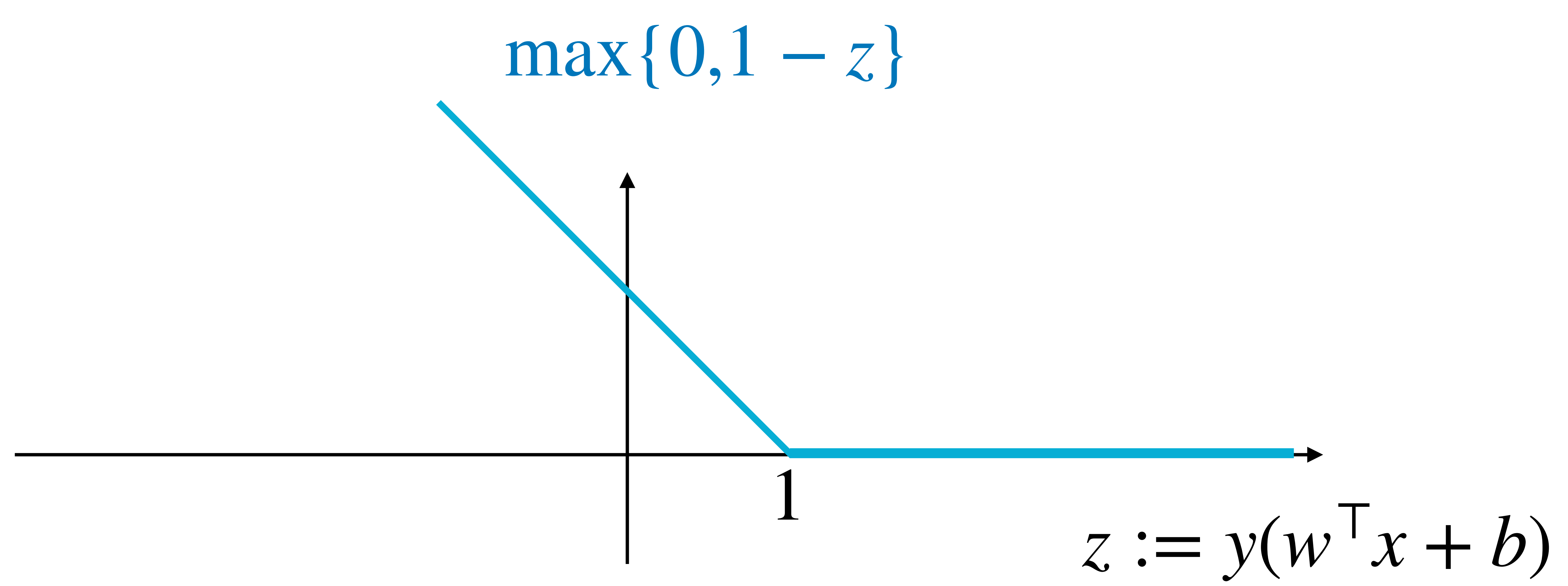

Hinge loss starts penalizing when functional margin falls below 1

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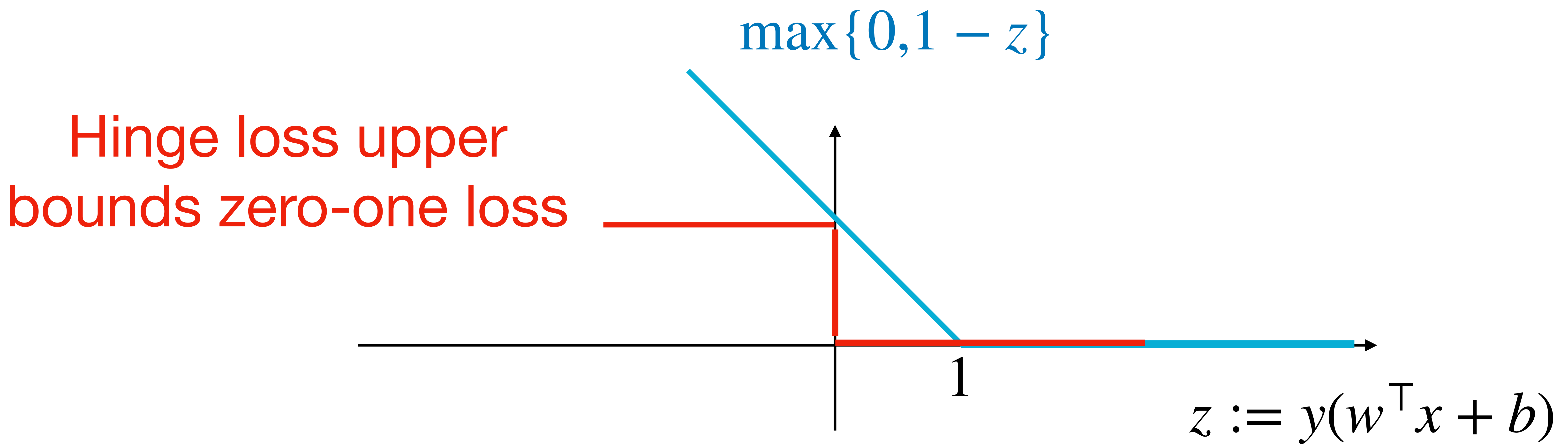

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When $c \to +\infty$:

forcing $y_i(w^\top x_i + b) \geq 1$ for as many data points as possible

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When $c \to 0^+$:

The solution $w \to \mathbf{0}$ (i.e., we do not care about hinge loss part)

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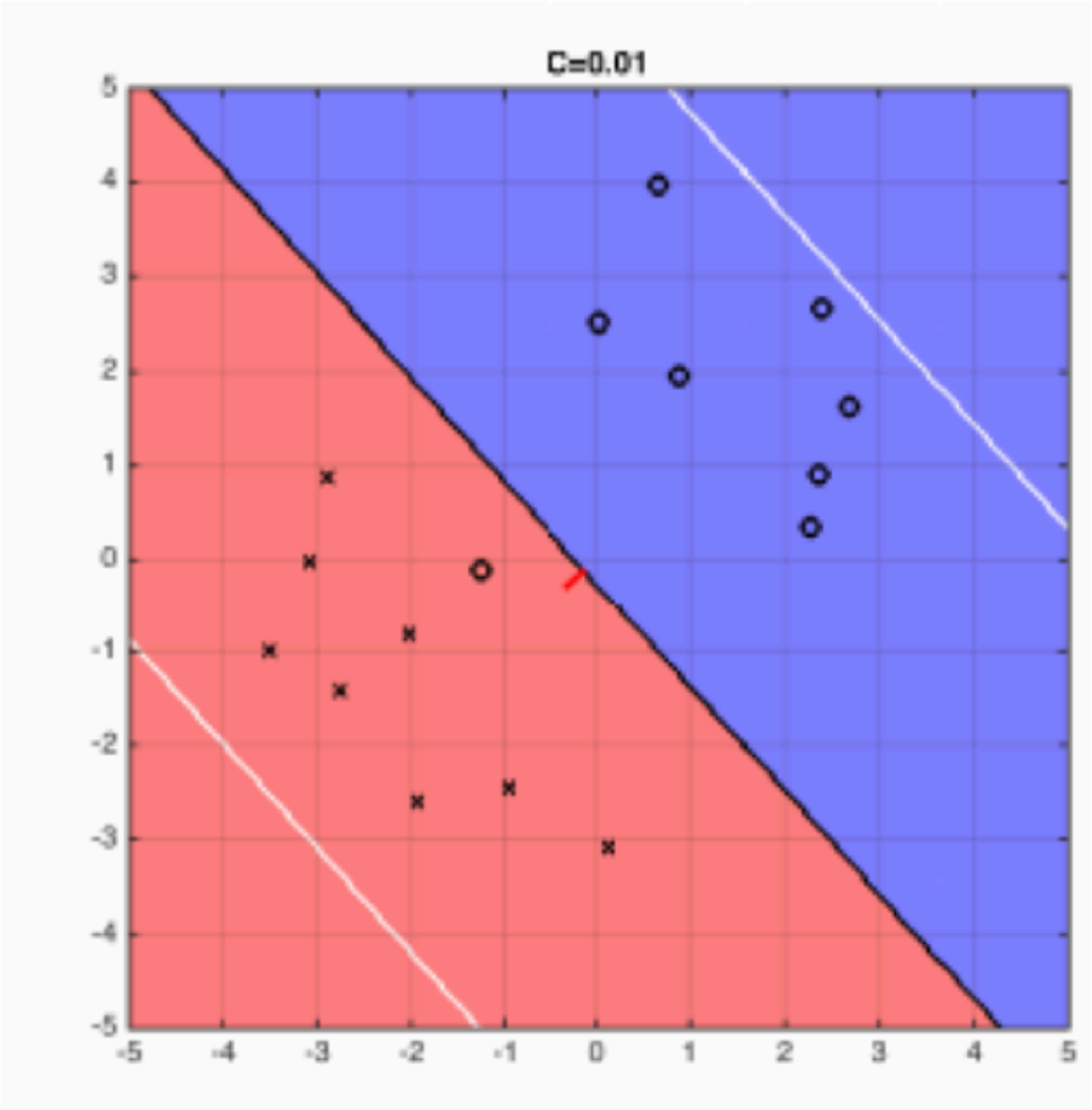
C = 0.01



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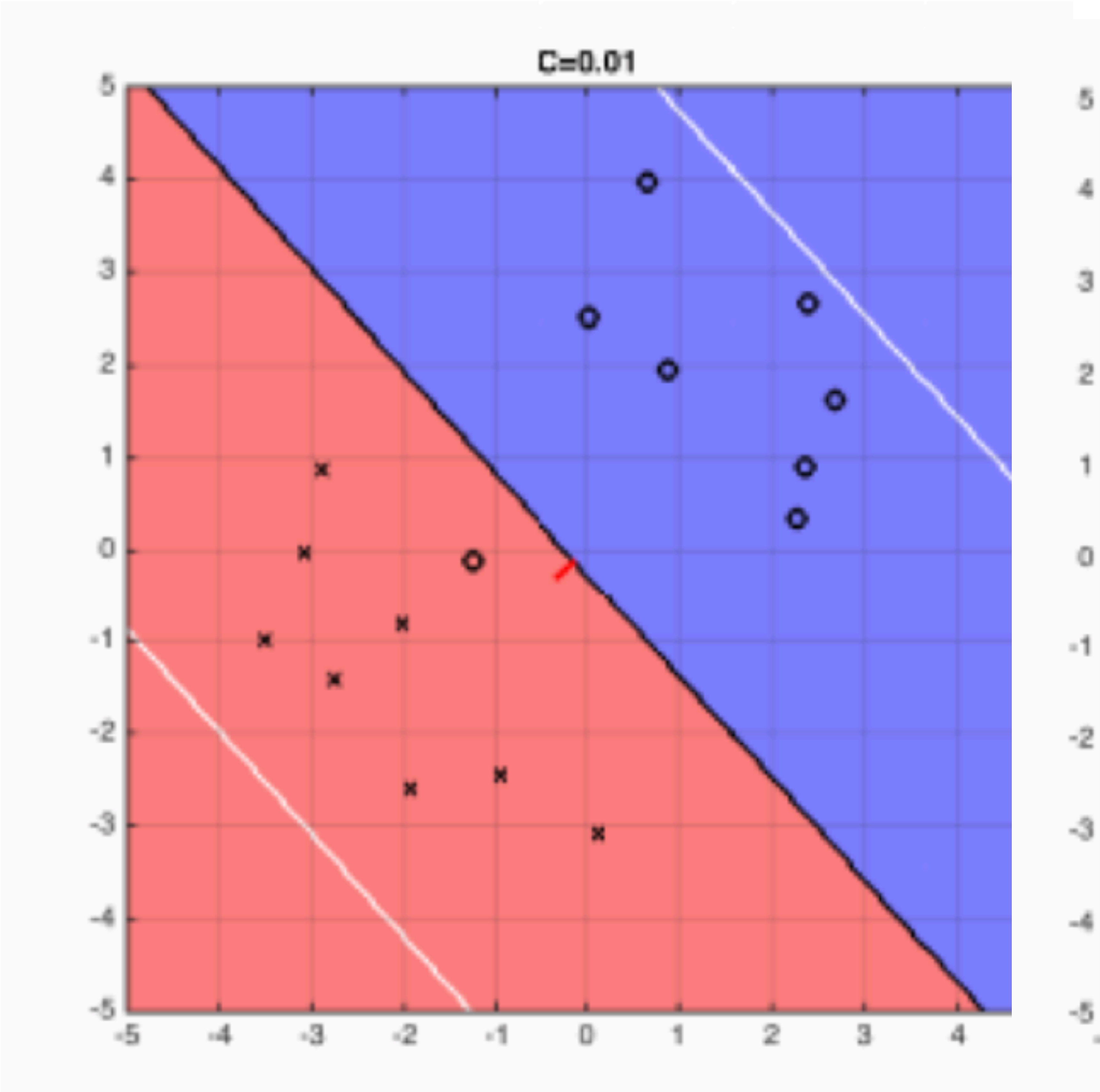
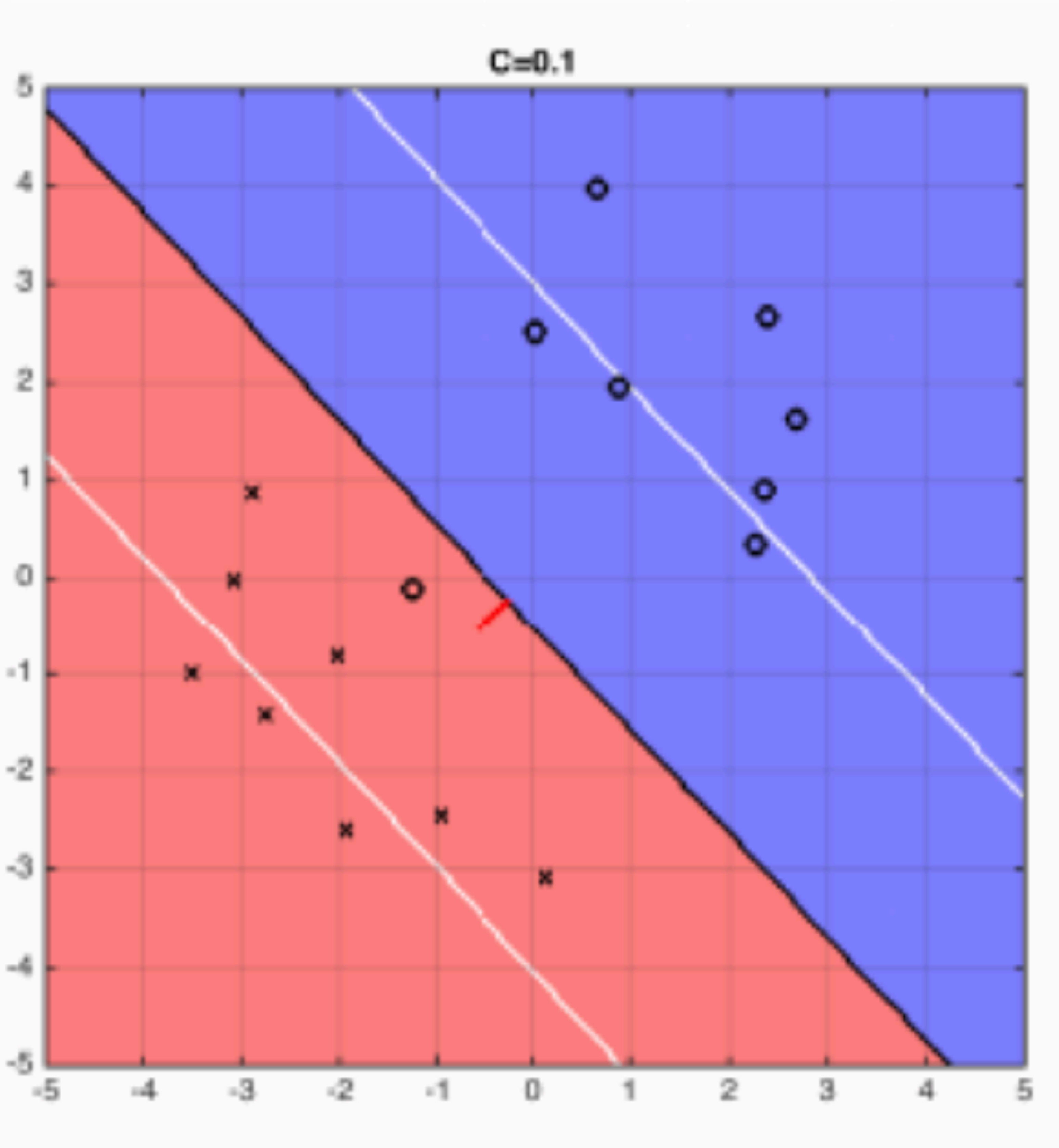


C = 1



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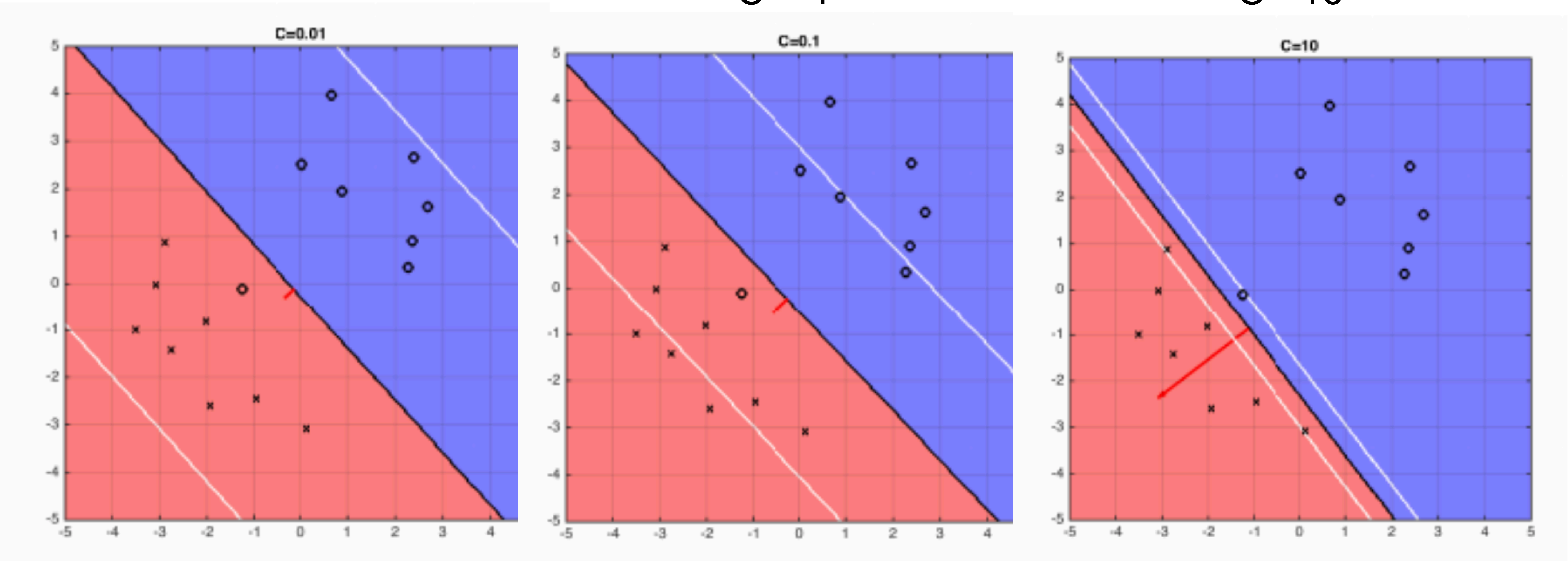
C = 0.01 C = 1 C =10



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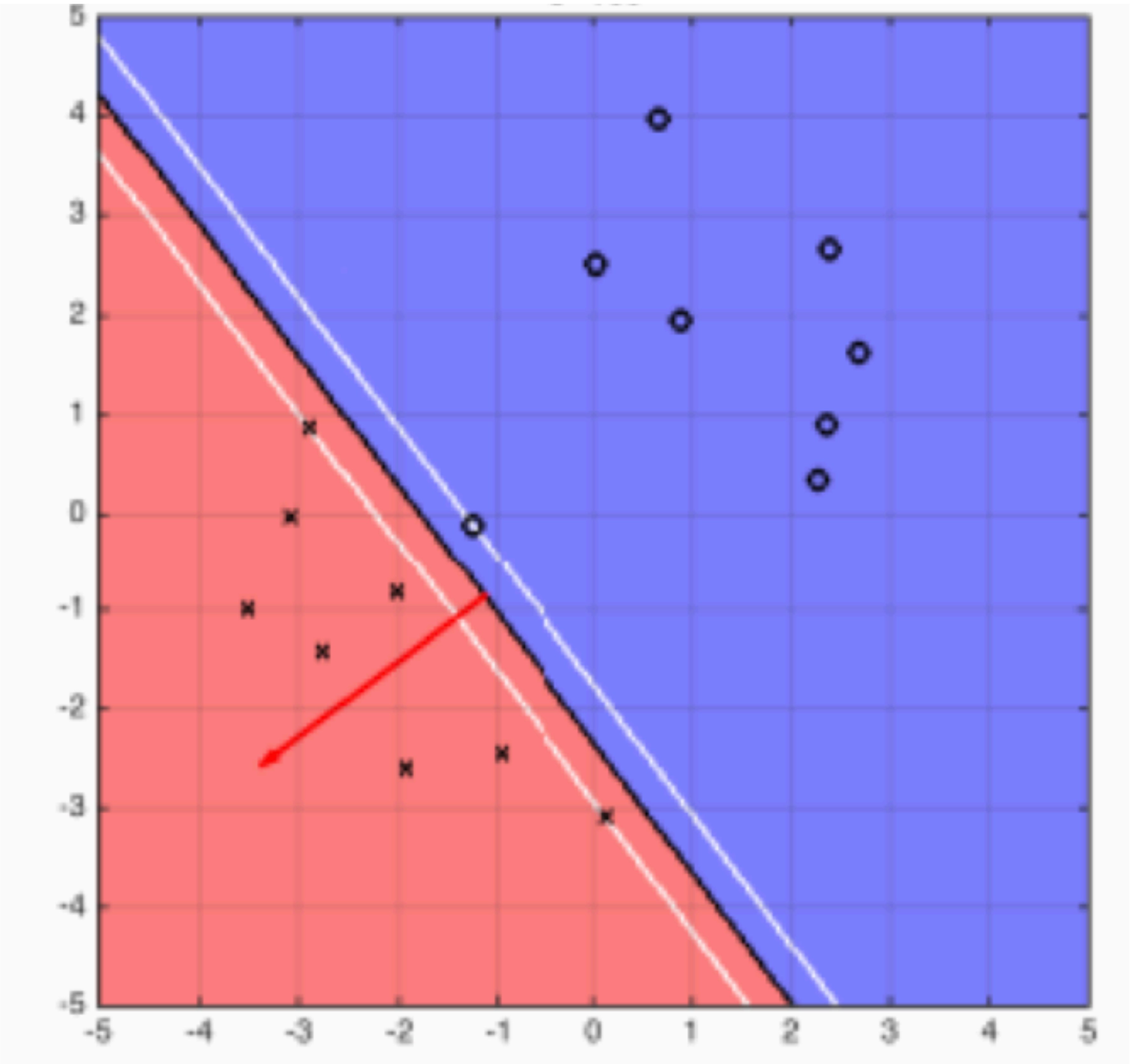

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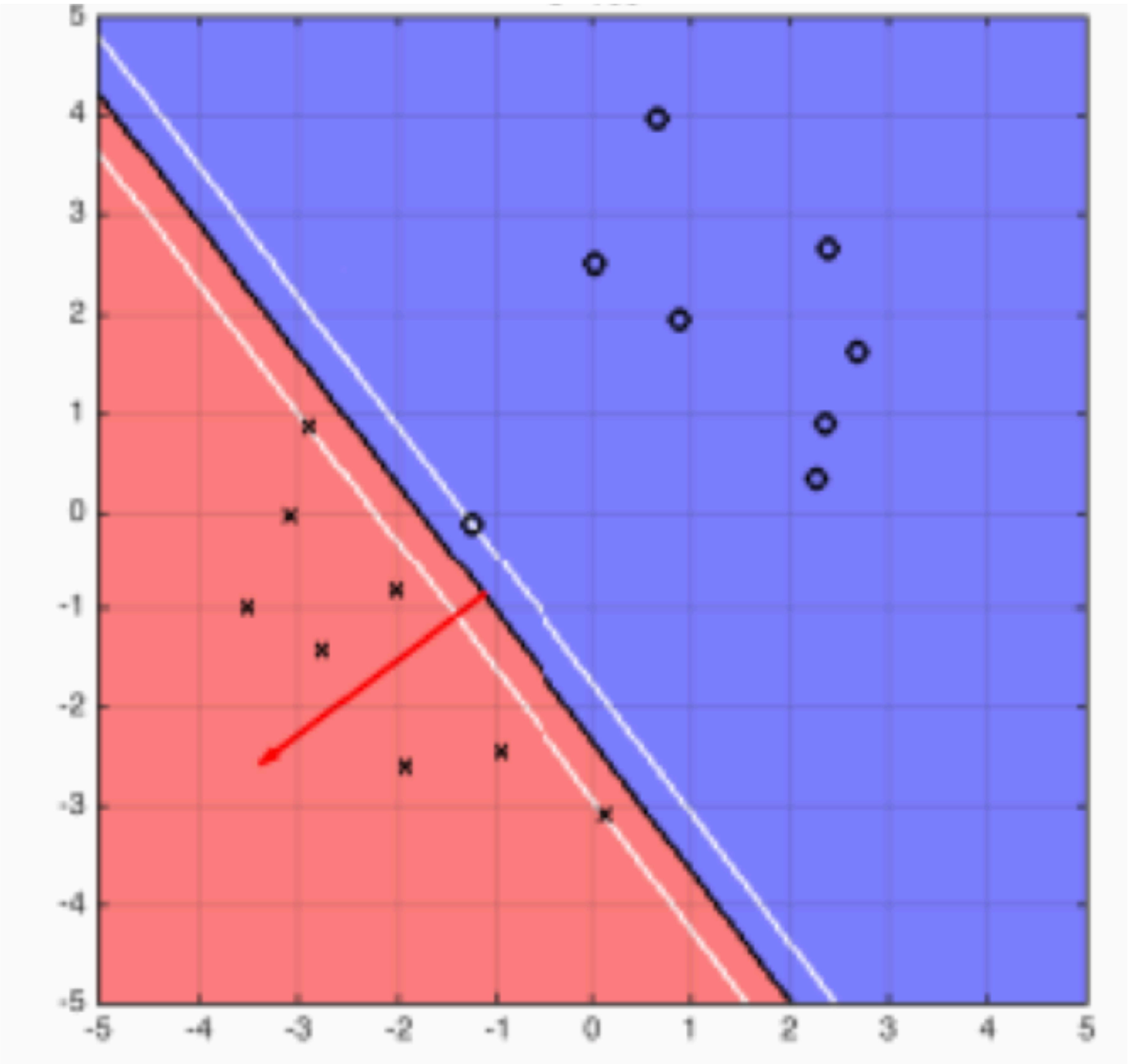

all examples have zero Hinge loss, but

w has large norm

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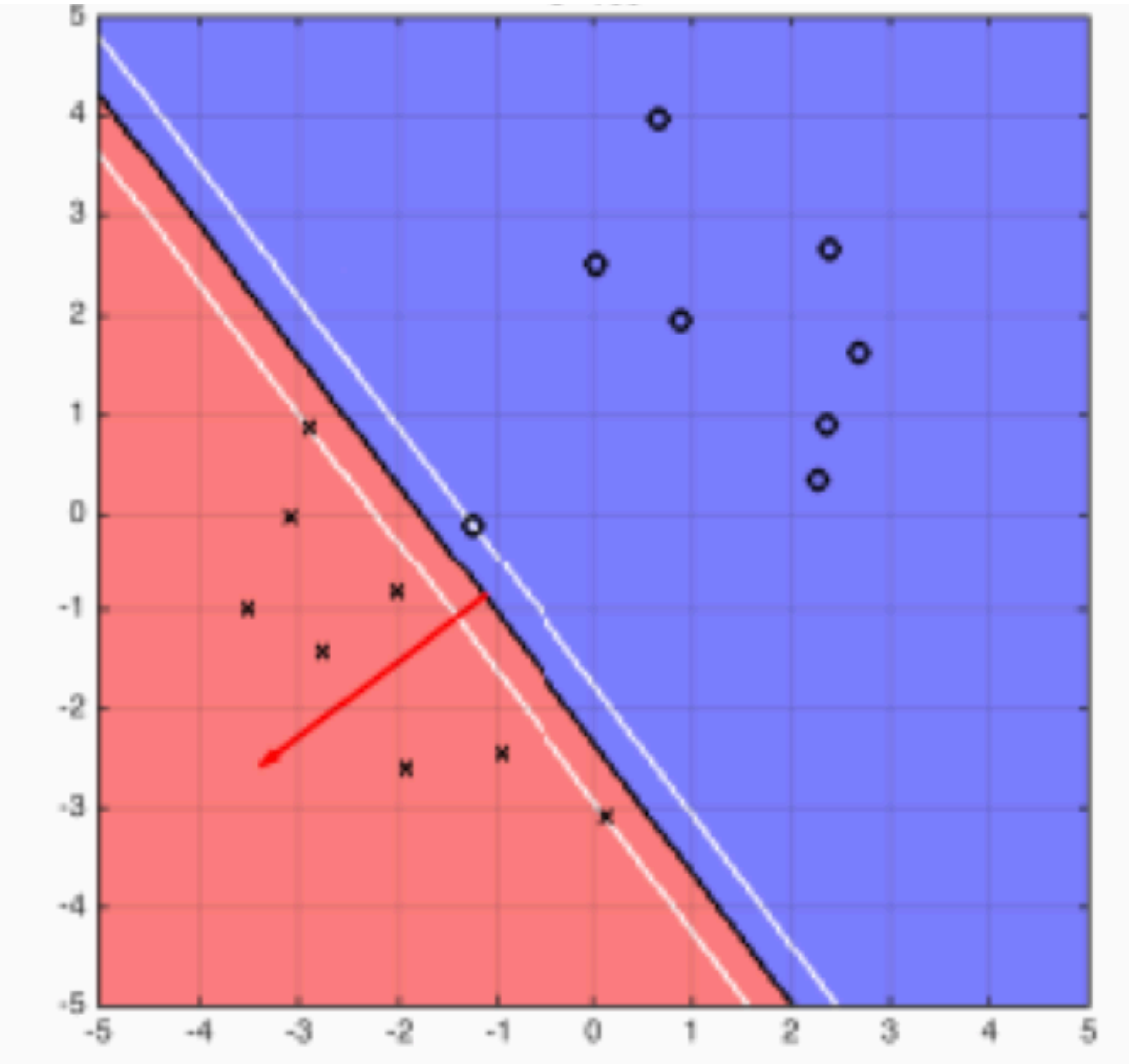

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Bad geometric margin but good functional margin (achieved by "cheating")

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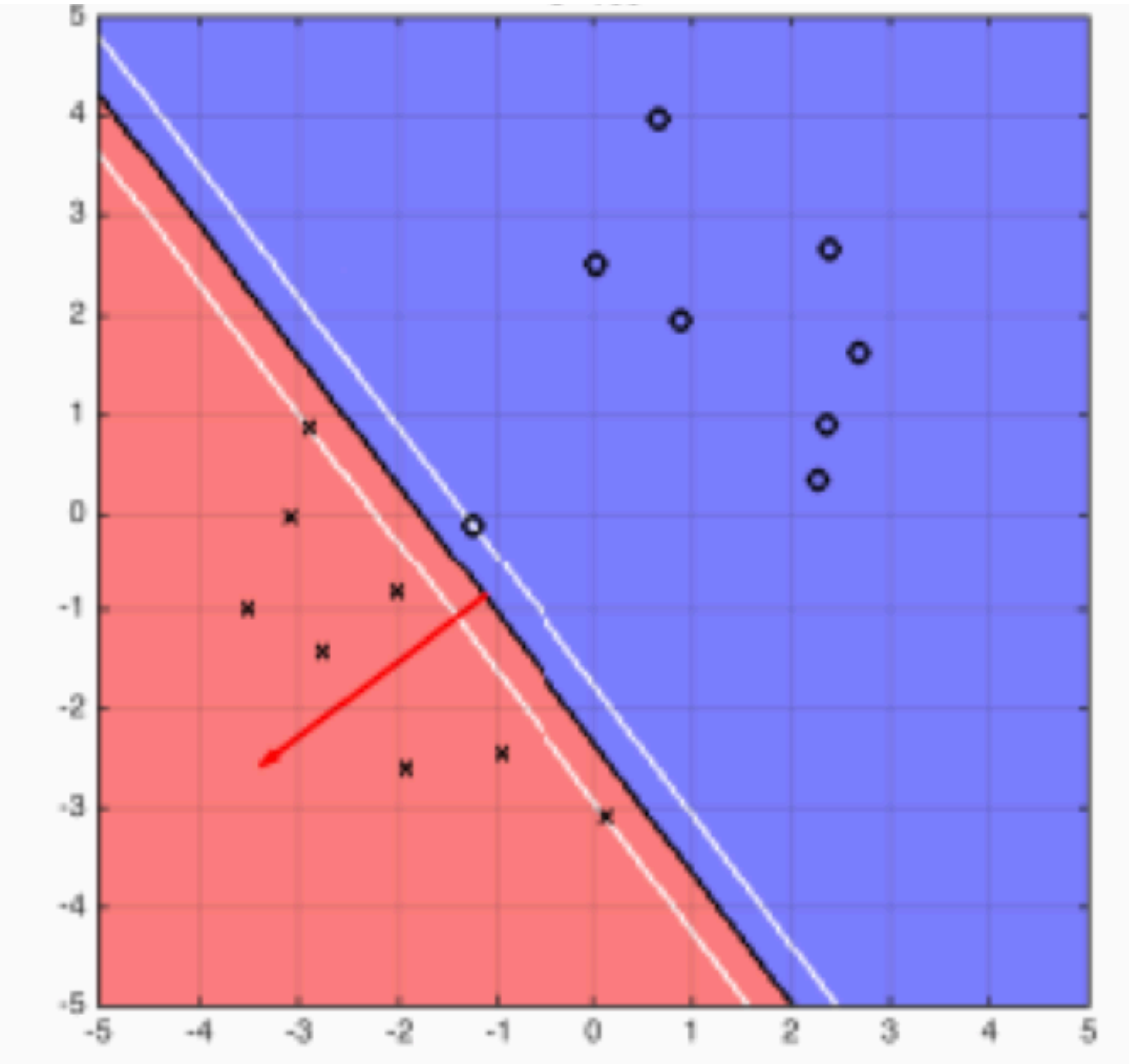

all examples have zero Hinge loss, but
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Bad geometric margin but good functional margin (achieved by "cheating")

Potentially overfitting to the noise, not a good classifier in test time maybe

Summary for today

1. SVM for linearly separable data

$$\min_{w,b} \|w\|_2^2$$

$$\forall i: \; y_i(w^\top x_i + b) \geq 1$$

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1. SVM for linearly separable data

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2. SVM for non-separable data

$$\min_{w,b} ||w||_2^2 + c \sum_{i=1}^{n} \boxed{\max \left\{0, 1 - y_i(w^\top x_i + b)\right\}}$$

Hinge loss