K-nearest Neighbor

Announcement:

1. HW1 will be out today / early tomorrow and Due Sep 13

Recap

Outline for Today

1. The K-NN Algorithm

2. Why/When does K-NN work

3. Curse of dimensionality (i.e., when it can fail)

The K-NN Algorithm

Input: classification training dataset $\{x_i, y_i\}_{i=1}^n$, and parameter $K \in \mathbb{N}^+$, and a distance metric $d(x, x')$ (e.g., $\|x - x'\|_2$ euclidean distance)

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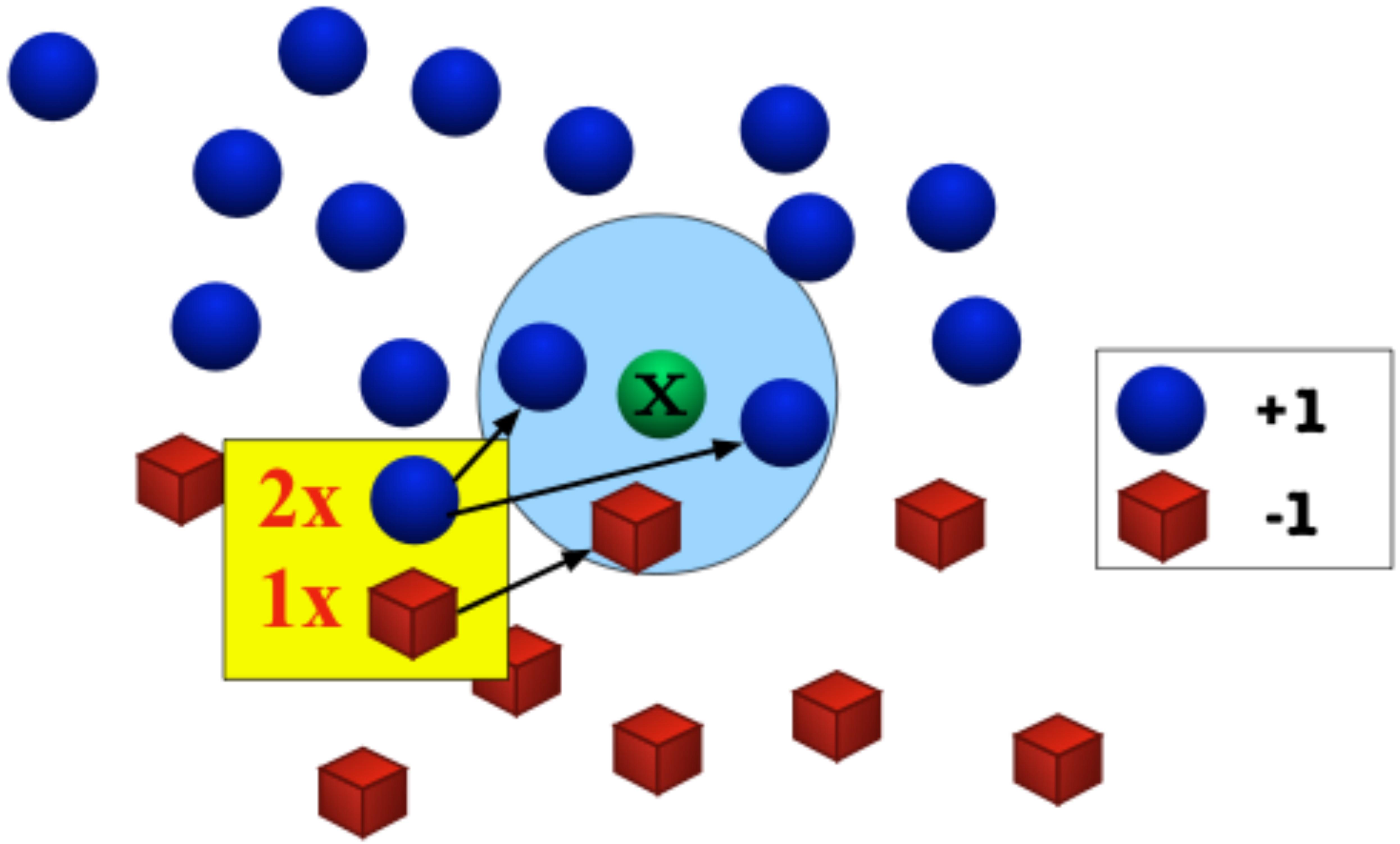
(If for regression, return the average value of the K neighbors)

The K-NN Algorithm

Example: 3-NN for binary classification using Euclidean distance



The choice of metric

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Another example: Manhattan distance (ℓ_1)

$$d(x, x') = \sum_{j=1}^{d} |x[j] - x'[j]|$$

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(What about the training error when K = 1?)

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$$\epsilon_{opt} = 1 - P(y_b \mid x) = 0.2$$

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Assume $x \in [-1,1]^2$, $P(x)$ has support everywhere $P(x) > 0, \forall x \in [-1,1]^2$

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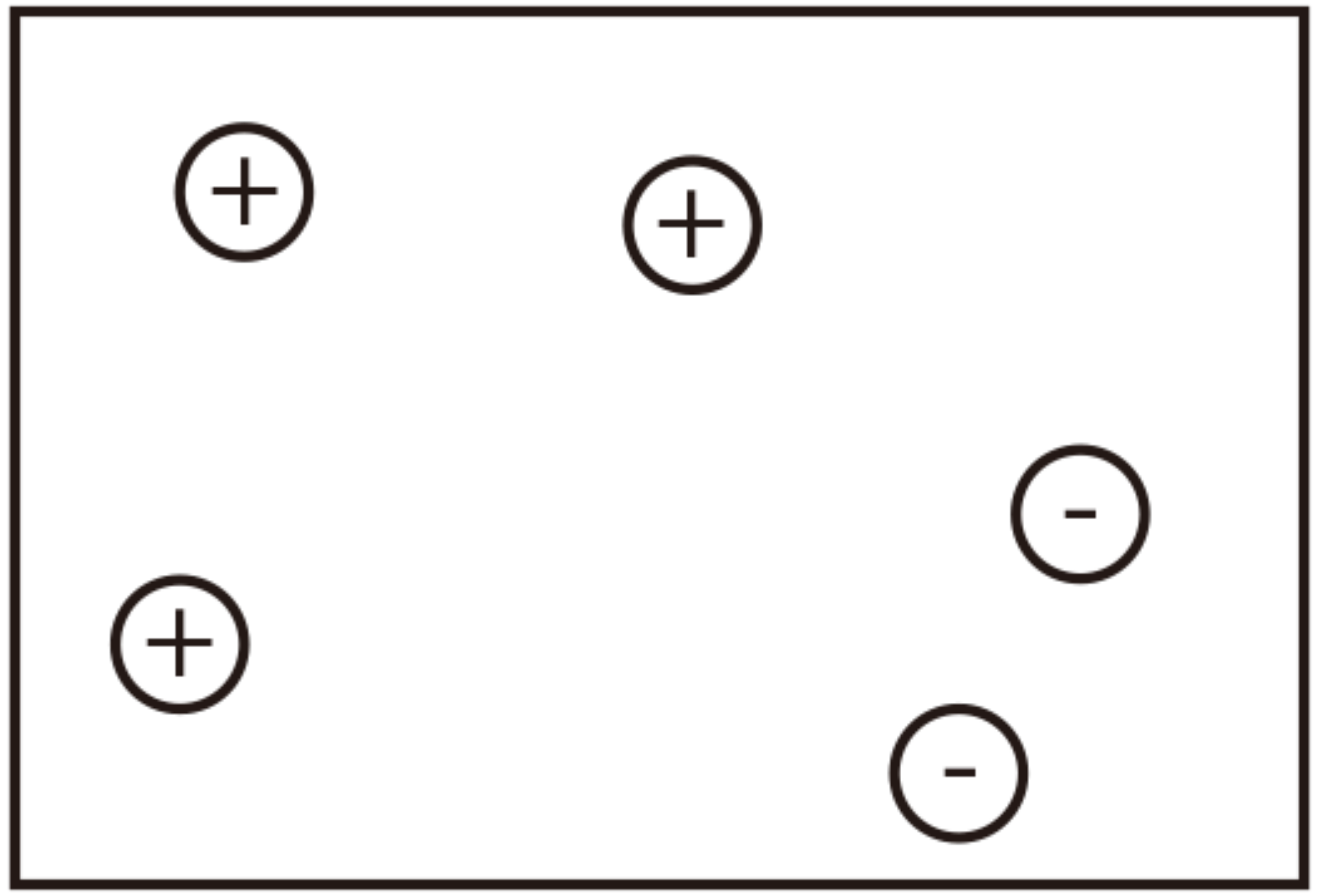

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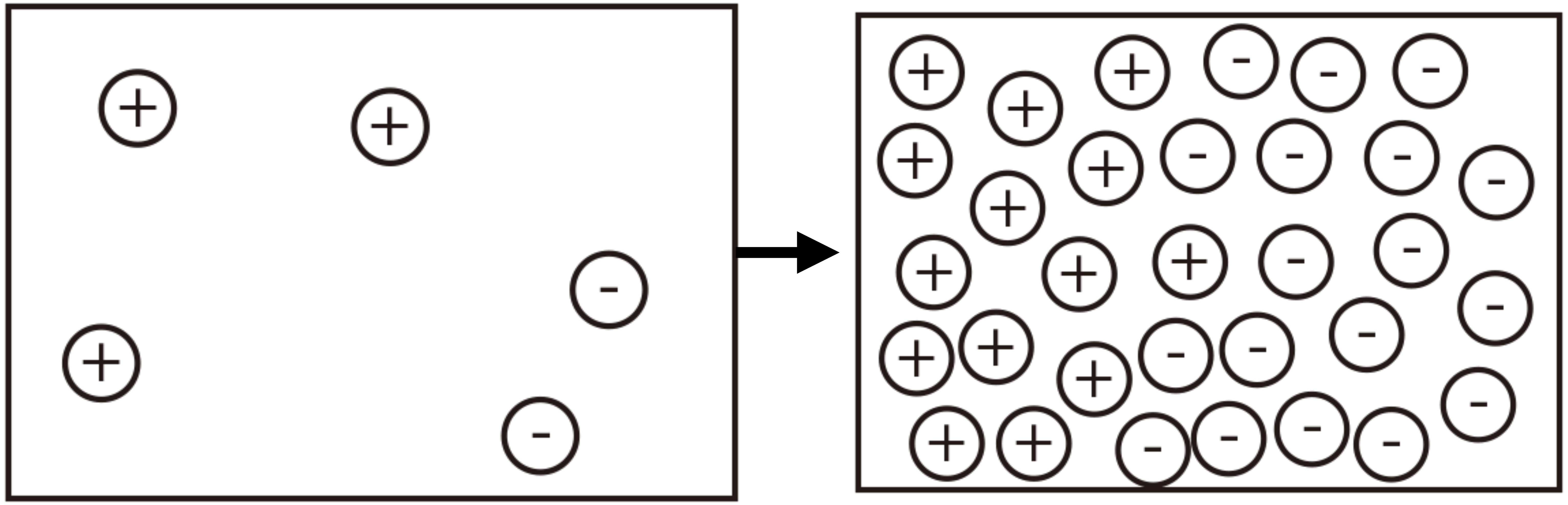

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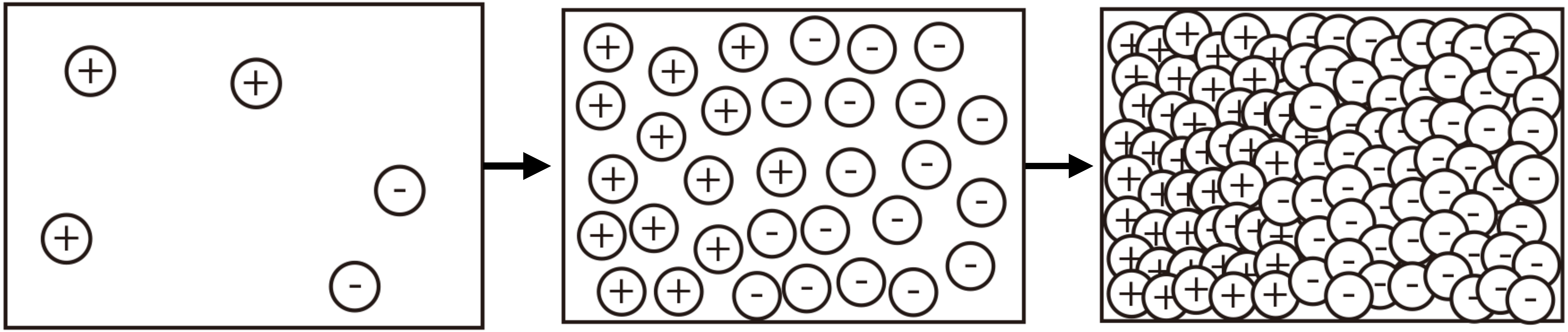

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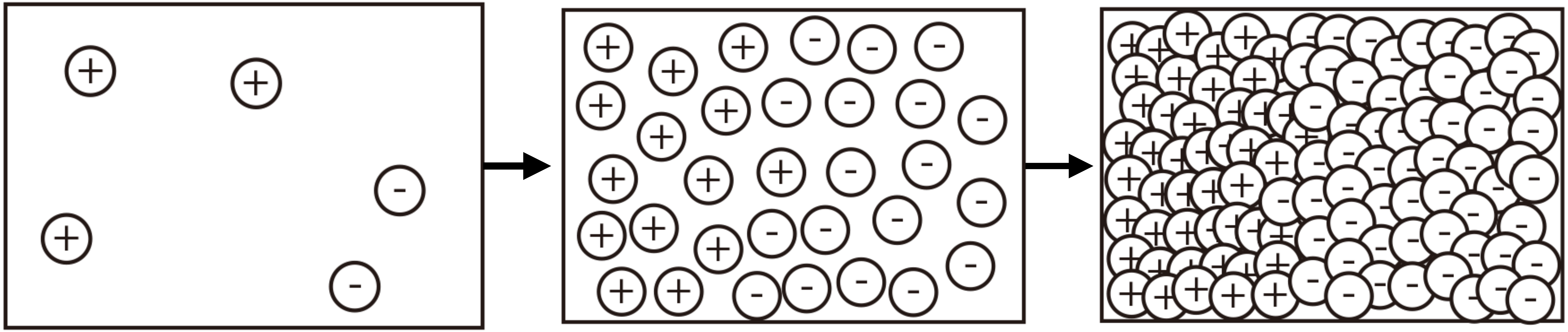

Given test x, as $n \to \infty$, its nearest neighbor x_{NN} is super close, i.e., $d(x, x_{NN}) \to 0$!

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Theorem: as $n \to \infty$, 1-NN prediction error is **no more than twice** of the error of the Bayes optimal classifier

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A: Given any x, the K-NN should return the y_b — the solution of the Bayes optimal

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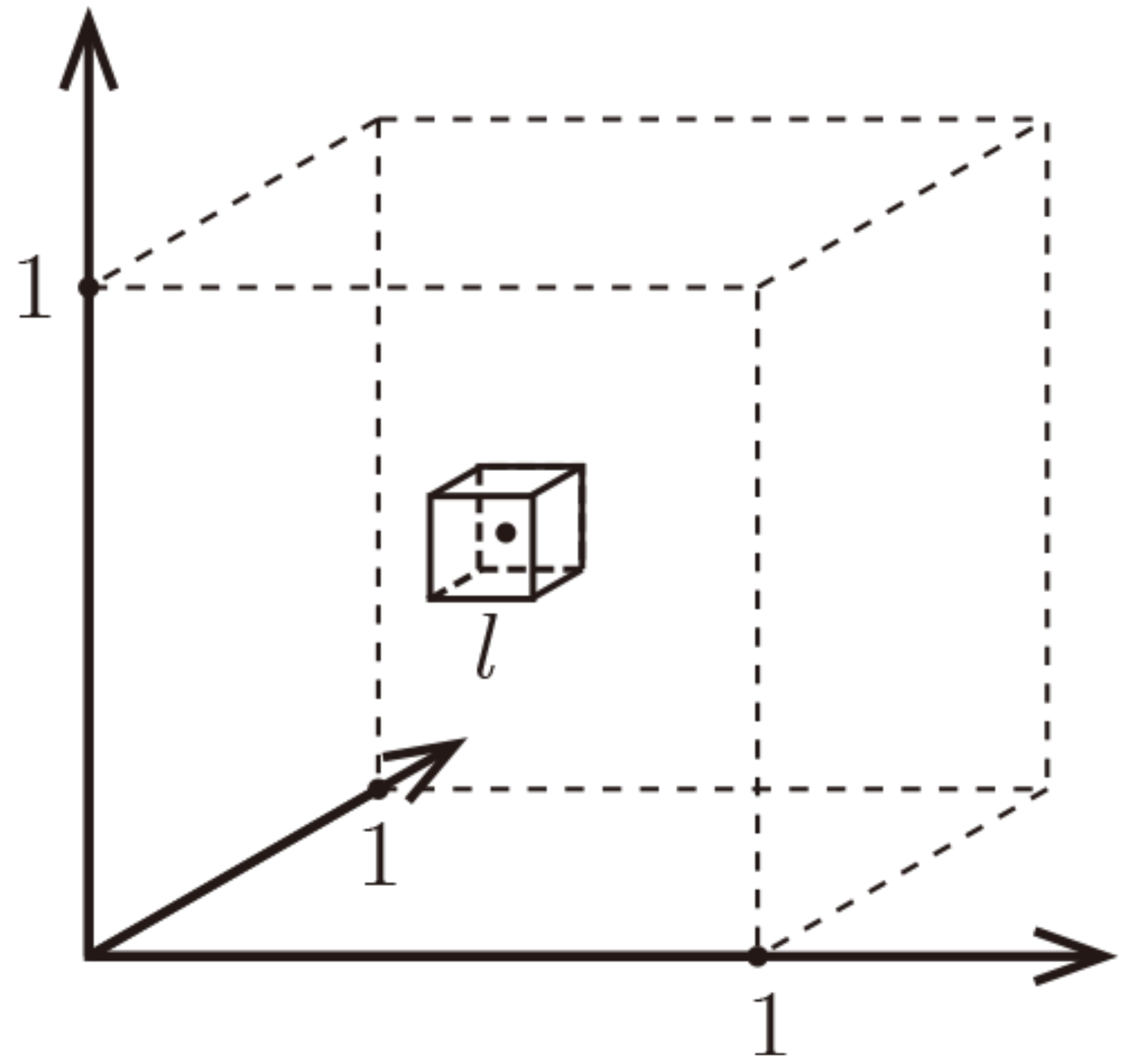
Curse of dimensionality!

Curse of Dimensionality Explanation

Key problem: in high dimensional space, points that are draw from a distribution tends to be far away from each other!



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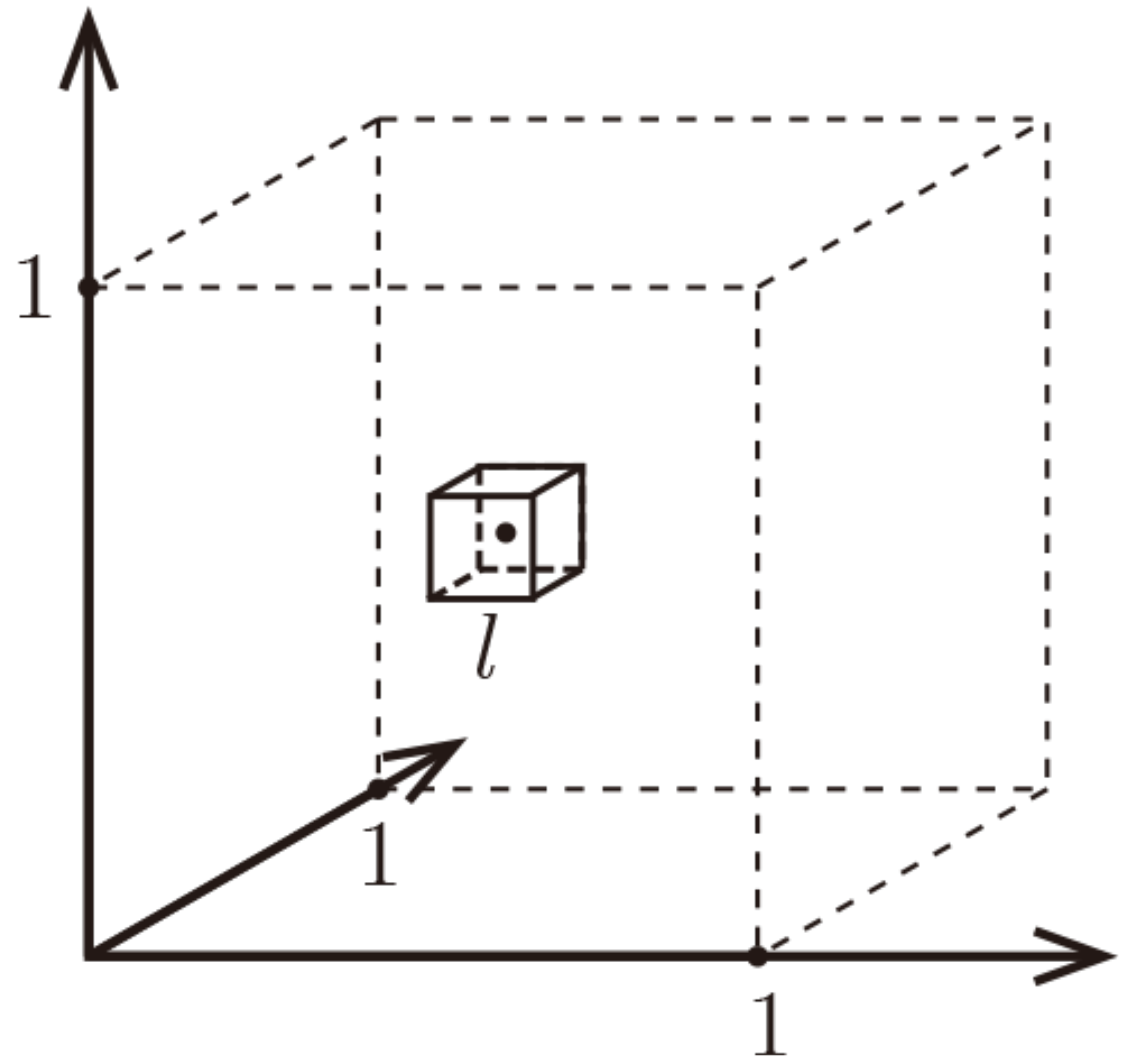

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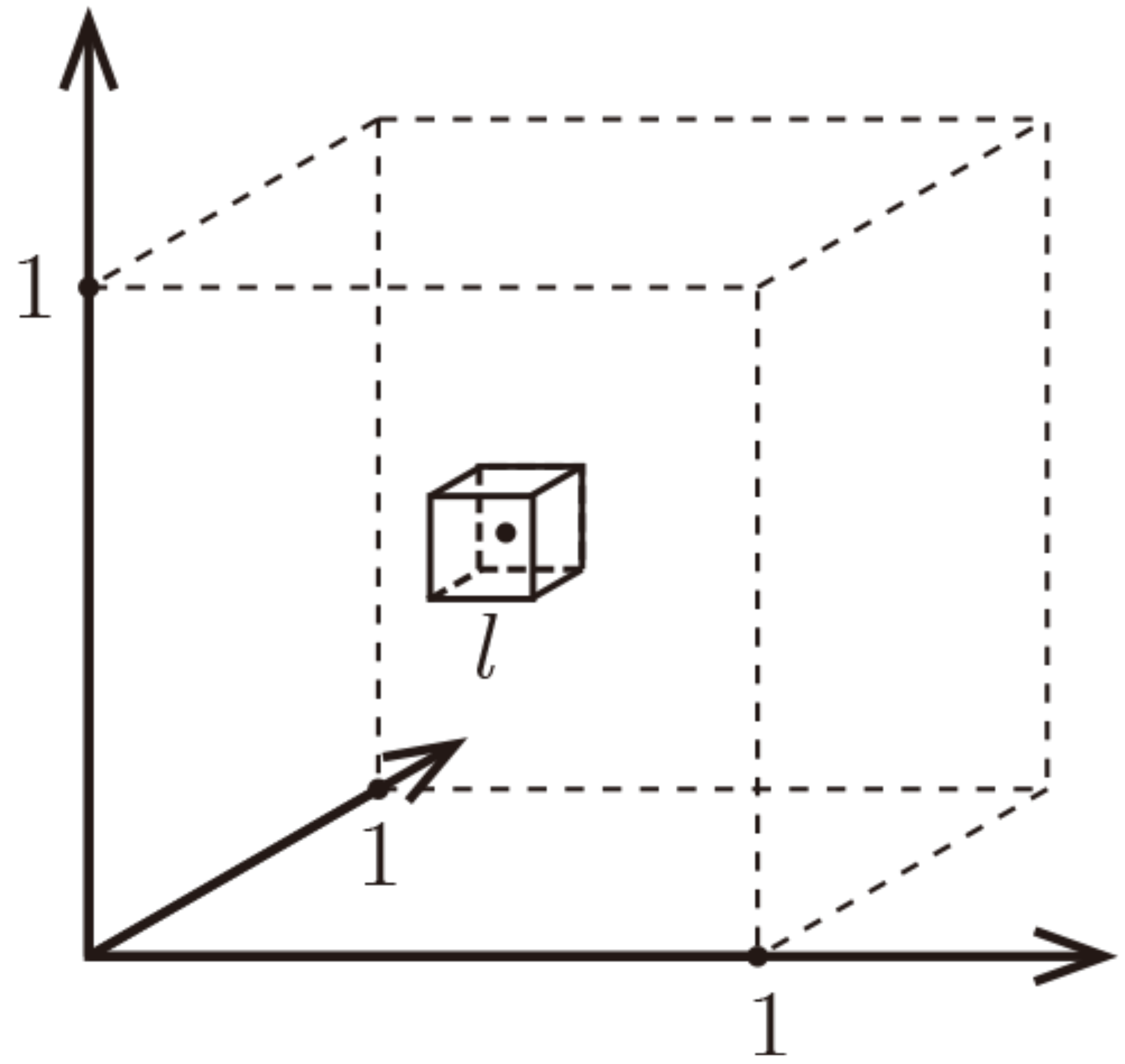

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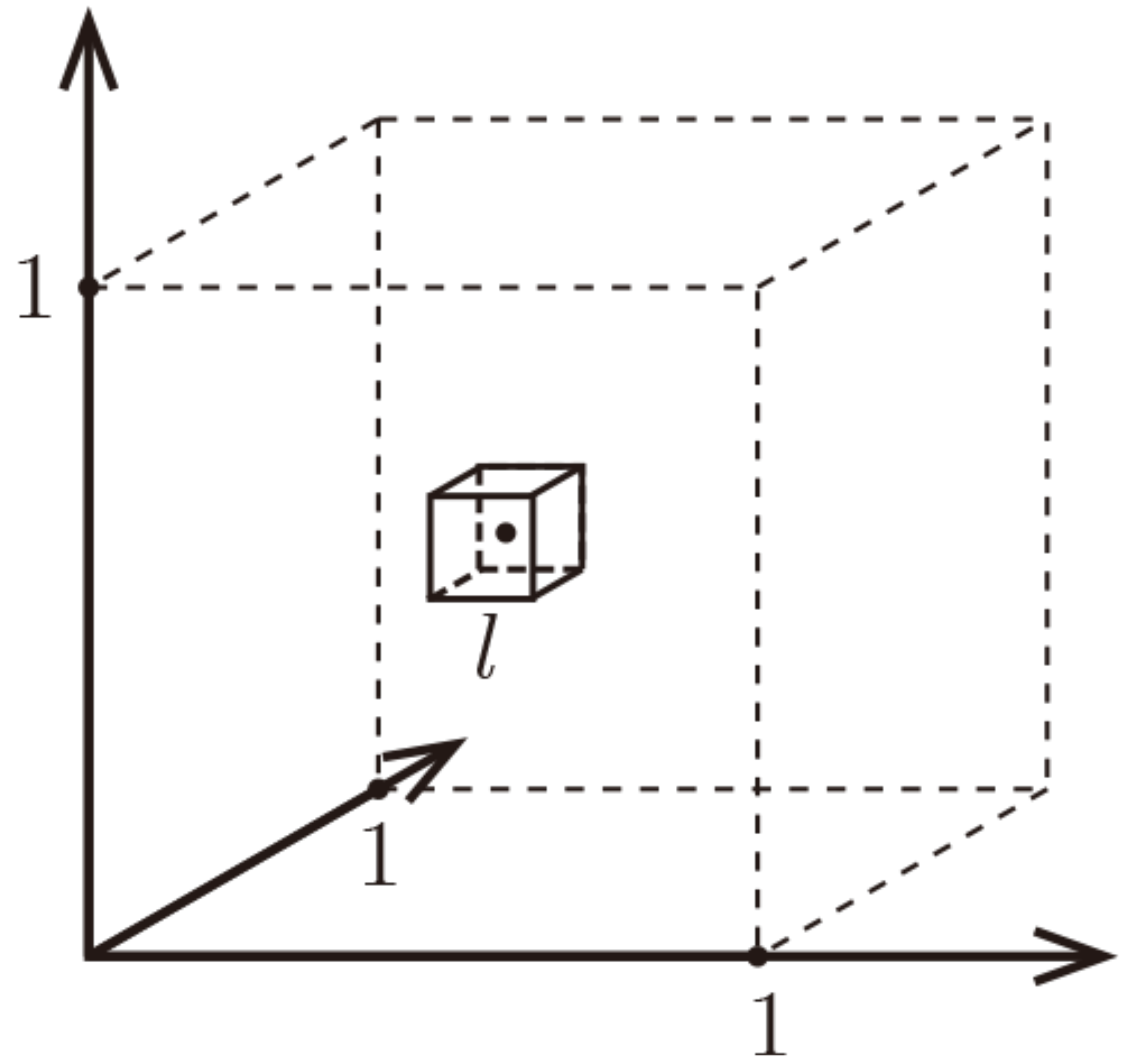

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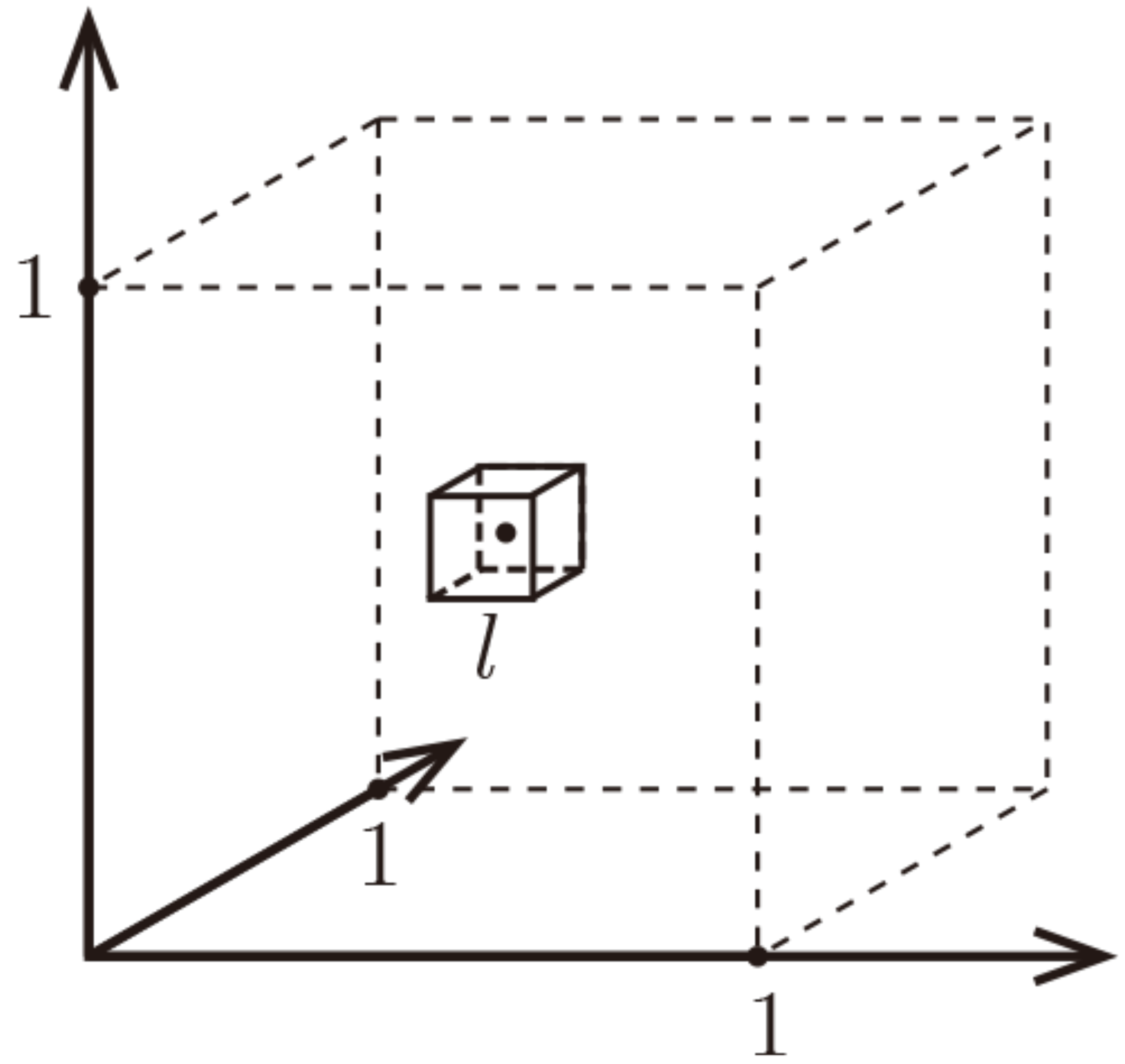

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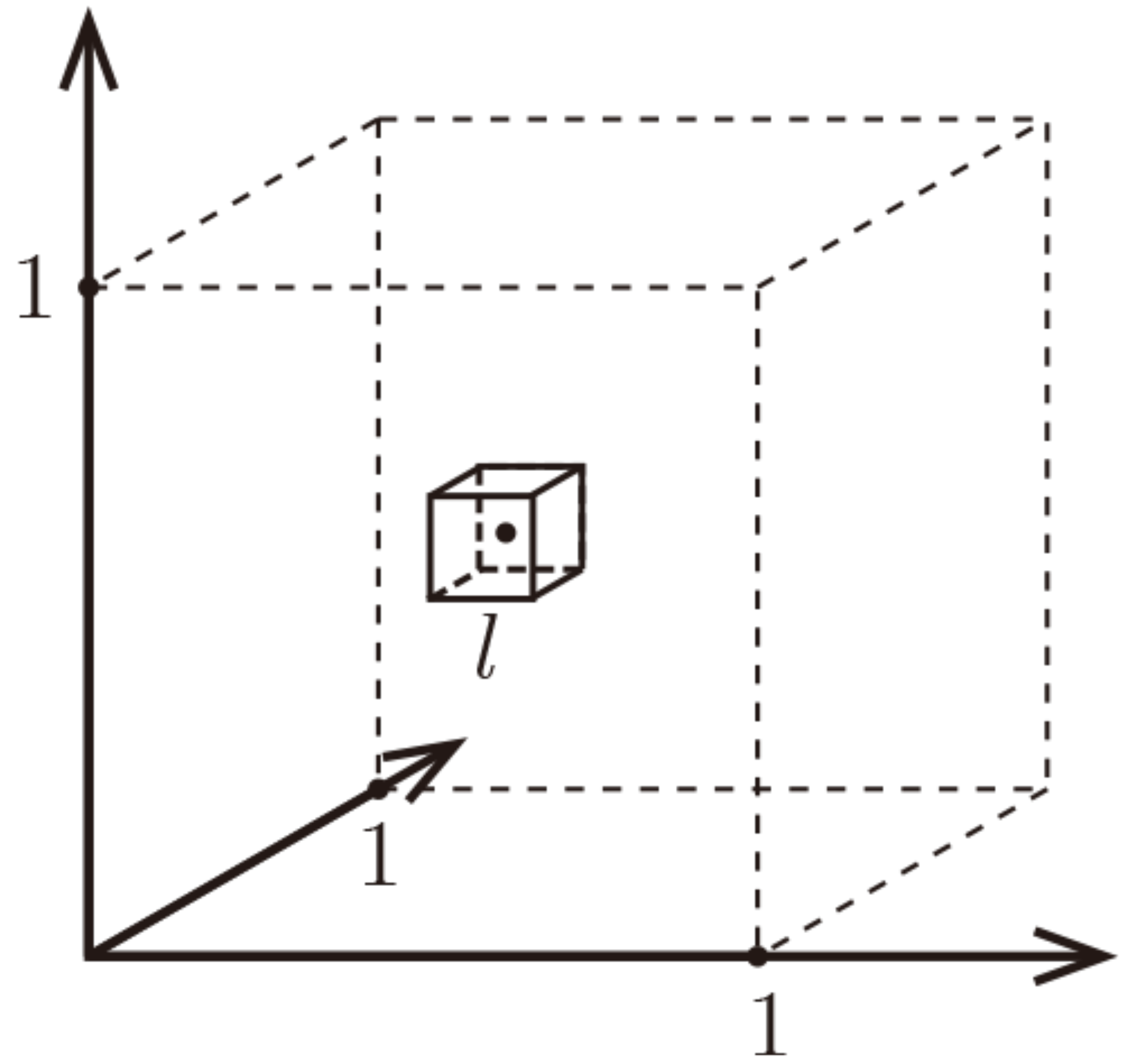

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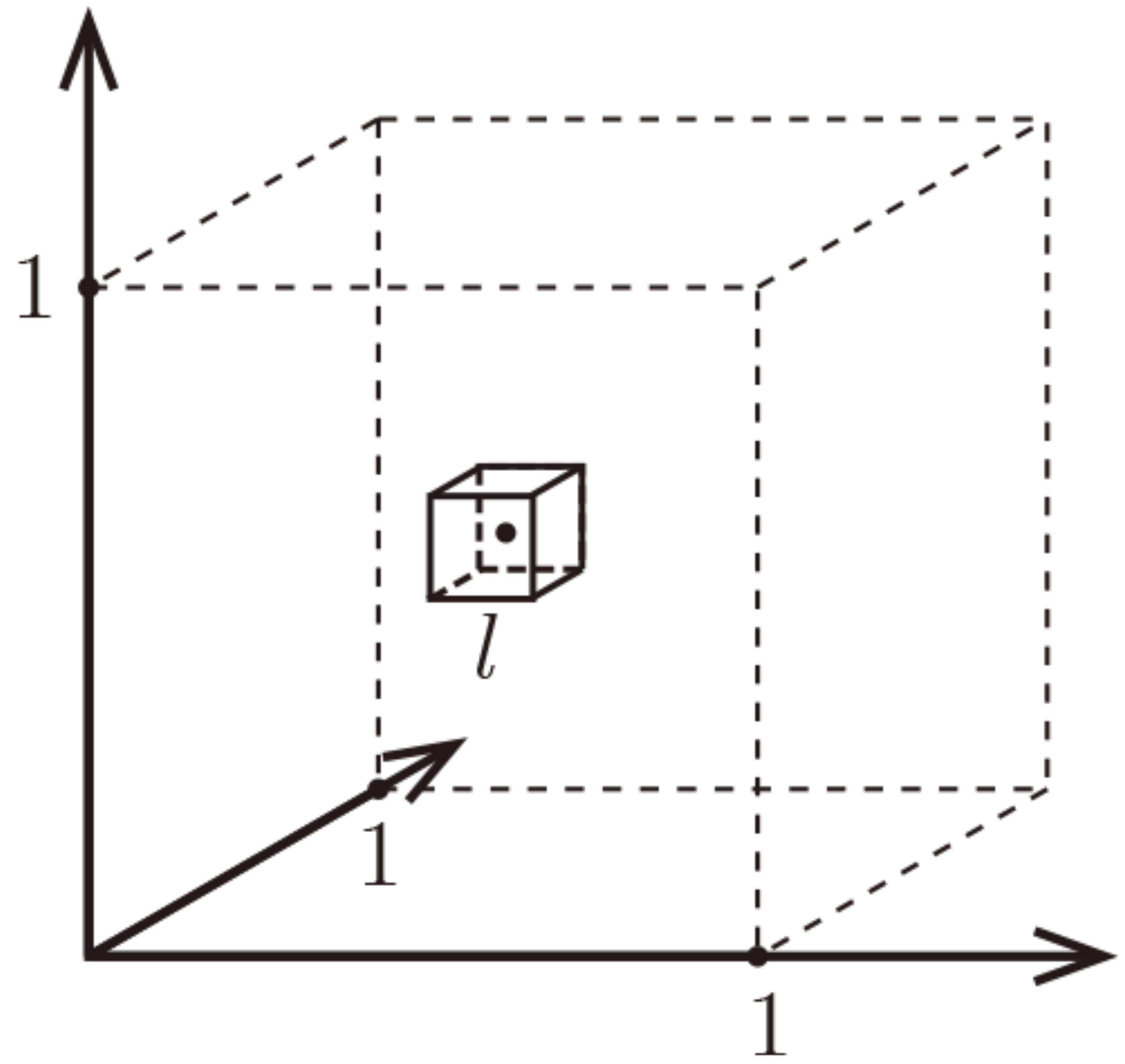

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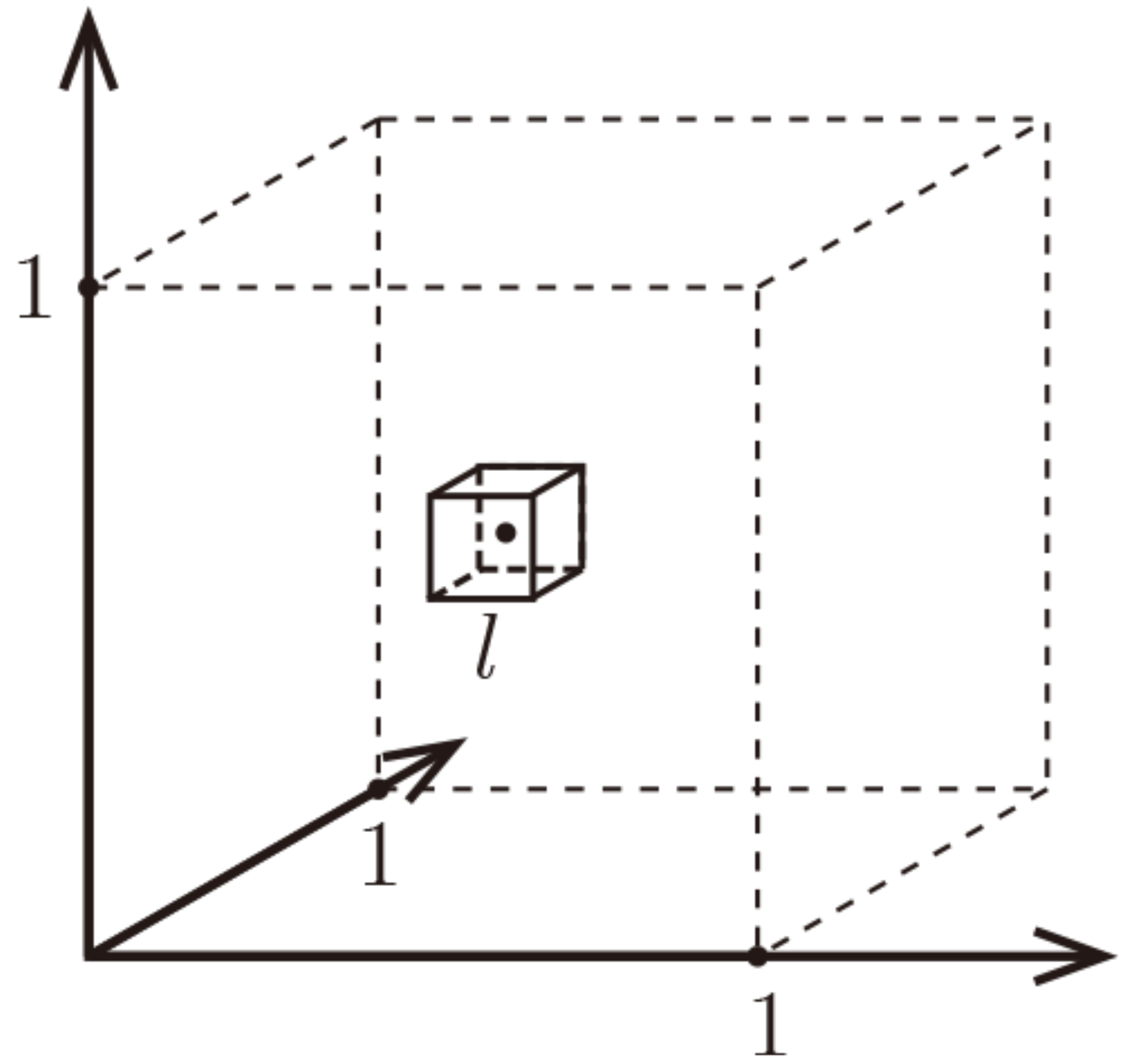

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Thus, we have $$l^d \approx \frac{K}{n}$$

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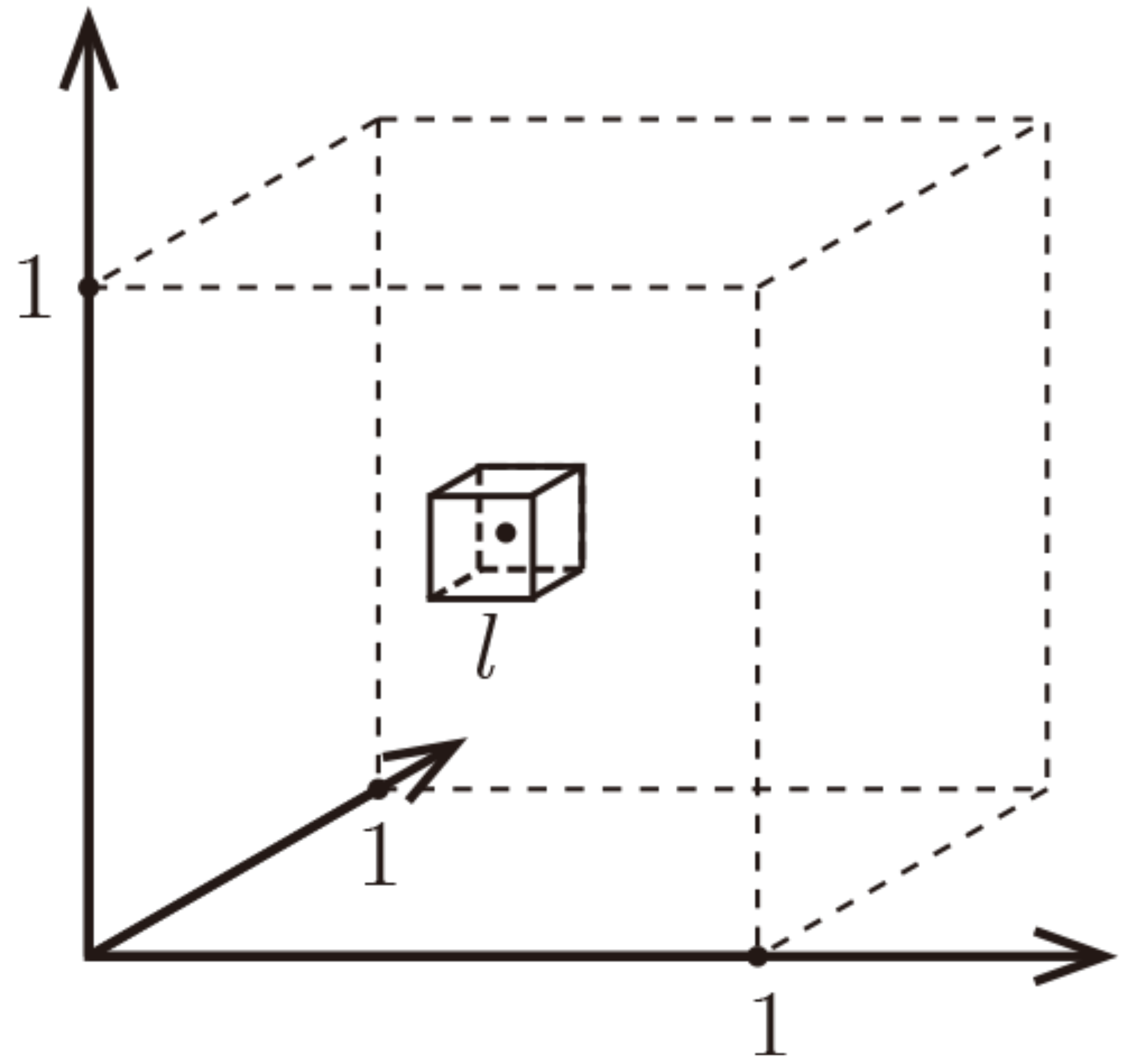

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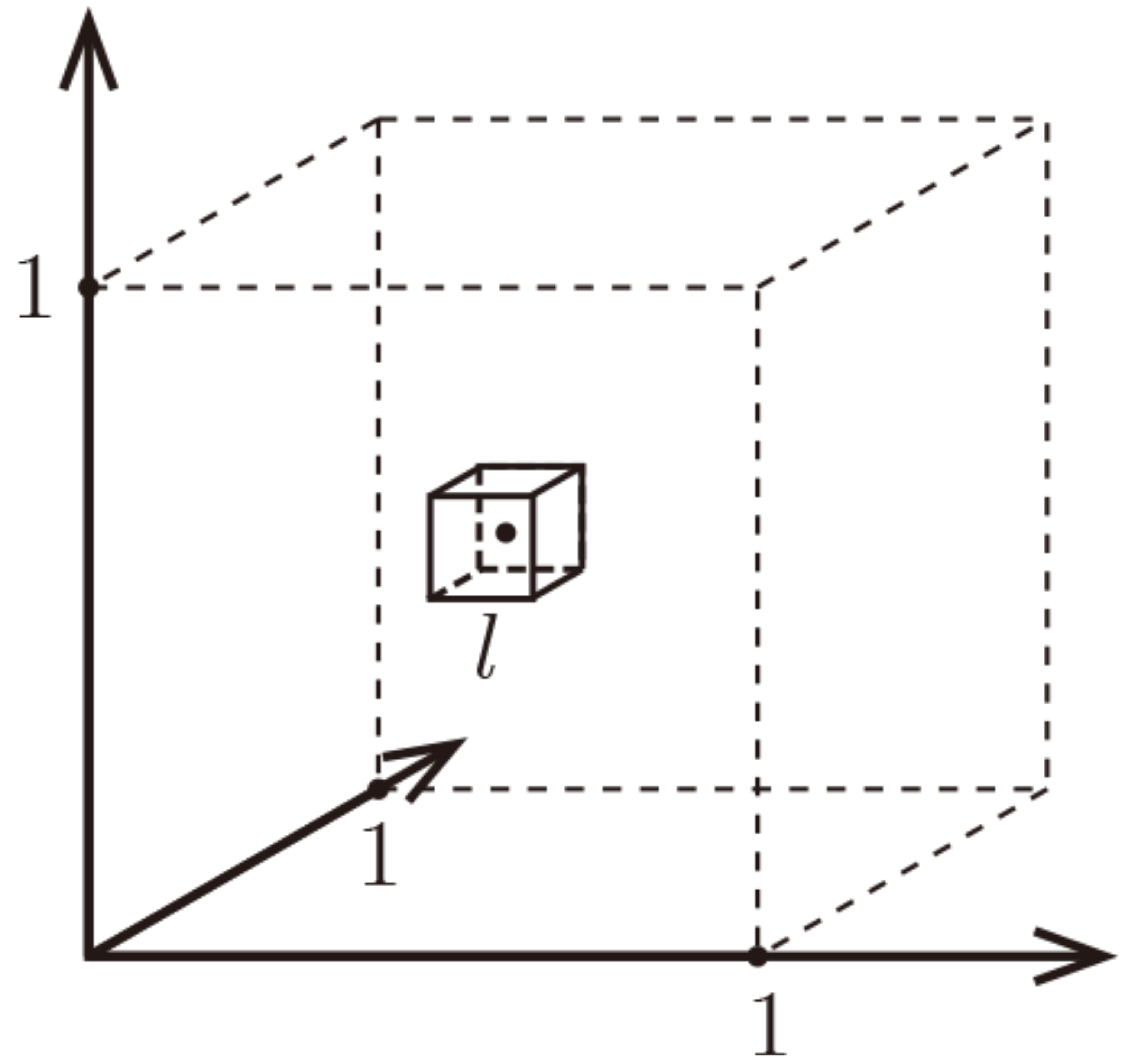

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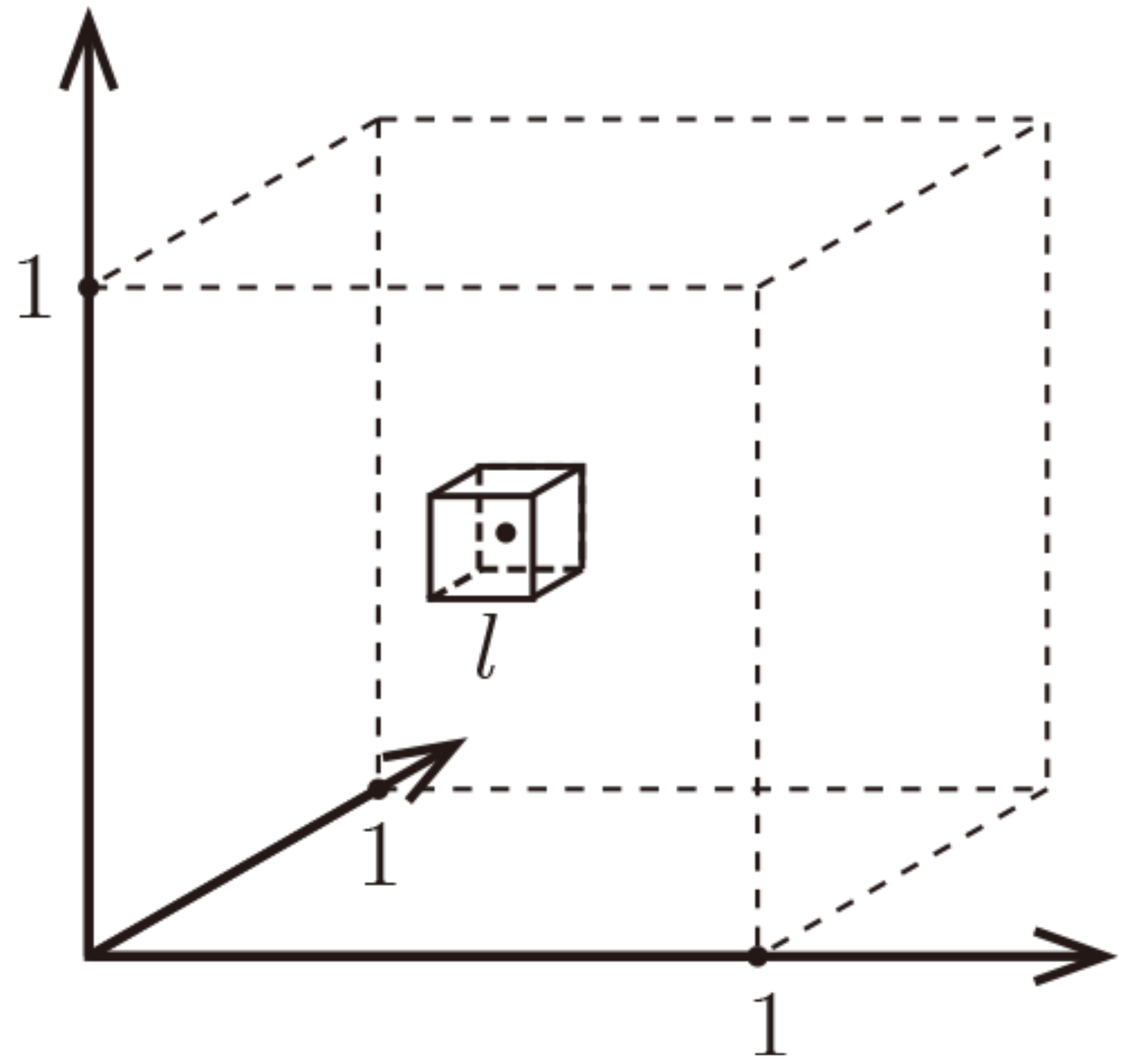

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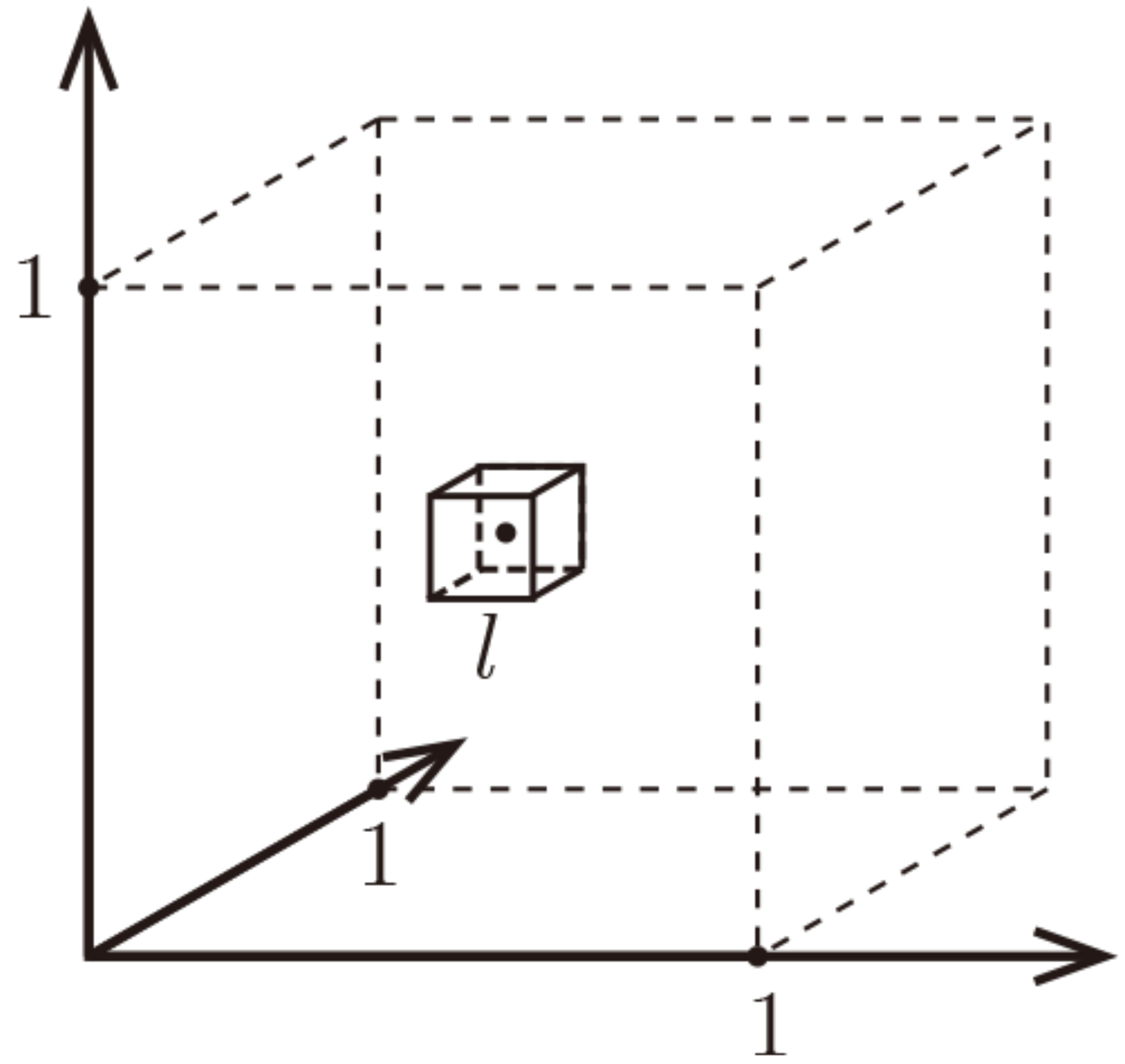

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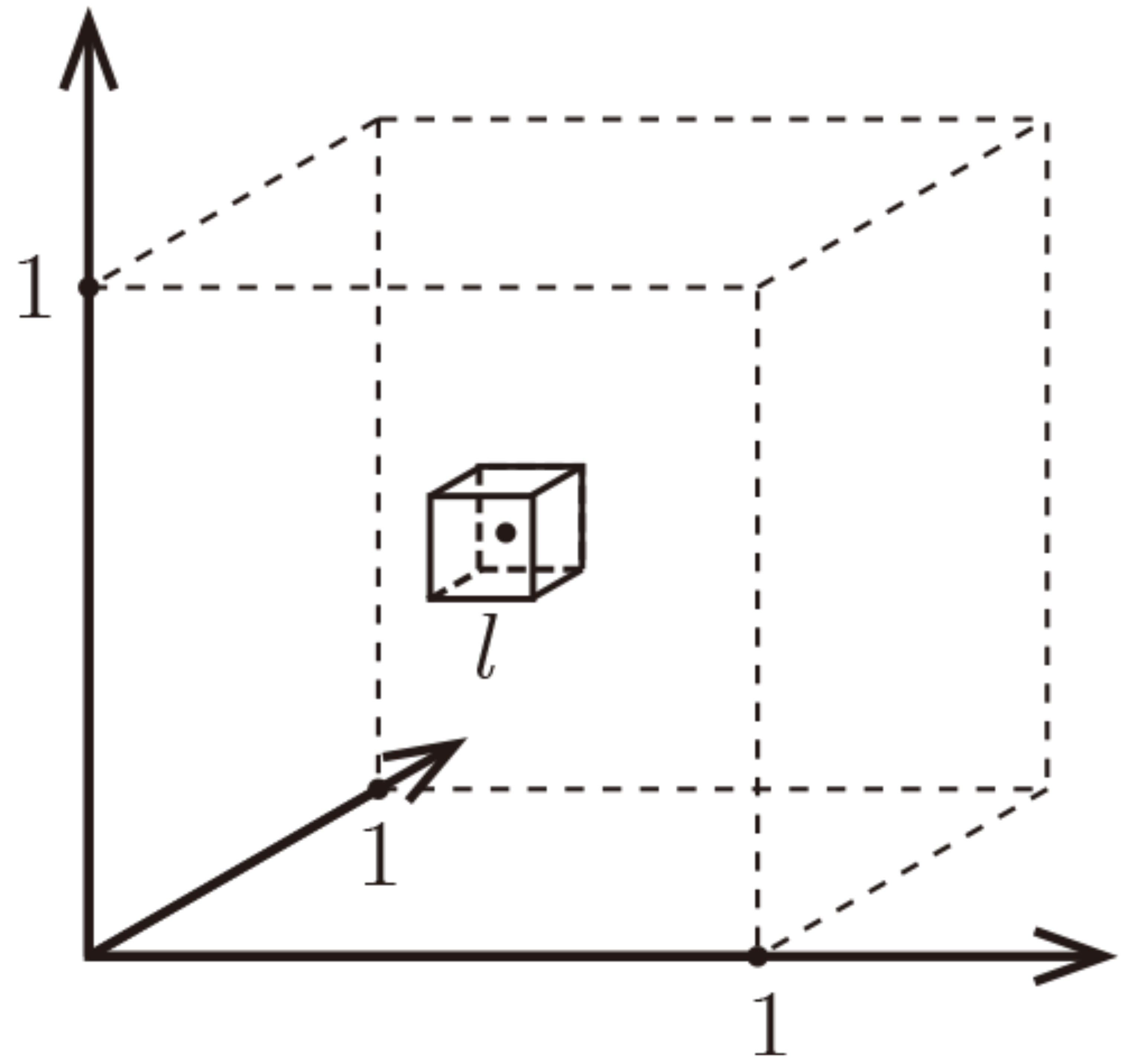

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Bad news: when $d \rightarrow \infty$, the K nearest neighbors will be all over the place! (Cannot trust them, as they are not nearby points anymore!)

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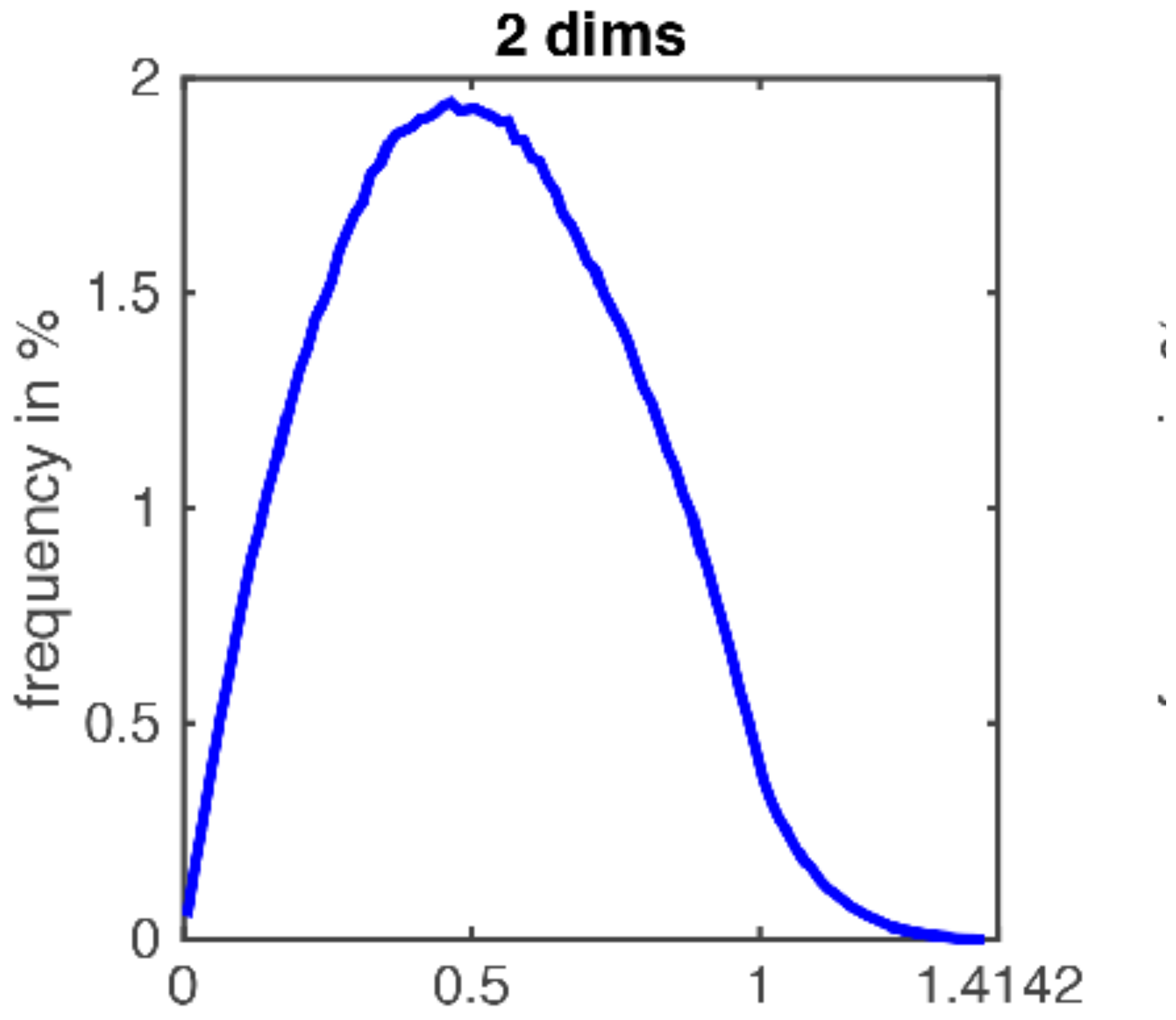

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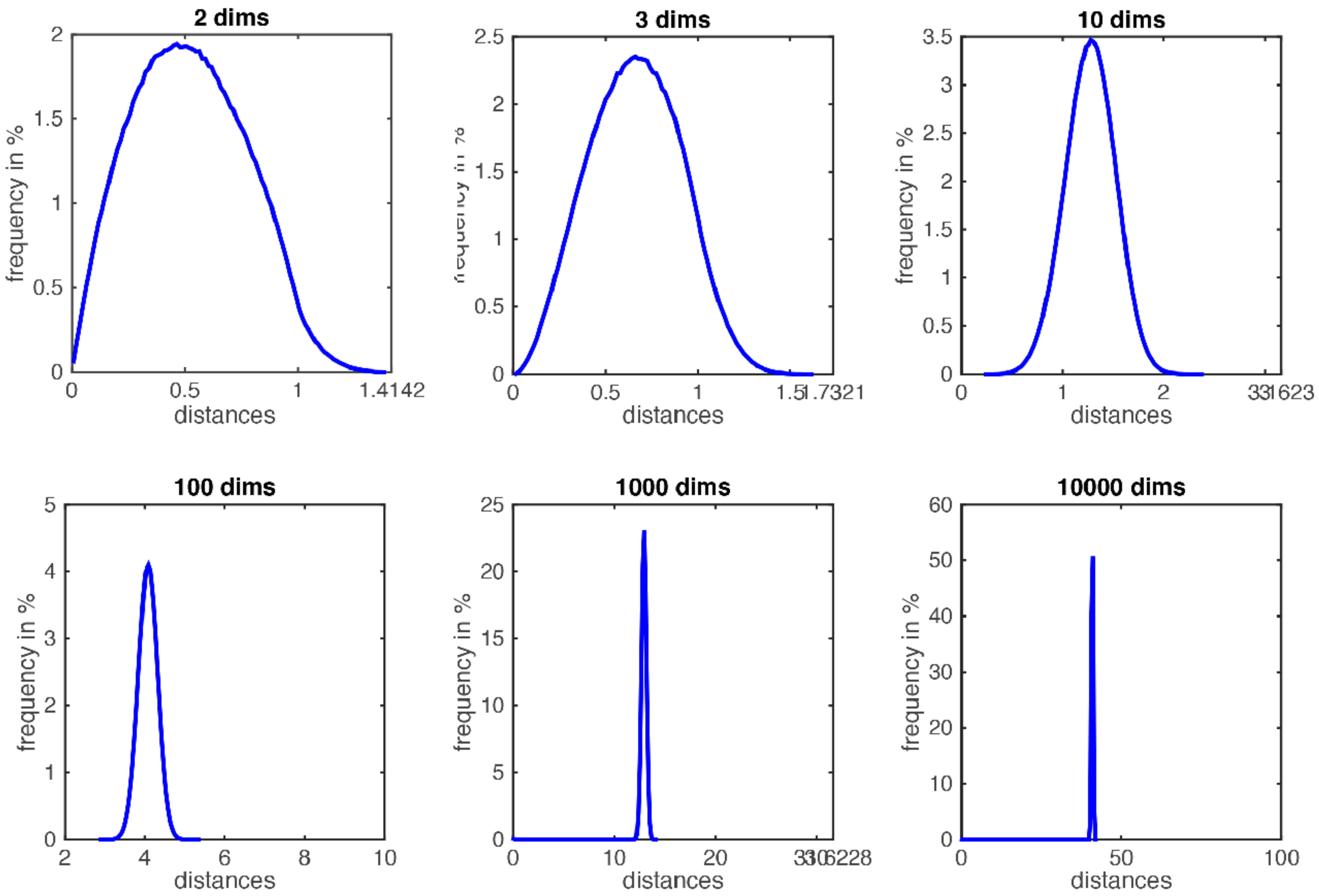

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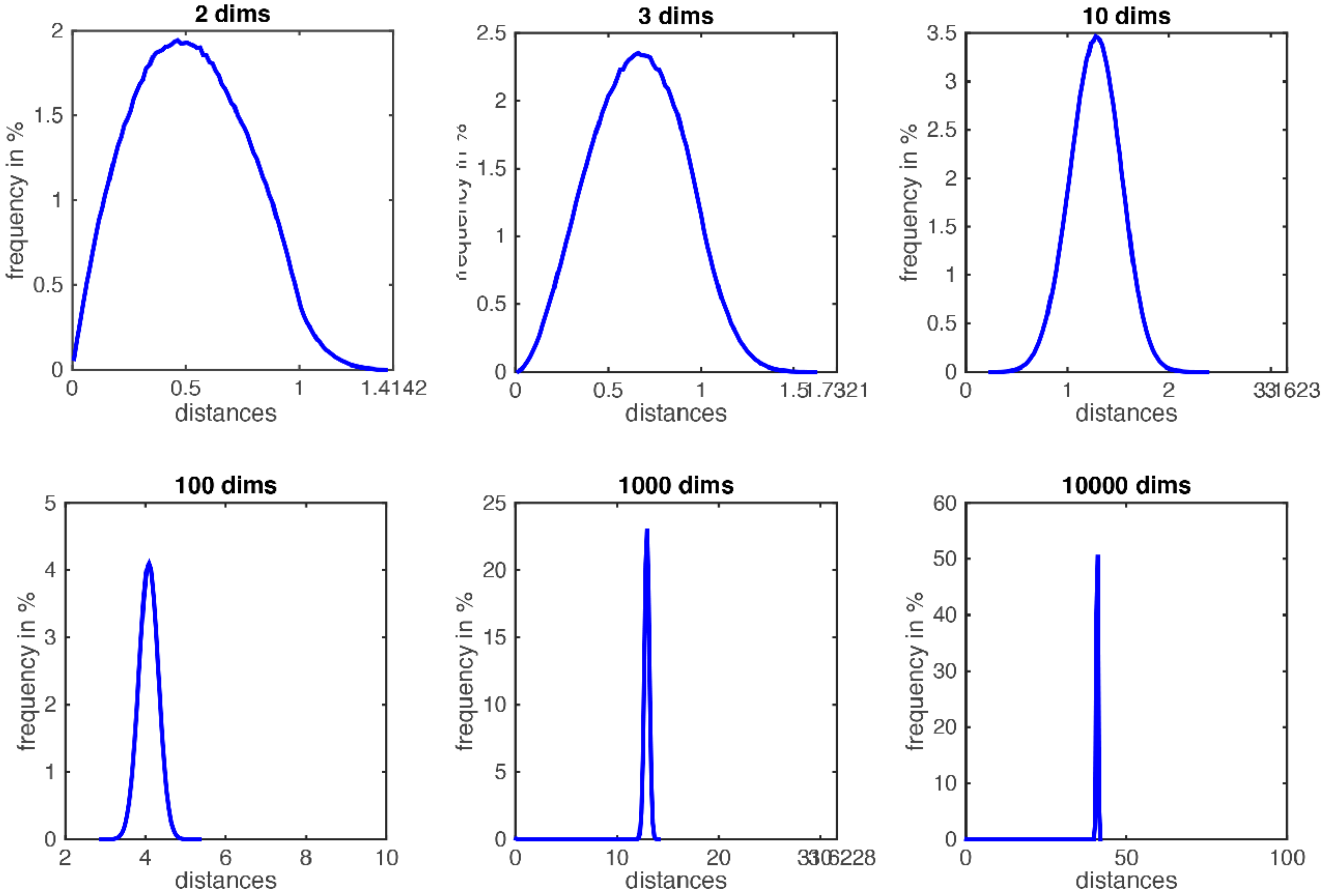
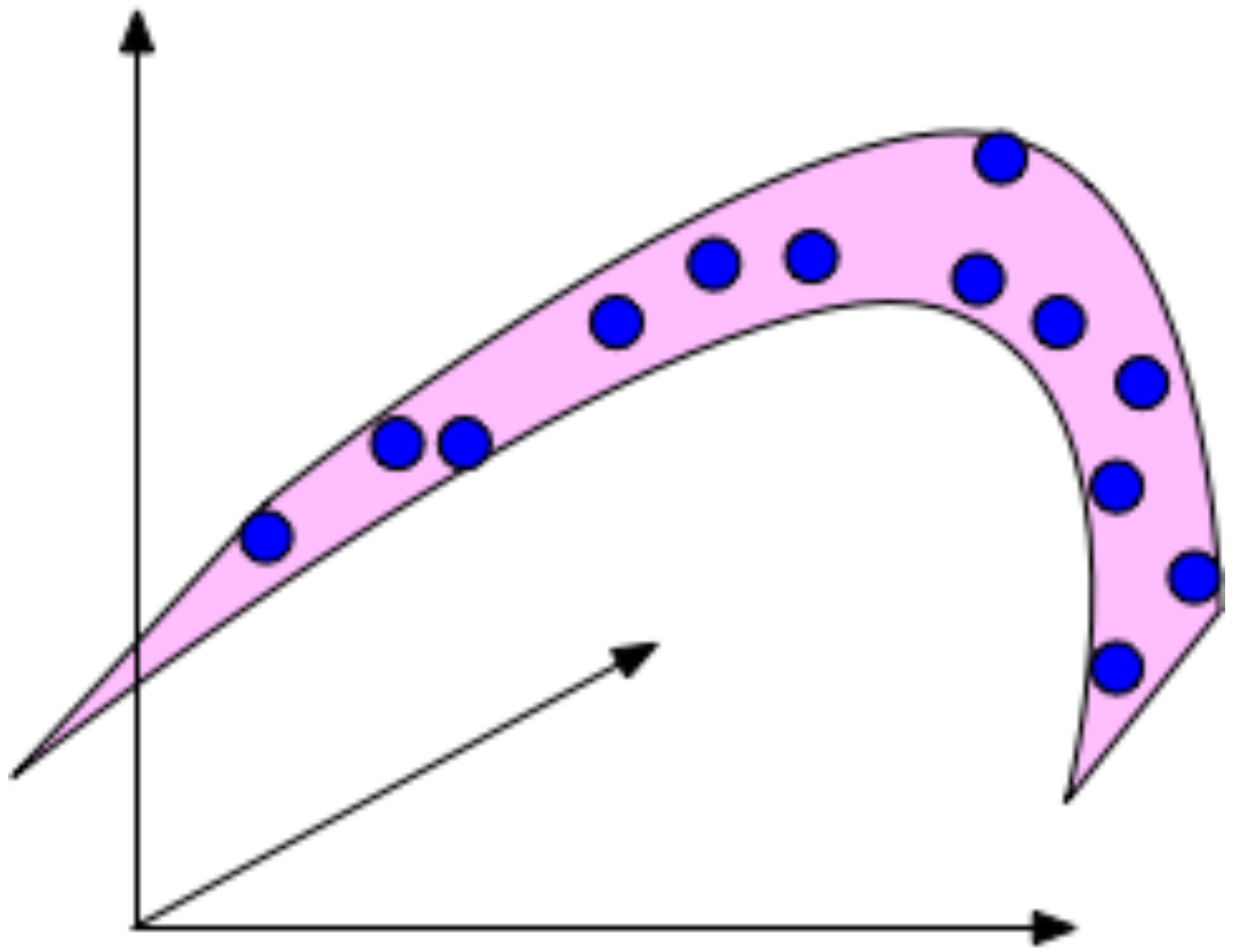


Distance increases as $d \to \infty$

Luckily, real world data often has low-dimensional structure!



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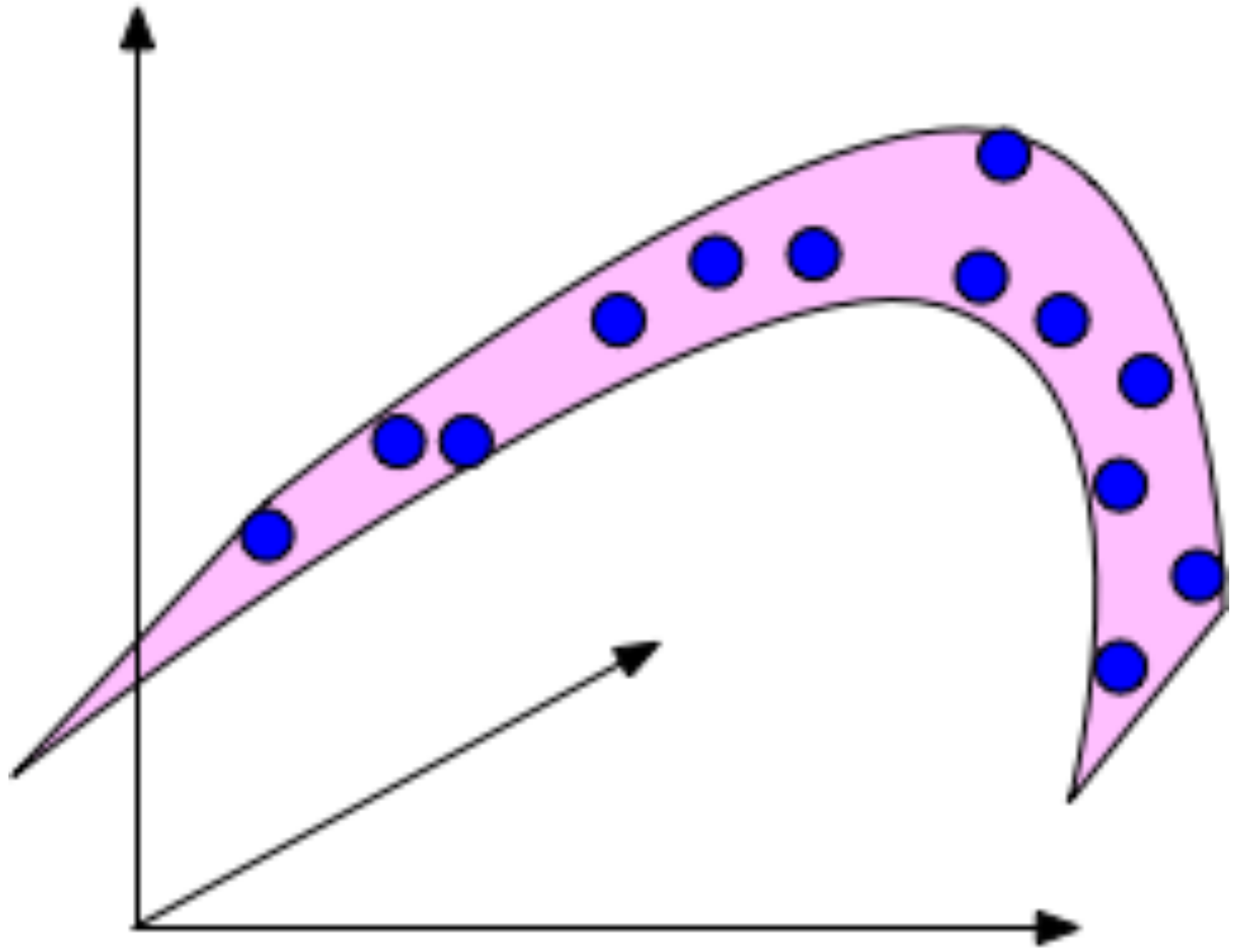

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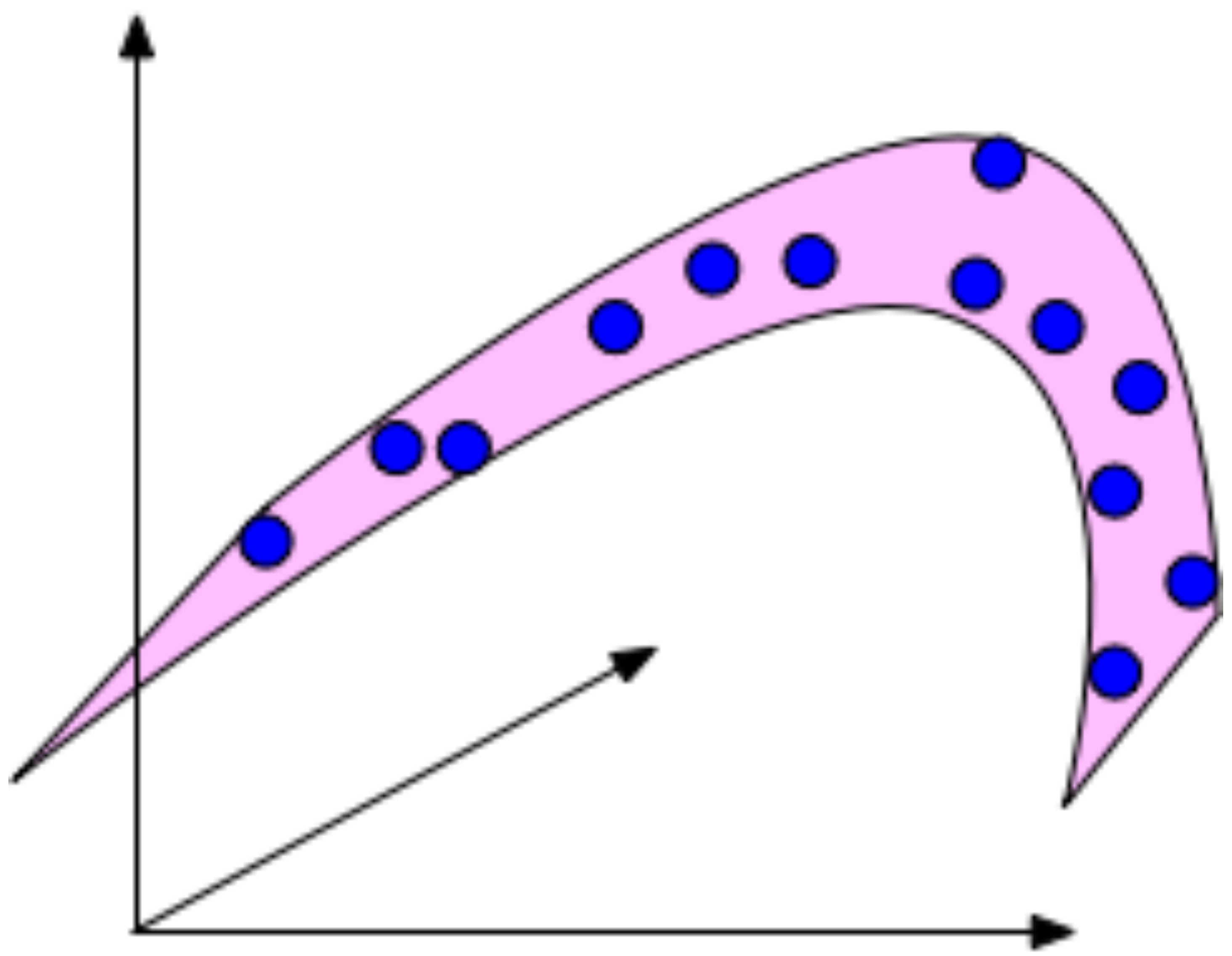

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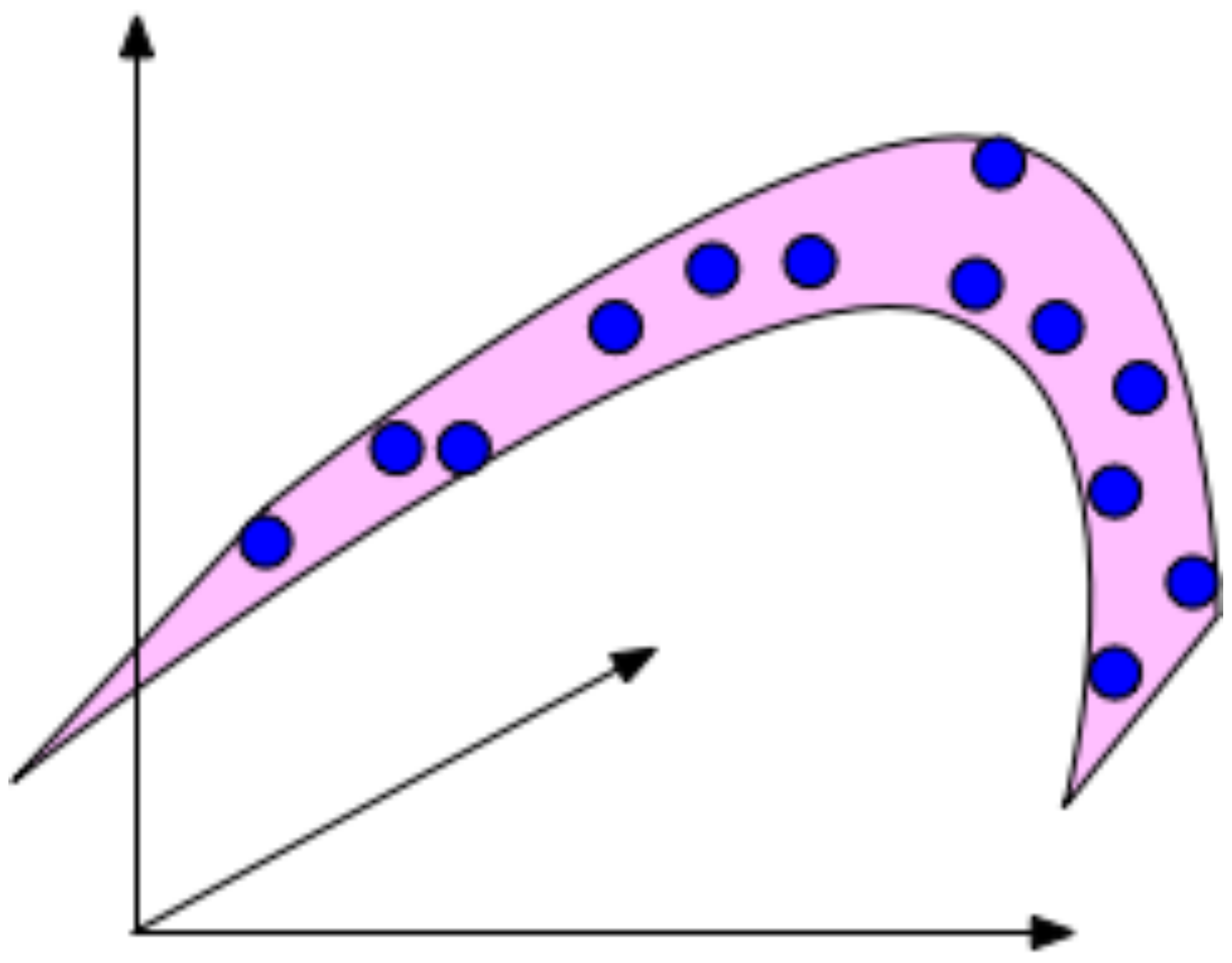

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Next week: we will see that these faces approximately live in 100-d space!

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2. Works well when data is low-dimensional (e.g., can compare against the Bayes optimal)

3. Suffer when data is high-dimensional, due to the fact that in high-dimension space, data tends to spread far away from each other