# Bias-Variance Tradeoff
# & Model Selection

# Announcements

HW5 and P5 are coming out

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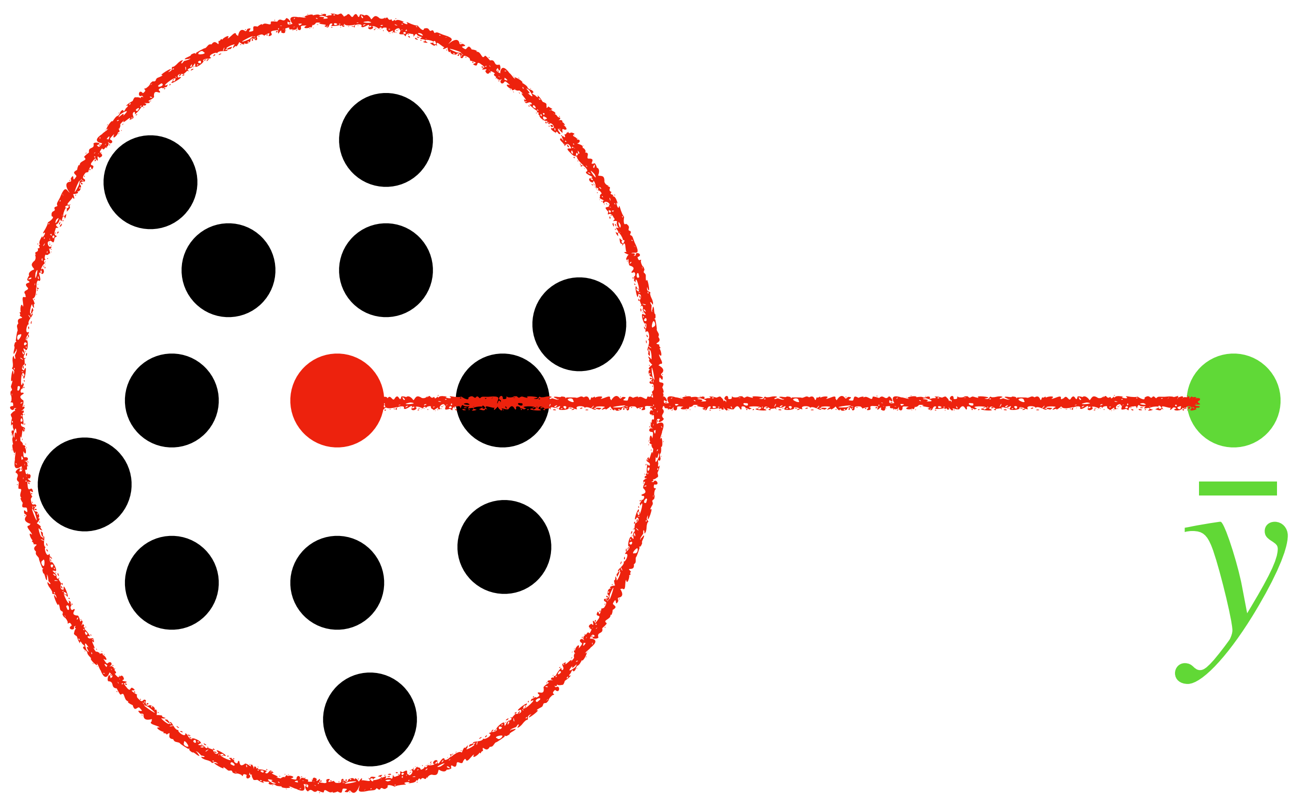

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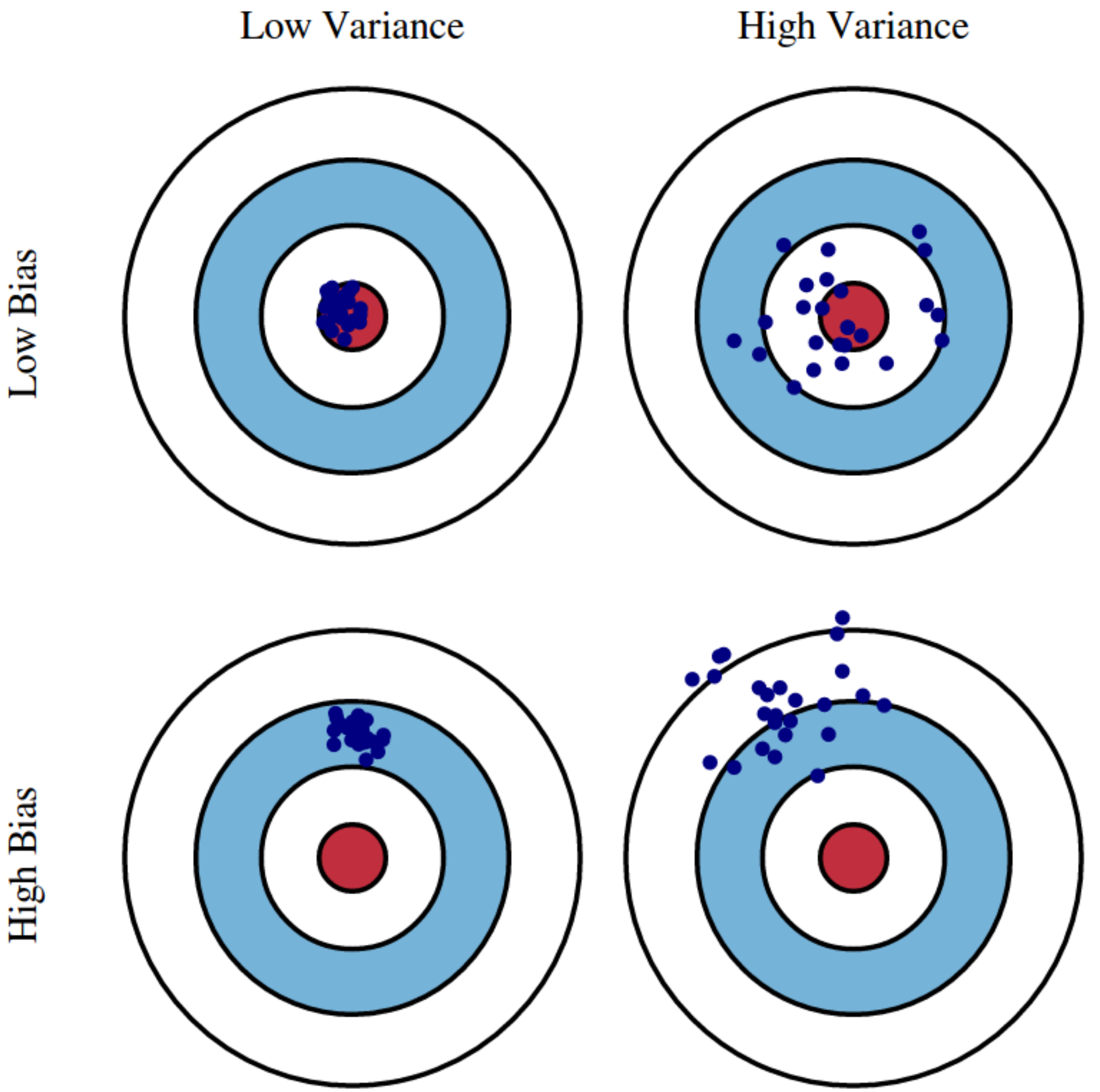

# Outline of Today

1. Bias & Variance tradeoff demo on Ridge Linear Regression

2. Derivation of Bias / Variance for Ridge LR

2. Model selection in practice (re-visit Cross Validation)

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(So the only randomness of our dataset $\mathcal{D} = \{x_i, y_i\}$ is coming from the noises $\epsilon_i$)

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(Q: think about the case where $\lambda \to \infty$, what happens to $\hat{w}$?)

# Ridge Linear regression

## Demonstration for 2d ridge linear regression

1. We create 5000 datasets: $\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_{5000}$,

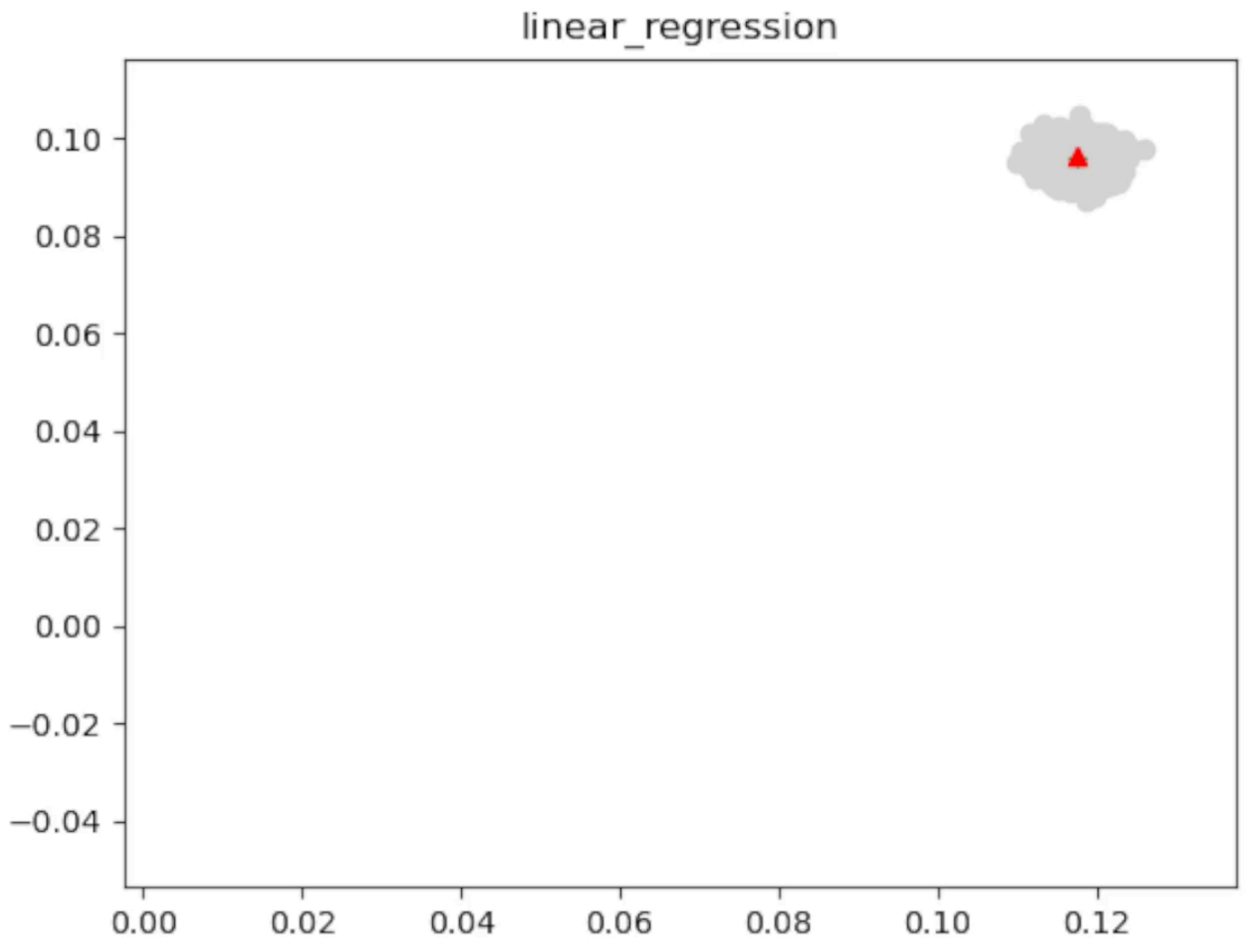
2. For a given $\lambda$, solve Ridge LR for each dataset, get $\hat{w}_1, \ldots, \hat{w}_{5000}$

3. Estimate the mean $\bar{w} = \sum_i \hat{w}_i / 5000$

4. Plot $\hat{w}_1, \ldots, \hat{w}_{5000}$, and mean $\bar{w}$, and the optimal $w^\star$

# Ridge Linear regression

We start with $\lambda = 0$, and gradually increase $\lambda$ to $+\infty$:



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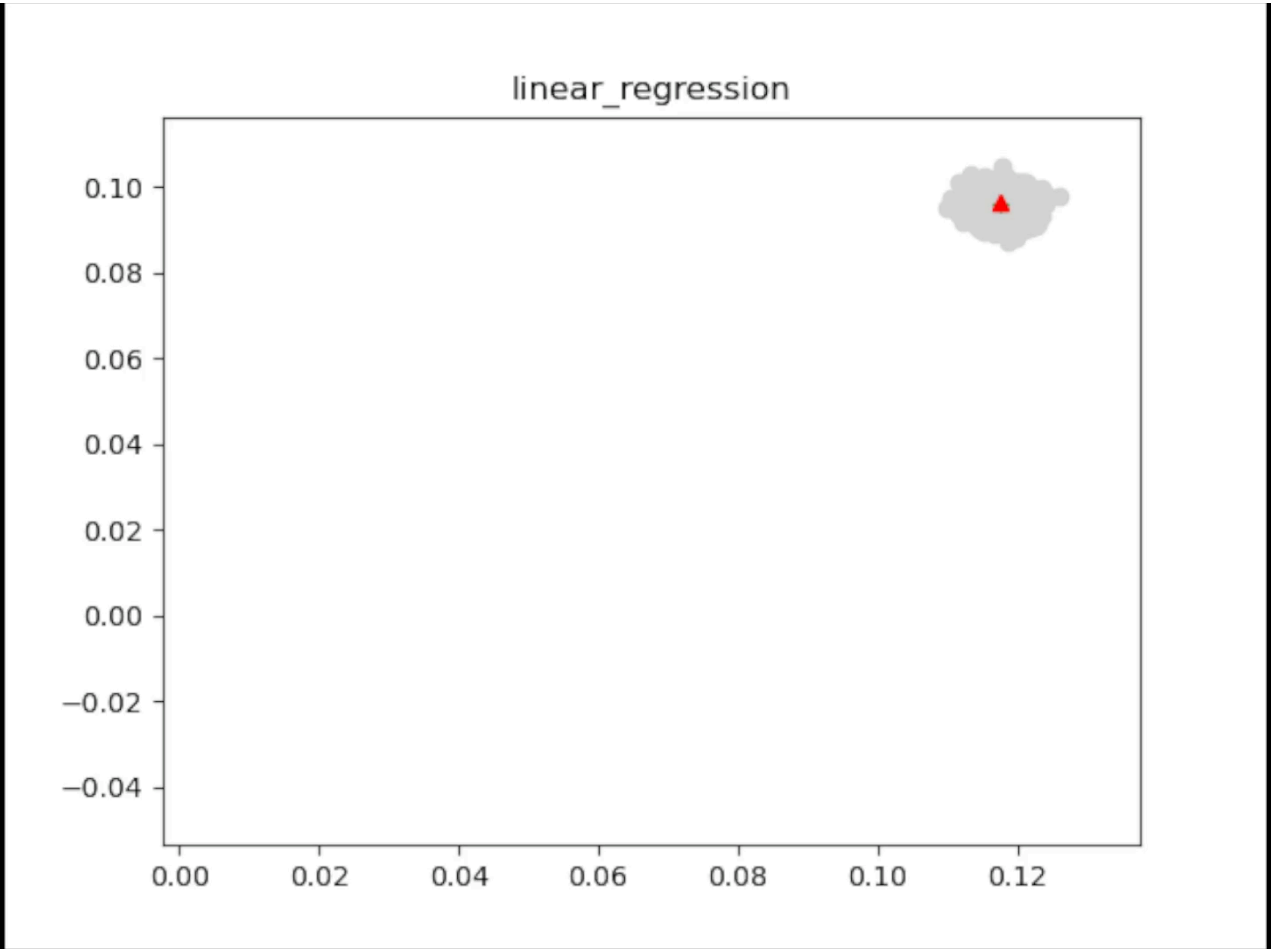

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# Derivation of Bias and Variance for Ridge Linear regression

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Since $y_i = (w^\star)^\top x_i + \epsilon_i$ we have $Y = X^\top w^\star + \epsilon$

# The Expectation of the Ridge LR solution

Recall we have closed form solution for Ridge LR

$$\hat{w} = (XX^\top + \lambda I)^{-1} XY \; = (XX^\top + \lambda I)^{-1} X(X^\top w^\star + \epsilon)$$

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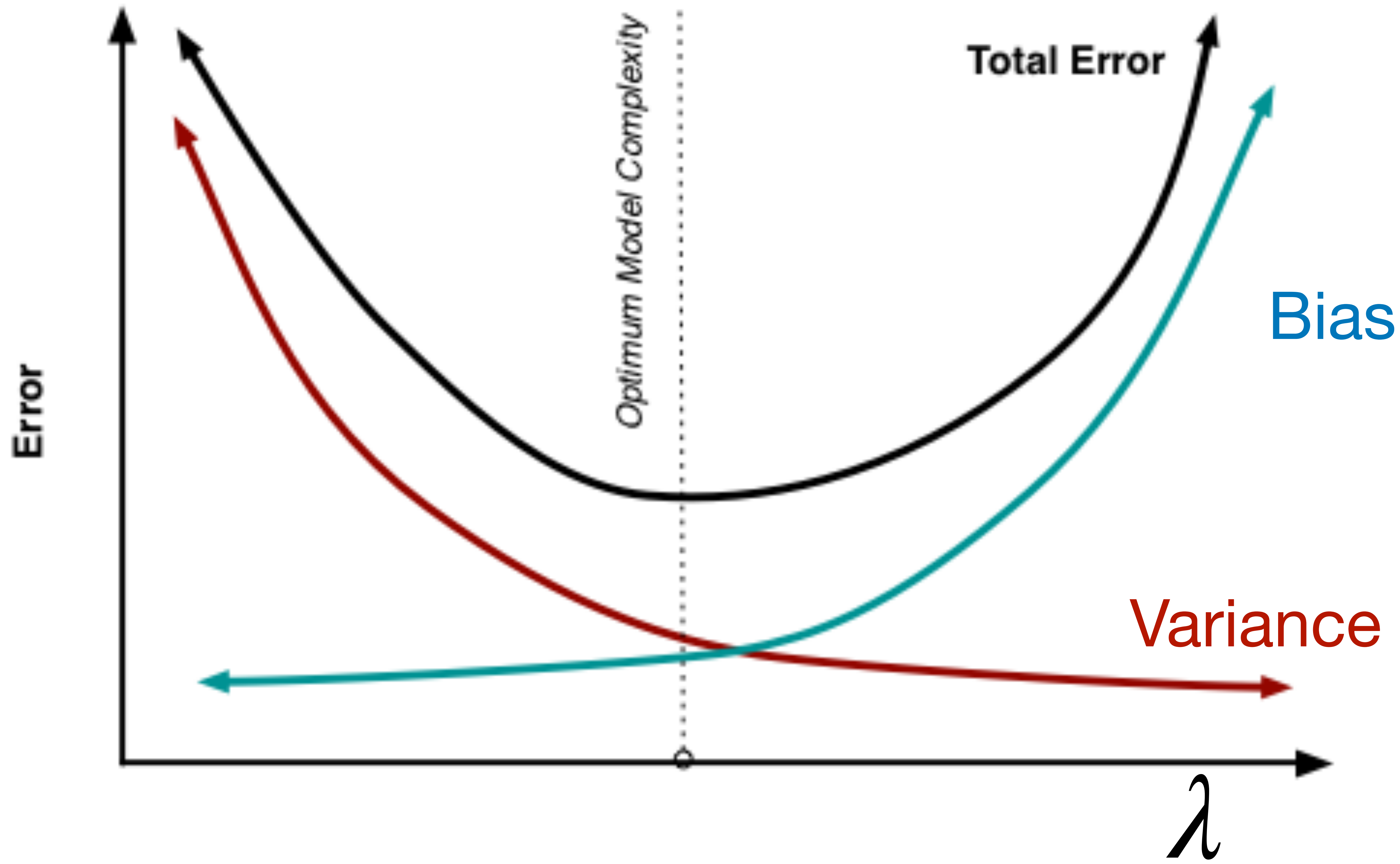
# Ridge Linear regression

Tuning $\lambda$ allows us to control the generalization error of Ridge LR solution:

$$\mathbb{E}(\hat{w}^\top x - y)^2 = \text{Variance} + \text{Bias} + \text{Inherent noise}$$



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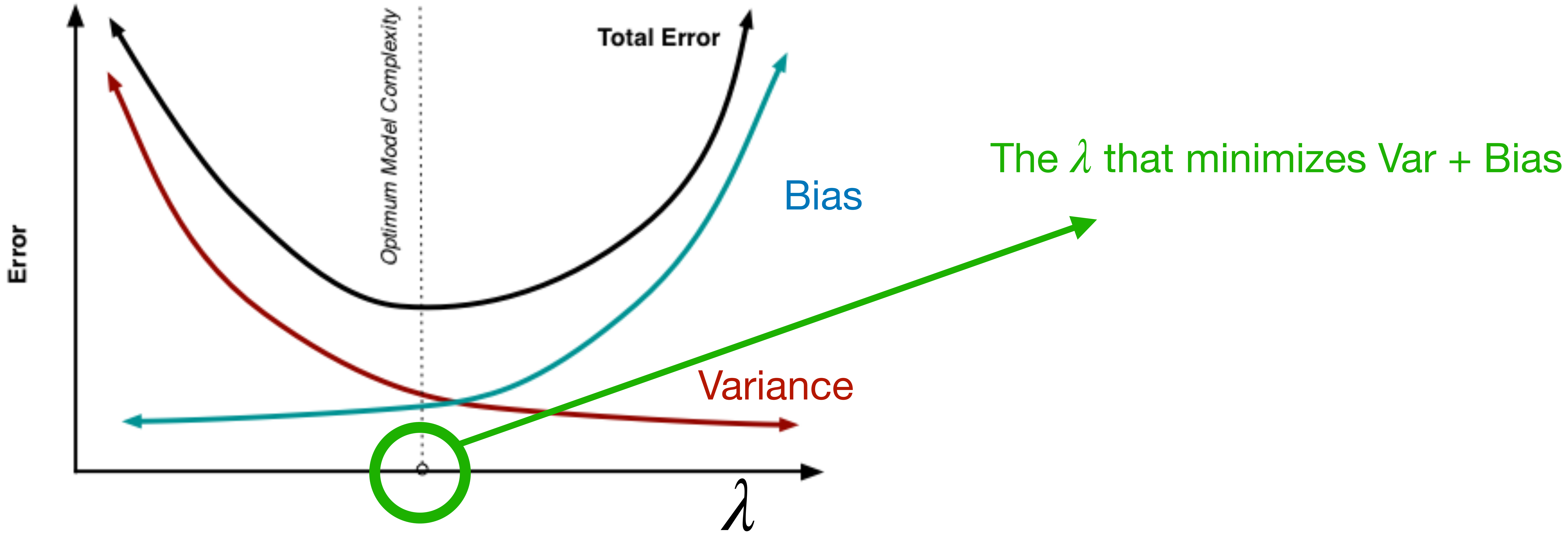

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Split the data into K folds

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Generalization error of Ridge LR w/ $\lambda$

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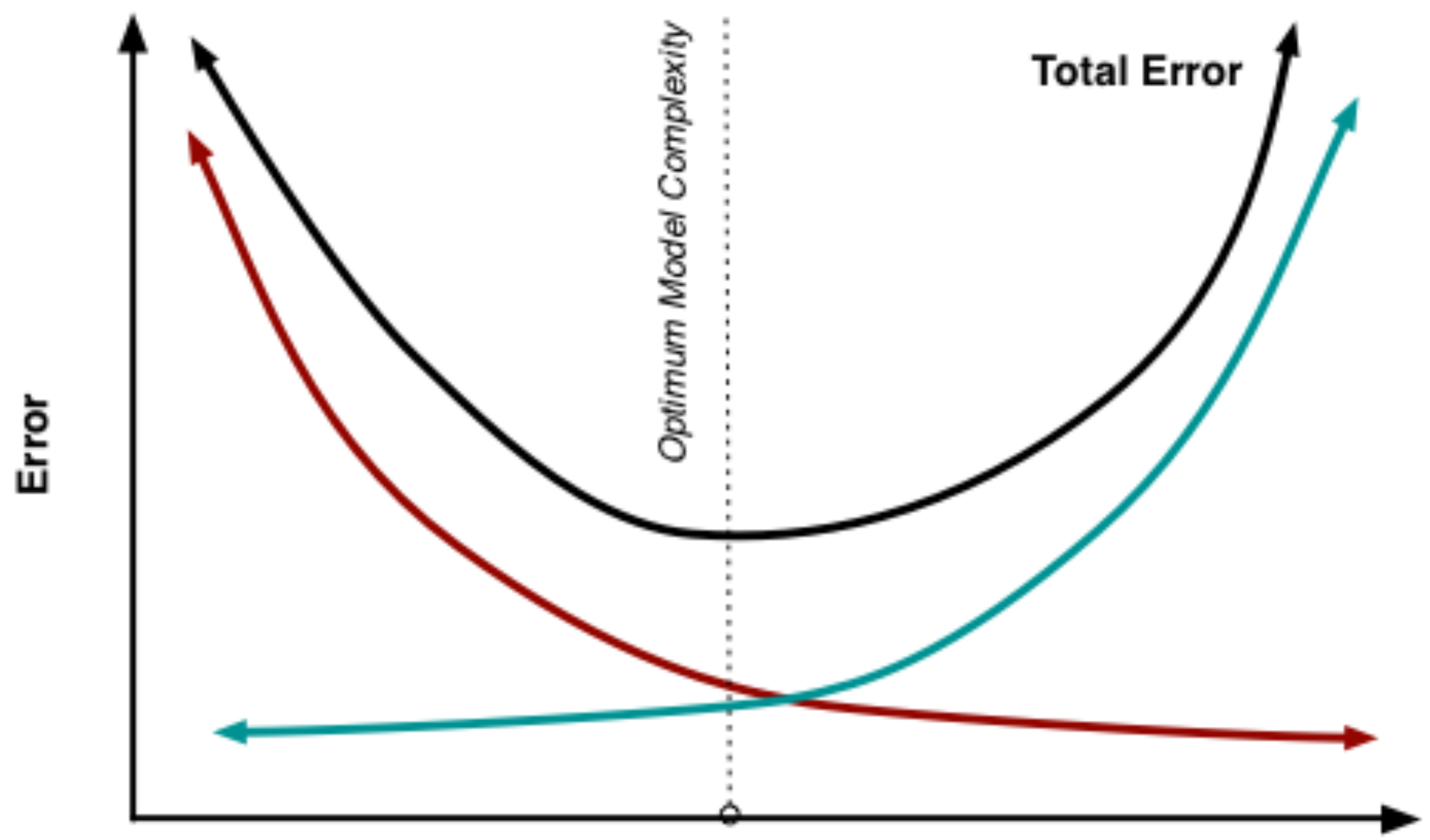

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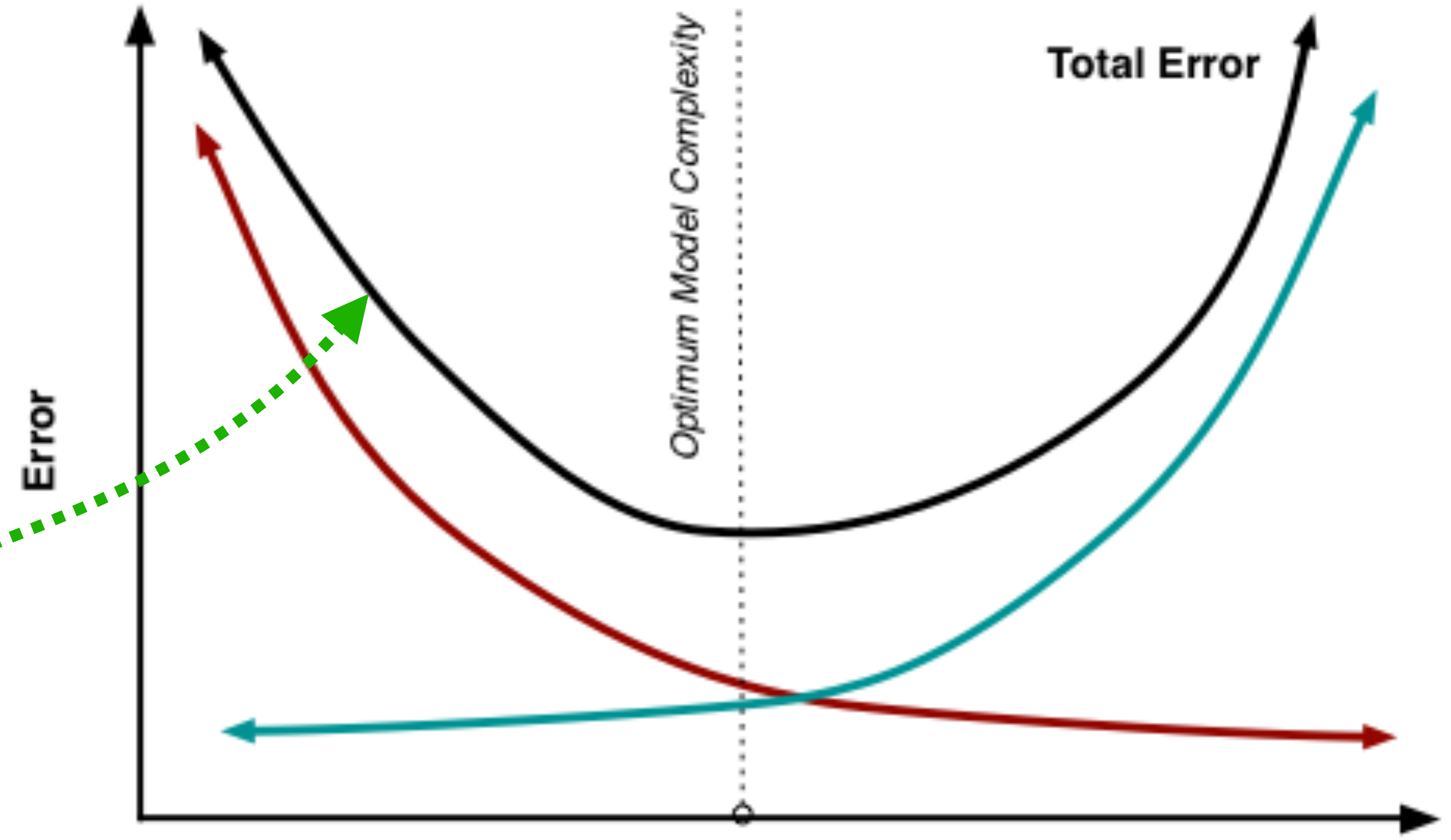
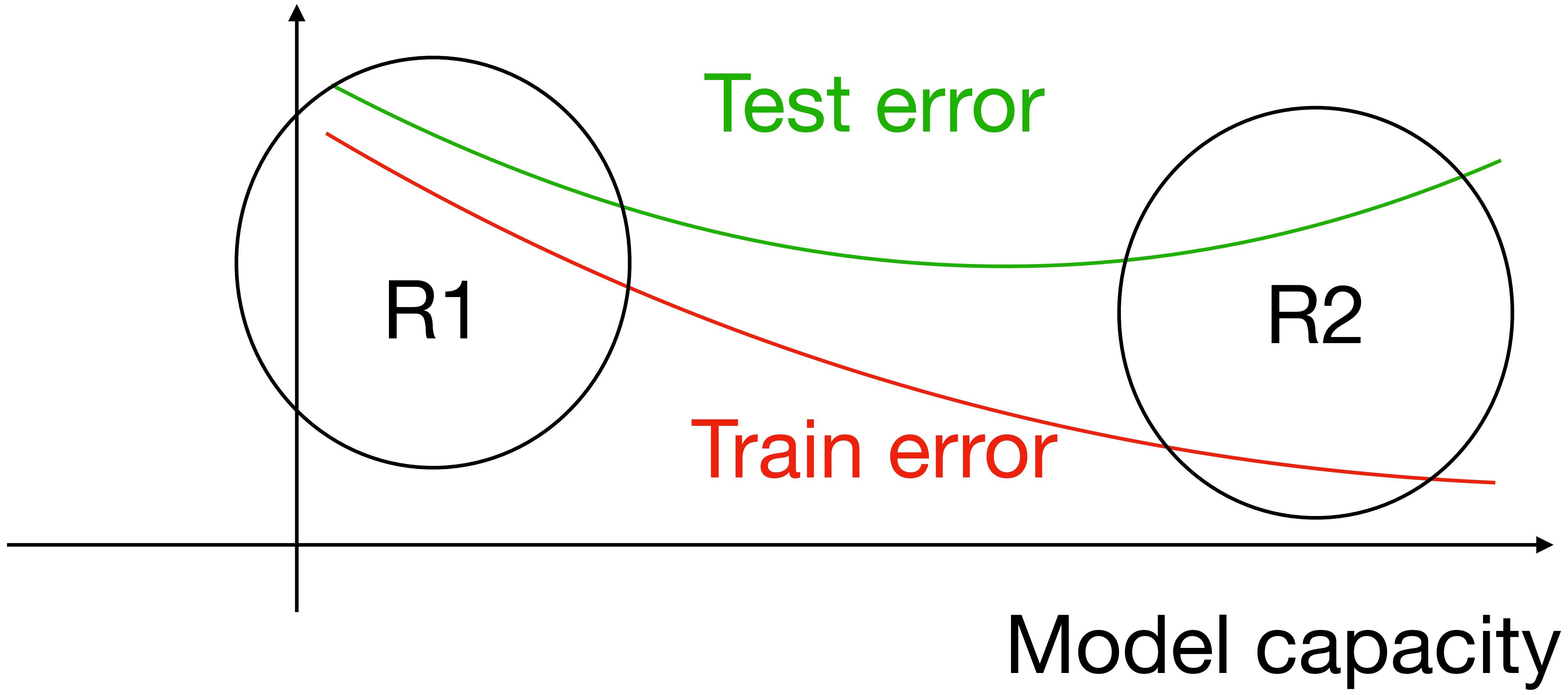


# Practical Suggestions for combating over/under fitting



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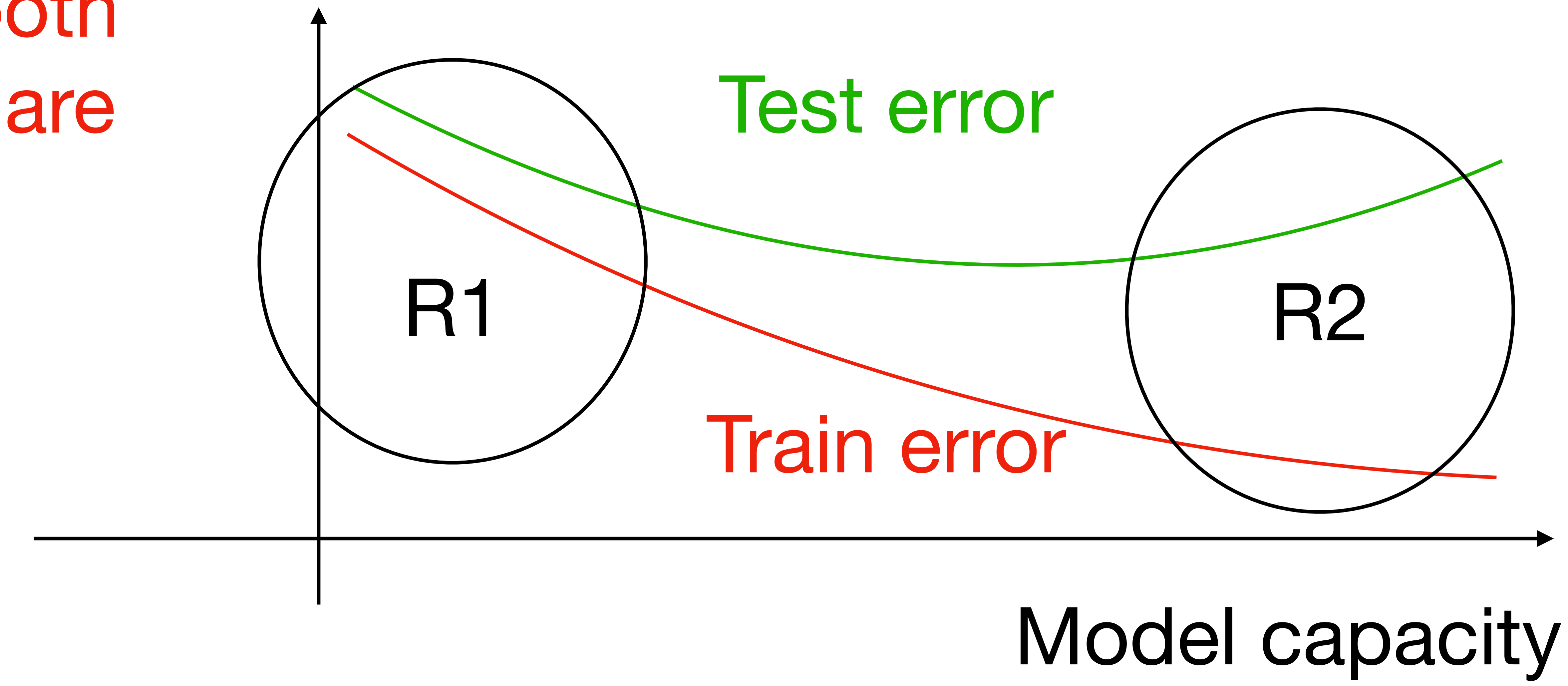
R1: Underfitting (both train and test errs are large)



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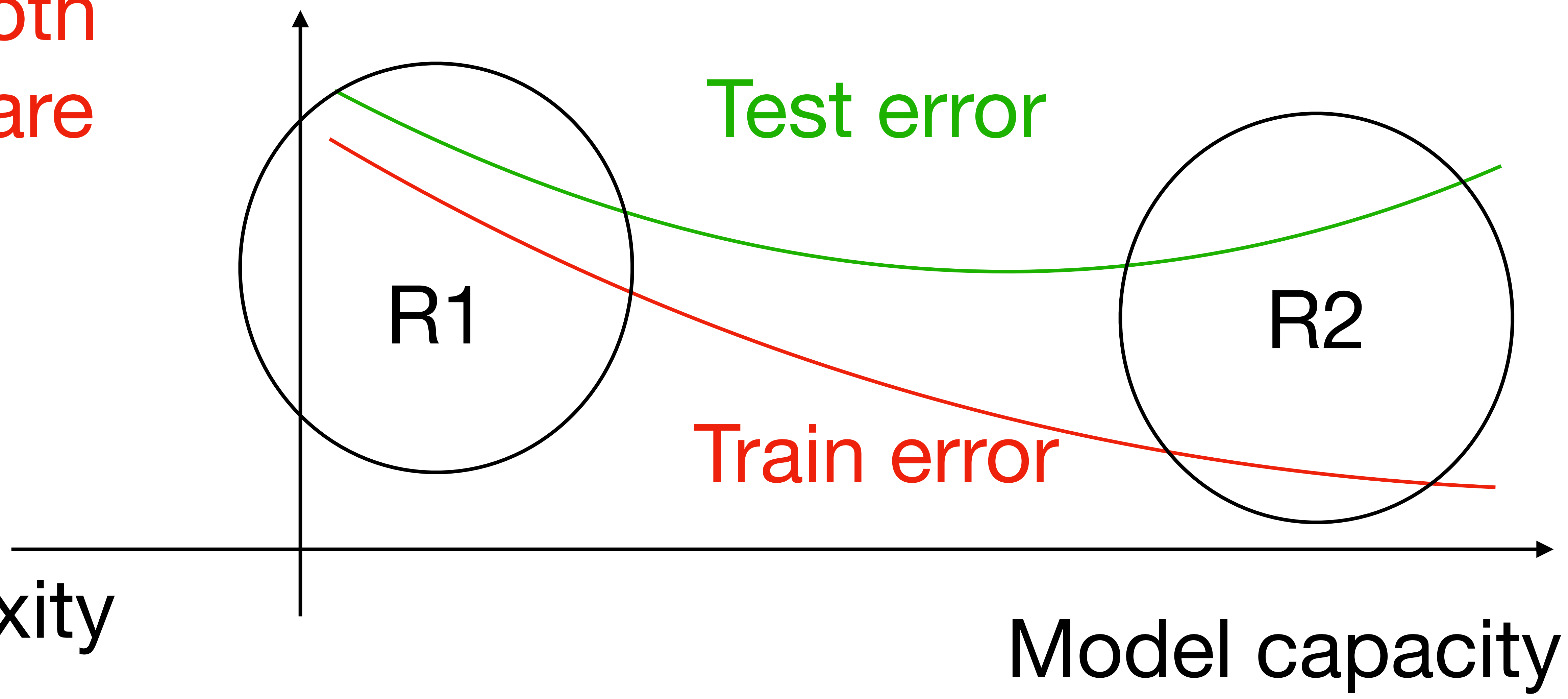

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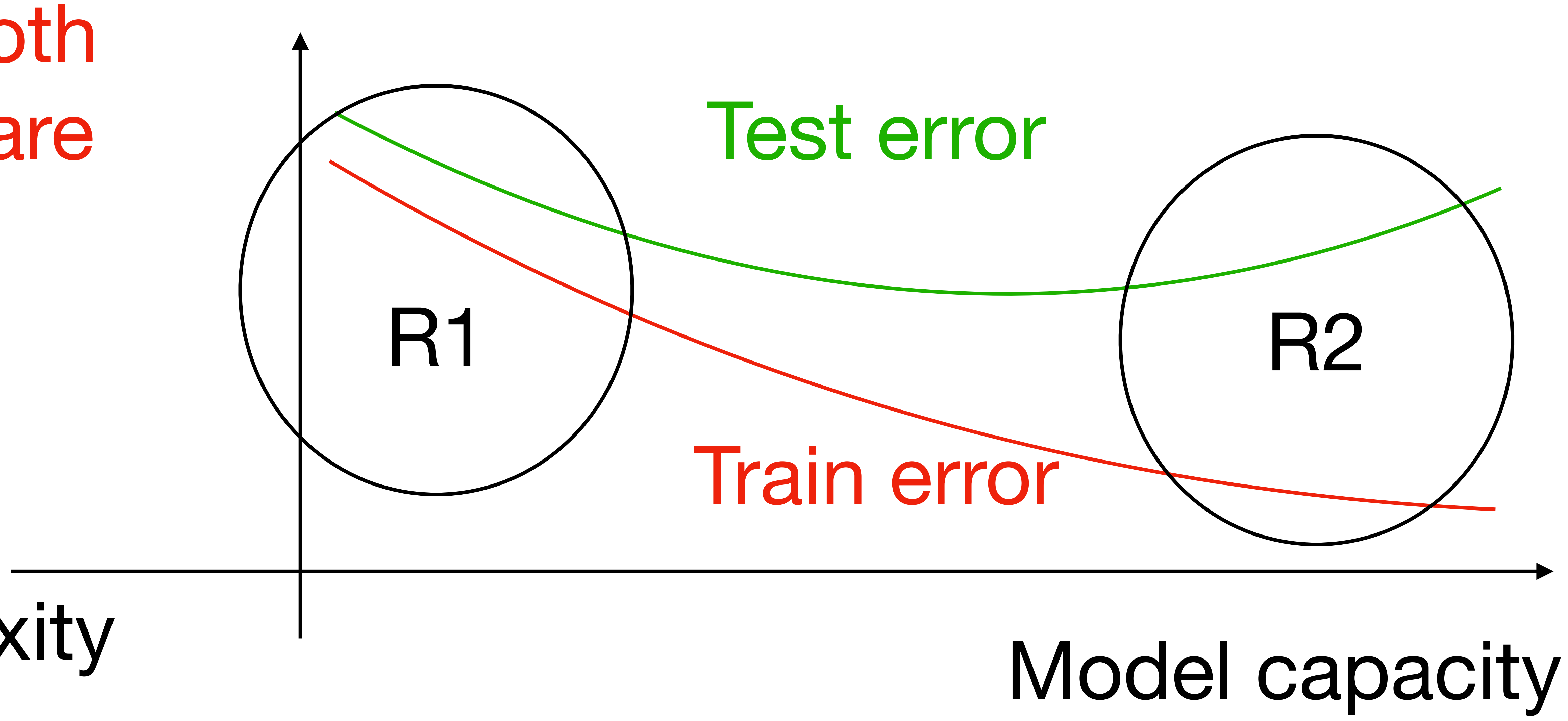

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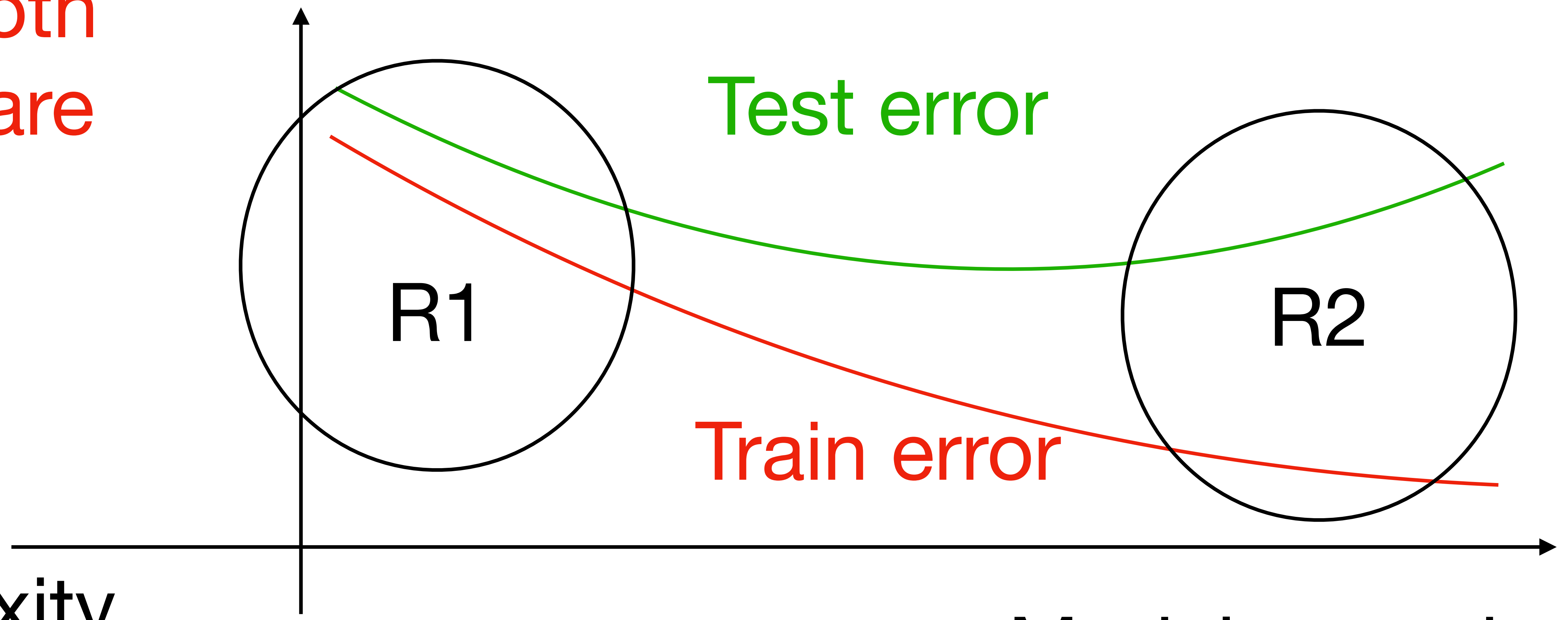
3. Using Boosting (we will see it later)



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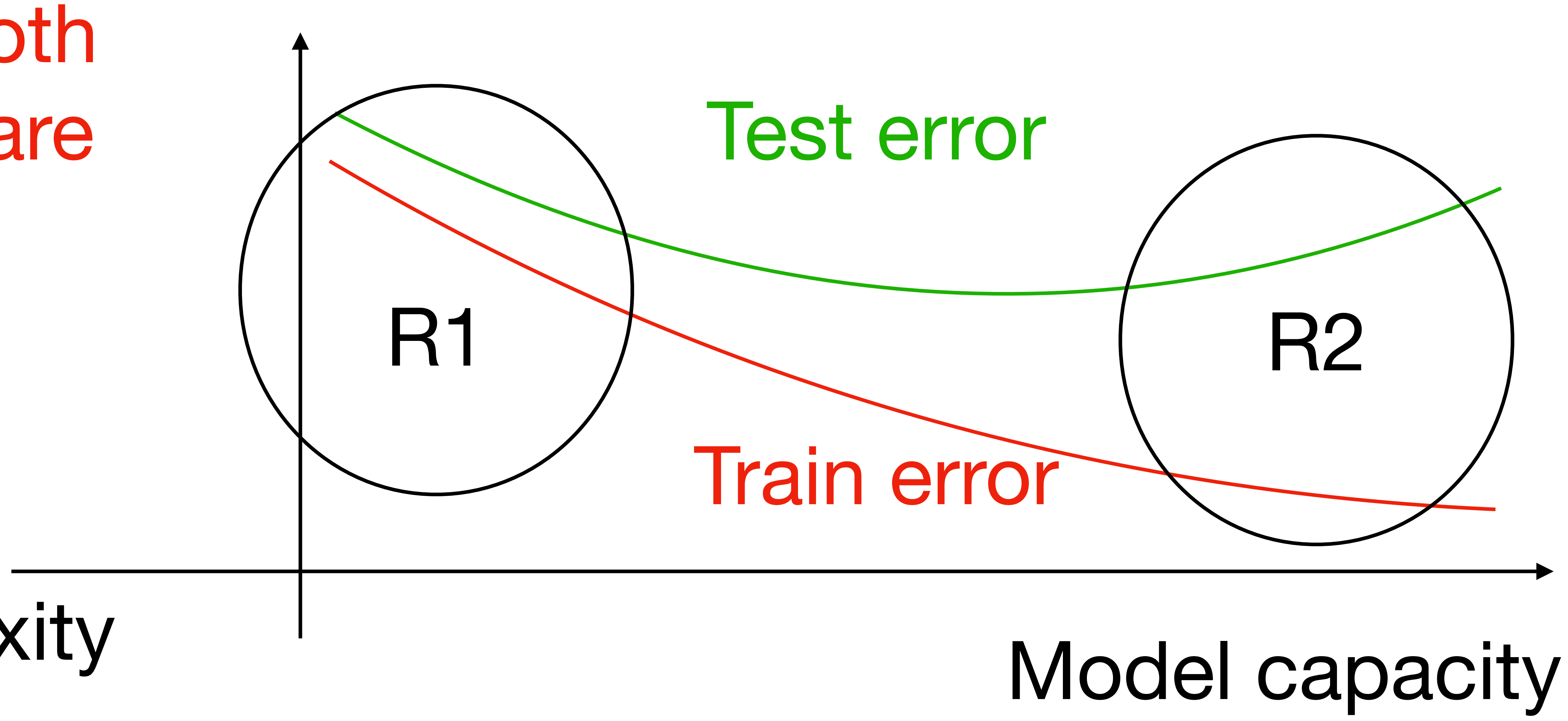


R2: overfitting (small train err but large test err)

Suggestions:

1. Increase complexity of models

2. More features

3. Using Boosting (we will see it later)

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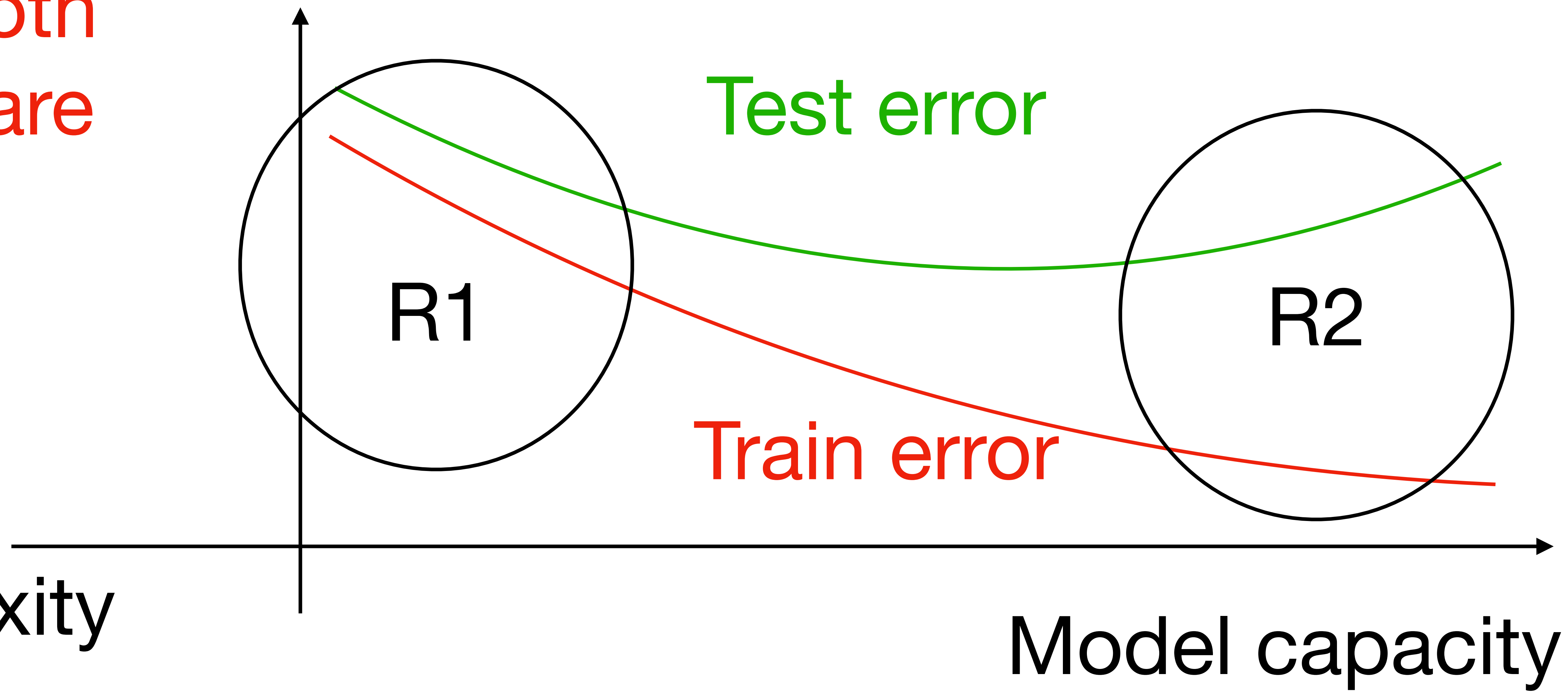


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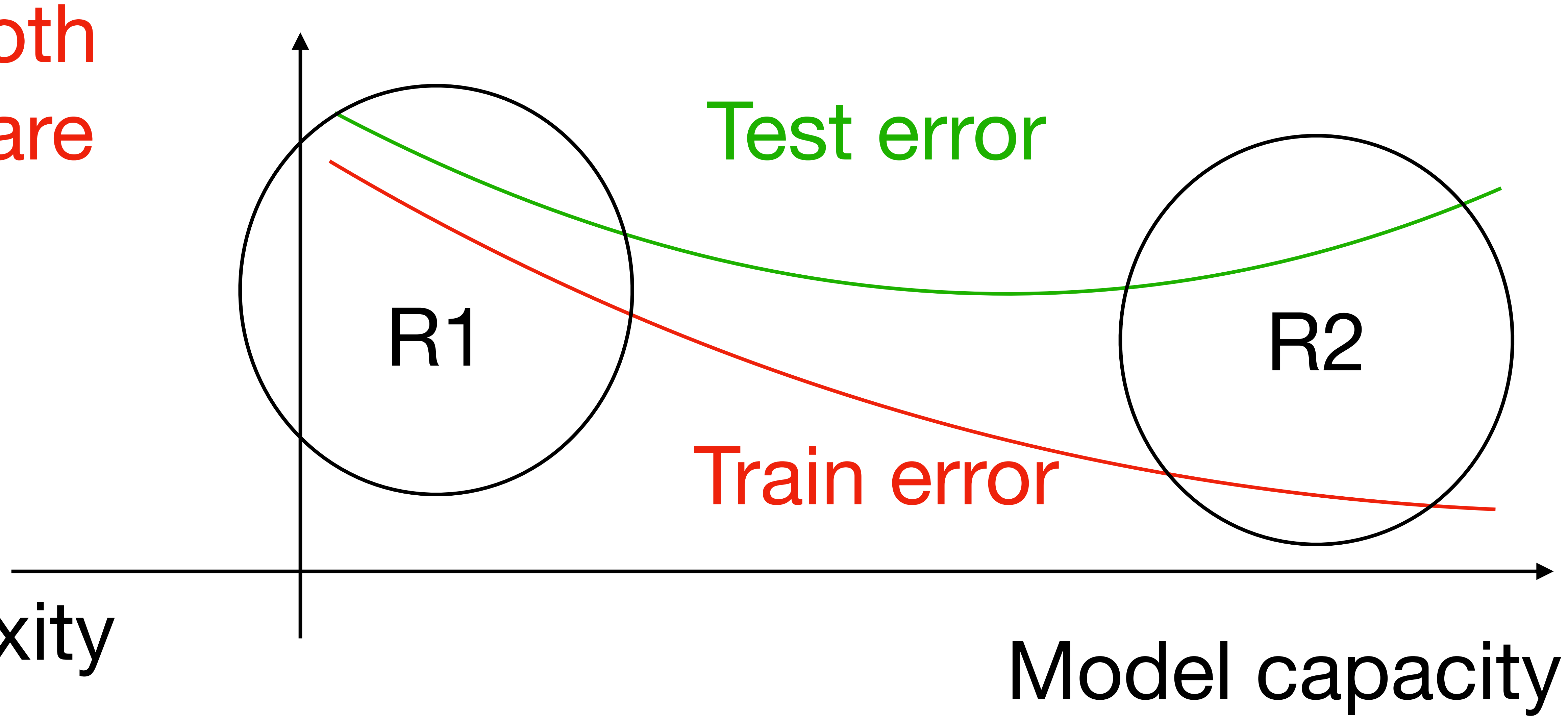


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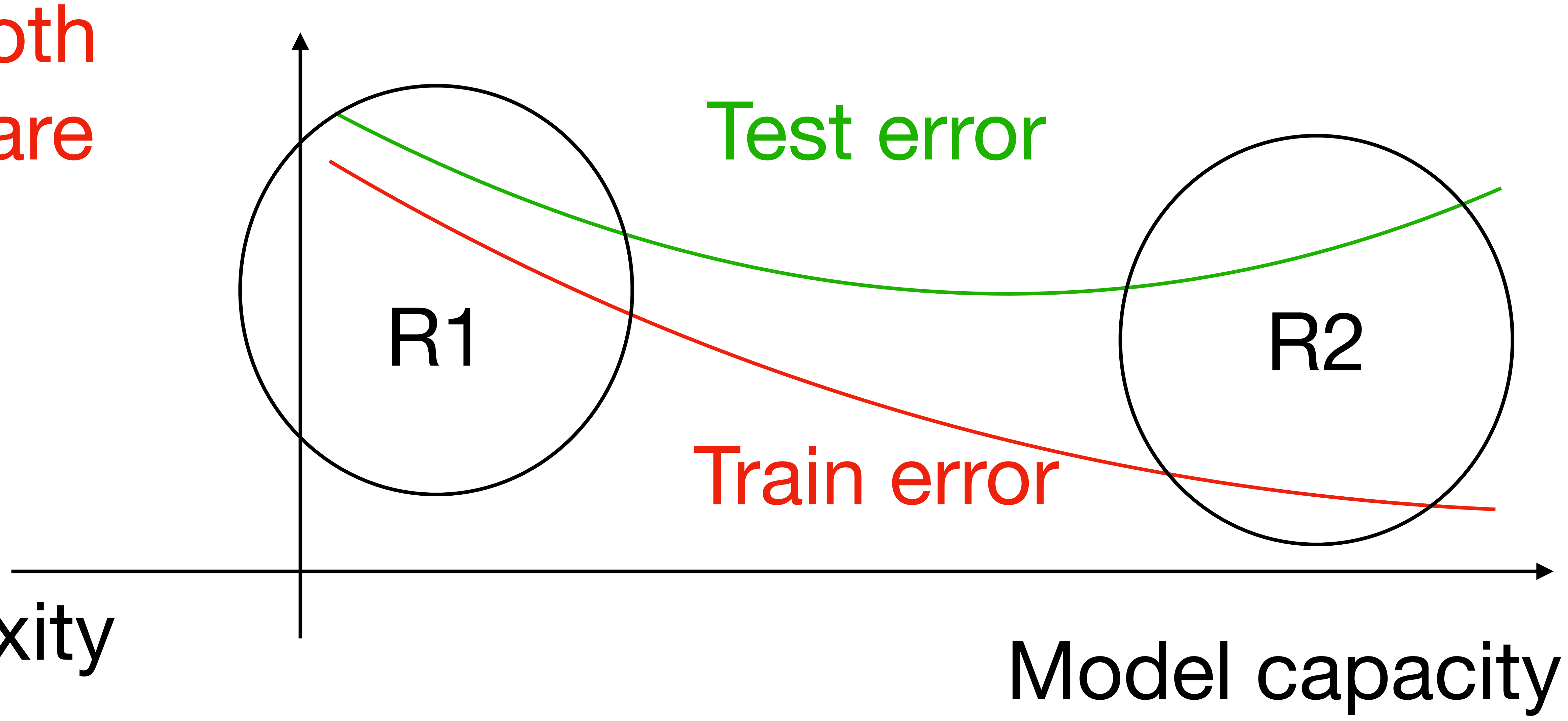


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