

## K-Nearest Neighbor (KNN)

- Given: Training data $\left(\left(\vec{x}_{1}, y_{1}\right), \ldots,\left(\overrightarrow{\mathrm{x}}_{\mathrm{n}}, y_{\mathrm{n}}\right)\right)$
- Attribute vectors: $\vec{x}_{i} \in X$
- Labels: $\quad y_{i} \in Y$
- Parameter:
- Similarity function: $K: X \times X \rightarrow \Re$
- Number of nearest neighbors to consider: $k$
- Prediction rule
- New example $x$,
- K-nearest neighbors: $k$ train examples with largest $K\left(\vec{x}_{i}, \vec{x}^{\prime}\right)$

Binary Classification Example

|  | correct <br> (complete, <br> partial, guessing) | color <br> (yes, no) | original <br> (yes, no) | presentation <br> (clear, unclear) | latex <br> (yes, no) | A+ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | complete | yes | yes | clear | no | yes |
| 2 | complete | no | yes | clear | no | yes |
| 3 | partial | yes | no | unclear | no | no |
| 4 | complete | yes | yes | clear | yes | yes |

Instance Space $\boldsymbol{X}$ : Set of all possible instances $x$ describable by attributes (often called features).
Target Attribute $Y$ : Label $y \in\{+1,-1\}$ (or yes/no, or $0 / 1$ ) for each instance.
Target Function $f$ : Function that assigns true label for each x ( $f$ is unknown).
Example ( $\boldsymbol{x}, \boldsymbol{y}$ ): Instance x with label $y=f(x)$.
Training Data S: Collection of examples observed by learning algorithm. -

## Supervised Learning

- Supervised Learning for Binary Classification:

Acquire an operational classification rule given positive and negative training examples.

Also called: concept learning.
Also called: concept learning....

## Learning a Binary Function

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- Learn (to imitate) a function $f: X \quad\{+1,-1\}$
- Learning algorithm is given the correct value of the function for particular inputs $\rightarrow$ training examples
- An example is a pair $(x, y)$, where x is the input and $y=f(x)$ is the output of the target function applied to $x$.

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$h: X\{+1,-1\}$
$f: X \quad\{+1,-1\}$

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\begin{aligned}
& \text { - Task: } \\
& \text { - Learn (to imitate) a } \\
& \text { - Training Examples: } \\
& \text { - Learning algorithm i } \\
& \text { for particular inputs } \\
& \text { - An example is a pair } \\
& \text { is the output of the } \\
& \text { - Goal: } \\
& \text { - Find a function } \\
& \text { that approximates } \\
& \text { as well as possible. }
\end{aligned}
$$ -

## Weighted K-Nearest Neighbor

- Given: Training datadata $\left(\left(\vec{x}_{1}, y_{1}\right), \ldots,\left(\vec{x}_{n}, y_{n}\right)\right)$
- Attribute vectors: $\vec{x}_{i} \in X$
- Target attribute: $\quad y_{i} \in Y$
- Parameter:
- Similarity function: $K: X \times X \rightarrow \Re$
- Number of nearest neighbors to consider: $k$
- Prediction rule
- New example $x$,
- K-nearest neighbors: k train examples with largest $K\left(\vec{x}_{i}, \vec{x}^{\prime}\right)$

```
h(\mp@subsup{\vec{x}}{}{\prime})=\operatorname{arg}\mp@subsup{\operatorname{max}}{y\inY}{{}{\mp@subsup{\sum}{i\in\operatorname{knn}(\mp@subsup{\vec{x}}{}{\prime})}{}\mp@subsup{1}{[\mp@subsup{y}{i}{}=y]}{}K(\mp@subsup{\vec{x}}{i}{},\mp@subsup{\vec{x}}{}{\prime})}
```

Types of Attributes

- Symbolic (nominal)
- EyeColor \{brown, blue, green\}
- Boolean
- Alive \{TRUE,FALSE\}
- Numeric
- Integer: age [0, 105]
- Real: height
- Structured
- Natural language sentence: parse tree
- Protein: sequence of amino acids


Supervised Learning

- Task:


## ?

- Learn (to imitate) a function $f: X \quad Y$
- Training Examples:
- Learning algorithm is given the correct value of the function for particular inputs $\rightarrow$ training examples
- An example is a pair $(x, f(x))$, where $x$ is the input and $f(x)$ is the output of the function applied to $x$.
- Goal:

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- Find a function
$h: X{ }^{\text {B }}$
that approximates
$f: X \quad Y$
as well as possible.

Weighted K-NN for Regression

- Given: Training data $\left(\left(\vec{x}_{1}, y_{1}\right), \ldots,\left(\vec{x}_{n}, y_{n}\right)\right)$
- Attribute vectors: $\vec{x}_{i} \in X$
- Target attribute: $\quad y_{i} \in \Re$
- Parameter:
- Similarity function: $K: X \times X \rightarrow$ R
- Number of nearest neighbors to consider: $k$
- Prediction rule
- New example $x$
- K-nearest neighbors: k train examples with largest $K\left(\vec{x}_{i}, \vec{x}^{\prime}\right)$


