

Modeling Sequence Data: Markov Models

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Thorsten Joachims
Cornell University

Reading:

Manning/Schuetze, Sections 9.1-9.3 (except 9.3.1)

Leeds Online HMM Tutorial (except Forward and Forward/Backward Algorithm)

(http://www.comp.leeds.ac.uk/roger/HiddenMarkovModels/html_dev/main.html)

“Less Naïve” Bayes Classifier

- Example: Classify sentences as insulting / not insulting

text	Insult?
$\bar{x}_1 = (Peter, is, nice, and, not, stupid)$	-1
$\bar{x}_2 = (Peter, is, not, nice, and, stupid)$	+1

- Assumption (I words in document)

- $P(X = x|Y = +1)$

- $= P(W_1 = w_1|Y = +1) \prod_{i=2}^l P(W_i = w_i|W_{i-1} = w_{i-1}, Y = +1)$

- $P(X = x|Y = -1)$

- $= P(W_1 = w_1|Y = -1) \prod_{i=2}^l P(W_i = w_i|W_{i-1} = w_{i-1}, Y = -1)$

- Decision Rule

$$h_{less}(x) = \operatorname{argmax}_{y \in \{+1, -1\}} \left\{ P(Y = y) P(W_1 = w_1|Y = y) \prod_{i=2}^l P(W_i = w_i|W_{i-1} = w_{i-1}, Y = y) \right\}$$

Markov Model

- Definition
 - Set of States: s_1, \dots, s_k
 - Start probabilities: $P(S_1=s)$
 - Transition probabilities: $P(S_i=s \mid S_{i-1}=s')$
- Random walk on graph
 - Start in state s with probability $P(S_1=s)$
 - Move to next state with probability $P(S_i=s \mid S_{i-1}=s')$
- Assumptions
 - Limited dependence: Next state depends only on previous state, but no other state (i.e. first order Markov model)
 - Stationary: $P(S_i=s \mid S_{i-1}=s')$ is the same for all i

Part-of-Speech Tagging Task

- Assign the correct part of speech (word class) to each word in a document

“The/DT planet/NN Jupiter/NNP and/CC its/PRP moons/NNS are/VBP in/IN effect/NN a/DT mini-solar/JJ system/NN ,/, and/CC Jupiter/NNP itself/PRP is/VBZ often/RB called/VBN a/DT star/NN that/IN never/RB caught/VBN fire/NN ./.”

- Needed as an initial processing step for a number of language technology applications
 - Information extraction
 - Answer extraction in QA
 - Base step in identifying syntactic phrases for IR systems
 - Critical for word-sense disambiguation (WordNet apps)
 - ...

Why is POS Tagging Hard?

- Ambiguity
 - He will **race**/VB the car.
 - When will the **race**/NN end?
 - I **bank**/VB at CFCU.
 - Go to the **bank**/NN!
- Average of ~2 parts of speech for each word
 - The number of tags used by different systems varies a lot. Some systems use < 20 tags, while others use > 400.

The POS Learning Problem

- Example

sentence	POS
$\bar{x}_1 = (I, bank, at, CFCU)$	$\bar{y}_1 = (PRP, V, PREP, N)$
$\bar{x}_2 = (Go, to, the, bank)$	$\bar{y}_2 = (V, PREP, DET, N)$

Hidden Markov Model for POS Tagging

- States
 - Think about as nodes of a graph
 - One for each POS tag
 - special start state (and maybe end state)
- Transitions
 - Think about as directed edges in a graph
 - Edges have transition probabilities
- Output
 - Each state also produces a word of the sequence
 - Sentence is generated by a walk through the graph

Hidden Markov Model

- States: $y \in \{s_1, \dots, s_k\}$
 - Outputs symbols: $x \in \{o_1, \dots, o_m\}$
 - Starting probability $P(Y_1 = y_1)$
 - Specifies where the sequence starts
 - Transition probability $P(Y_i = y_i \mid Y_{i-1} = y_{i-1})$
 - Probability that one states succeeds another
 - Output/Emission probability $P(X_i = x_i \mid Y_i = y_i)$
 - Probability that word is generated in this state
- => Every output+state sequence has a probability

$$\begin{aligned} P(x, y) &= P(x_1, \dots, x_l, y_1, \dots, y_l) \\ &= P(y_1)P(x_1|y_1) \prod_{i=2}^l P(x_i|y_i)P(y_i|y_{i-1}) \end{aligned}$$

Estimating the Probabilities

- Given: Fully observed data
 - Pairs of output sequence with their state sequence
- Estimating transition probabilities $P(Y_i | Y_{i-1})$

$$P(Y_i = a | Y_{i-1} = b) = \frac{\# \text{ of times state } a \text{ follows state } b}{\# \text{ of times state } b \text{ occurs}}$$

- Estimating emission probabilities $P(X_i | Y_i)$

$$P(X_i = a | Y_i = b) = \frac{\# \text{ of times output } a \text{ is observed in state } b}{\# \text{ of times state } b \text{ occurs}}$$

- Smoothing the estimates
 - Laplace smoothing -> uniform prior
 - See naïve Bayes for text classification
- Partially observed data
 - Expectation Maximization (EM)