CS 4758/6758: Robot Learning: Homework 5

Due: May 3, 2012 (In class)

1 Support Vector Machines(40 pts.)

1.1 RBF Kernel (20 pts)

Which of the following are valid kernels? If yes, use the given properties to prove that they are kernels.

(a) $K(x, z) = xz \exp(x + z)$
(b) $K(x, z) = \cos^2(x - z)$
(c) $K(x, z) = (x - z)^2$
(d) $K(x, z) = \exp(-||x - y||_2^2)$

Properties that can be used: Let $K_1(\mathbf{x}, \mathbf{z})$ and $K_2(\mathbf{x}, \mathbf{z})$ be kernels. Then all of the following are also kernels:

(a) $aK_1 + bK_2, \quad \forall a, b \in \mathbb{R}, a > 0, b > 0$
(b) $f(\mathbf{x})^T f(\mathbf{z}), \quad f : \mathbb{R}^n \to \mathbb{R}^m$
(c) $K_1 K_2$
(d) $e^{K_1(\mathbf{x},\mathbf{z})}$
(e) $K_1(x, z) = x^T z$ (linear kernel).

1.2 Kernels and SVMs (20 pts)

A robot has collected data with its sensor. We want to use the data to build a classifier. In order to make our classifer robust, we would like to find the max-margin hyperplane (the hyperplane that has the largest margin). Currently, the feature space is a one dimensional space $X \in R$. The desired classification output is $Y = \{+, -\}$, as shown in Figure 1. The training set contains three positive examples, $x_1 = 0$, $x_3 = 3$, and $x_4 = 4$, and one negative example $x_2 = 1$,

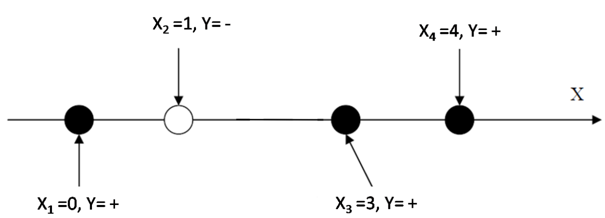

Figure 1: Training set of four examples with their desired classifications

Currently, the data points are unseparable, and so therefore we must define a transformation (i.e. a feature mapping function) that maps the data into a projected space in R^2. Consider the mapping $\Phi(X) = (X, (X - 1)^2)$.

(a) First, draw the data points after the transformation to the $2D$ domain. Draw the separation plane and indicate the margin. Also write which examples (out of x_1, x_2, x_3, x_4) are support vectors.

(b) If we get one more datapoint $x_5 = 2$, would it affect the margin? Justify. (Don't draw for this.)

2 Particle Filters (10 pts.)

Consider the following proposition:
There is no point in using a particle filter if the number of distinct states at any time is no more than the number of particles one plans to allocate to the particle filter.

Justify or refute this statement.

3 Particle Filters (50 pts.)

In this question, you will implement a particle filter for the non-linear system defined over three state variables, and given by a deterministic state transition:

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x + \cos\theta \\ y + \sin\theta \\ \theta \end{pmatrix}$$

The initial state estimate was:

$$\mu = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 10000 \end{pmatrix}$$

A. Give a suitable initial estimate for the particle prior, which reflects the state of knowledge in the gaussian prior.

B. Implement a particle filter and run its prediction step. Compare the resulting prior with the one from your intuitive analysis. What can be said about the resolution of the $x - y$ co-ordinates and the orientation θ in your particle filter?

C. Now let us add a measurement to our estimate. The measurement is a noisy projection of the x-coordinate of the robot, with covariance Q=0.01. Implement the step, compute the result and plot it. Compare with the result of your intuitive drawing.