CS 4758/6758 Robot Learning: Homework 4

Due April 12 at 5:00 PM

1 Kalman Filters: Multiple Sensors (30 points)

Suppose we have a continuous system with state $x \in \Re^N$. The state at time k is determined by the equation:

$$x_{k+1} = x_k + w_{k+1} \quad \text{where } w_{k+1} \sim N(0, Q)$$

Now we have two sensors z_k and y_k. One sensor gives data at $k = 1, 3, 5, \ldots$, and the other sensor gives data at $k = 2, 4, 6, \ldots$.

$$z_k = x_k + v_k \quad \text{where } v_k \sim N(0, R_1)$$

$$y_k = x_k + e_k \quad \text{where } e_k \sim N(0, R_2)$$

Derive the time and measurement update equations for this situation.

2 Kalman Filter Implementation (50 points)

You are designing a ground robot that will operate autonomously and use a Kalman filter for localization. The robot moves according to $x_k = Ax_{k-1} + w_k$, $w_k \sim \mathcal{N}(0, Q)$ and makes measurements $z_k = Hx_k + v_k$, $v_k \sim \mathcal{N}(0, R)$. You can choose between three chassis/dynamics and two sensors arrays but need to decide which pairing is best. Your options are:

Dynamics

$$(a): \quad A = 1.01 \begin{bmatrix} \cos .1 & -\sin .1 \\ \sin .1 & \cos .1 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$(b): \quad A = \begin{bmatrix} .99 & 0 \\ 0 & .99 \end{bmatrix} \quad Q = \begin{bmatrix} .4 & -.2 \\ -.2 & .4 \end{bmatrix}$$

$$(c): \quad A = \begin{bmatrix} 1.01 & 0 \\ 0 & 1.01 \end{bmatrix} \quad Q = \begin{bmatrix} .1 & -.05 \\ -.05 & .25 \end{bmatrix}$$

Sensors

$$(d): \quad H = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad R = \begin{bmatrix} .1 & 0 \\ 0 & .1 \end{bmatrix}$$

$$(e): \quad H = \begin{bmatrix} 1 & .5 \\ .5 & -1 \\ 1 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 0 & -.5 \\ 0 & 2 & .5 \\ -.5 & .5 & 1 \end{bmatrix}$$

(a) The expected squared error of the Kalman filter estimate $\hat{x}_k$ is given by $\mathbb{E}\left[(x_k - \hat{x}_k)^T (x_k - \hat{x}_k)\right]$. Express this quantity in terms of the estimate covariance P_k. (10 points)

(b) For each of the six combinations of dynamics and sensors, plot the expected squared error with respect to time. Assume you know the robot's starting position; that is, $P_0 = 0$. Attach the code you used for your calculations. (20 points)

(c) If you only plan on operating the robot for $t < 50$, which combination has the lowest error? What is its expected error at $t = 50$? (5 points)

(d) If you plan to operate the robot indefinitely, which combination is best? What is its expected error as $t \to \infty$? (5 points)

(e) If the robot instead starts from a completely unknown position, how do your answers to (c) and (d) change? Repeat them for this case. (10 points)

3 Potential Fields (12 points)

Each of the scenarios below represents a potential field.

The attractive component of the field points towards the circled dot, with magnitude:

$$k_{att}\left((x - x_{goal})^2 + (y - y_{goal})^2\right)$$

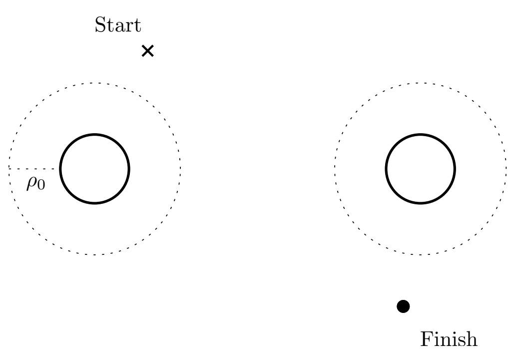
There are two repulsive components, each pointing away from one of the circular walls, with magnitude:

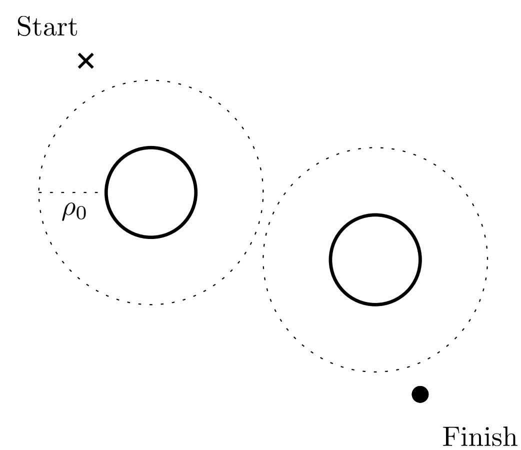
$$\begin{cases} k_{rep}\left(\frac{1}{\rho(x,y)} - \frac{1}{\rho_0}\right)^2 & \rho(x,y) \le \rho_0 \\ 0 & else \end{cases} \quad \text{(where } \rho(x,y) \text{ is the distance to the wall)}$$

Assume $k_{rep} >> k_{att}$.

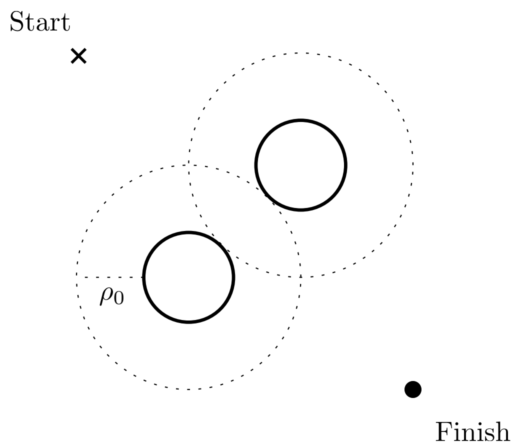
Questions:

(a) For each of the two cases below, draw the path taken from the starting position to the goal.





(b) In the scenario below, this method may not work. One way to make it work is to change ρ_0. Would you increase it or decrease it to make it work? Justify in a line.



(c) Is there some other potential you could add to make it work? Explain briefly.

4 Proportional Control (8 points)

Consider the linear system

$$x(t+1) = \begin{bmatrix} 10 & 1 \\ 1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & .1 \\ .1 & 1 \end{bmatrix} u(t)$$

where $u(t)$ is a control signal:

$$u(t) = \begin{bmatrix} K_{p1} & 0 \\ 0 & K_{p2} \end{bmatrix} (x_{desired} - x(t))$$

For each of the following cases, say whether or not the resulting system is stable. Show your work. (4 points each)

(a) $K_{p1} = 1$**,** $K_{p2} = 1$

(b) $K_{p1} = 10.5$**,** $K_{p2} = .5$