



# CS 4758/6758: Robot Learning

Spring 2010: Lecture 9

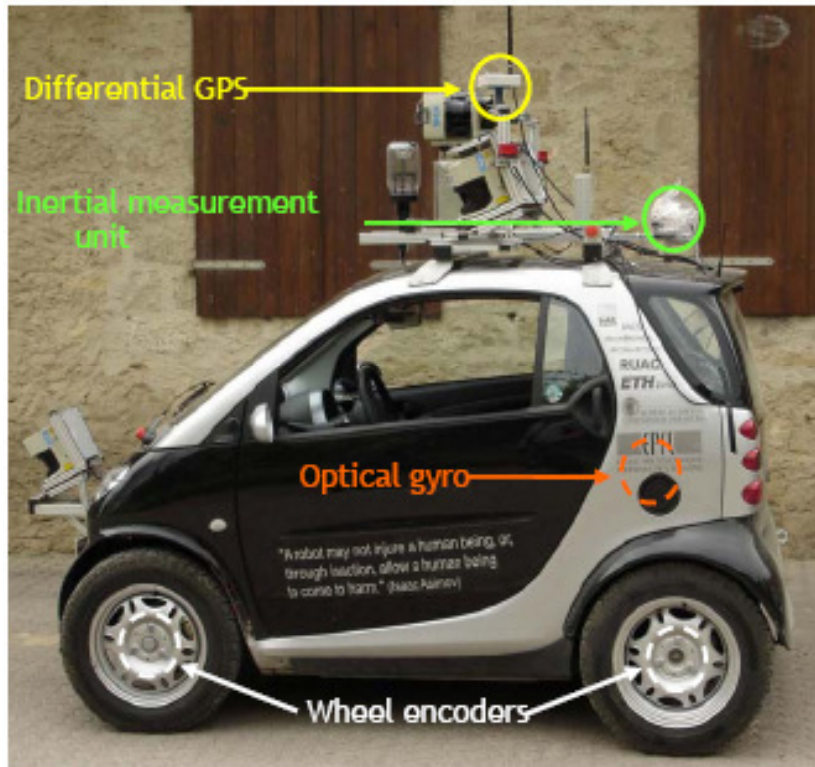
Ashutosh Saxena



# Why planning and control?

[Video](#)

# Typical Architecture



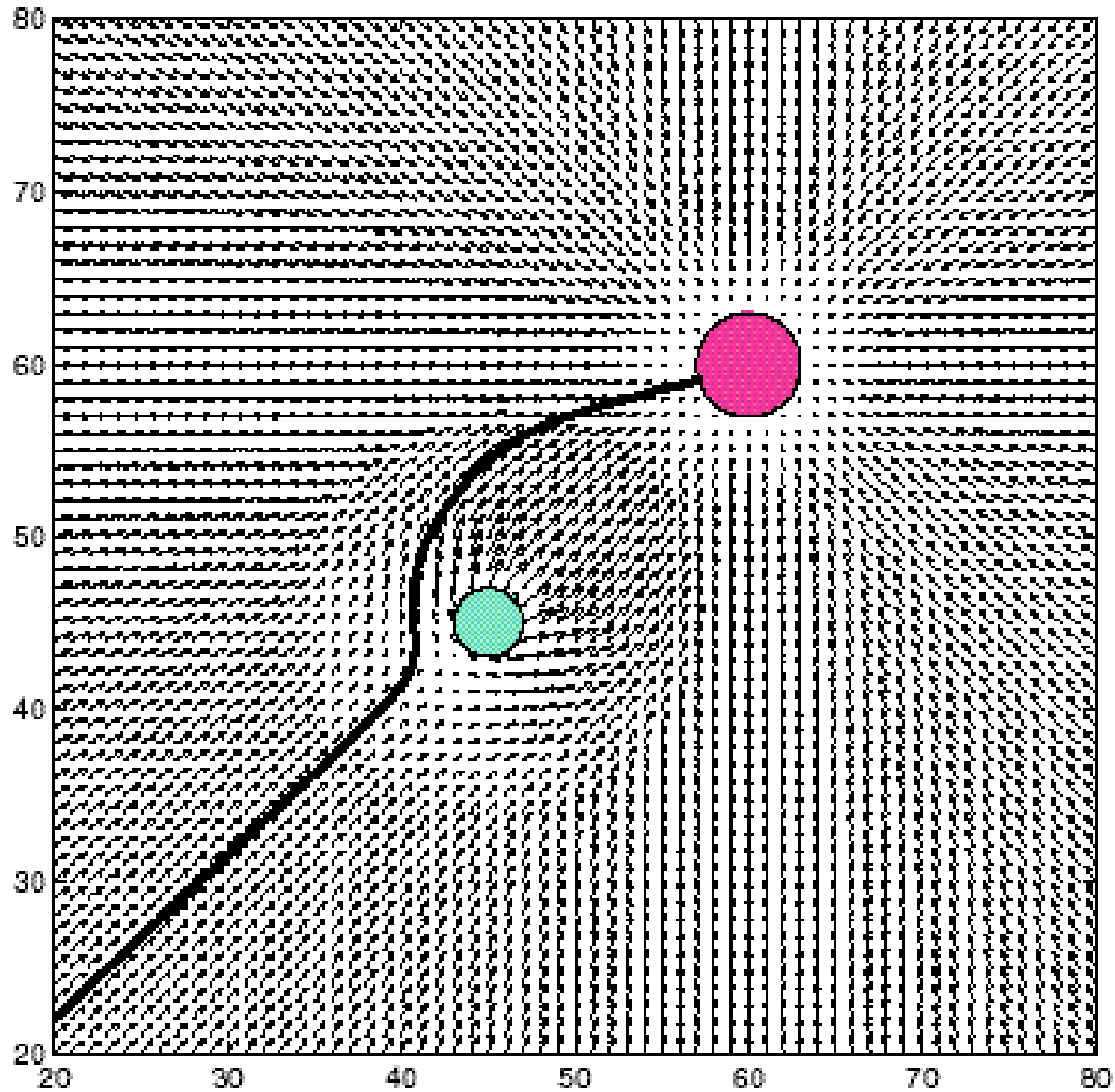
Planning  
0.1 Hz

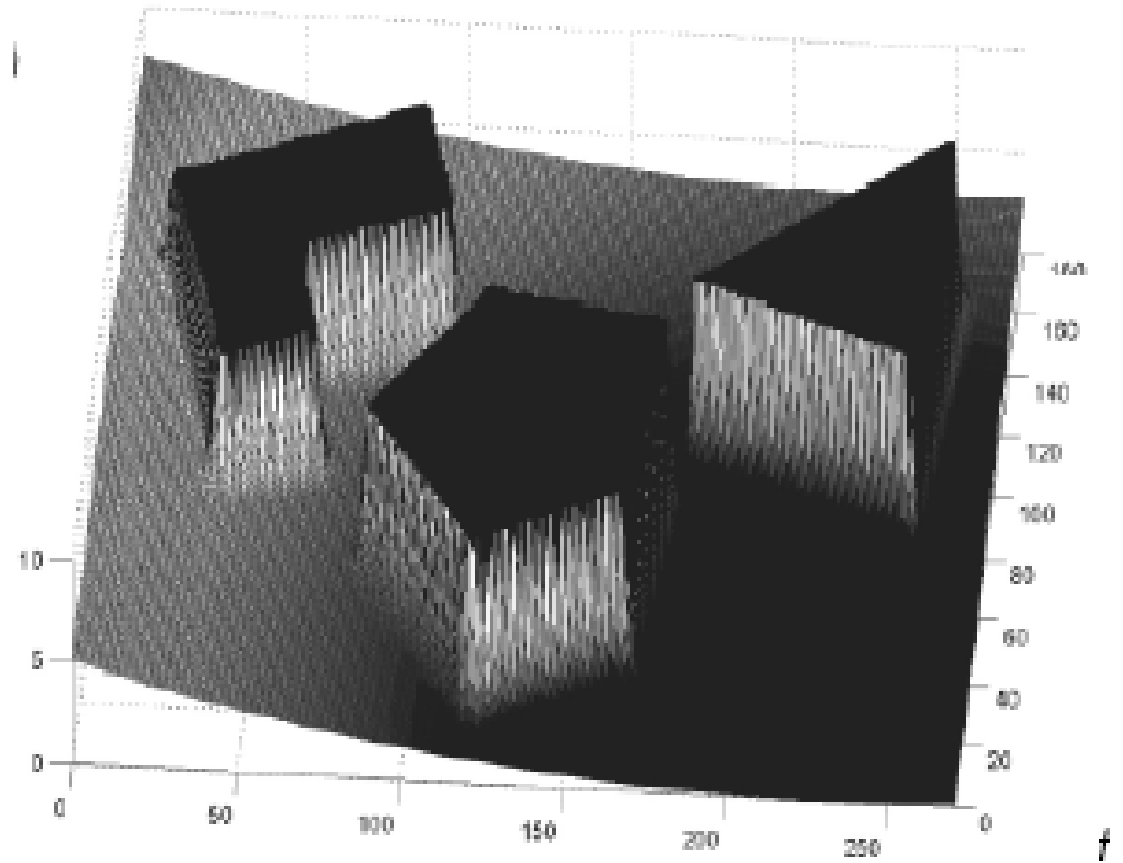
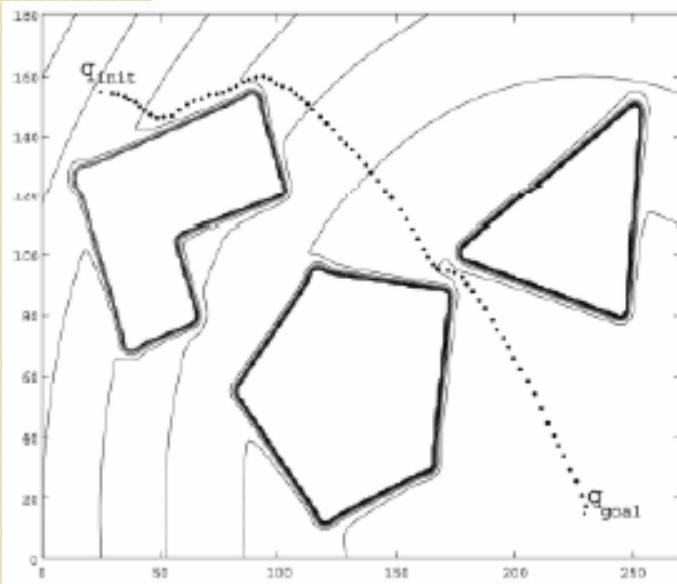
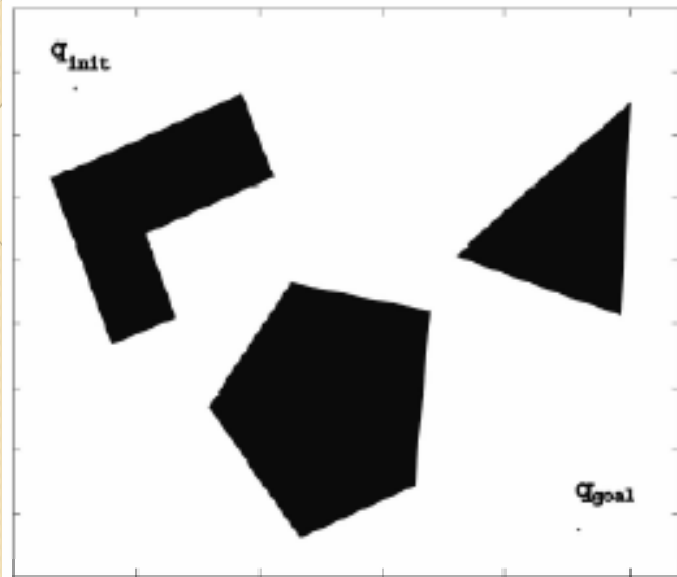


Control  
50 Hz

Does it apply to all robots and all scenarios?

# Previous Lecture: Potential Field

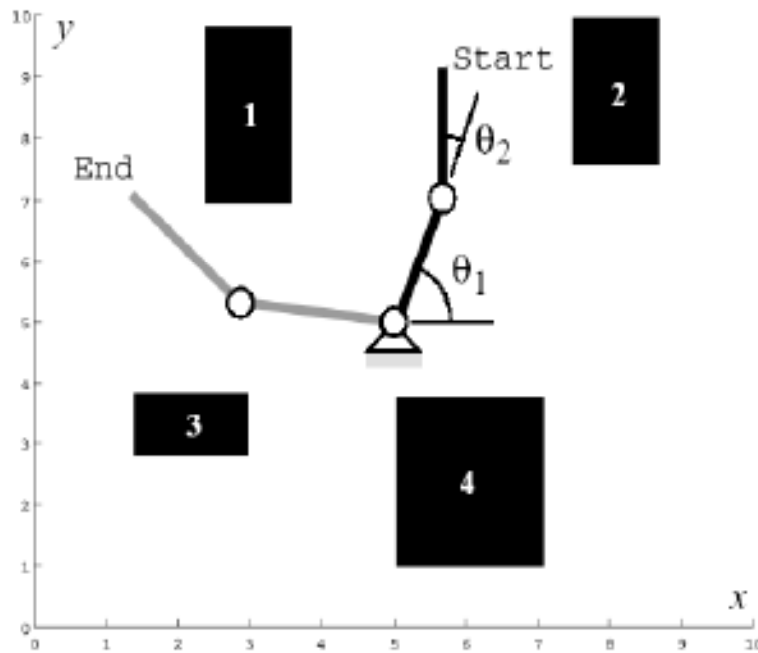




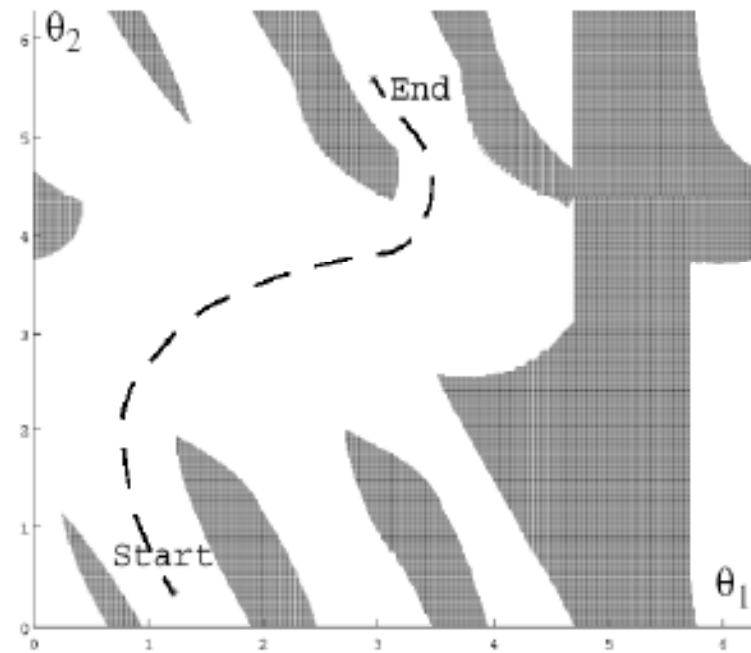
# Planning: Articulated Robots



# Path Planning



**Work Space**



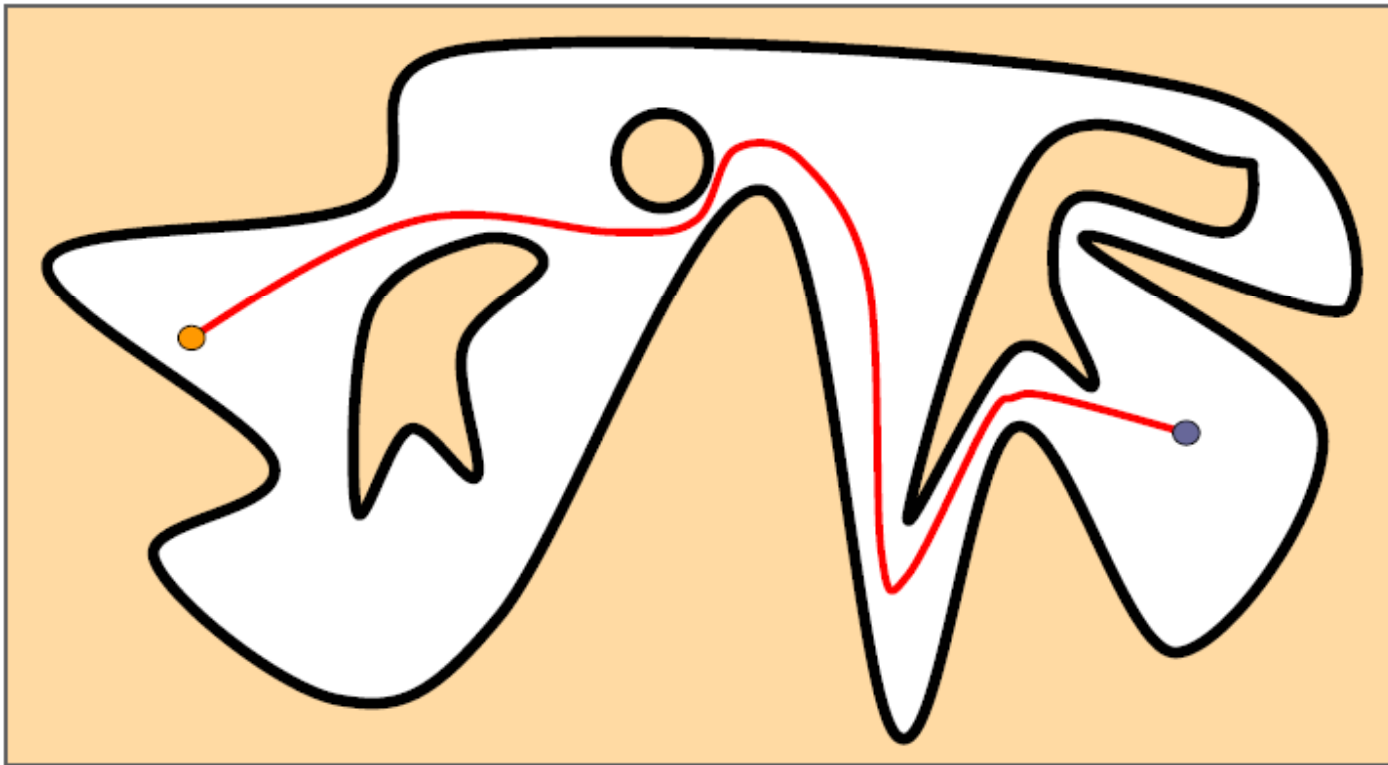
**Configuration Space:**  
the dimension of this space is equal to  
the Degrees of Freedom (DoF) of  
the robot

# Planning: Goals

- **Compute** motion strategies, e.g.,
  - **Geometric paths**
  - **Time-parameterized trajectories**
  - **Sequence of sensor-based motion commands**
- **Achieve** high-level goals, e.g.,
  - **Go to the door and do not collide with obstacles**
  - **Assemble/disassemble the IKEA bookshelf.**



# Fundamental Question

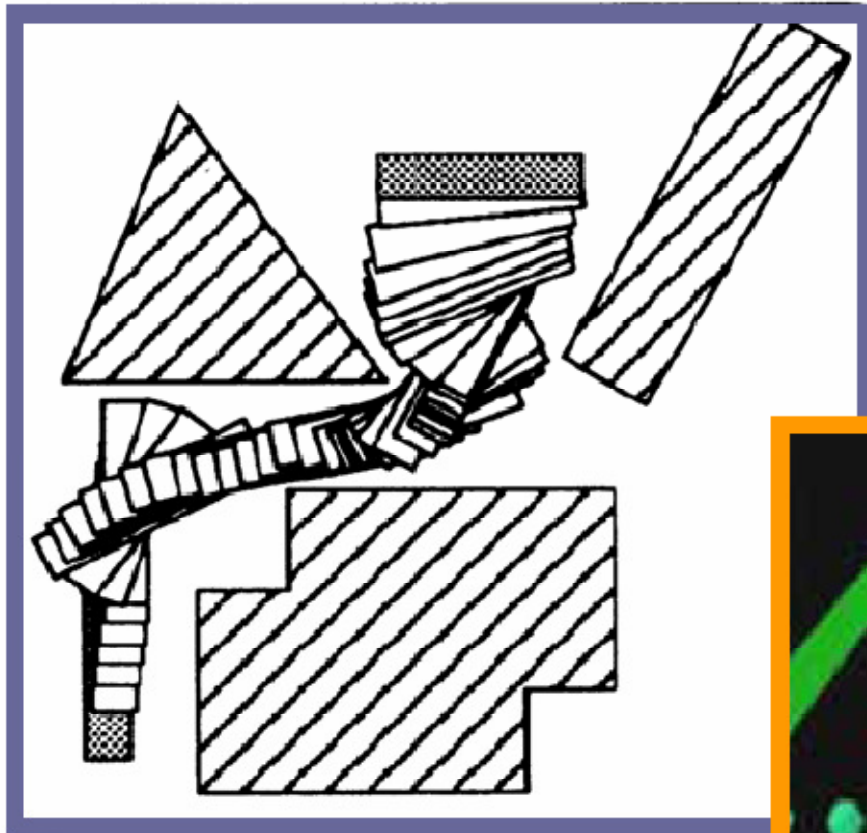


Are two given points connected by a path?

# Motion Planning: Basic Problem

- Problem statement:  
**Compute a collision-free path for a rigid or articulated moving object among static obstacles.**
- Input
  - Geometry of a moving object, robot, and obstacles
  - How does the robot move?
  - Kinematics of the robot (degrees of freedom)
  - Initial and goal robot configurations (positions & orientations)
- Output  
Continuous sequence of collision-free robot configurations connecting the initial and goal configurations

# Example: Rigid Objects

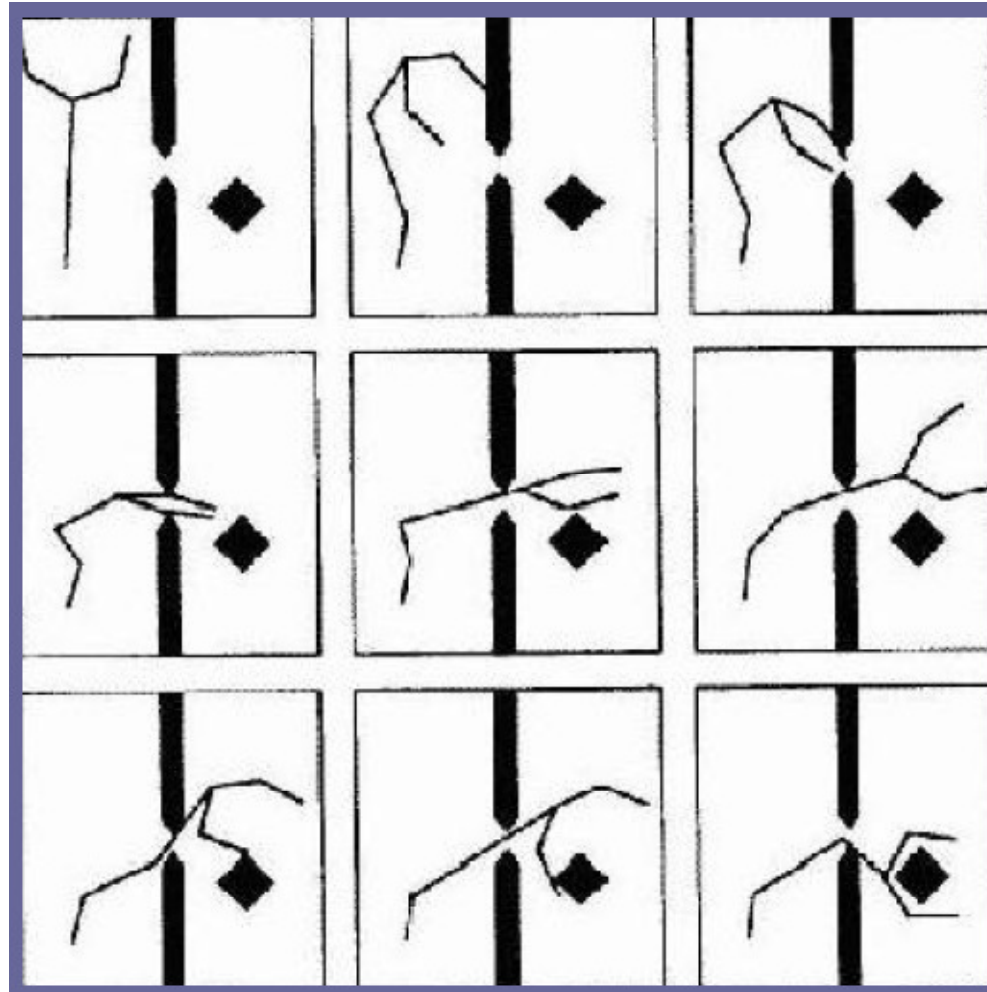


→ Ladder problem

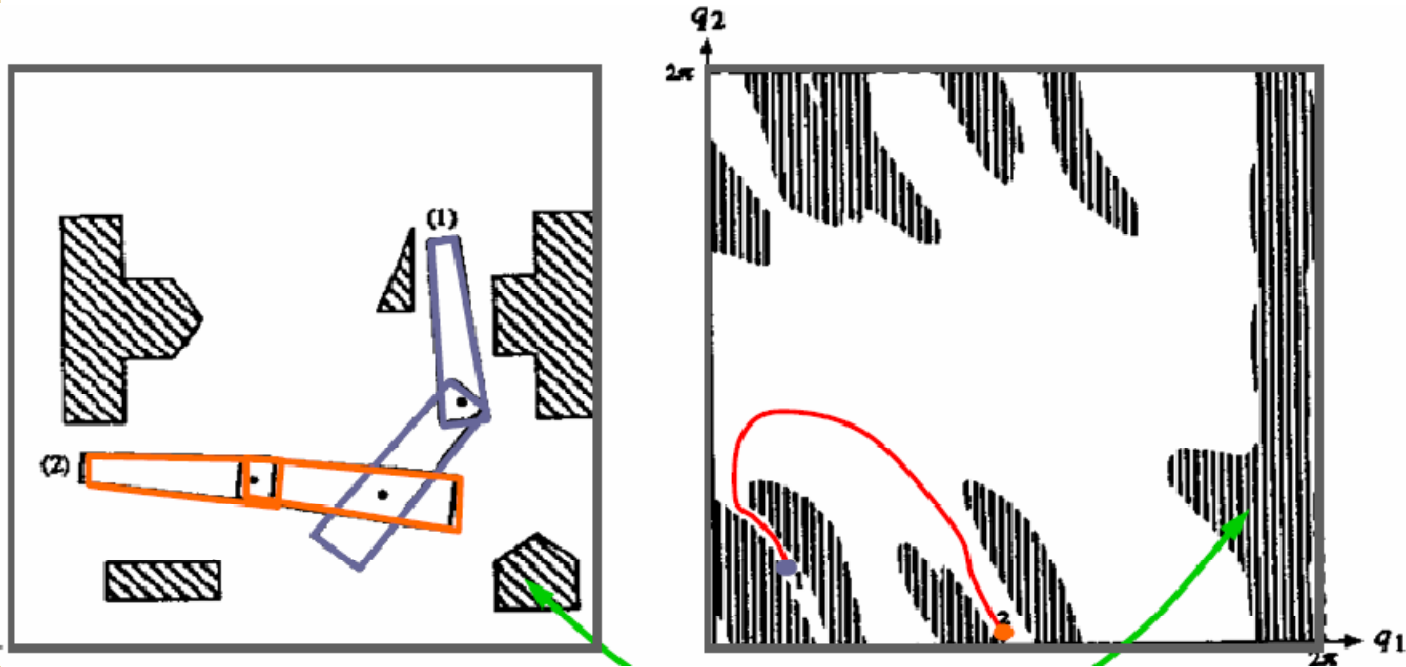
Piano mover's problem ←



# Example: Articulated Robot



# Tool: Configuration Space



## Difficulty

- Number of degrees of freedom (dimension of configuration space)
- Geometric complexity



# Extensions of the Basic Problem

- More complex robots
  - **Multiple robots**
  - **Movable objects**
  - **Nonholonomic & dynamic constraints**
  - **Physical models and deformable objects**
  - **Sensorless motions (exploiting task mechanics)**
  - **Uncertainty in control**



# Extensions of the Basic Problem

- More complex environments
  - **Moving obstacles**
  - **Uncertainty in sensing**
- More complex objectives
  - **Integration of planning and control**
  - **Sensing the environment**
    - **Model/map building**
    - **Target finding, tracking**



# Practical Algorithms

- **A complete motion planner always returns a solution when one exists and indicates that no such solution exists otherwise.**
- **Most motion planning problems are hard, meaning that complete planners take exponential time in the number of degrees of freedom, moving objects, etc.**

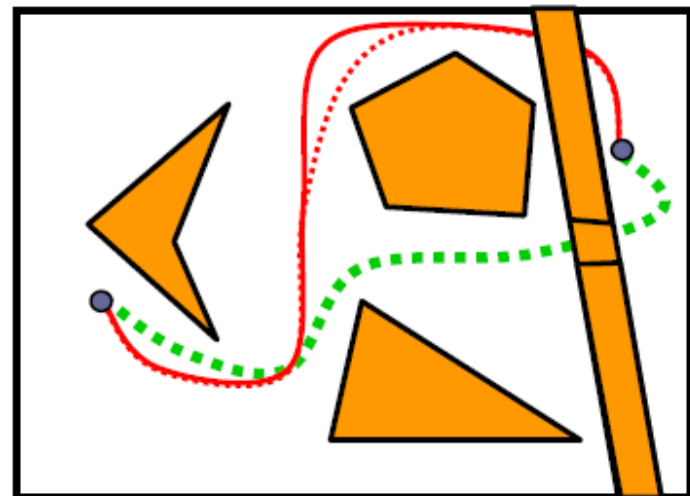


# Practical Algorithms

- **Theoretical algorithms strive for completeness and low worst-case complexity**
  - **Difficult to implement**
  - **Not robust**
- **Heuristic algorithms strive for efficiency in commonly encountered situations.**
  - **No performance guarantee**
- **Practical algorithms with performance guarantees**
  - **Weaker forms of completeness**
  - **Simplifying assumptions on the space: “exponential time” algorithms that work in practice**

# Problem Formulation for Point Robot

- **Input**
  - **Robot represented as a point in the plane**
  - **Obstacles represented as polygons**
  - **Initial and goal positions**
- **Output**
  - **A collision-free path between the initial and goal positions**



# Framework

**continuous representation**

(configuration space formulation)



**discretization**

(random sampling, processing critical geometric events)

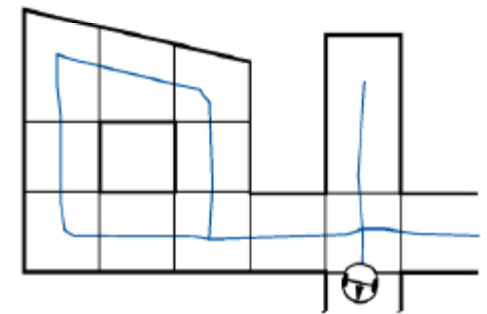
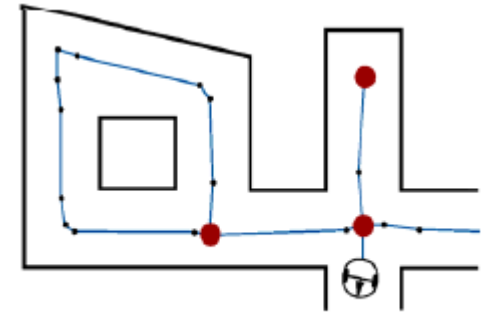


**graph searching**

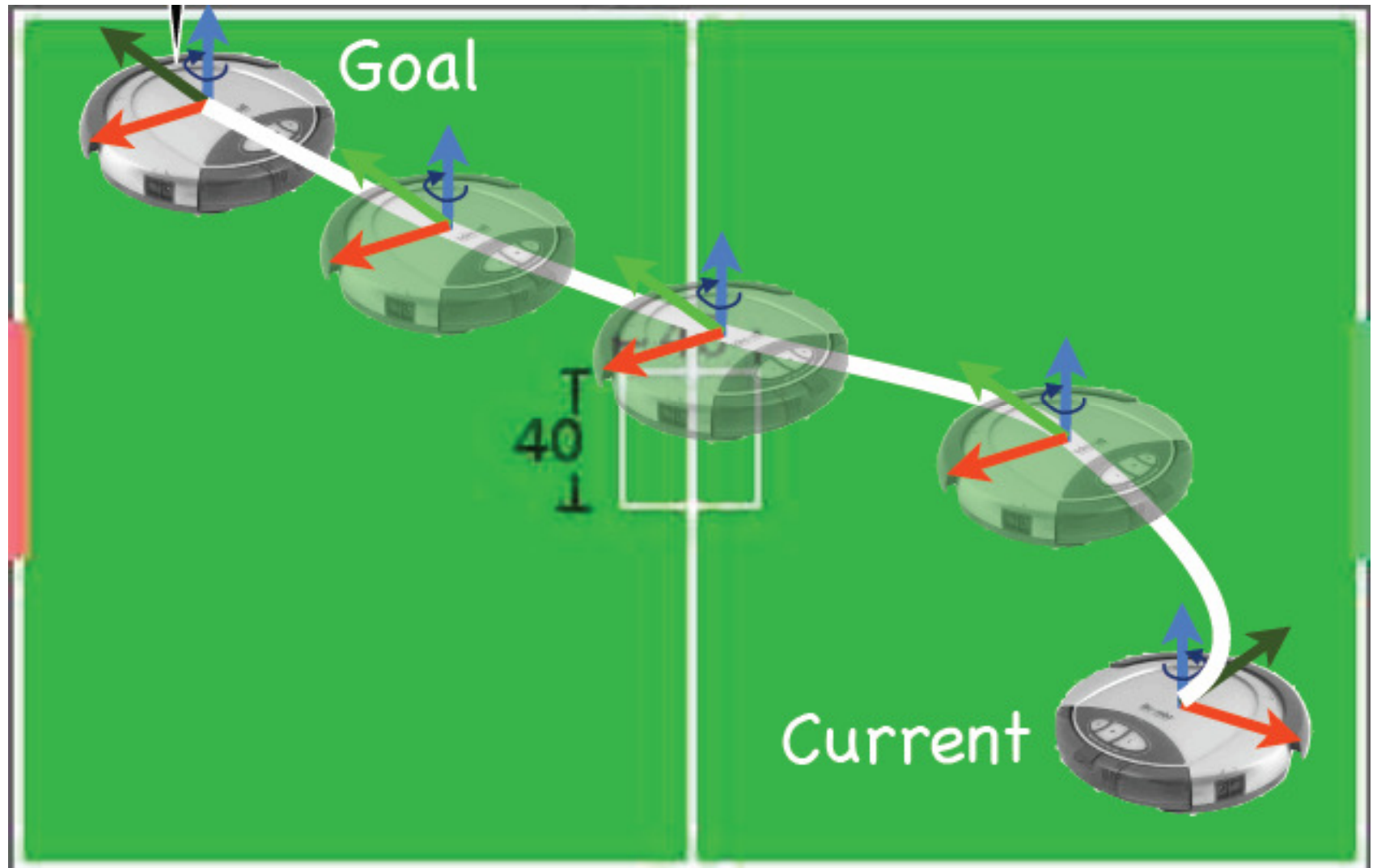
(breadth-first, best-first, A\*)

# Discretization Methods

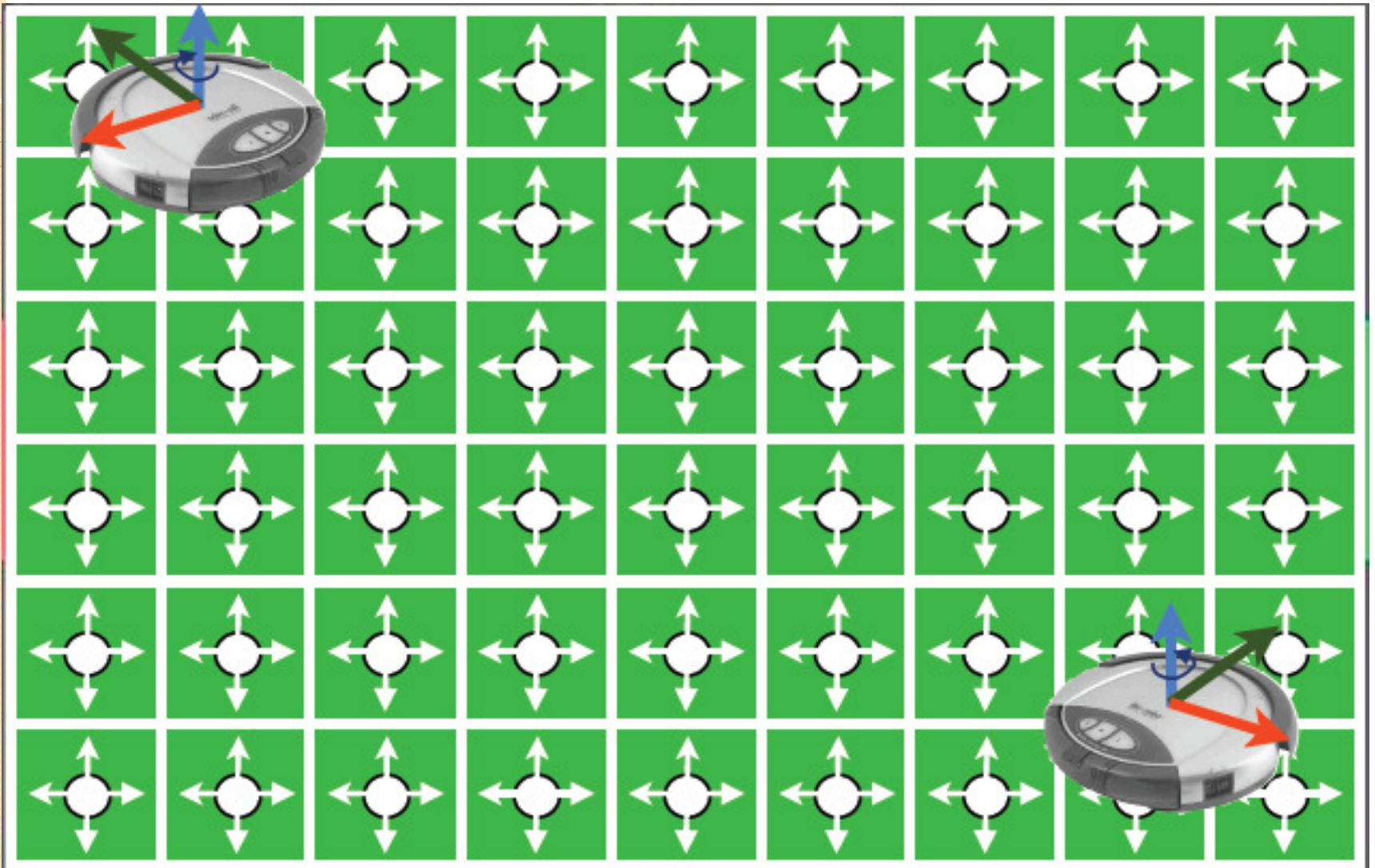
- Full Grid
- Visibility Graph
- Voronoi Diagram
- Cell Decomposition
- Probabilistic Roadmaps



# Full Grid

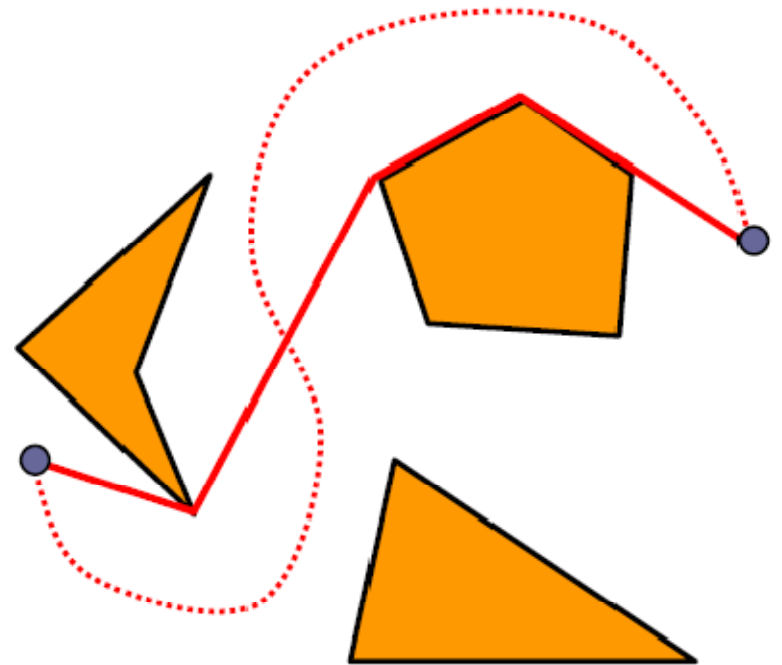


# Full Grid

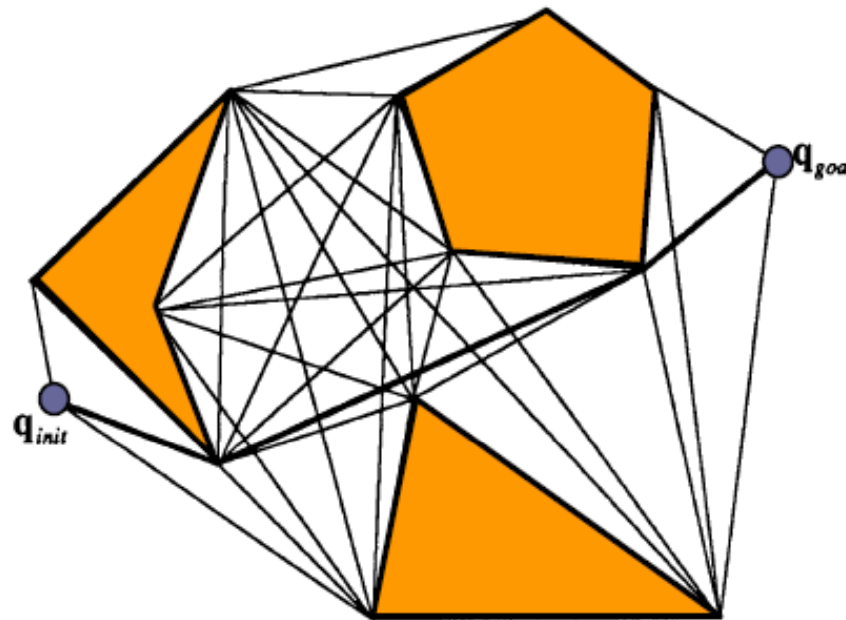


# Visibility Graph Method

- **Observation: If there is a collision-free path between two points, then there is a polygonal path that bends only at the obstacles vertices.**
- **Why?**
  - **Any collision-free path can be transformed into a polygonal path that bends only at the obstacle vertices.**
- **A polygonal path is a piecewise linear curve.**



# Visibility Graph



- **A visibility graph is a graph such that**
  - **Nodes:**  $q_{init}$ ,  $q_{goal}$ , or an obstacle vertex.
  - **Edges:** An edge exists between nodes  $u$  and  $v$  if the line segment between  $u$  and  $v$  is an obstacle edge or it does not intersect the obstacles.



# A Simple Algorithm for Building Visibility Graphs

**Input:**  $q_{init}$ ,  $q_{goal}$ , polygonal obstacles

**Output:** visibility graph  $G$

```
1: for every pair of nodes  $u, v$ 
2:   if segment( $u, v$ ) is an obstacle edge then
3:     insert edge( $u, v$ ) into  $G$ ;
4:   else
5:     for every obstacle edge  $e$ 
6:       if segment( $u, v$ ) intersects  $e$ 
7:         go to (1);
8:     insert edge( $u, v$ ) into  $G$ .
```

# Computational Efficiency

```
1: for every pair of nodes u,v
2:   if segment(u,v) is an obstacle edge then
3:     insert edge(u,v) into G;
4:   else
5:     for every obstacle edge e
6:       if segment(u,v) intersects e
7:         go to (1);
8:     insert edge(u,v) into G.
```

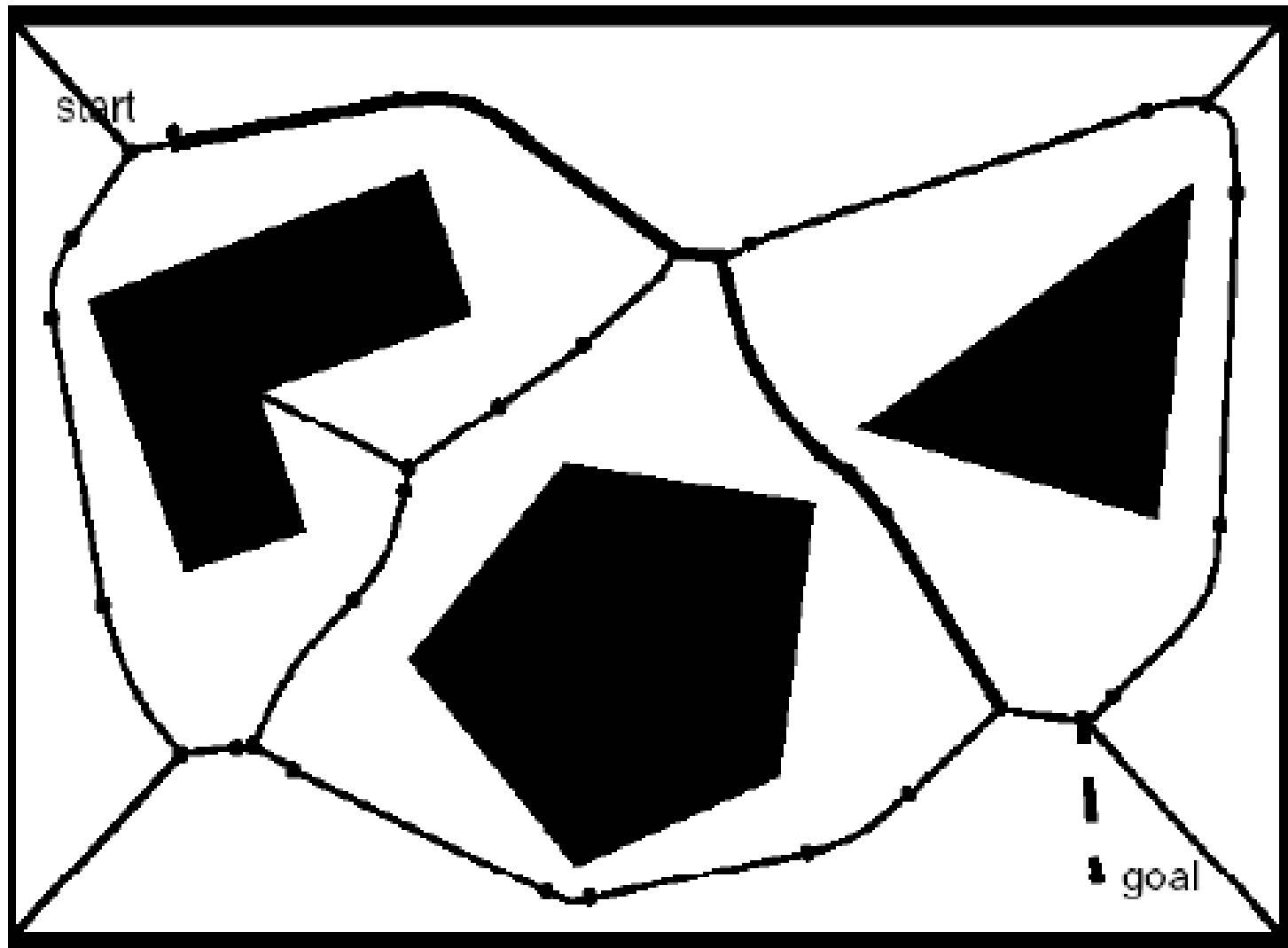
$O(n^2)$

$O(n)$

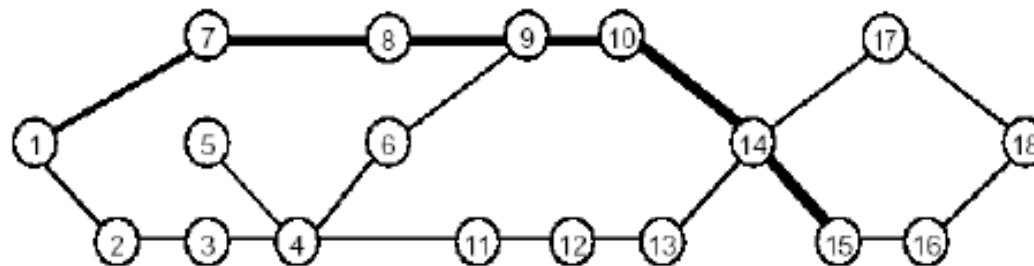
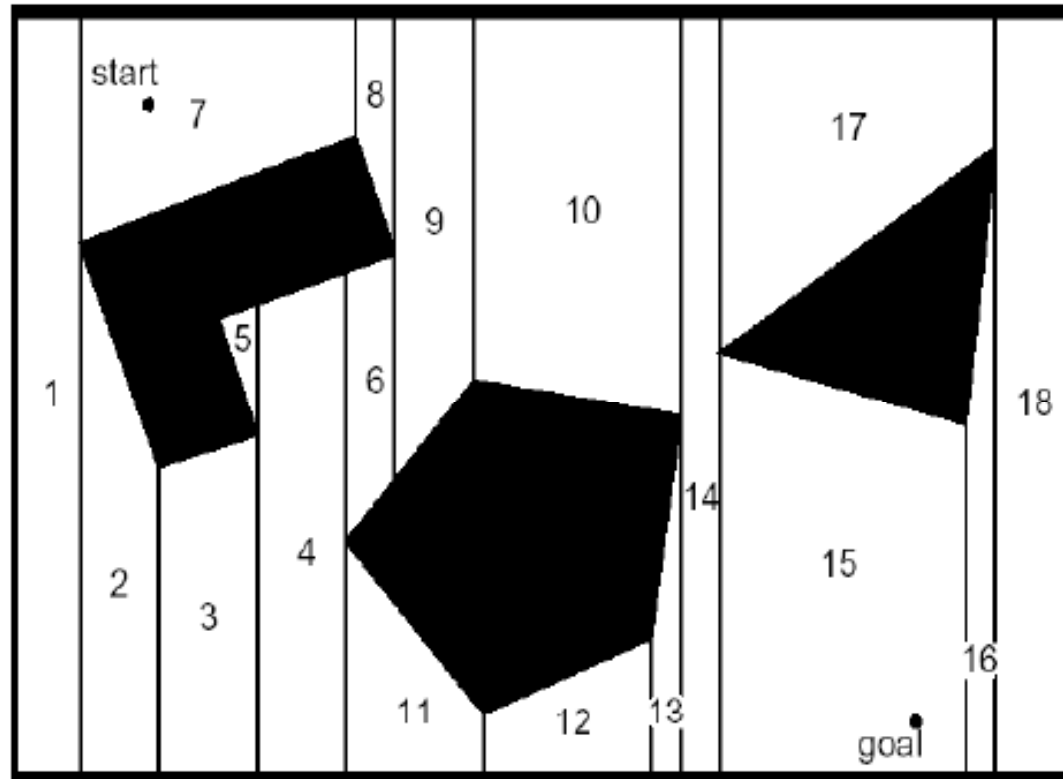
$O(n)$

- **Simple algorithm  $O(n^3)$  time**
- **More efficient algorithms**
  - **Rotational sweep  $O(n^2 \log n)$  time**
  - **Optimal algorithm  $O(n^2)$  time**
  - **Output sensitive algorithms**
- **$O(n^2)$  space**

# Voronoi Diagram

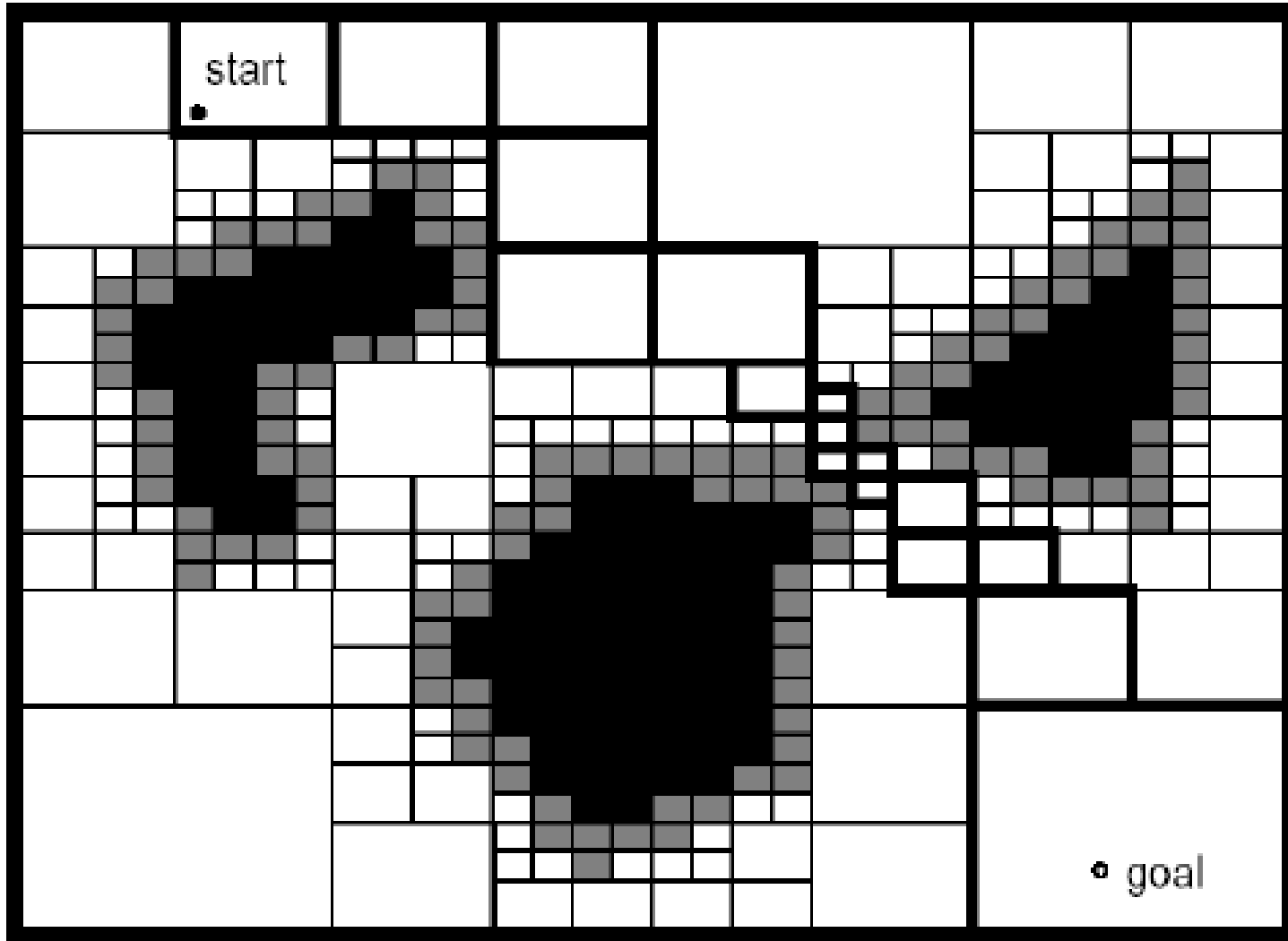


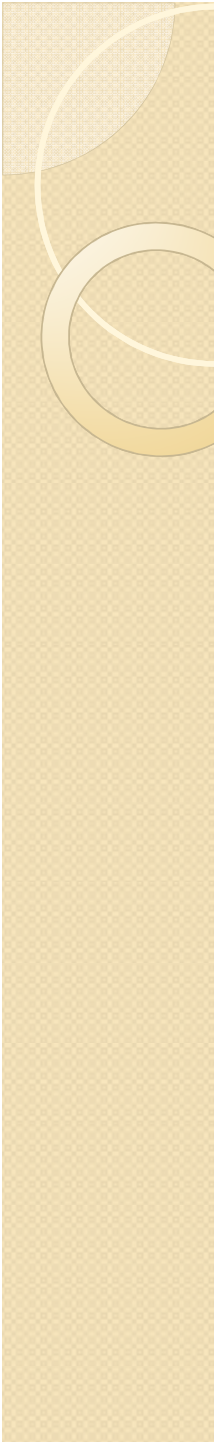
# Cell Decomposition



VTMF

# Approximate Cell Decomposition



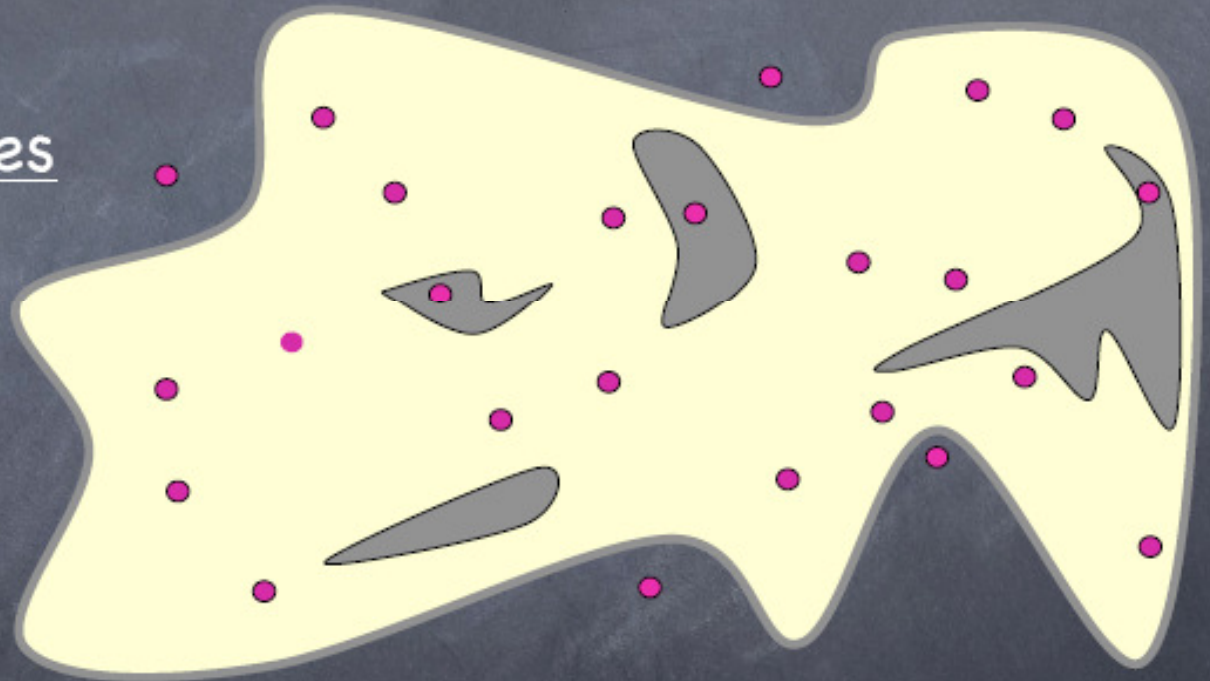


# Problem Formulation for General Robot

- What is the state space?
- Does it include other objects
  - E.g., when interacting with other objects
- Full Grid discretization is huge (consider 7-dof arm,  $100^7$  possibilities)
- How should I discretize?

# PRM: construction phase

- Select sample poses at random
- Eliminate invalid poses
- Connect neighboring poses



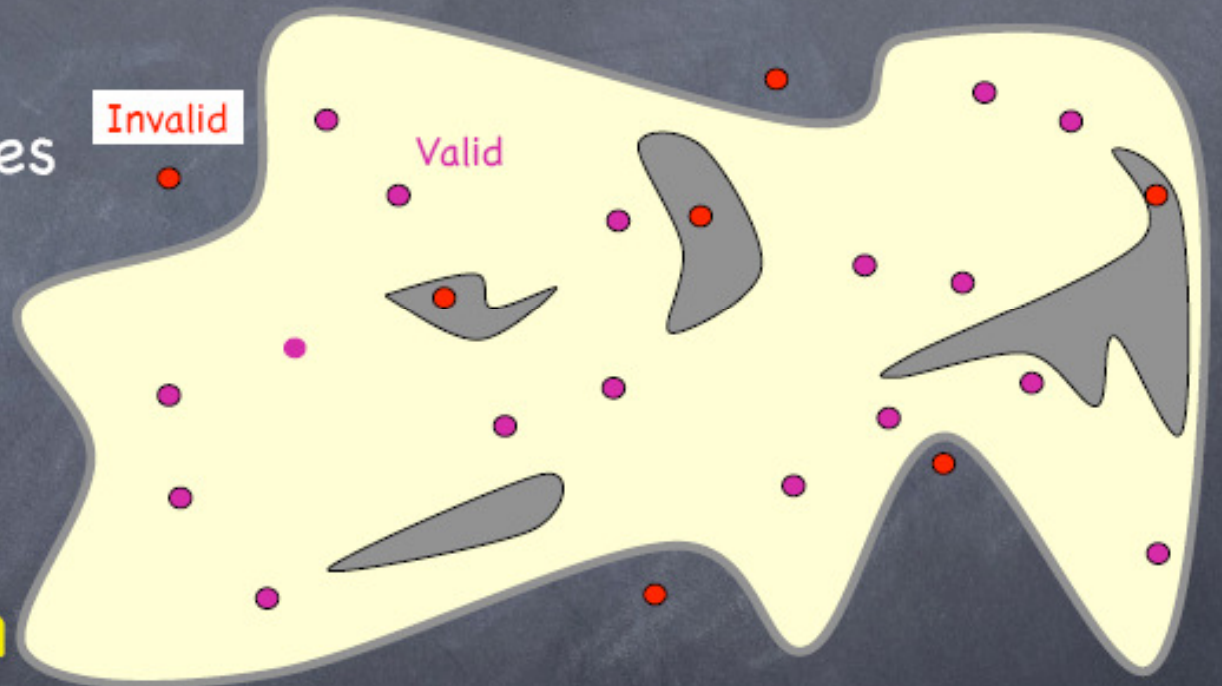
# PRM: construction phase

- Select sample poses at random

- Eliminate invalid poses

## Collision detection

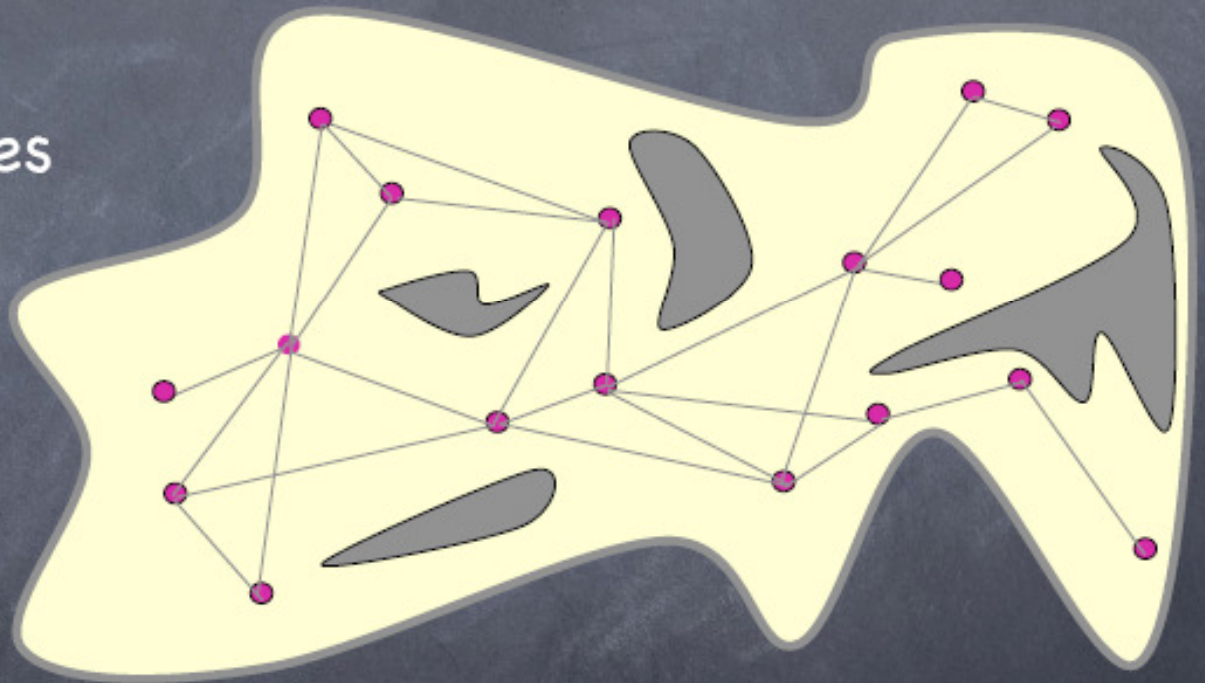
- Connect neighboring poses





# PRM: construction phase

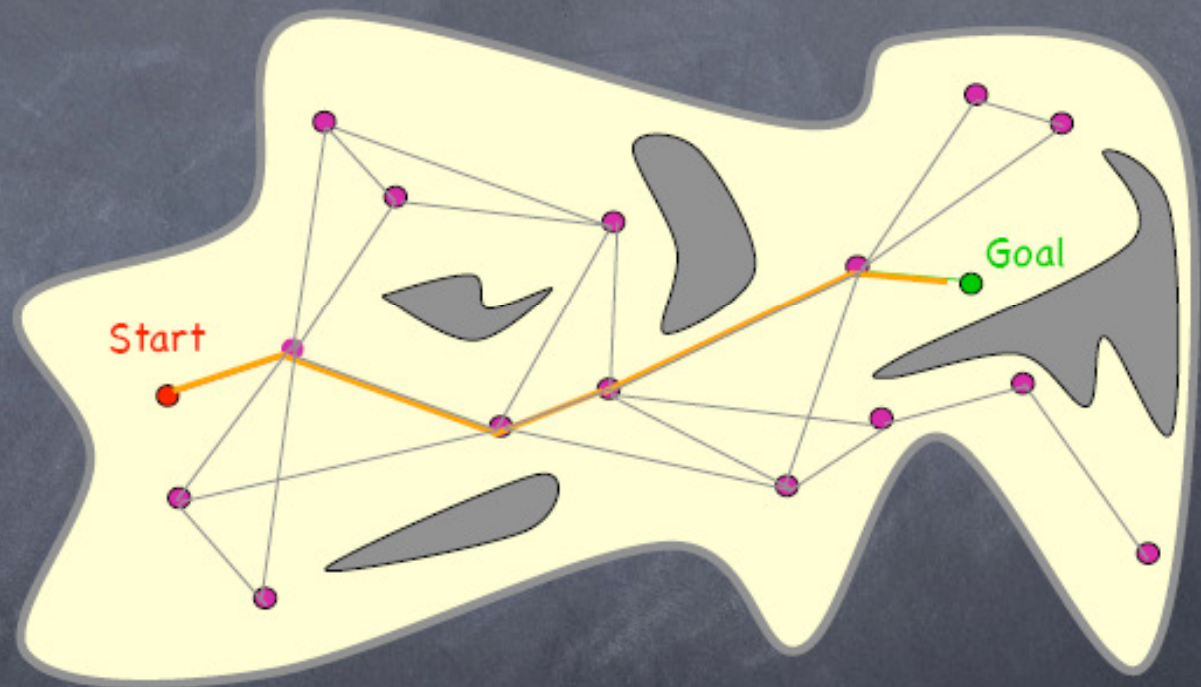
- Select sample poses at random
- Eliminate invalid poses
- Connect neighboring poses



Threshold neighborhood radius or population

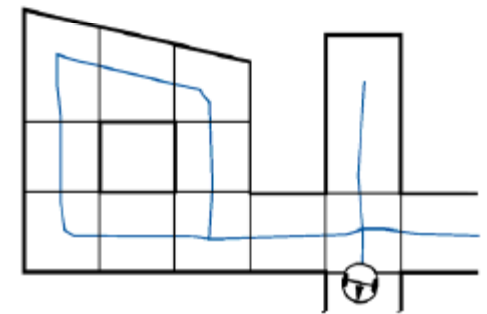
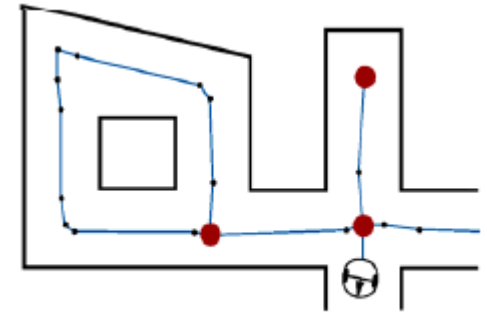
# PRM: query phase

- Given constructed roadmap
- Find path in roadmap between two poses
- Search on an undirected graph



# Discretization Methods

- Full Grid
- Visibility Graph
- Voronoi Diagram
- Cell Decomposition
- Probabilistic Roadmaps



# Framework

**continuous representation**

(configuration space formulation)



**discretization**

(random sampling, processing critical geometric events)



**graph searching**

(breadth-first, best-first, A\*)



# Summary

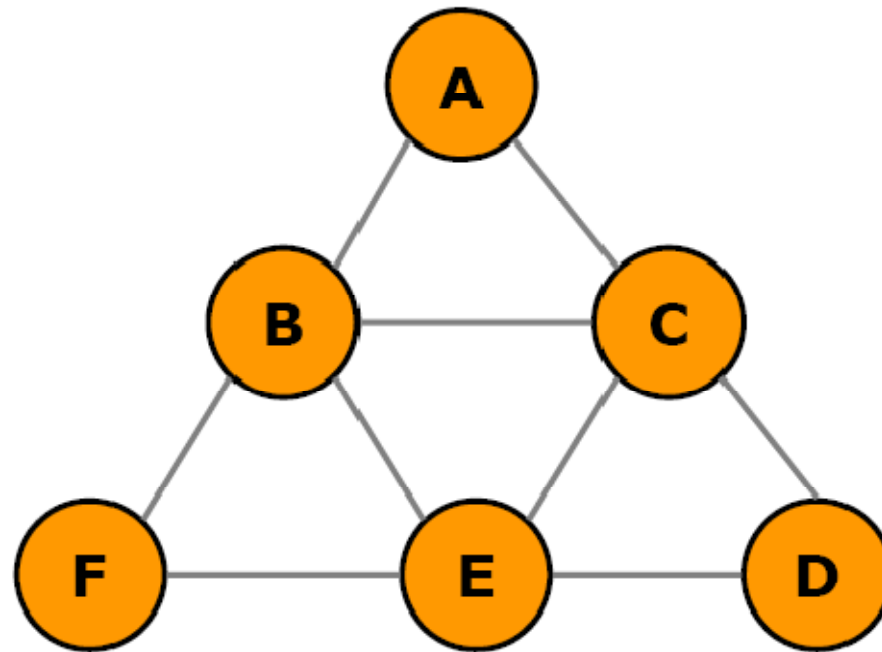
- **Discretize the space by constructing visibility graph**
- **Search the visibility graph with breadth-first search**



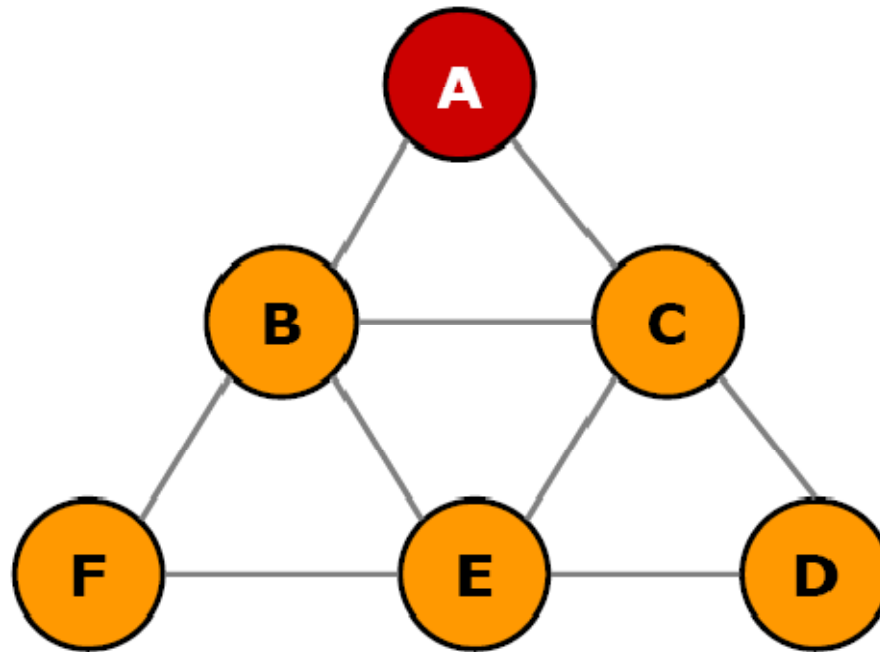
# Search: Overview

- Breadth-first search
- Depth-first search
- A\* star search
- Dijkstra

# Breadth-First Search

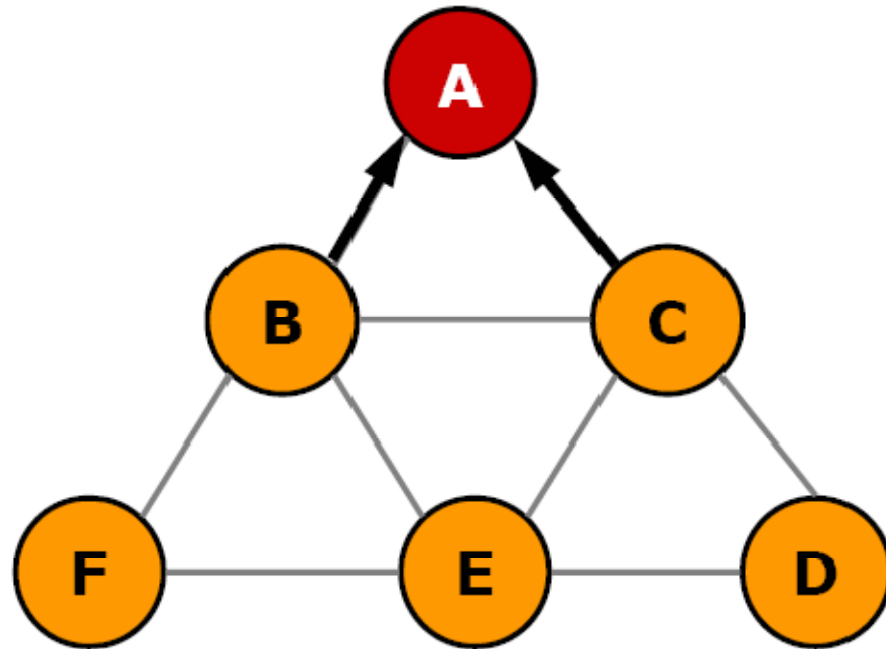


# Breadth-First Search

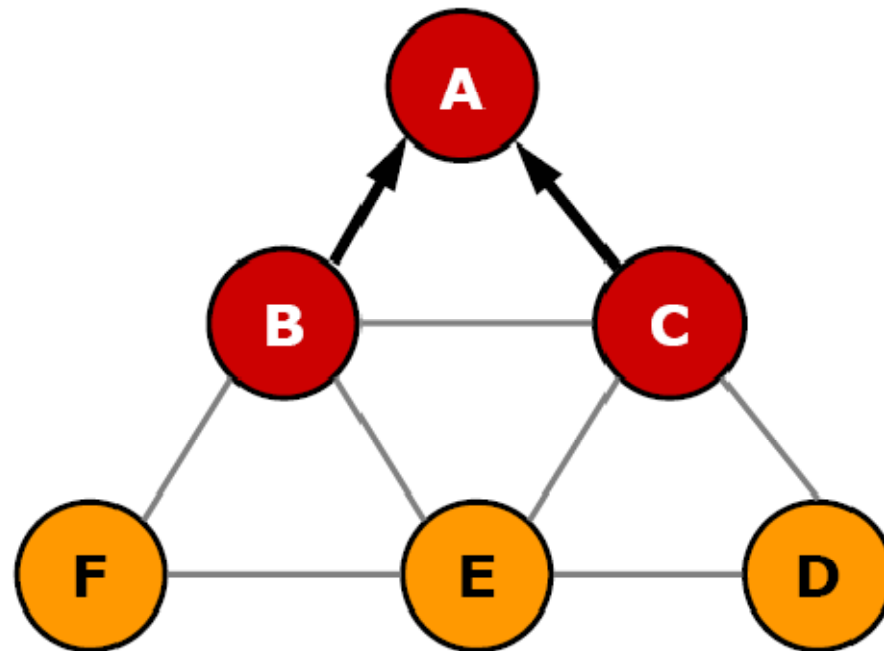




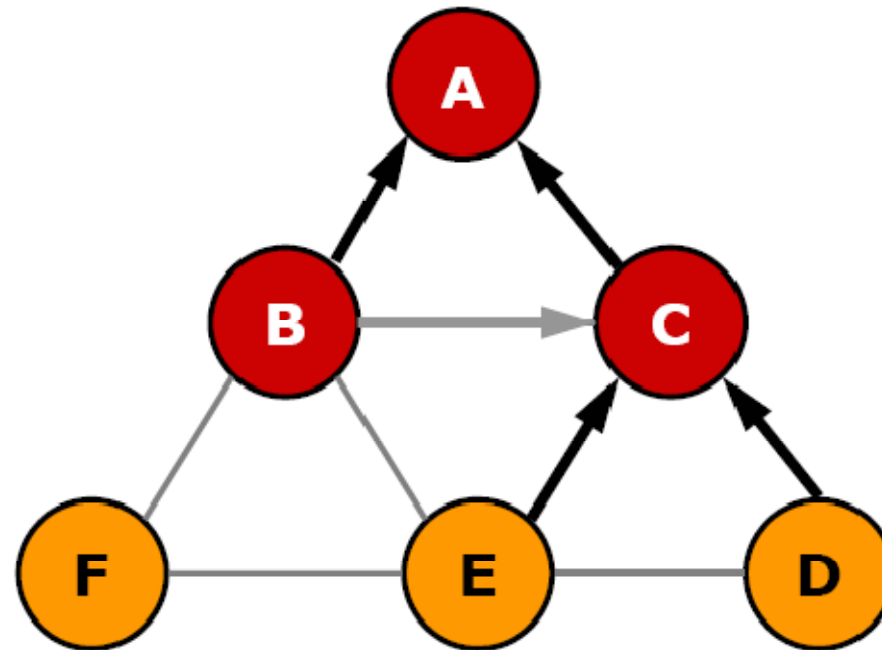
# Breadth-First Search



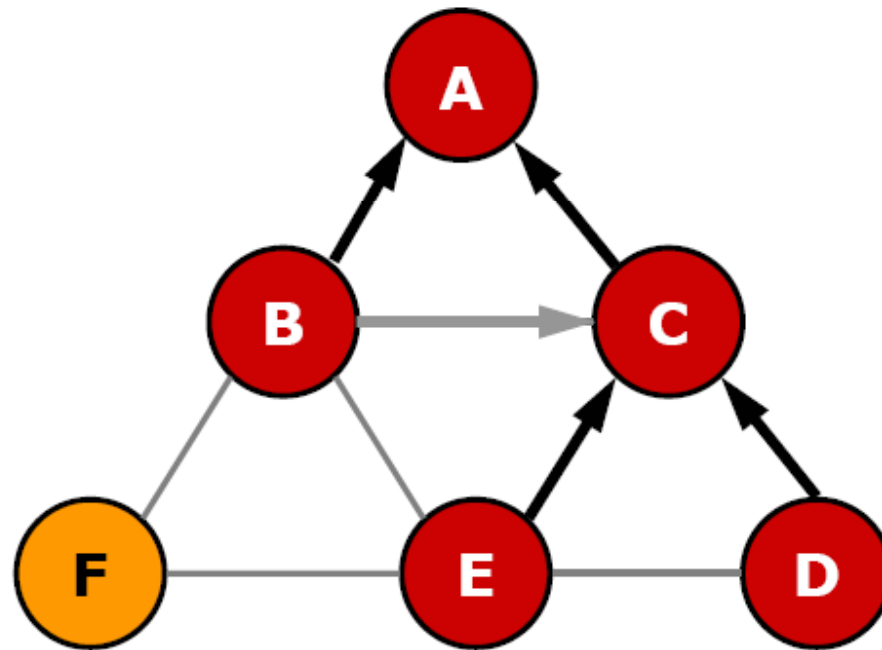
# Breadth-First Search



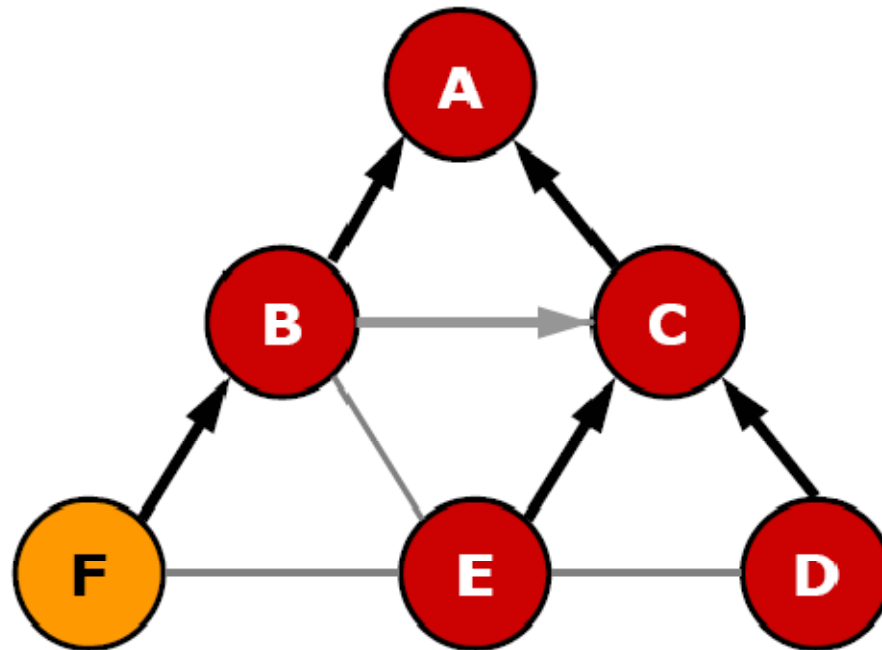
# Breadth-First Search



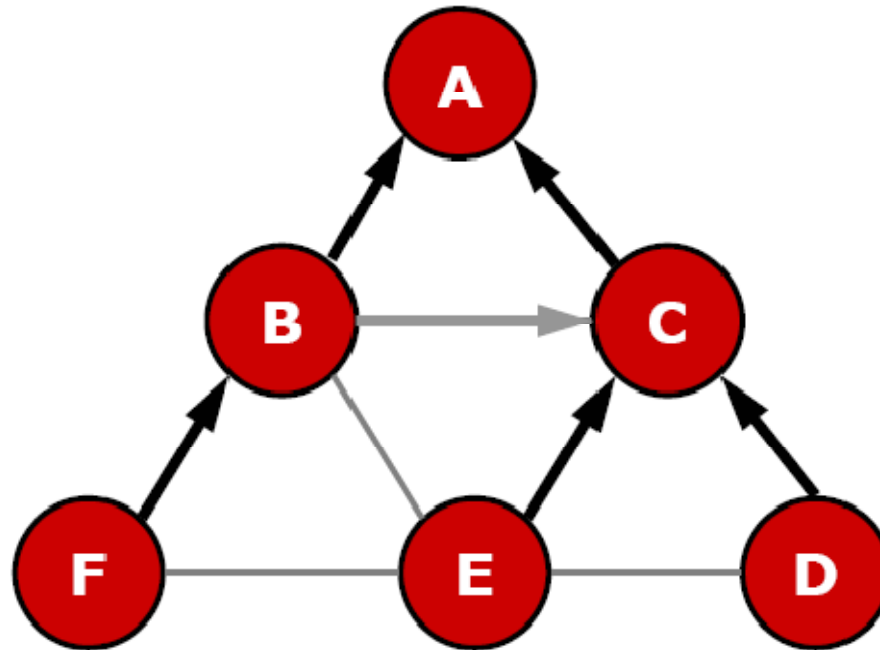
# Breadth-First Search



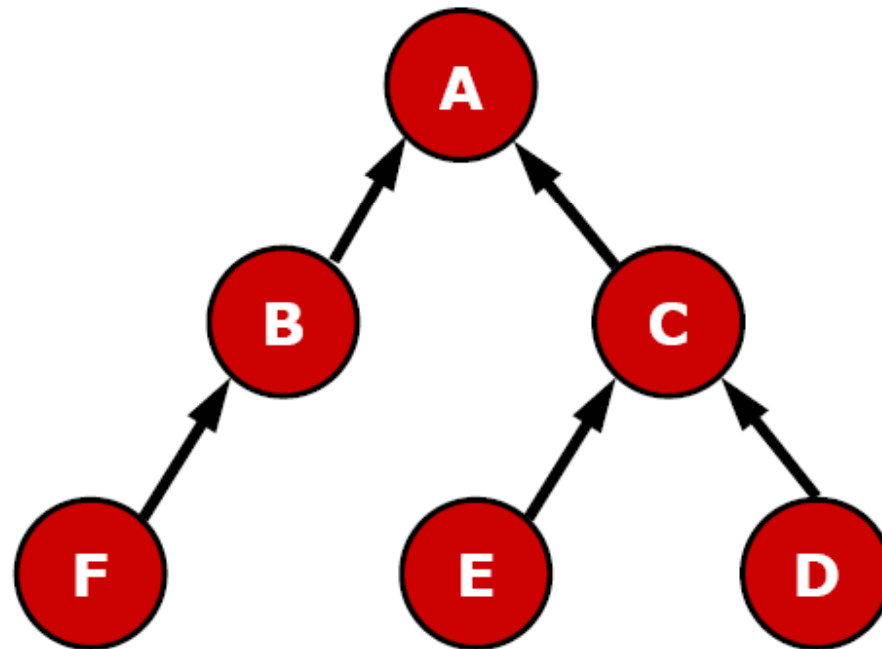
# Breadth-First Search



# Breadth-First Search



# Breadth-First Search

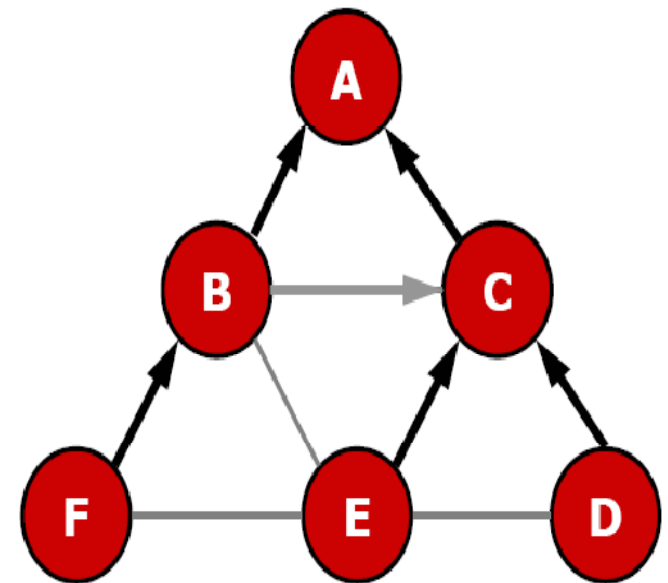


# Breadth-First Search

**Input:**  $q_{init}$ ,  $q_{goal}$ , visibility graph  $G$

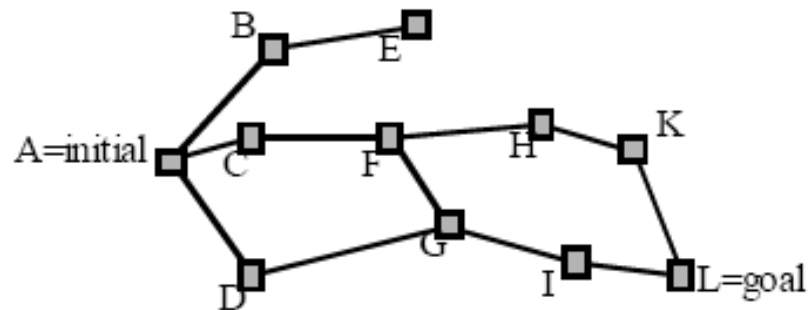
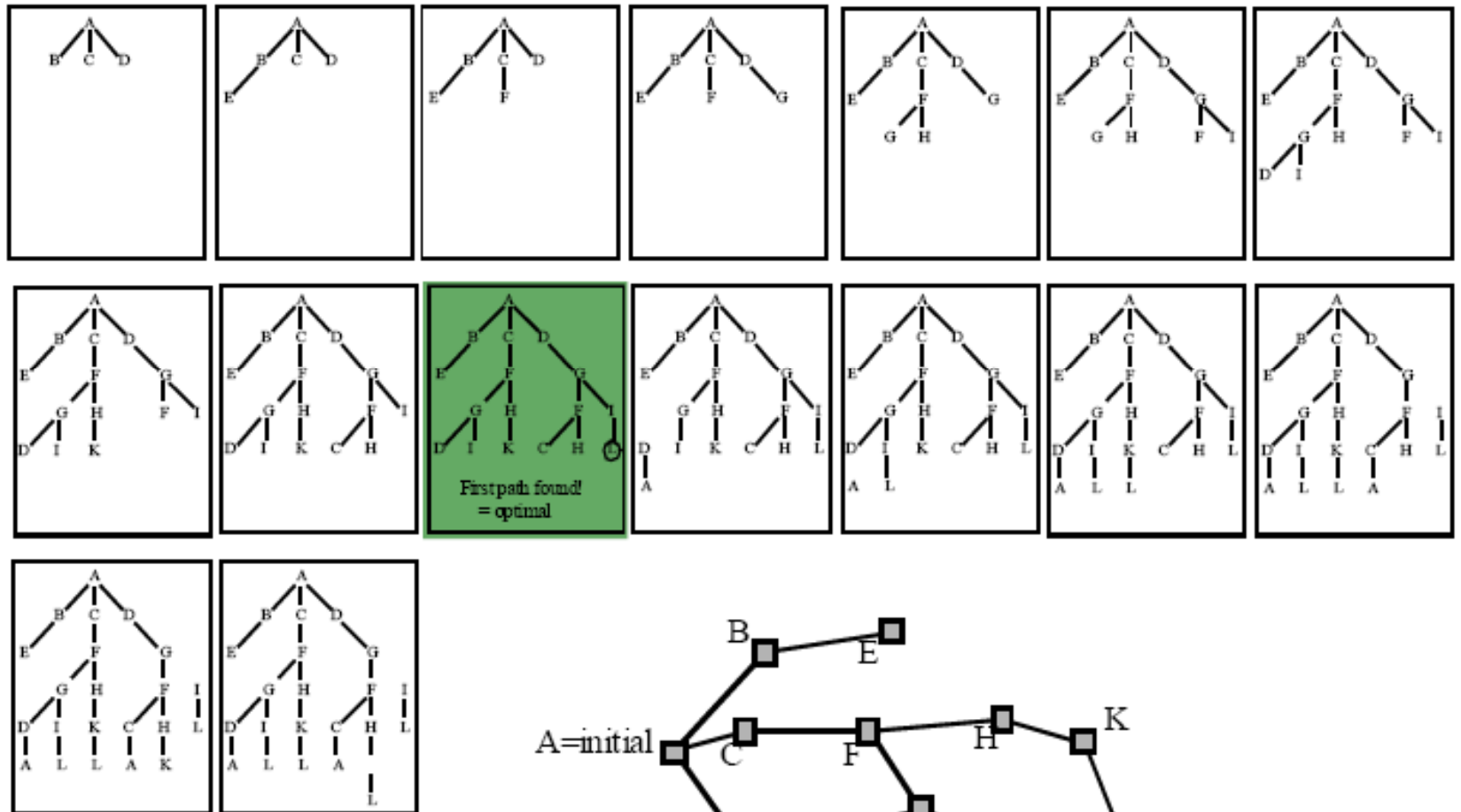
**Output:** a path between  $q_{init}$  and  $q_{goal}$

```
1: Q = new queue;
2: Q.enqueue( $q_{init}$ );
3: mark  $q_{init}$  as visited;
4: while Q is not empty
5:   curr = Q.dequeue();
6:   if curr ==  $q_{goal}$  then
7:     return curr;
8:   for each w adjacent to curr
10:    if w is not visited
11:      w.parent = curr;
12:      Q.enqueue(w)
13:    mark w as visited
```

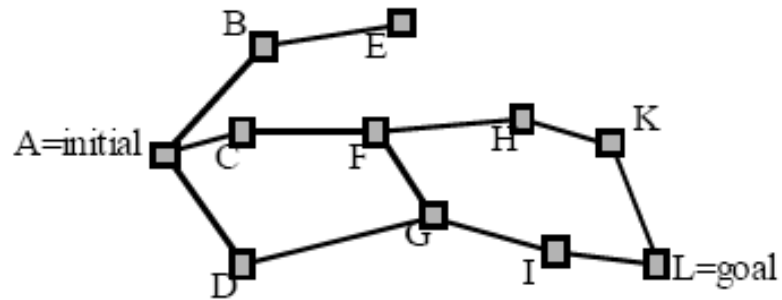
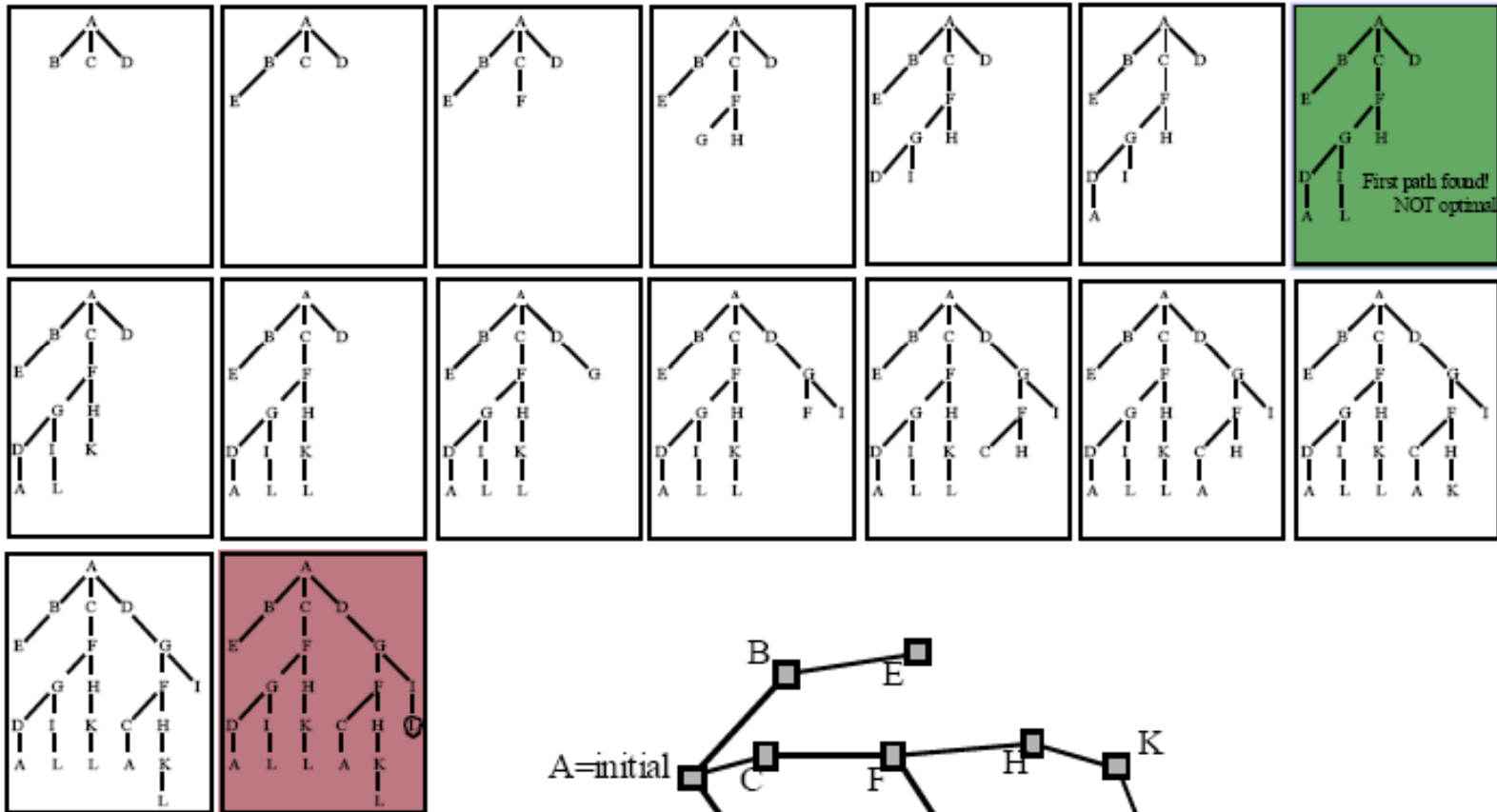




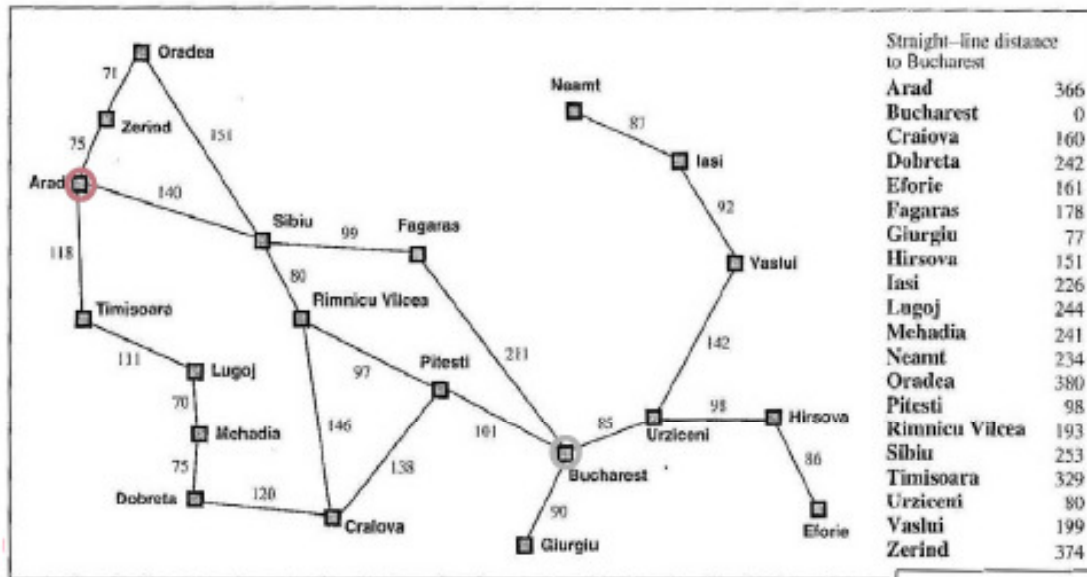
# Breadth-first Search



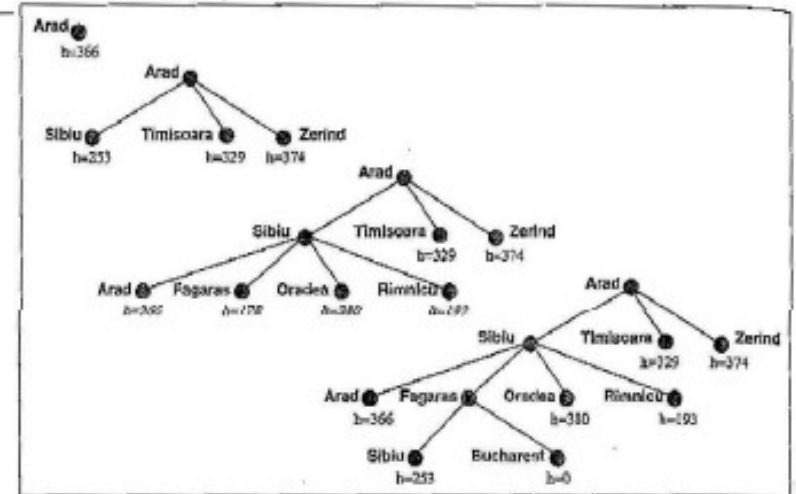
# Depth-First Search



# A\* Search

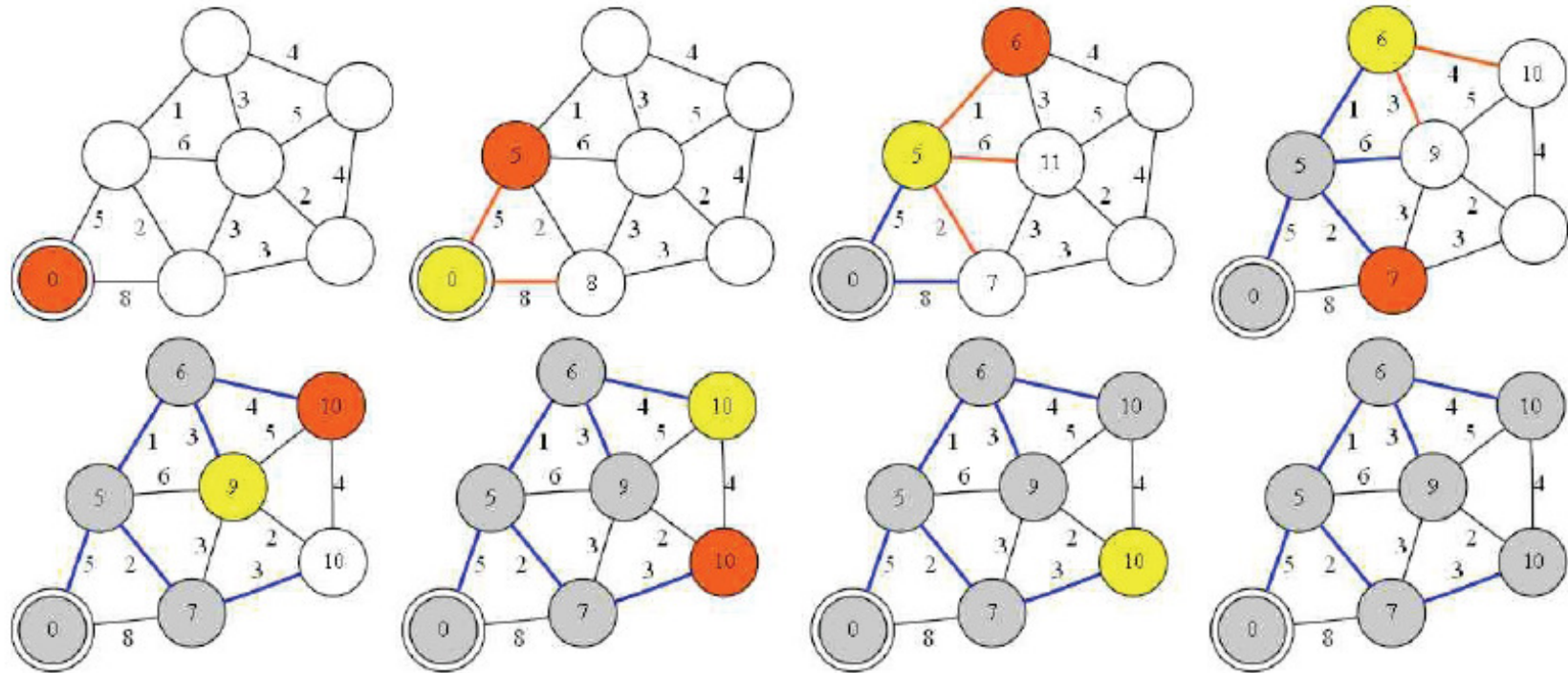


- cost estimate of the cheapest path from state at node n to the goal



# Dijkstra's Algorithm:

Given weighted graph and start node, find shortest path back to start for all nodes



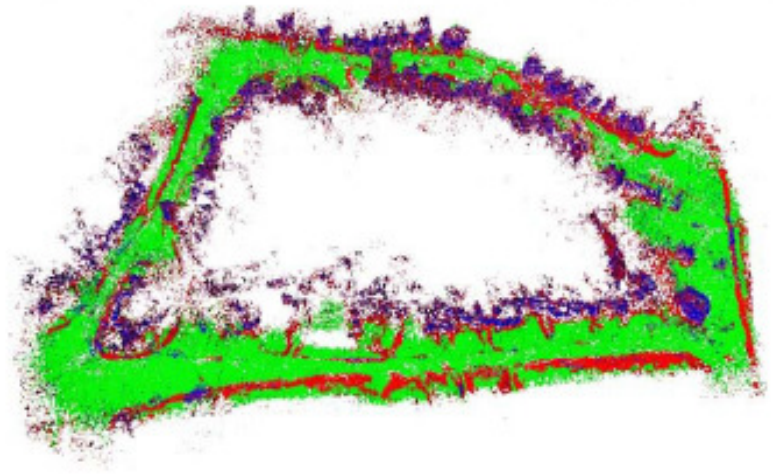
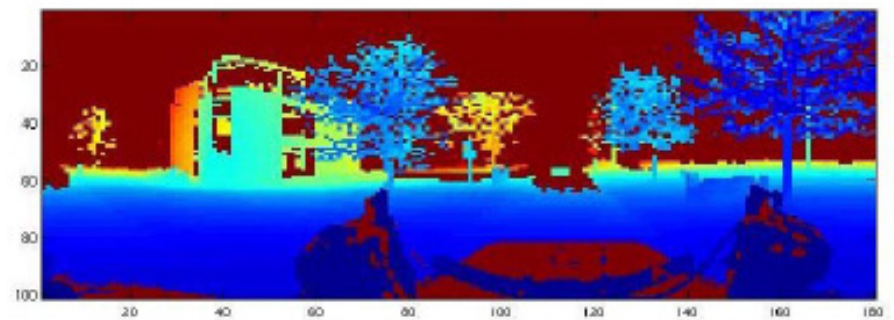
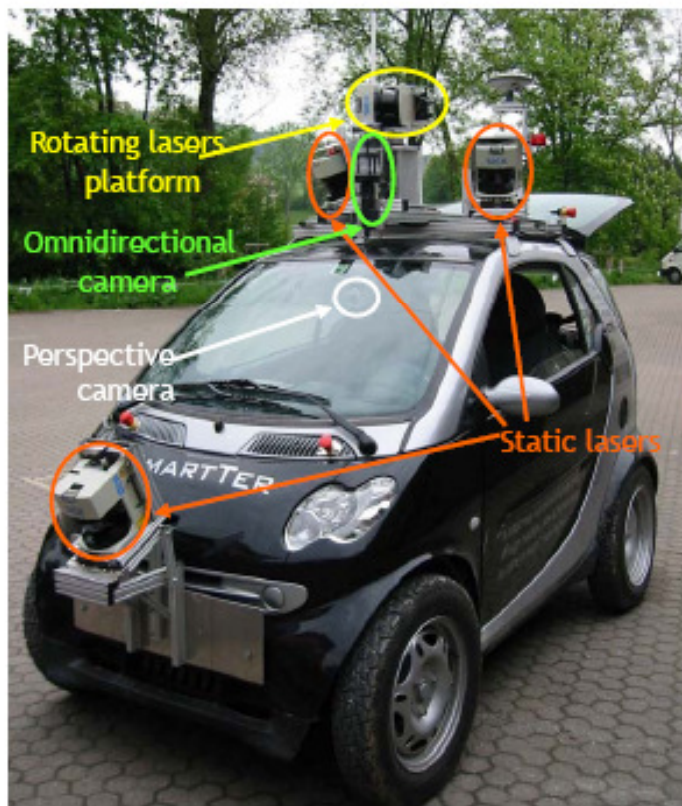
Explore each node one-by-one, calculate path distance from explored tree

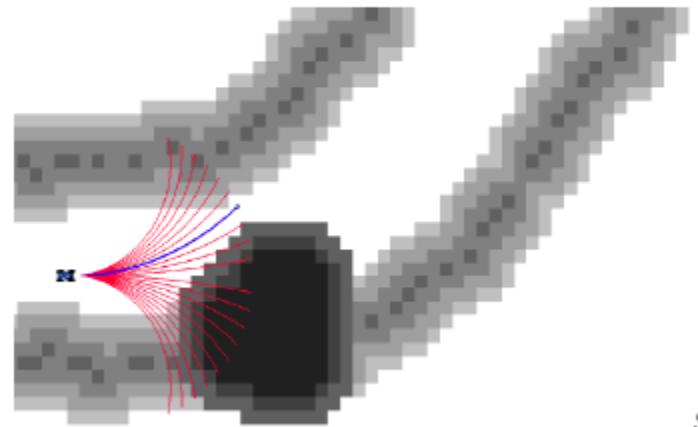
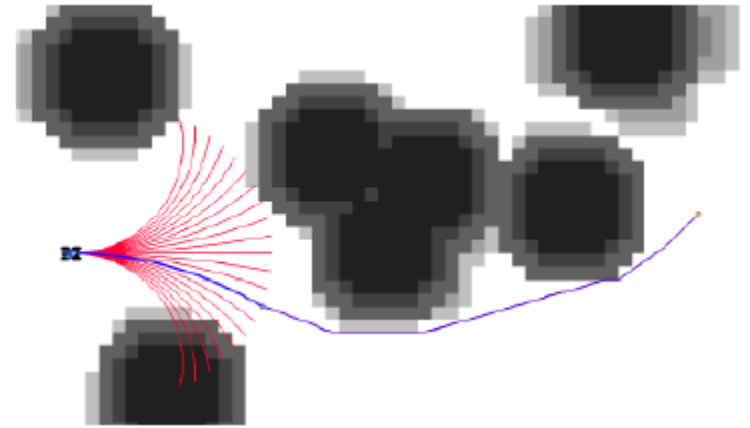
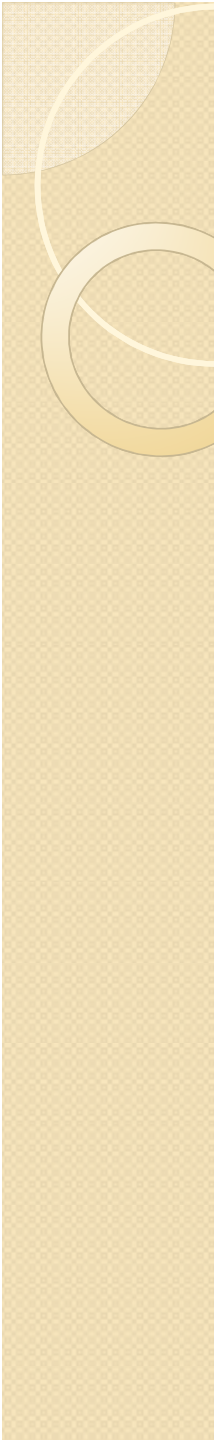


Given *perfect* sensing and map, one can do:

1. Planning
  1. Discretize
  2. Search
2. Control
  1. Kinematics
  2. PID control

[Example video](#)





SL

# Rapidly-exploring Random Trees

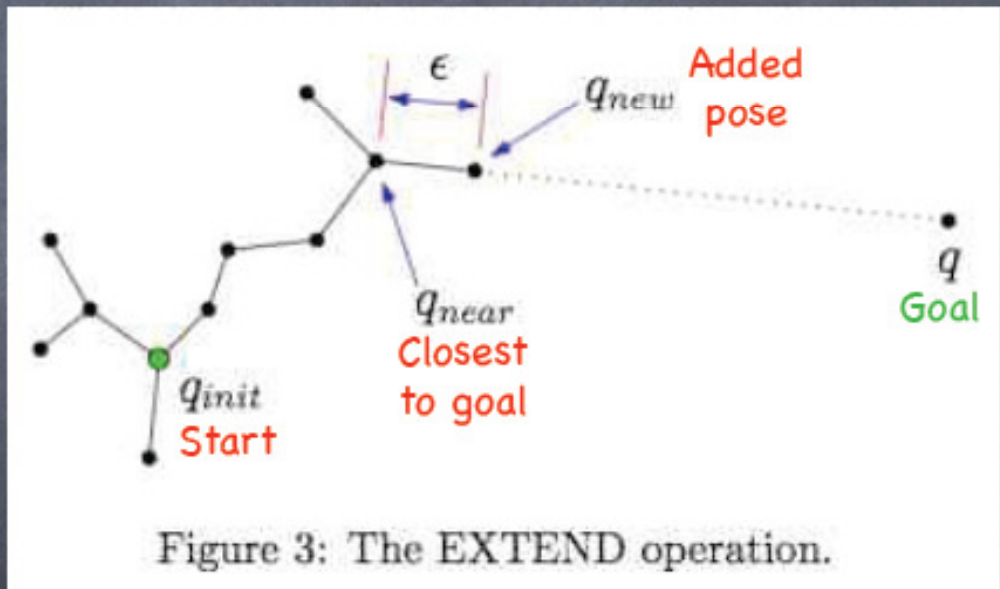
Explore incrementally along  
straight line path to goal

```
BUILD_RRT( $q_{init}$ )
1   $\mathcal{T}.init(q_{init});$ 
2  for  $k = 1$  to  $K$  do
3     $q_{rand} \leftarrow \text{RANDOM\_CONFIG}();$ 
4     $\text{EXTEND}(\mathcal{T}, q_{rand});$ 
5  Return  $\mathcal{T}$ 

EXTEND( $\mathcal{T}, q$ )
1   $q_{near} \leftarrow \text{NEAREST\_NEIGHBOR}(q, \mathcal{T});$ 
2  if  $\text{NEW\_CONFIG}(q, q_{near}, q_{new})$  then
3     $\mathcal{T}.add\_vertex(q_{new});$ 
4     $\mathcal{T}.add\_edge(q_{near}, q_{new});$ 
5    if  $q_{new} = q$  then
6      Return Reached;
7    else
8      Return Advanced;
9  Return Trapped;
```

Figure 2: The basic RRT construction algorithm.

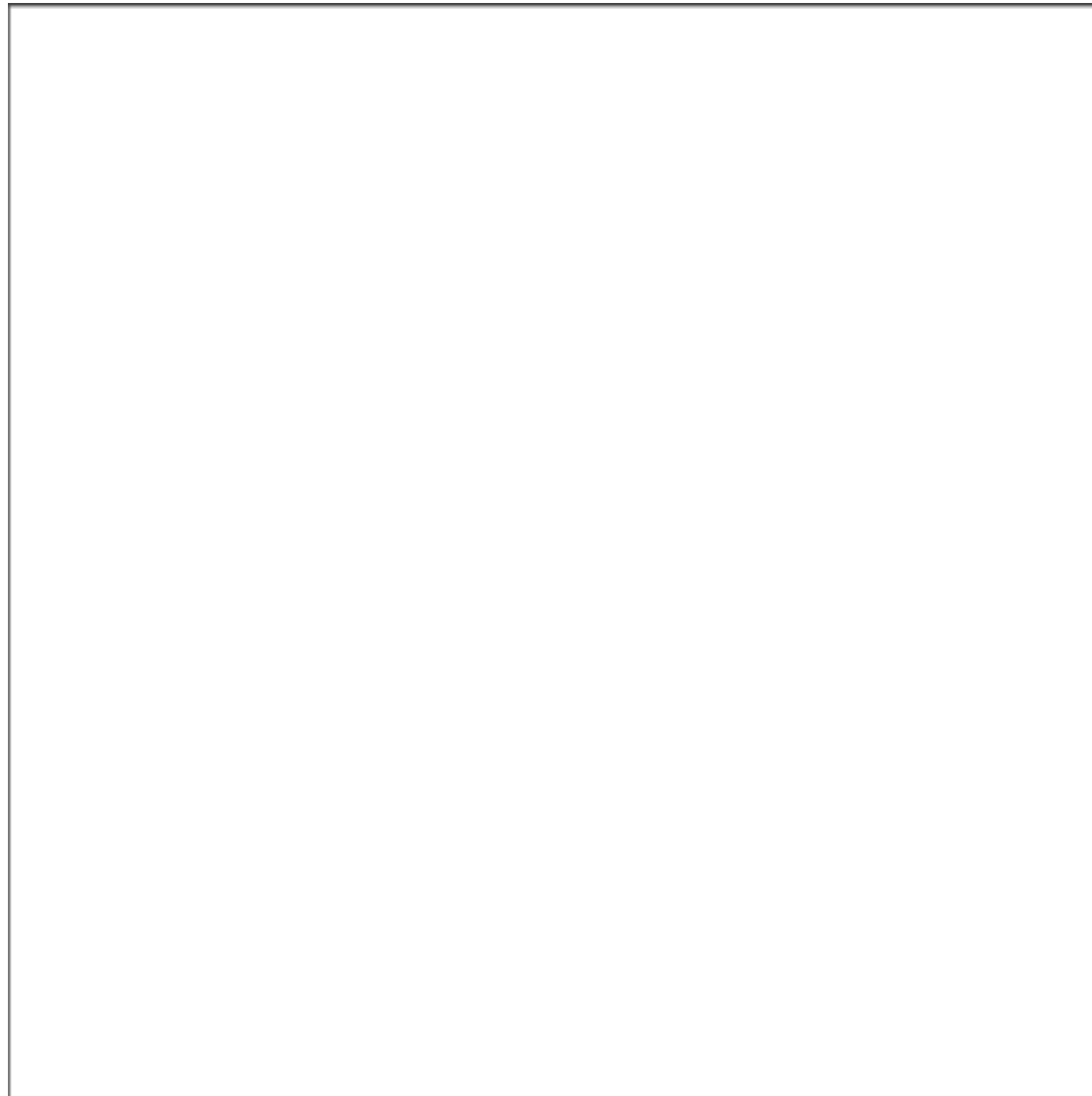
[Kuffner, LaValle 2000]



Single vs. multiple query

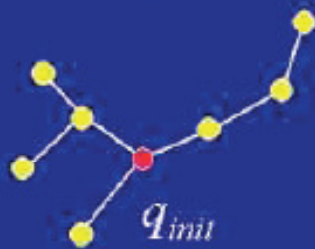


# RRT



# Kuffner's RRT Animations

Existing RRT is "grown" as follows...



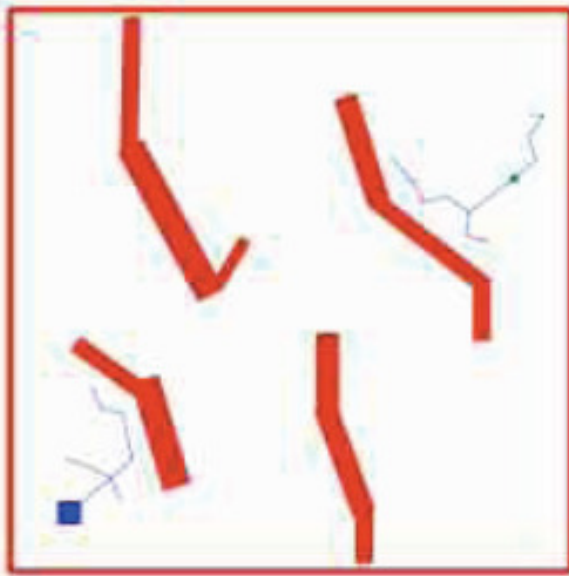
Extending roadmap

Bidirectional exploration

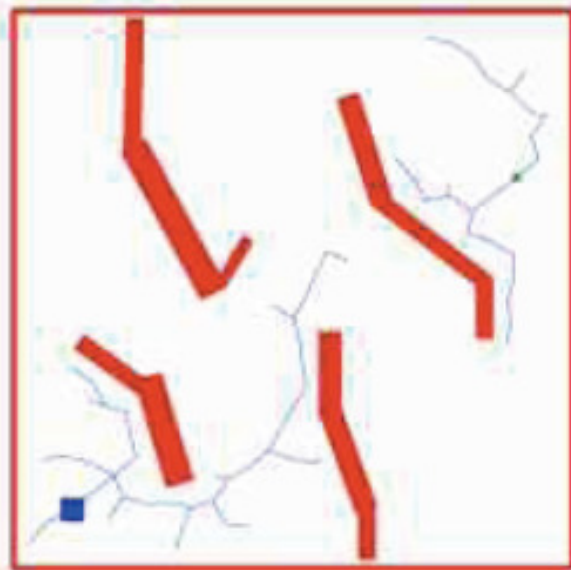
A single RRT-Connect iteration...



# RRT bidirectional search example



extend graph from both start and goal

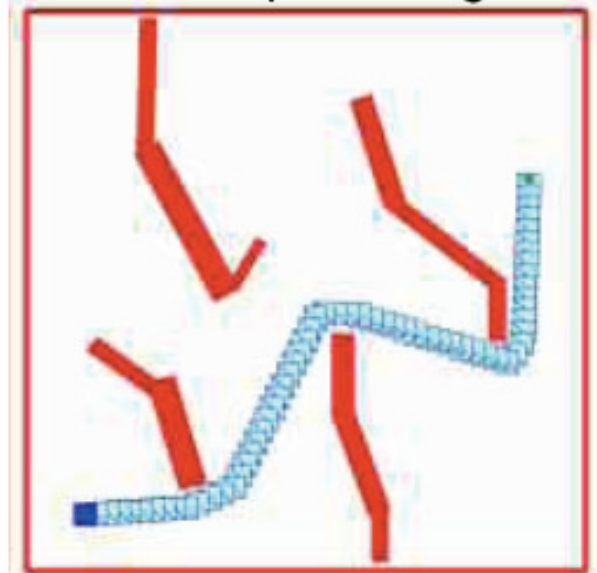


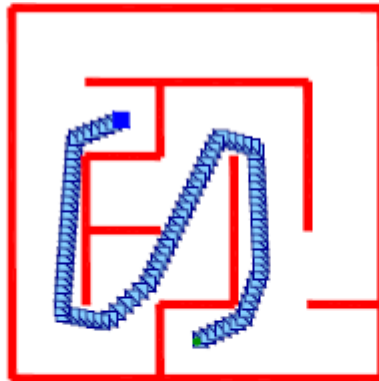
continue exploration until start and goal connected

once connected, search for shortest path

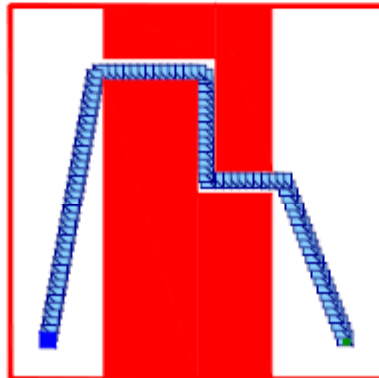
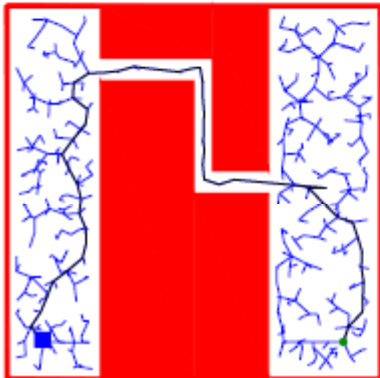


execute path to goal



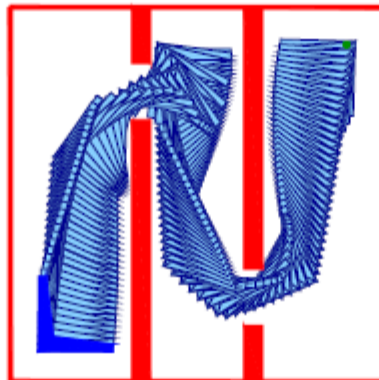
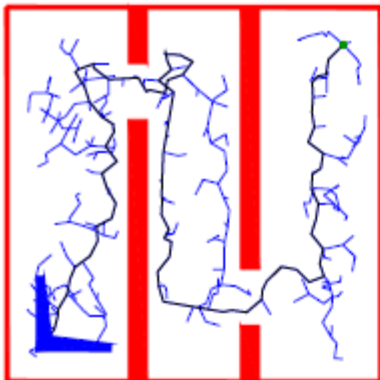


2 DOF maze



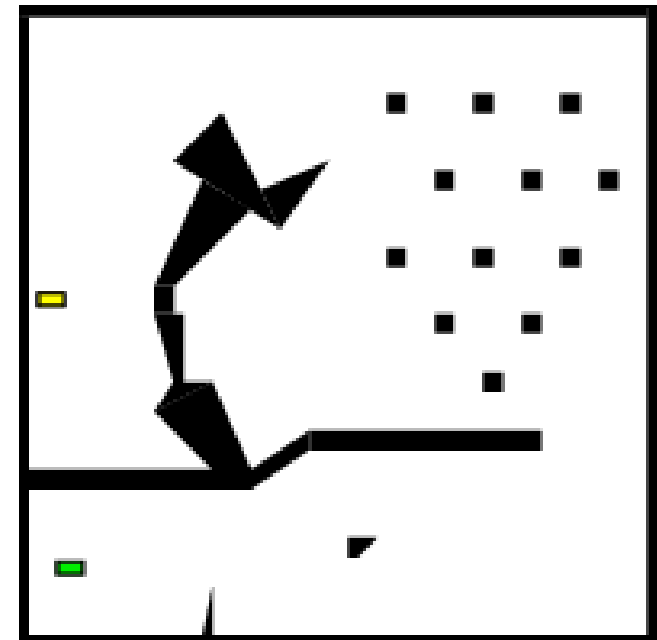
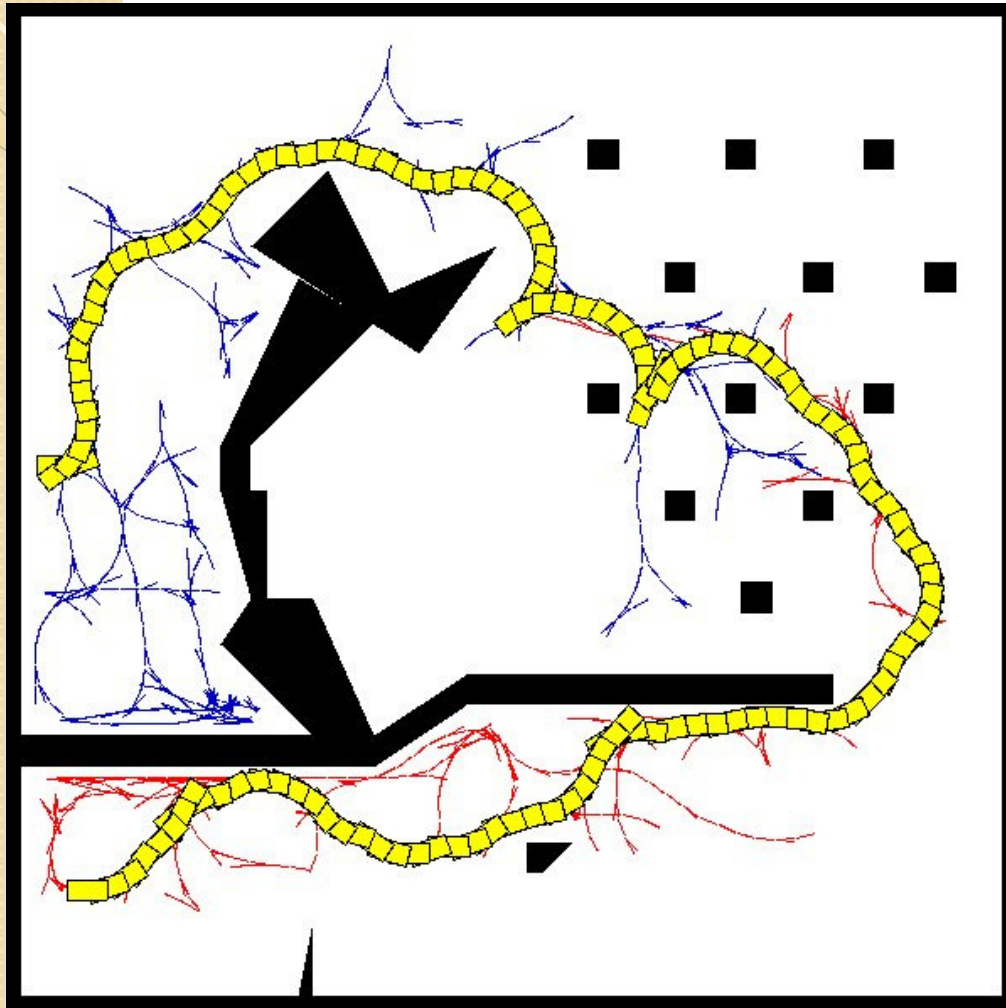
2 DOF single passway

## RRT Examples

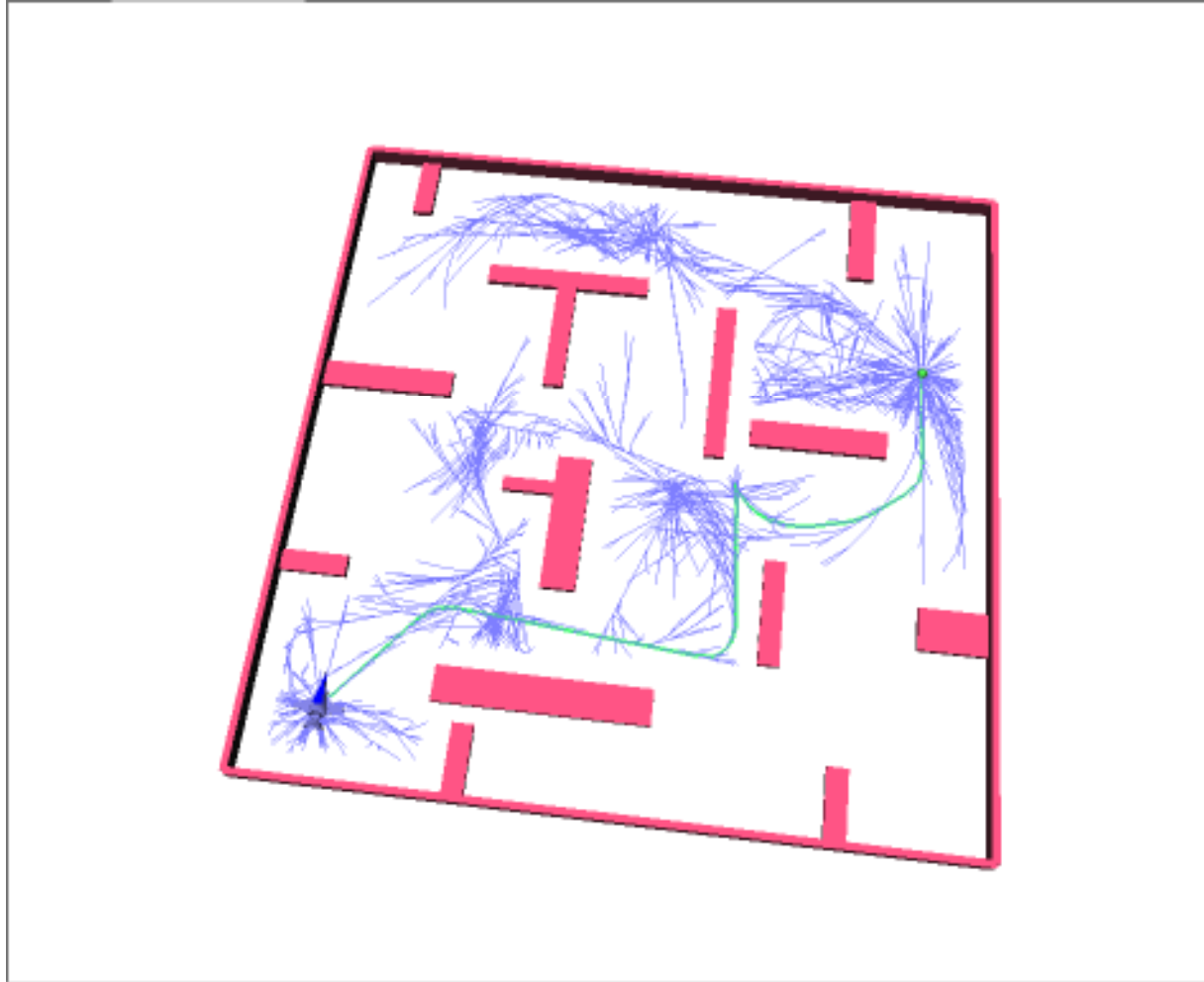


3 DOF single passway

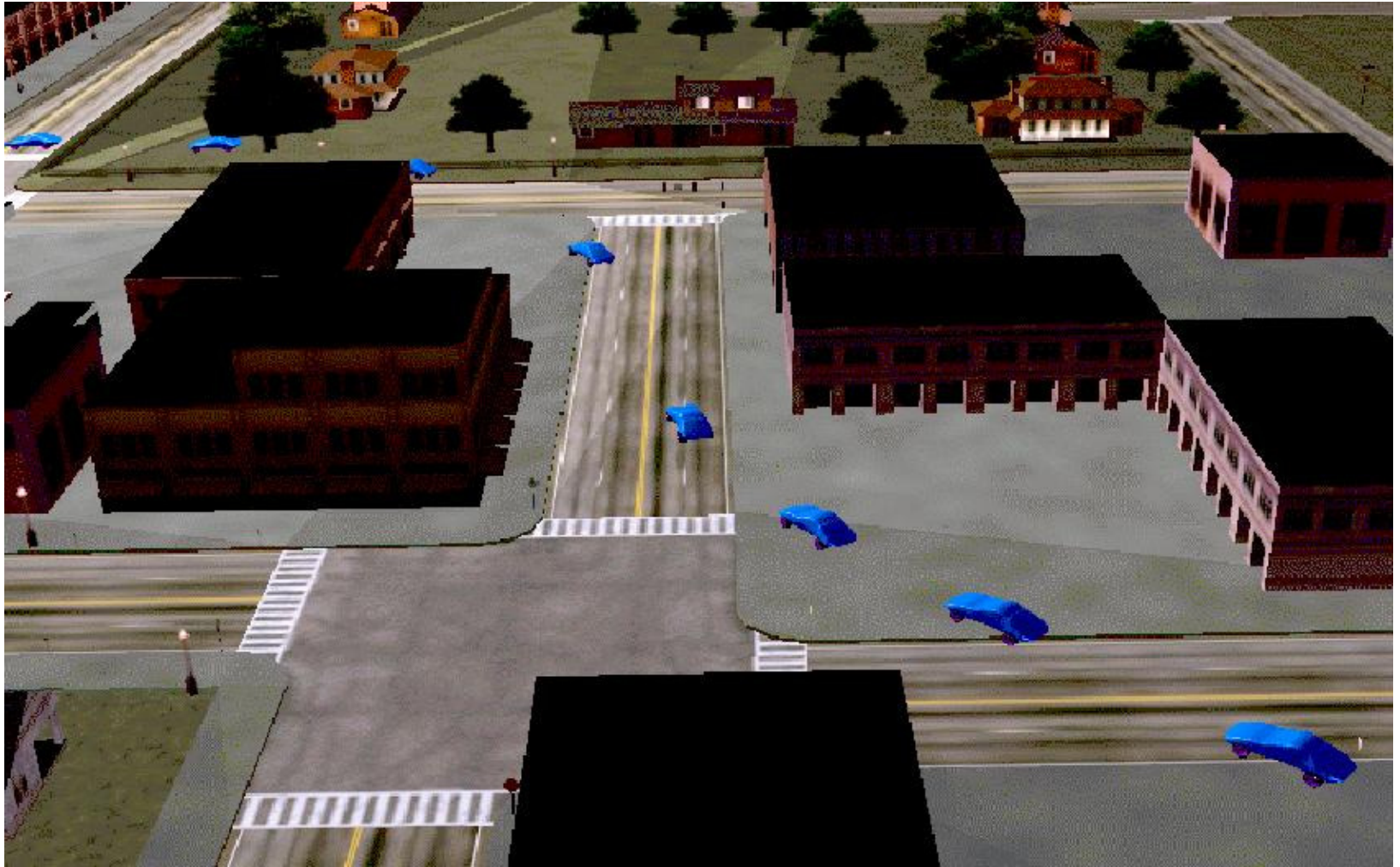
# RRT: Car like robot



# Hovercraft



# RRT Reckless Driver





## Driver's perspective



# Plans in ROS

[packages](#)   [repositories](#)     

## Found 22 packages relating to 'planner'

Name	Description
<a href="#">base_local_planner</a>	A local planner for a mobile base
<a href="#">base_planner_cu</a>	A 2D Path Planning System
<a href="#">carrot_planner</a>	carrot_planner
<a href="#">chomp_motion_planner</a>	CHOMP - Covariant Hamiltonian Optimization
<a href="#">doors_planner_core</a>	Door planning
<a href="#">interpolated_ik_motion_planner</a>	interpolated_ik_motion_planner
<a href="#">kipla</a>	Cram based kimp planner.
<a href="#">move_arm</a>	A general arm planning and control interface
<a href="#">move_base</a>	A general navigation stack
<a href="#">move_base_topo</a>	move_base_topo
<a href="#">mpbench</a>	tools for comparing motion planners
<a href="#">mpglue</a>	wrappers and tools for generically handling mot
<a href="#">navfn</a>	A fast interpolated navigation function
<a href="#">nav_core</a>	This package provides common interfaces for
<a href="#">orrosplanning</a>	OpenRAVE Plugin for ROS Planning
<a href="#">person_following_planner</a>	person_following_planner
<a href="#">sbpl_arm_planner</a>	Motion Planning Research for a Robotic Manip
<a href="#">sbpl_arm_planner_node</a>	A node to use the sbpl arm planner for the PR2
<a href="#">sbpl_door_planner</a>	Doorway Planning Research
<a href="#">sbpl_door_planner_action</a>	Doorway Planner Action
<a href="#">sbpl_global_planner</a>	sbpl_global_planner
<a href="#">sbpl_planner_node</a>	ROS ified SBPL planning

- OpenRave
- [planar\\_patch\\_map](#)  
(to convert point-cloud to polygonal structures)