CS 4700: Foundations of Artificial Intelligence
Spring 2020
Prelim Prep Questions
Quiz 7

1. True/False Questions:
   g. If a sentence in propositional logic is unsatisfiable then its negation is a tautology.
   h. If α and β are clauses in propositional logic, and all the literals in α are contained in β, then $\alpha \models \beta$.
   i. For any propositional sentences α, β, and γ, if $\alpha \models (\beta \land \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.
   j. If you know $(P \lor Q \lor R)$ and $(\neg P \lor \neg Q \lor R)$, resolution allows you to conclude R.

12. Resolution is *refutation complete*, in that if $KB \models \phi$ then if you negate ϕ and convert KB and the negated ϕ to CNF you are guaranteed to be able to generate the empty clause. But how about if you simply start with KB and apply resolution repeatedly until you get ϕ? Give an example of a KB and ϕ for which $KB \models \phi$ but you cannot prove ϕ directly from KB in this fashion.

13. Convert $\neg\,([(P \land \neg Q) \Rightarrow (R \lor S)] \land T)$ to CNF.

14. Fill in the missing information to make the following application of resolution correct:

15. Consider the following sentence in propositional logic.

$$P \land \neg Q \land (P \Rightarrow R) \land (\neg Q \lor W) \land (W \Rightarrow P) \land (\neg R \lor W)$$

    Show W using resolution.

16. Imagine you are using the hillclimbing approach to find a truth assignment that satisfies a given Boolean formula: You pick an initial random assignment, and at each step you flip one assignment guided by the difference in how many clauses are satisfied by the assignment. Is it possible that this algorithm will fail to find a truth assignment for some satisfiable sentences? If yes, give an example where this happens. If not, explain why not.

17. What is the result if you unify P(F(G(x)),x) with P(F(y),F(G(A)))?

18. What is the result if you unify P(F(G(x)),x) with P(y,F(G(y)))?

19. Imagine you have a scene with three blocks D, E, and F whose colors and relative positions are written in first-order logic:

    Green(D)
    ¬Green(F)
    On(D,E)
    On(E,F)

    Furthermore, you are also given:

    $\forall xyz\ [(On(x,y) \land On(y,z)) \Rightarrow On(x,z)]$

    Show using resolution that $\exists x\ \exists y\ [On(x,y) \land Green(x) \land \neg Green(y)]$.

Solutions

1. True/False Questions:
   g. If a sentence in propositional logic is unsatisfiable then its negation is a tautology.
      True. If it unsatisfiable it means its truth value is False for all truth assignments. This means the truth assignment for the negation of the sentence will be True for all assignments. That's the definition of being a tautology.
   h. If α and β are clauses in propositional logic, and all the literals in α are contained in β, then $\alpha \models \beta$.
      True. β has everything that's in α only with additional disjuncts. If α is true then all the truth assignments that make it true will trivially make β true as well – a disjunction can't be converted to False by adding more disjuncts to it.
   i. For any propositional sentences α, β, and γ, if $\alpha \models (\beta \wedge \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.
      True. To state it succinctly, $\alpha \models (\beta \wedge \gamma)$ and $(\beta \wedge \gamma) \models \beta$. By the definition of entailment this means, by transitivity, $\alpha \models \beta$ (and similarly for γ).
   j. If you know $(P \vee Q \vee R)$ and $(\neg P \vee \neg Q \vee R)$, resolution allows you to conclude R.
      False. You can't resolve on two different propositional symbols (like both P and Q) simultaneously.

12. Resolution is *refutation complete*, in that if $KB \models \phi$ then if you negate ϕ and convert KB and the negated ϕ to CNF you are guaranteed to be able to generate the empty clause. But how about if you simply start with KB and apply resolution repeatedly until you get ϕ? Give an example of a KB and ϕ for which $KB \models \phi$ but you cannot prove ϕ directly from KB in this fashion.

    There are many, here's one: KB = P, $\phi = (P \vee Q)$. $KB \models \phi$, but you can't derive ϕ from KB with any number of resolution inference steps.

13. Convert $\neg ([(P \wedge \neg Q) \Rightarrow (R \vee S)] \wedge T)$ to CNF.

    $\neg ([(P \wedge \neg Q) \Rightarrow (R \vee S)] \wedge T)$

    $\neg ([\neg (P \wedge \neg Q) \vee (R \vee S)] \wedge T)$

    $(\neg [\neg (P \wedge \neg Q) \vee (R \vee S)] \vee \neg T)$

    $([\neg\neg (P \wedge \neg Q) \wedge \neg (R \vee S)] \vee \neg T)$

    $((P \wedge \neg Q \wedge \neg R \wedge \neg S) \vee \neg T)$

    $(P \vee \neg T) \wedge (\neg Q \vee \neg T) \wedge (\neg R \vee \neg T) \wedge (\neg S \vee \neg T)$

14. Fill in the missing information to make the following application of resolution correct:

    $\neg P \vee \neg T \vee$ __¬S__

    $S \vee Q \vee$ __¬R__ $\vee U$

    ____

    $\neg P \vee \neg T \vee Q \vee \neg R \vee U$

15. Consider the following sentence in propositional logic.

$$P \wedge \neg Q \wedge (P \Rightarrow R) \wedge (\neg Q \vee W) \wedge (W \Rightarrow P) \wedge (\neg R \vee W)$$

Show W using resolution.

Converting the initial sentence and the negated goal to CNF gives the following result:

1	P	KB
2	$\neg Q$	KB
3	$(\neg P \vee R)$	KB
4	$(\neg Q \vee W)$	KB
5	$(\neg W \vee P)$	KB
6	$(\neg R \vee W)$	KB
7	$\neg W$	Negated goal
8	$\neg R$	6,7
9	$\neg P$	3,8
10	{}	1,9

Notice that none of lines 2, 4, or 5 are used in the proof. There is nothing wrong with that.

16. Imagine you are using the hillclimbing approach to find a truth assignment that satisfies a given Boolean formula: You pick an initial random assignment, and at each step you flip one assignment guided by the difference in how many clauses are satisfied by the assignment. Is it possible that this algorithm will fail to find a truth assignment for some satisfiable sentences? If yes, give an example where this happens. If not, explain why not.

It can happen, and here's one example.

Sentence: $(P \vee Q) \wedge (\neg P \vee \neg R) \wedge R$

Initial truth assignment: P=True, Q=False, R=False

The initial assignment satisfies two clauses.

There are three variables, so three values that might be flipped:

- If you flip P you get P=False, Q=False, R=False, which satisfies 1 clause
- If you flip Q you get P=True, Q=True, R=False, which satisfies 2 clauses
- If you flip R you get P=True, Q=False, R=True, which satisfies 2 clauses

None of these improves on the number of sentences satisfied, so hillclimbing would stop without finding a satisfying assignment.

17. What is the result if you unify P(F(G(x)),x) with P(F(y),F(G(A)))?

[y/G(F(G(A))), x/F(G(A))] or [y/G(x), x/F(G(A))]

18. What is the result if you unify P(F(G(x)),x) with P(y,F(G(y)))?

Fail

19. Imagine you have a scene with three blocks D, E, and F whose colors and relative positions are written in first-order logic:

Green(D)
¬Green(F)
On(D,E)
On(E,F)

Furthermore, you are also given:

∀xyz [(On(x,y) ∧ On(y,z)) ⇒ On(x,z)]

Show using resolution that ∃x ∃y [On(x,y) ∧ Green(x) ∧ ¬Green(y)].

i. Convert KB to CNF: This is trivial for the first four facts, you get what you started with:

Green(D)
¬Green(F)
On(D,E)
On(E,F)

For ¬∀xyz [(On(x,y) ∧ On(y,z)) ⇒ On(x,z)] you get

(¬On(x1,y1) ∨ ¬On(y1,z1) ∨ On(x1,z1))

ii. Negate the goal and convert to CNF:

¬ ∃x ∃y [On(x,y) ∧ Green(x) ∧ ¬Green(y)]
∀x ∀y ¬ [On(x,y) ∧ Green(x) ∧ ¬Green(y)]
∀x ∀y (¬On(x,y) ∨ ¬Green(x) ∨ Green(y))

This leaves us with:

(¬On(x2,y2) ∨ ¬Green(x2) ∨ Green(y2))

iii. Finally, the resolution proof (just one of multiple options):

1	Green(D)	KB
2	¬Green(F)	KB
3	On(D,E)	KB
4	On(E,F)	KB
5	(¬On(x1,y1) ∨ ¬On(y1,z1) ∨ On(x1,z1))	KB
6	(¬On(x2,y2) ∨ ¬Green(x2) ∨ Green(y2))	Negated Goal
7	(¬On(E,z2) ∨ On(D,z2))	[3, 5, x1/D, y1/E]
8	On(D,F)	[4, 7, z2/F]
9	(¬Green(D) ∨ Green(F))	[6, 8, x2/D, y2/F]
10	Green(F)	[1, 9]
11	()	[2,10]