CS 4700, Foundations of Artificial Intelligence
Spring 2020
Solutions to Quiz 8

There were three possible questions. They all were identical except for the last row of the table and the specifics of the new example to label:

Consider the following set of training data. The first attribute x_1 will take on one of the values "A", "B", or "C". The second attribute x_2 will take on one of the values "X", "Y", or "Z". The label will take on one of the values "1", "2", or "3".

x_1	x_2	y
A	X	1
B	Y	2
C	Z	3
<see below>	<see below>	<see below>

You are then given a new example to label for which x_1=<see below> and x_2=<see below>. (If you have a tie between two choices pick whichever comes alphabetically earliest or is numerically lowest.)

What category would Naive Bayes assign to this example if Laplace smoothing were not used (answer 1, 2, or 3)?
Please specify the values you computed for category 1: ____ category 2: _____ category 3: _____
(These values will only be consulted in case questions arise about the grading of this question.)

What category would Naive Bayes assign if Laplace smoothing is used for the two attributes, where α=1 (answer 1, 2, or 3)?
Please specify the values you computed for category 1: ____ category 2: _____ category 3: _____
(These values will only be consulted in case questions arise about the grading of this question.)

Version 1:

Last line of table:

x_1	x_2	y
A	Z	1

Values of the example being asked about:

x_1	x_2
B	Y

$P(c=1) = 2/4$ $P(c=2) = 1/4$ $P(c=3) = 1/4$
$P(x_1=B|c=1) = 0$ $P(x_1=B|c=2) = 1$ $P(x_1=B|c=3) = 0$
$P(x_2=Y|c=1) = 0$ $P(x_2=Y|c=2) = 1$ $P(x_2=Y|c=3) = 0$

$P(c=1)P(x_1=B|c=1)P(x_2=Y|c=1) = 2/4 \times 0 \times 0 = 0$
$P(c=2)P(x_1=B|c=2)P(x_2=Y|c=2) = 1/4 \times 1 \times 1 = 1/4$
$P(c=3)P(x_1=B|c=3)P(x_2=Y|c=3) = 1/4 \times 0 \times 0 = 0$

So Naïve Bayes would label it with category 2.

If we add Laplace smoothing in with α=1:

$P(c=1) = 2/4$ $P(c=2) = 1/4$ $P(c=3) = 1/4$
$P(x_1=B|c=1) = 1/5$ $P(x_1=B|c=2) = 2/4$ $P(x_1=B|c=3) = 1/4$
$P(x_2=Y|c=1) = 1/5$ $P(x_2=Y|c=2) = 2/4$ $P(x_2=Y|c=3) = 1/4$

$P(c=1)P(x_1=B|c=1)P(x_2=Y|c=1) = 2/4 \times 1/5 \times 1/5 = 1/50$
$P(c=2)P(x_1=B|c=2)P(x_2=Y|c=2) = 1/4 \times 2/4 \times 2/4 = 1/16$
$P(c=3)P(x_1=B|c=3)P(x_2=Y|c=3) = 1/4 \times 1/4 \times 1/4 = 1/64$

So Naïve Bayes would still label it with category 2.

Version 2:

Last line of table:

x_1	x_2	y
B	Z	3

Values of the example being asked about:

x_1	x_2
A	X

$P(c=1) = 1/4$ $P(c=2) = 1/4$ $P(c=3) = 2/4$
$P(x_1=A|c=1) = 1$ $P(x_1=A|c=2) = 0$ $P(x_1=A|c=3) = 0$
$P(x_2=X|c=1) = 1$ $P(x_2=X|c=2) = 0$ $P(x_2=X|c=3) = 0$

$P(c=1)P(x_1=A|c=1)P(x_2=X|c=1) = 1/4 \times 1 \times 1 = 1/4$
$P(c=2)P(x_1=A|c=2)P(x_2=X|c=2) = 1/4 \times 0 \times 0 = 0$
$P(c=3)P(x_1=A|c=3)P(x_2=X|c=3) = 2/4 \times 0 \times 0 = 0$

So Naïve Bayes would label it with category 1.

If we add Laplace smoothing in with α=1:

$P(c=1) = 1/4$ $P(c=2) = 1/4$ $P(c=3) = 2/4$
$P(x_1=A|c=1) = 2/4$ $P(x_1=A|c=2) = 1/4$ $P(x_1=A|c=3) = 1/5$
$P(x_2=X|c=1) = 2/4$ $P(x_2=X|c=2) = 1/4$ $P(x_2=X|c=3) = 1/5$

$P(c=1)P(x_1=A|c=1)P(x_2=X|c=1) = 1/4 \times 2/4 \times 2/4 = 1/16$
$P(c=2)P(x_1=A|c=2)P(x_2=X|c=2) = 1/4 \times 1/4 \times 1/4 = 1/64$
$P(c=3)P(x_1=A|c=3)P(x_2=X|c=3) = 2/4 \times 1/5 \times 1/5 = 1/50$

So Naïve Bayes would still label it with category 1.

Version 3:

Last line of table:

x_1	x_2	y
A	Y	2

Values of the example being asked about:

x_1	x_2
C	Z

$P(c=1) = 1/4$ $P(c=2) = 2/4$ $P(c=3) = 1/4$
$P(x_1=C|c=1) = 0$ $P(x_1=C|c=2) = 0$ $P(x_1=C|c=3) = 1$
$P(x_2=Z|c=1) = 0$ $P(x_2=Z|c=2) = 0$ $P(x_2=Z|c=3) = 1$

$P(c=1)P(x_1=C|c=1)P(x_2=Z|c=1) = 1/4 \times 0 \times 0 = 0$
$P(c=2)P(x_1=C|c=2)P(x_2=Z|c=2) = 2/4 \times 0 \times 0 = 0$
$P(c=3)P(x_1=C|c=3)P(x_2=Z|c=3) = 1/4 \times 1 \times 1 = 1/4$

So Naïve Bayes would label it with category 3.

If we add Laplace smoothing in with $\alpha=1$:

$P(c=1) = 1/4$ $P(c=2) = 1/4$ $P(c=3) = 2/4$
$P(x_1=A|c=1) = 2/4$ $P(x_1=A|c=2) = 1/4$ $P(x_1=A|c=3) = 1/5$
$P(x_2=X|c=1) = 2/4$ $P(x_2=X|c=2) = 1/4$ $P(x_2=X|c=3) = 1/5$

$P(c=1)P(x_1=A|c=1)P(x_2=X|c=1) = 1/4 \times 2/4 \times 2/4 = 1/16$
$P(c=2)P(x_1=A|c=2)P(x_2=X|c=2) = 1/4 \times 1/4 \times 1/4 = 1/64$
$P(c=3)P(x_1=A|c=3)P(x_2=X|c=3) = 2/4 \times 1/5 \times 1/5 = 1/50$

$P(c=1) = 1/4$ $P(c=2) = 2/4$ $P(c=3) = 1/4$
$P(x_1=C|c=1) = 1/4$ $P(x_1=C|c=2) = 1/5$ $P(x_1=C|c=3) = 2/4$
$P(x_2=Z|c=1) = 1/4$ $P(x_2=Z|c=2) = 1/5$ $P(x_2=Z|c=3) = 2/4$

$P(c=1)P(x_1=C|c=1)P(x_2=Z|c=1) = 1/4 \times 1/4 \times 1/4 = 1/64$
$P(c=2)P(x_1=C|c=2)P(x_2=Z|c=2) = 2/4 \times 1/5 \times 1/5 = 1/50$
$P(c=3)P(x_1=C|c=3)P(x_2=Z|c=3) = 1/4 \times 2/4 \times 2/4 = 1/16$

So Naïve Bayes would still label it with category 3.