# CS 4700:
# Foundations of Artificial Intelligence

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**Module: Knowledge, Reasoning, and Planning**

## Logical Agents
## Representing Knowledge and Inference

**R&N: Chapter 7**

# Illustrative example: Wumpus World

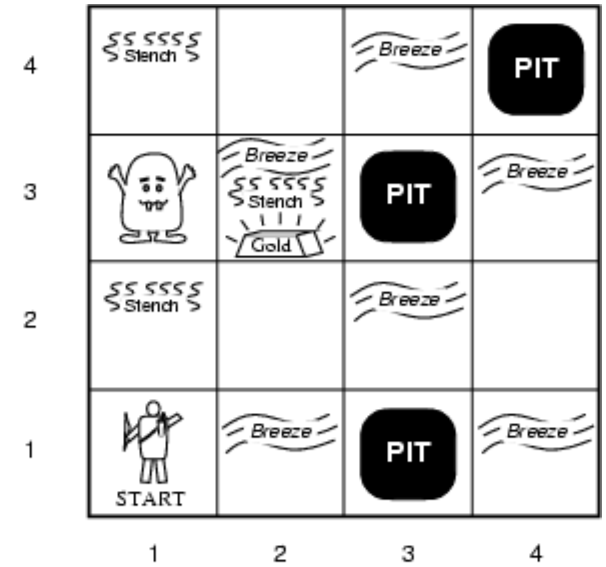
**Performance measure**

- **gold +1000,**
- **death -1000**

**(falling into a pit or being eaten by the wumpus)**

- **-1 per step, -10 for using the arrow**

(Somewhat whimsical!)




**Environment**

- **Rooms / squares connected by doors.**
- **Squares adjacent to wumpus are smelly**
- **Squares adjacent to pit are breezy**
- **Glitter iff gold is in the same square**
- **Shooting kills wumpus if you are facing it**
- **Shooting uses up the only arrow**
- **Grabbing picks up gold if in same square**
- **Releasing drops the gold in same square**
- **World randomly generated at start of game.**
- ***Wumpus only senses current room.***

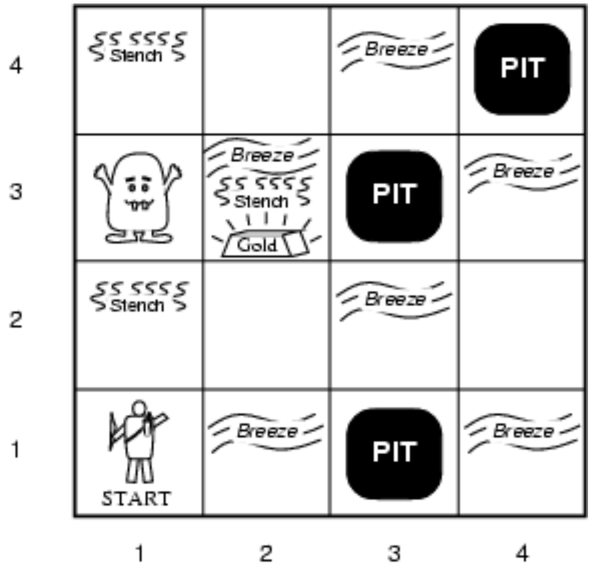
**Sensors:** **Stench, Breeze, Glitter, Bump, Scream [perceptual inputs]**

**Actuators:** **Left turn, Right turn, Forward, Grab, Release, Shoot**

# Wumpus world characterization

| | |
|---|---|
| **Fully Observable** | **No – only local perception** |
| **Deterministic** | **Yes – outcomes exactly specified** |
| **Static** | **Yes – Wumpus and Pits do not move** |
| **Discrete** | **Yes** |
| **Single-agent?** | **Yes – Wumpus is essentially a "natural feature."** |

# Exploring a wumpus world



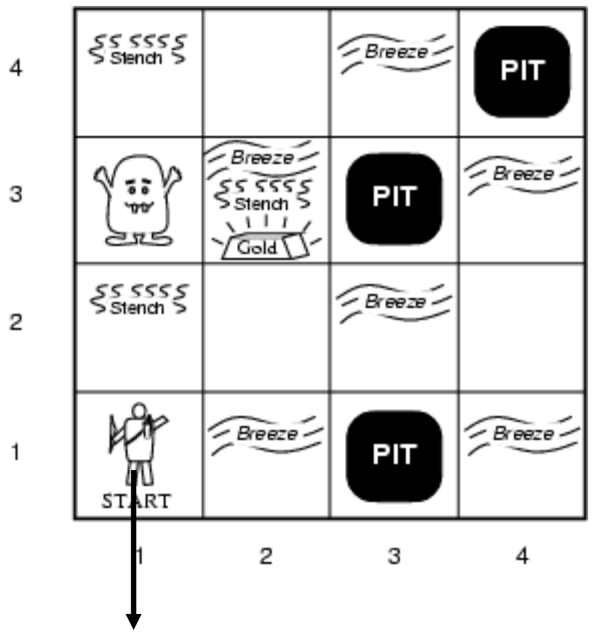

The knowledge base of the agent consists of the rules of the Wumpus world plus the percept "nothing" in [1,1]

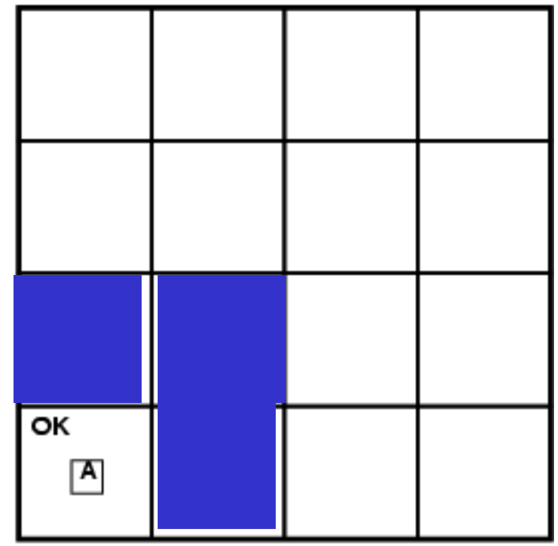
None, none, none, none, none

Stench, Breeze, Glitter, Bump, Scream

Boolean percept feature values:
<0, 0, 0, 0, 0>

## World “known” to agent at time = 0.



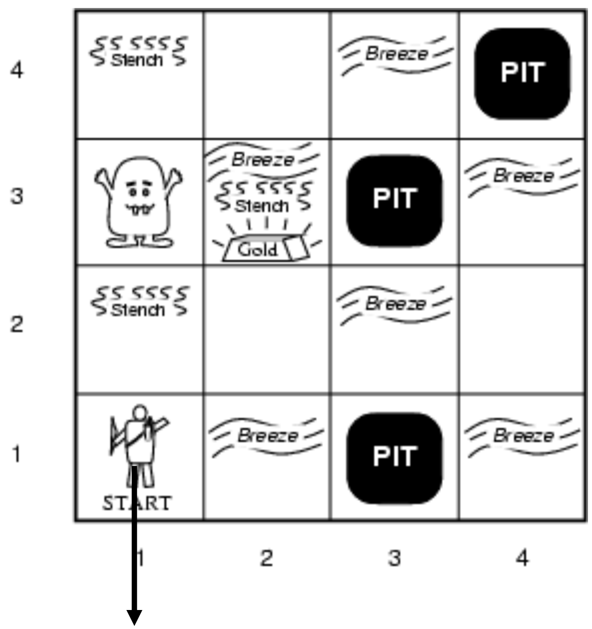

None, none, none, none, none

Stench, Breeze, Glitter, Bump, Scream

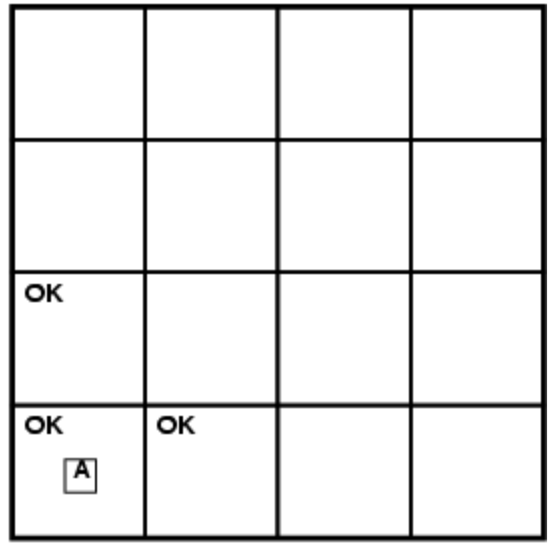
**T=0 The KB of the agent consists of the rules of the Wumpus world plus the percept “nothing” in [1,1].**
***By inference, the agent’s knowledge base also has the information that [1,2] and [2,1] are okay. (Why?)***
**Added as propositions.**

# Further exploration

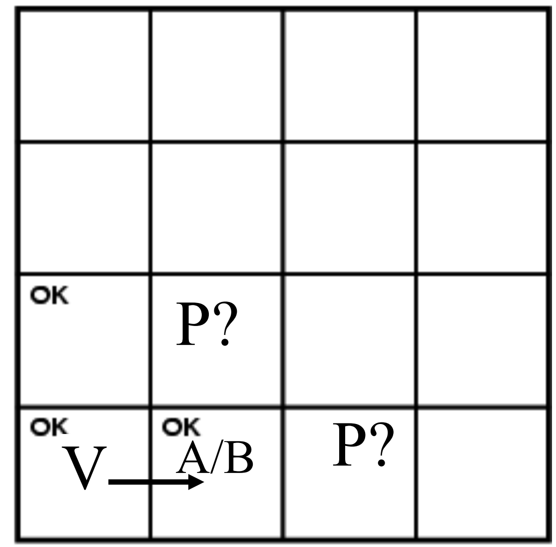
T = 0




None, none, none, none, none




T = 1




None, breeze, none, none, none

**A – agent**

**V – visited**

**B - breeze**

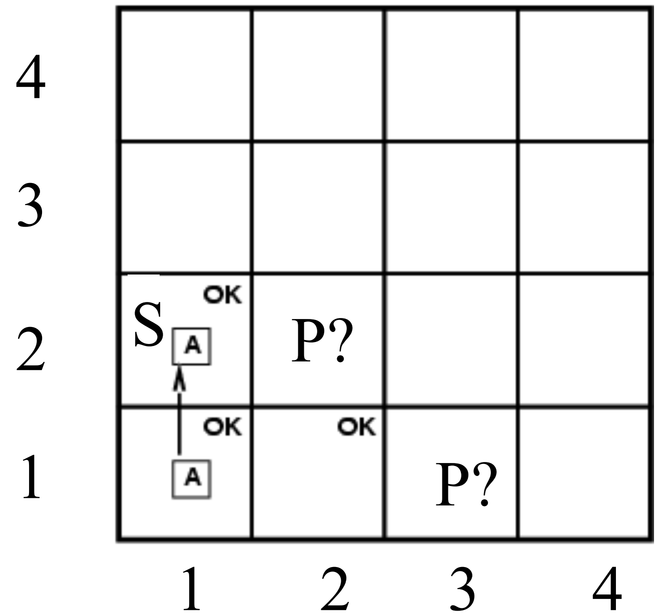
Stench, Breeze, Glitter, Bump, Scream

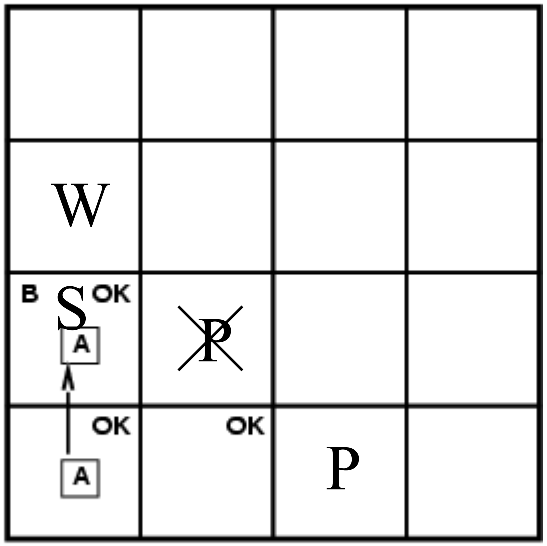
**Where next?**

**@ T = 1 What follows?**

**Pit(2,2) or Pit(3,1)**

## T=3



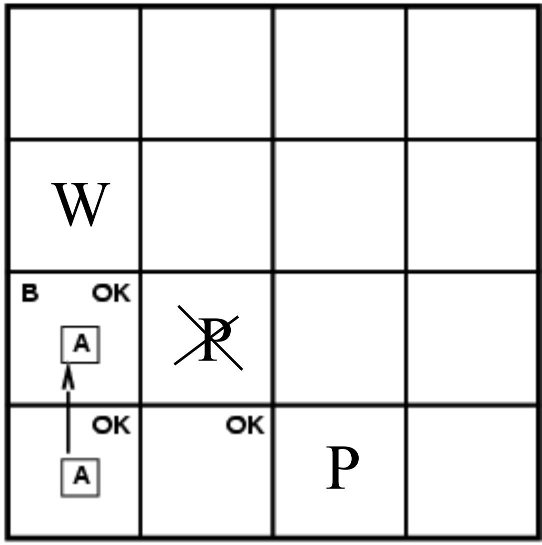

**Stench, none, none, none, none**

Stench, Breeze, Glitter, Bump, Scream

**Where is Wumpus?**

**Wumpus cannot be in (1,1) or in (2,2) (Why?)➔ Wumpus in (1,3)**
**Not breeze in (1,2) ➔ no pit in (2,2); but we know there is pit in (2,2) or (3,1) ➔ pit in (3,1)**

**We reasoned about the possible states the Wumpus world can be in, given our percepts and our knowledge of the rules of the Wumpus world.**
**I.e., the content of KB at T=3.**




**What follows is what holds true in all those worlds that satisfy what is known at that time T=3 about the particular Wumpus world we are in.**

Example property: P_in_(3,1)

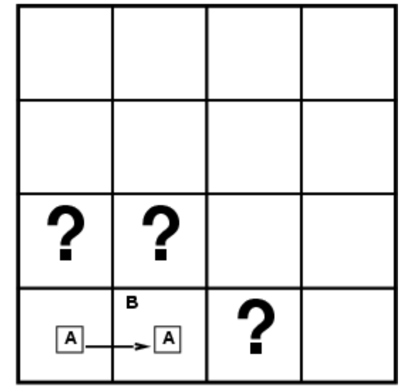
**Models(KB) $\subseteq$ Models(P_in_(3,1))**

**Essence of logical reasoning:**
**Given *all we know*, Pit_in_(3,1) holds.**
**(“The world cannot be different.”)**

# Formally: Entailment

**Knowledge Base (KB) in the Wumpus World → Rules of the wumpus world + new percepts**

**Situation after detecting nothing in [1,1], moving right, breeze in [2,1]. I.e. T=1.**



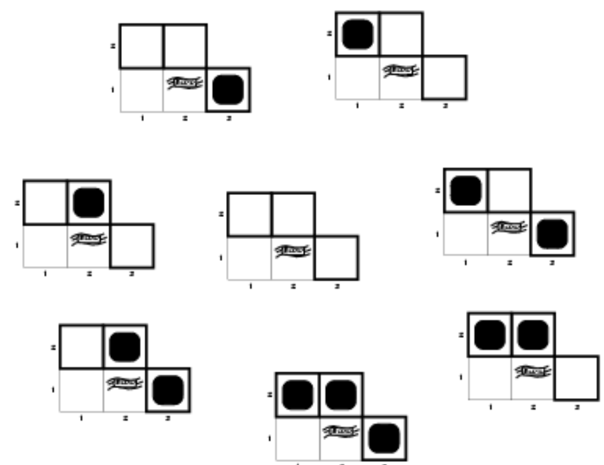

T = 1

**Consider possible models for *KB* with respect to the cells (1,2), (2,2) and (3,1), with respect to the existence or non existence of pits**

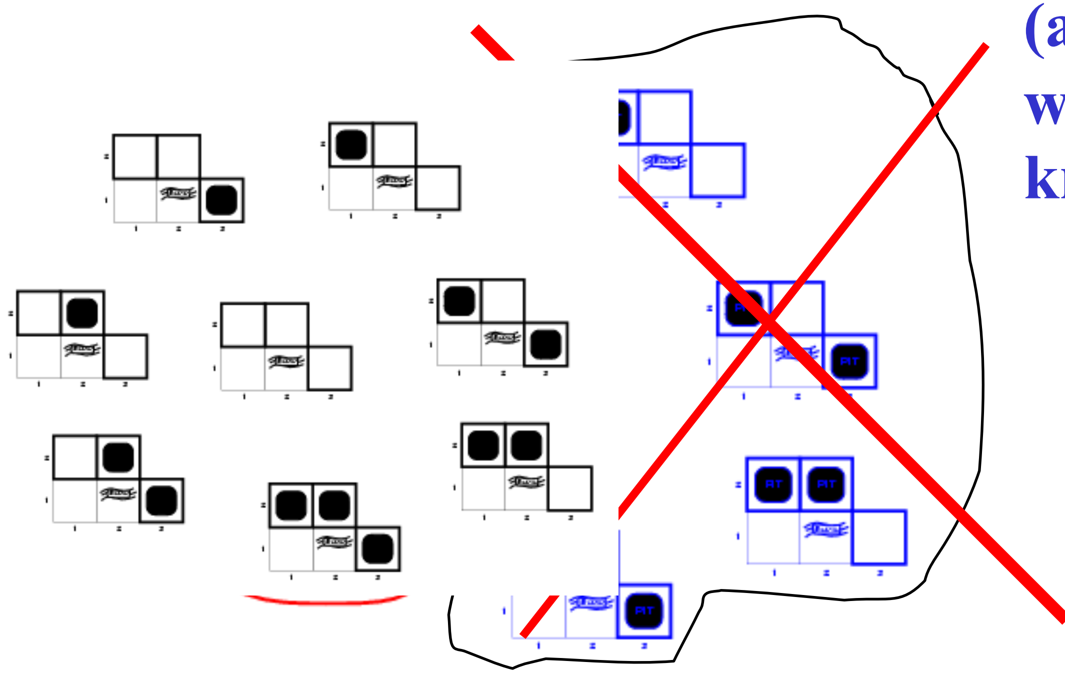
**3 Boolean choices ⇒**

**8 possible interpretations**

**(enumerate all the models or "possible worlds" wrt Pit location)**

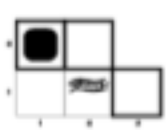


Is KB consistent with all
8 possible worlds?



Worlds
that violate KB
(are inconsistent
with what we
know)

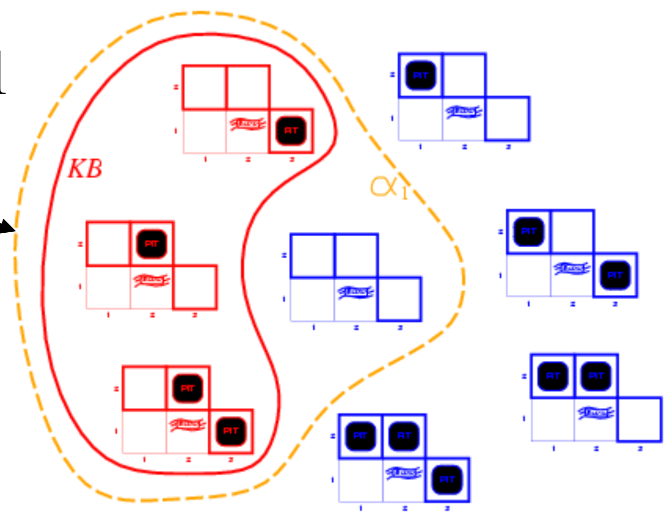
*KB* = Wumpus-world rules + observations (T=1)

Q: Why does world  violate KB?

# Entailment in Wumpus World

**So, KB defines all worlds that we hold possible.**
**Queries: we want to know the properties of those worlds.**
**That's how the semantics of logical entailment is defined.**

Models of the KB and α1




Note: \alpha_1 holds in more models than KB. That's OK, but we don't care about those worlds.

**$KB$ = Wumpus-world rules + observations**

**$\alpha_1$ = "[1,2] has no pit", $KB \models \alpha_1$**

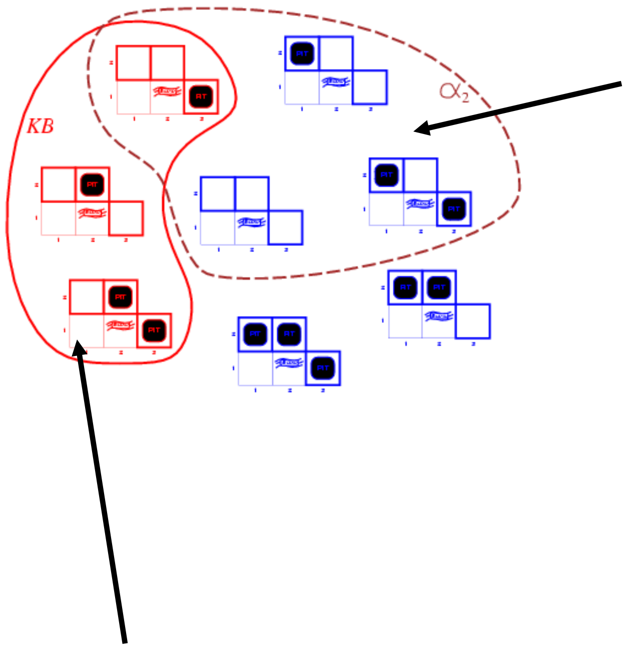
- **In every model in which KB is true, $\alpha_1$ is True (proved by "model checking")**

# Wumpus models

**KB = wumpus-world rules + observations**

**$\alpha 2$ = "[2,2] has no pit", this is only True in some of the models for which KB is True, therefore KB $\not\models$ $\alpha 2$**

Model Checking




Models of $\alpha 2$

**A model of KB where $\alpha 2$ does NOT hold!**

# Entailment via "Model Checking"

**Inference by Model checking –**

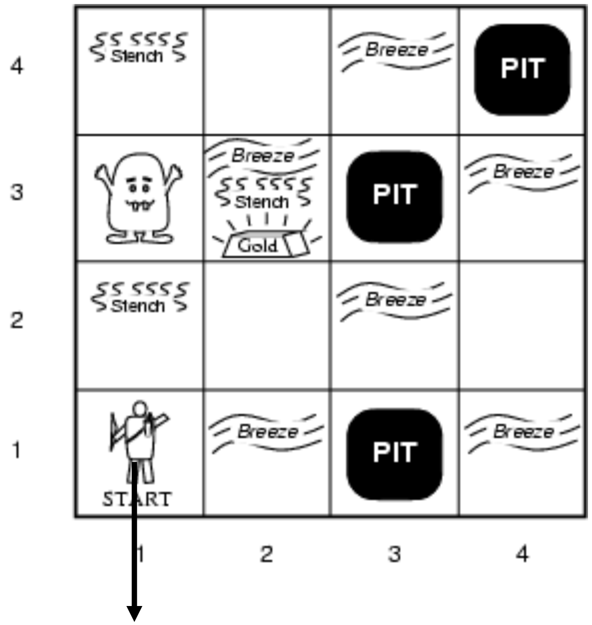
**We enumerate all the KB models and check if $\alpha_1$ and $\alpha_2$ are True in all the models (which implies that we can only use it when we have a finite number of models).**

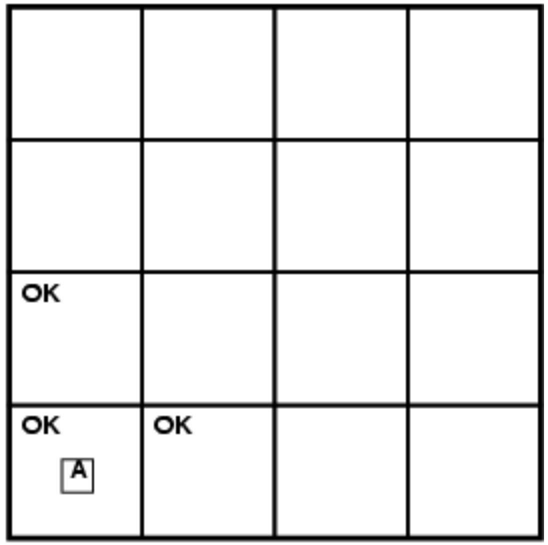
**I.e. using semantics directly.**

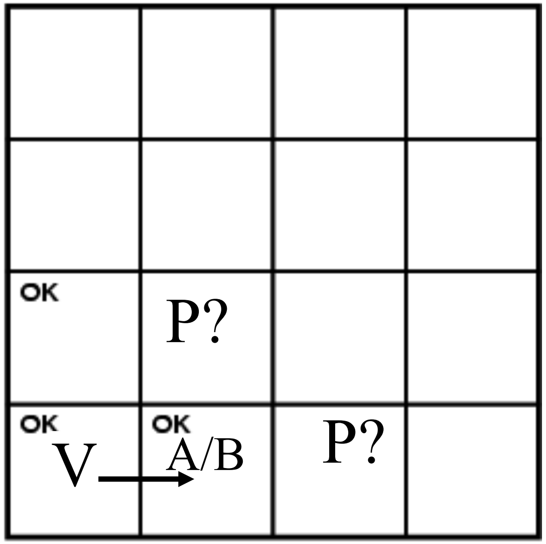
$$\text{Models(KB)} \subseteq \text{Models}(\alpha)$$

$$KB \models \alpha$$

# Example redux: More formal




**None, none, none, none, none**

**None, breeze, none, none, none**

Stench, Breeze, Glitter, Bump, Scream

**A – agent**

**V – visited**

**B - breeze**

**How do we actually encode background knowledge and percepts in formal language?**

# Wumpus World KB

**Define propositions:**

**Let $P_{i,j}$ be true if there is a pit in [i, j].**

**Let $B_{i,j}$ be true if there is a breeze in [i, j].**

**Sentence 1 (R1):** $\neg P_{1,1}$ [Given.]

**Sentence 2 (R2):** $\neg B_{1,1}$ [Observation T = 0.]

**Sentence 3 (R3):** $B_{2,1}$ [Observation T = 1.]

**"Pits cause breezes in adjacent squares"**

**Sentence 4 (R4):** $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

**Sentence 5 (R5):** $B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

**etc.**

**Notes: (1) one such statement about Breeze for each square.**

**(2) similar statements about Wumpus, and stench and Gold and glitter. (Need more propositional letters.)**

# What about Time? What about Actions?

**Is Time represented?**

**No!**

**Can include time in propositions:**

- **Explicit time** $P_{i,j,t}$ $B_{i,j,t}$ $L_{i,j,t}$ **etc.**
- **Many more props: $O(TN^2)$ ($L_{i,j,t}$ for agent at (i,j) at time t)**

**Now, we can also model actions, use props: Move(i, j, k, l ,t)**

- **E.g. Move(1, 1, 2, 1, 0)**

**What knowledge axiom(s) capture(s) the effect of an Agent move?**

$$\text{Move}(i, j, k, l, t) \Rightarrow (\neg L(i, j, t+1) \wedge L(k, l, t+1))$$

**Is this it?**

**What about i, j, k, and l?**

**What about Agent location at time t?**

**Improved:** *Move implies a change in the world state;*
*a change in the world state, implies a move occurred!*

**Move(i, j, k, l ,t) $\Leftrightarrow$ (L(i, j, t) $\wedge \neg$ L(i, j, t+1) $\wedge$ L(k, l, t+1))**

**For all tuples (i, j, k, l) that represent legitimate possible moves.**
**E.g. (1, 1, 2, 1) or (1, 1, 1, 2)**

**Still, some remaining subtleties when representing time and actions. What happens to propositions at time t+1 compared to at time t, that are *not* involved in any action?**

**E.g. P(1, 3, 3) is derived at some point.**
**What about P(1, 3, 4), True or False?**

**R&N suggests having P as an "atemporal var" since it cannot change over time. Nevertheless, we have many other vars that can change over time, called "fluents".**

**Values of propositions not involved in any action should not change! "The Frame Problem" / Frame Axioms R&N 7.7.1**

# Successor-State Axioms

## Axiom schema:

**F is a fluent (prop. that can change over time)**

For example:

$$L_{1,1}^{t+1} = (L_{1,1}^{t} \wedge (\neg Forward^{t} \vee Bump^{t+1}))$$
$$\vee (L_{1,2}^{t} \wedge (South^{t} \wedge Forward^{t}))$$
$$\vee (L_{2,1}^{t} \wedge (West^{t} \wedge Forward^{t}))$$

**i.e. L_1,1 was "as before" with [no movement action or bump into wall] or resulted from some action (movement into L_1,1).**

## Actions and inputs up to time 6

### Note: includes turns!

## Some example inferences
## Section 7.7.1 R&N

$$\neg Stench^0 \wedge \neg Breeze^0 \wedge \neg Glitter^0 \wedge \neg Bump^0 \wedge \neg Scream^0 \; ; \; Forward^0$$
$$\neg Stench^1 \wedge Breeze^1 \wedge \neg Glitter^1 \wedge \neg Bump^1 \wedge \neg Scream^1 \; ; \; TurnRight^1$$
$$\neg Stench^2 \wedge Breeze^2 \wedge \neg Glitter^2 \wedge \neg Bump^2 \wedge \neg Scream^2 \; ; \; TurnRight^2$$
$$\neg Stench^3 \wedge Breeze^3 \wedge \neg Glitter^3 \wedge \neg Bump^3 \wedge \neg Scream^3 \; ; \; Forward^3$$
$$\neg Stench^4 \wedge \neg Breeze^4 \wedge \neg Glitter^4 \wedge \neg Bump^4 \wedge \neg Scream^4 \; ; \; TurnRight^4$$
$$\neg Stench^5 \wedge \neg Breeze^5 \wedge \neg Glitter^5 \wedge \neg Bump^5 \wedge \neg Scream^5 \; ; \; Forward^5$$
$$Stench^6 \wedge \neg Breeze^6 \wedge \neg Glitter^6 \wedge \neg Bump^6 \wedge \neg Scream^6$$

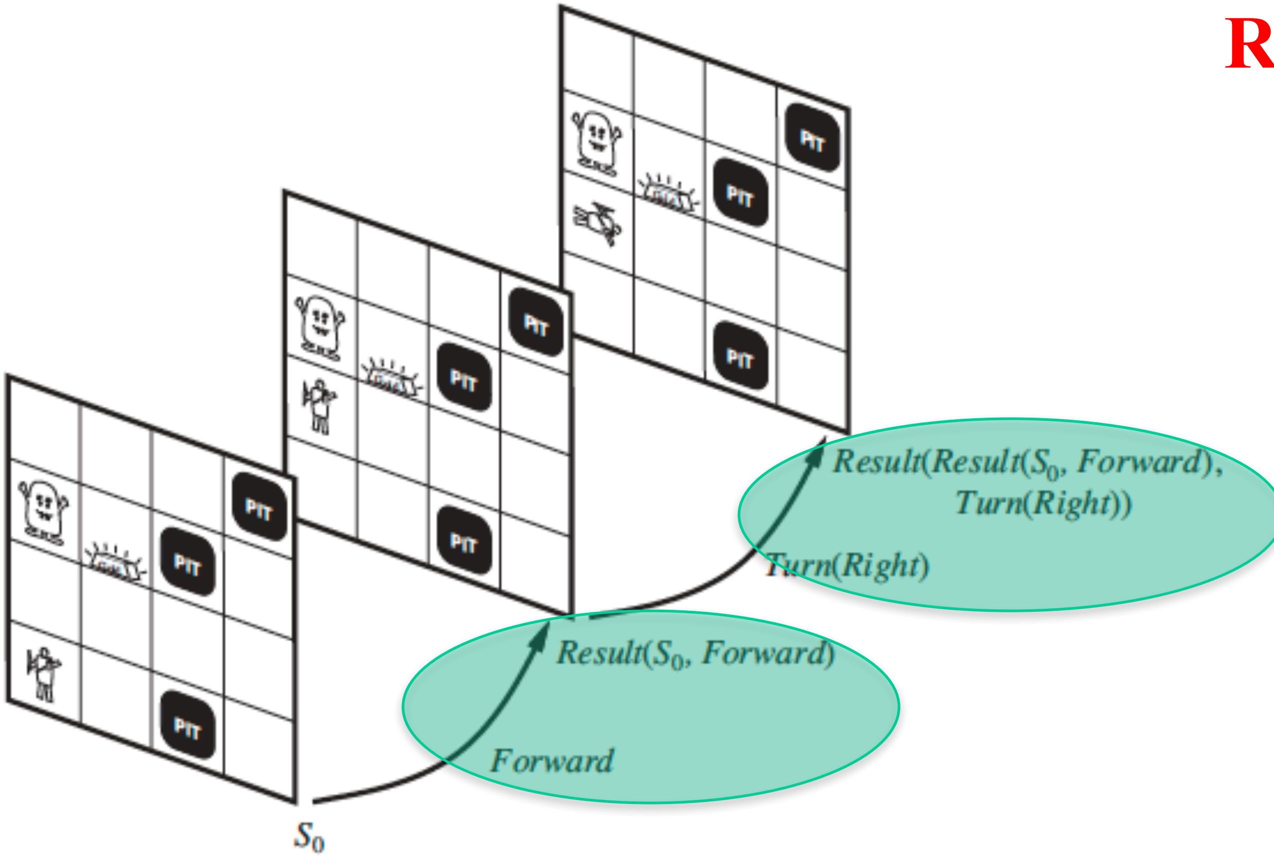
$\textsc{Ask}(KB, P_{3,1}) = true$ $\textsc{Ask}(KB, W_{1,3}) = true$

Define "OK": $OK^t_{x,y} \Leftrightarrow \neg P_{x,y} \wedge \neg(W_{x,y} \wedge WumpusAlive^t)$

$\textsc{Ask}(KB, OK^6_{2,2}) = true$, so the square $[2,2]$ is OK

In milliseconds, with modern SAT solver.

# Alternative formulation: Situation Calculus R&N 10.4.2




No explicit time. Actions are what changes the world from "situation" to "situation". More elegant, but still need frame axioms to capture what stays the same. Inherent with many representation formalisms: "physical" persistance does not come for free! (and probably shouldn't)

# Inference by enumeration / "model checking" Style I

**The goal of logical inference is to decide whether $KB \models \alpha$, for some $\alpha$.**

**For example, given the rules of the Wumpus World, is $P_{22}$ entailed?** Relevant propositional symbols:

R1: $\neg P_{1,1}$

R2: $\neg B_{1,1}$

R3: $B_{2,1}$

"Pits cause breezes in adjacent squares"

R4: $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

R5: $B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

$$\text{Models(KB)} \overset{?}{\subseteq} \text{Models}(P_{22})$$

**Inference by enumeration. We have 7 relevant symbols Therefore $2^7 = 128$ interpretations. Need to check if $P_{22}$ is true in all of the KB models (interpretations that satisfy KB sentences).**

**Q.: KB has many more symbols. Why can we restrict ourselves to these symbols here?**

But, be careful, typically we can't!!

All equivalent
Prop. / FO Logic

1) $\text{KB} \models \alpha$ **entailment**

# Proof techniques

$M(KB) \subseteq M(\alpha)$

by defn. / semantic proofs / truth tables
"model checking"
**(style I, R&N 7.4.4) Done.**

$KB \vdash \alpha$

soundness and completeness
logical deduction / symbol pushing
**proof by inference rules (style II)**
e.g. modus ponens (R&N 7.5.1)

$(KB \wedge \neg \alpha)$ is inconsistent

Proof by contradiction
use CNF / clausal form
**Resolution (style III, R&N 7.5)**
**SAT solvers (style IV, R&N 7.6)**
most effective

## Standard syntax and semantics for propositional logic. (CS-2800; see 7.4.1 and 7.4.2.)

Syntax:

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} & \rightarrow & (\ \mathit{Sentence}\ ) \mid [\ \mathit{Sentence}\ ] \\
& \mid & \neg\ \mathit{Sentence} \\
& \mid & \mathit{Sentence} \land \mathit{Sentence} \\
& \mid & \mathit{Sentence} \lor \mathit{Sentence} \\
& \mid & \mathit{Sentence} \ \Rightarrow\ \mathit{Sentence} \\
& \mid & \mathit{Sentence} \ \Leftrightarrow\ \mathit{Sentence} \\
\\
\textsc{Operator Precedence} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

## Semantics

Note: Truth value of a sentence is built from its parts "compositional semantics"

| $P$ | $Q$ | $\neg P$ | $P \wedge Q$ | $P \vee Q$ | $P \Rightarrow Q$ | $P \Leftrightarrow Q$ |
|---|---|---|---|---|---|---|
| $false$ | $false$ | $true$ | $false$ | $false$ | $true$ | $true$ |
| $false$ | $true$ | $true$ | $false$ | $true$ | $true$ | $false$ |
| $true$ | $false$ | $false$ | $false$ | $true$ | $false$ | $false$ |
| $true$ | $true$ | $false$ | $true$ | $true$ | $true$ | $true$ |

# Logical equivalences

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) && \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) && \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) && \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) && \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \lor \beta) && \text{implication elimination} \quad (*) \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) && \text{de Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) && \text{de Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) && \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) && \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

**(*) key to go to clausal (Conjunctive Normal Form)**
**Implication for "humans"; clauses for machines.**
**de Morgan laws also very useful in going to clausal form.**

KB at T = 1:

R1: $\neg P_{1,1}$

R2: $\neg B_{1,1}$

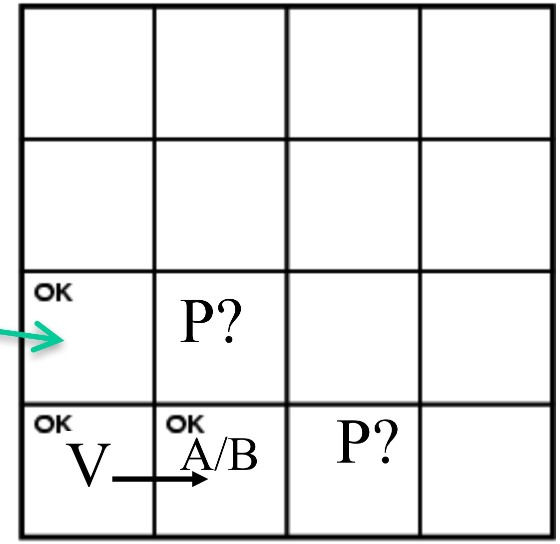
R3: $B_{2,1}$

R4: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R5: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

# Style II: Proof by inference rules
# Modus Ponens (MP)

**How can we show that KR $\models \neg P_{1,2}$ ?**




Wumpus world at T = 1

Note: In formal proof, every step needs to be justified.

So, we used R2 and R4.

# Length of Proofs

**Why bother with inference rules? We could always use a truth table to check the validity of a conclusion from a set of premises.**

**But, *resulting proof can be much shorter than truth table method.***

Consider KB:
p_1, p_1 → p_2, p_2 → p_3, …, p_(n-1) → p_n

To prove conclusion: p_n

Inference rules: n-1 MP steps    Truth table: $2^n$

**Key open question: Is there always a short proof for any valid conclusion? Probably not. The NP vs. co-NP question.**
**(The closely related: P vs. NP question carries a $1M prize.)**

# Style III: Resolution

**First, we need a conversion to Conjunctive Normal Form (CNF) or Clausal Form.**

**Let's consider converting R4 in clausal form:**

$\mathbf{R4: \; B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}$

**We have:**

$B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})$

**which gives (implication elimination):**

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1})$

**Also**

$(P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}$

**which gives:**

$(\neg (P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

**Thus,**

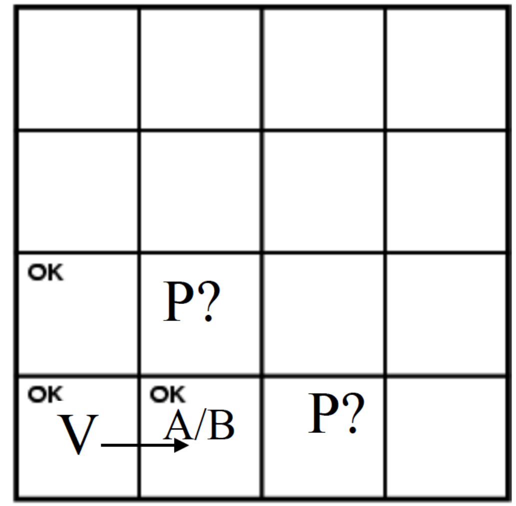
$(\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1}$

**leaving,**

$(\neg P_{1,2} \vee B_{1,1})$

$(\neg P_{2,1} \vee B_{1,1})$

**(note: clauses in red)**




Wumpus world at T = 1

## KB at T = 1:

R1: $\neg P_{1,1}$

R2: $\neg B_{1,1}$

R3: $B_{2,1}$

R4: $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

R5: $B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

## KB at T=1 in clausal form:

R1: $\neg P_{1,1}$

R2: $\neg B_{1,1}$

R3: $B_{2,1}$

R4a: $\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}$

R4b: $\neg P_{1,2} \lor B_{1,1}$

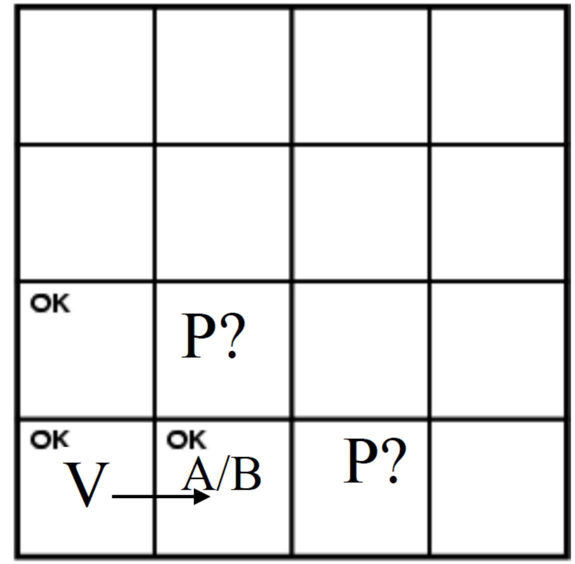
R4c: $\neg P_{2,1} \lor B_{1,1}$

R5a: $\neg B_{2,1} \lor P_{1,1} \lor P_{2,2} \lor P_{3,1}$

R5b: $\neg P_{1,1} \lor B_{2,1}$

R5c: $\neg P_{2,2} \lor B_{2,1}$

R5d: $\neg P_{3,1} \lor B_{2,1}$




Wumpus world at T = 1

## How can we show that KR $\models \neg P_{1,2}$ ?

Proof by contradiction:
Need to show that ($KB \wedge P_{1,2}$) is
**inconsistent (unsatisfiable).**

Resolution rule:

**($\alpha \vee p$) and ($\beta \vee \neg p$)**

**gives resolvent (logically valid conclusion):**

**($\alpha \vee \beta$)**

**If we can reach the empty clause, then KB is inconsistent. (And, vice versa.)**

**KB at T=1 in clausal form:**

**R1:** $\neg P_{1,1}$

**R2:** $\neg B_{1,1}$

**R3:** $B_{2,1}$

**R4a:** $\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}$

**R4b:** $\neg P_{1,2} \lor B_{1,1}$

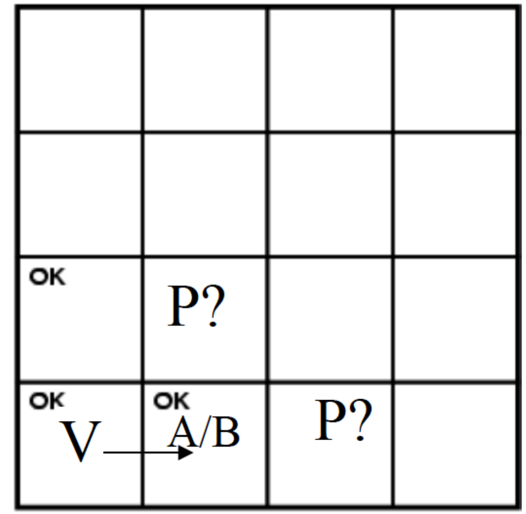
**R4c:** $\neg P_{2,1} \lor B_{1,1}$

**R5a:** $\neg B_{2,1} \lor P_{1,1} \lor P_{2,2} \lor P_{3,1}$

**R5b:** $\neg P_{1,1} \lor B_{2,1}$

**R5c:** $\neg P_{2,2} \lor B_{2,1}$

**R5d:** $\neg P_{3,1} \lor B_{2,1}$




Wumpus world at T = 1

Show that ($KB \land P_{1,2}$) is **inconsistent.** **(unsatisfiable)**

R4b with $P_{1,2}$ resolves to $B_{1,1}$,
which with R2, resolves to the empty clause, $\square$ .
So, we can conclude $KB \models \neg P_{1,2}$.
(make sure you use “what you want to prove.”)

**KB at T=1 in clausal form:**

**R1:** $\neg P_{1,1}$

**R2:** $\neg B_{1,1}$

**R3:** $B_{2,1}$

**R4a:** $\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}$

**R4b:** $\neg P_{1,2} \lor B_{1,1}$

**R4c:** $\neg P_{2,1} \lor B_{1,1}$

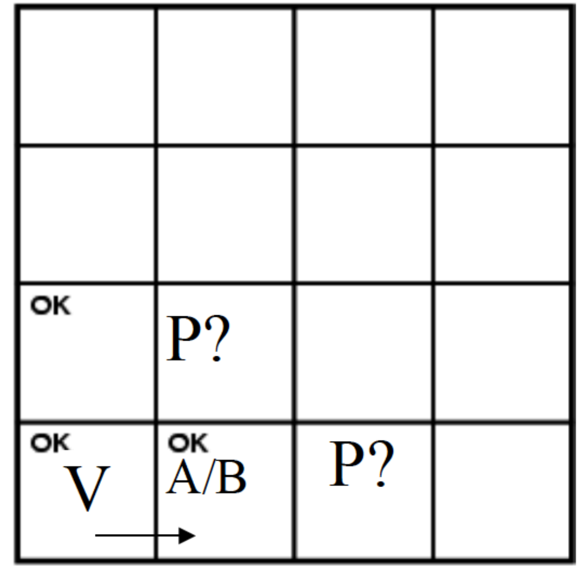
**R5a:** $\neg B_{2,1} \lor P_{1,1} \lor P_{2,2} \lor P_{3,1}$

**R5b:** $\neg P_{1,1} \lor B_{2,1}$

**R5c:** $\neg P_{2,2} \lor B_{2,1}$

**R5d:** $\neg P_{3,1} \lor B_{2,1}$

## Another example resolution proof




Wumpus world at T = 1

Note that R5a resolved with R1, and then resolved with R3, gives $(P_{2,2} \lor P_{3,1})$.

Almost there… to show KB $\models (P_{2,2} \lor P_{3,1})$ , we need to show KB $\land (\neg (P_{2,2} \lor P_{3,1}))$ is inconsistent. (Why? Semantically?)
So, show KB $\land \neg P_{2,2} \land \neg P_{3,1}$ is inconsistent.
This follows from $(P_{2,2} \lor P_{3,1})$; because in two more resolution steps, we get the empty clause (a contradiction).

## Length of Proofs

Consider KB:

p_1, p_1 → p_2, p_2 → p_3, …, p_(n-1) → p_n

To prove conclusion: p_n

**Resolution. Assert (¬ p_n)**
**with (¬ p_(n-1) ∨ p_n) gives (¬ p_(n-1))**
**with (¬ p_(n-2) ∨ p_(n-1) gives (¬ p_(n-2))**
**…**
**with (¬ p_1) ∨ p_2) gives (¬ p_1)**
**with (p_1) gives empty clause (contradiction).**
**QED**
**Note how resolution mimics Modus Ponens steps.**

Inference rules: n resolution steps  Truth table: $2^n$

**So, efficient on these proofs!**

# Length of Proofs

**What is hard for resolution?**

**Consider:**

**Given a fixed pos. int. N**

(P(i,1) ∨ P(i … N+1

(¬ P(i,j) ∨ ¬ … N;

… N+1;

…N+1; i =/= i'

What does this encode?

Think of: P(i,j) for "object i in location j"

Pigeon hole problem…

Provable requires exponential number of resolution steps to reach empty clause (Haken 1985). Method "can't count."

# Style IV: SAT Solvers

Instead of using resolution to show that

$KB \wedge \neg \alpha$ is inconsistent,

modern Satisfiability (SAT) solvers operating on the clausal form are *much* more effi

NOTE: SAT SOLVERS CAN BE VIEWED AS DOING A SPECIAL FORM OF RESOLUTION

The SAT solvers treat … nstraints
(disjunctions) on Boo … lem!
Current solvers are ve … lion+
variables and several millions of clauses.

Systematic: Davis Putnam (DPLL) + *series of improvements*
Stochastic local search: WalkSAT (issue?)

**See R&N 7.6. "Ironically," we are back to semantic model checking, but way more clever than basic truth assignment enumeration (exponentially faster)!**

# DPLL improvements

Backtracking + …

1) **Component analysis (disjoint sets of constraints? Problem decomposition?)**
2) **Clever variable and value ordering (e.g. degree heuristics)**
3) **Intelligent backtracking and clause learning (conflict learning)**
4) **Random restarts (heavy tails in search spaces…)**
5) **Clever data structures**

**1+ Million Boolean vars & 10+ Million clause/constraints are feasible nowadays. (e.g. Minisat solver)**

**Has changed the world of verification (hardware/software) over the last decade (incl. Turing award for Clarke). Widely used in industry, Intel, Microsoft, IBM etc.**

ENDS LOGIC PART

All equivalent
Prop. / FO Logic

entailment

1) $KB \models \alpha$