Reinforcement Learning

So far, we had a well-defined set of training examples.

What if feedback is not so clear?

E.g., when playing a game, only after many actions

final result: win, loss, or draw.

In general, agent exploring environment, delayed

feedback: survival or not ... (evolution)

Issue: **delayed rewards / feedback**.

Field: **reinforcement learing**

Main success: Tesauro's backgammon player
(TD Gammon).
start from random play; millions of games
world-level performance (changed game itself)

Chapter 20 R& N.

Imagine agent wandering around in environment.

How does it learn **utility** values of each state?

(i.e., what are good / bad states? avoid bad ones...)

Reinforcement learning will tell us how!

Compare: in backgammon game: states = boards.

only clear feedback in final states (win/loss).

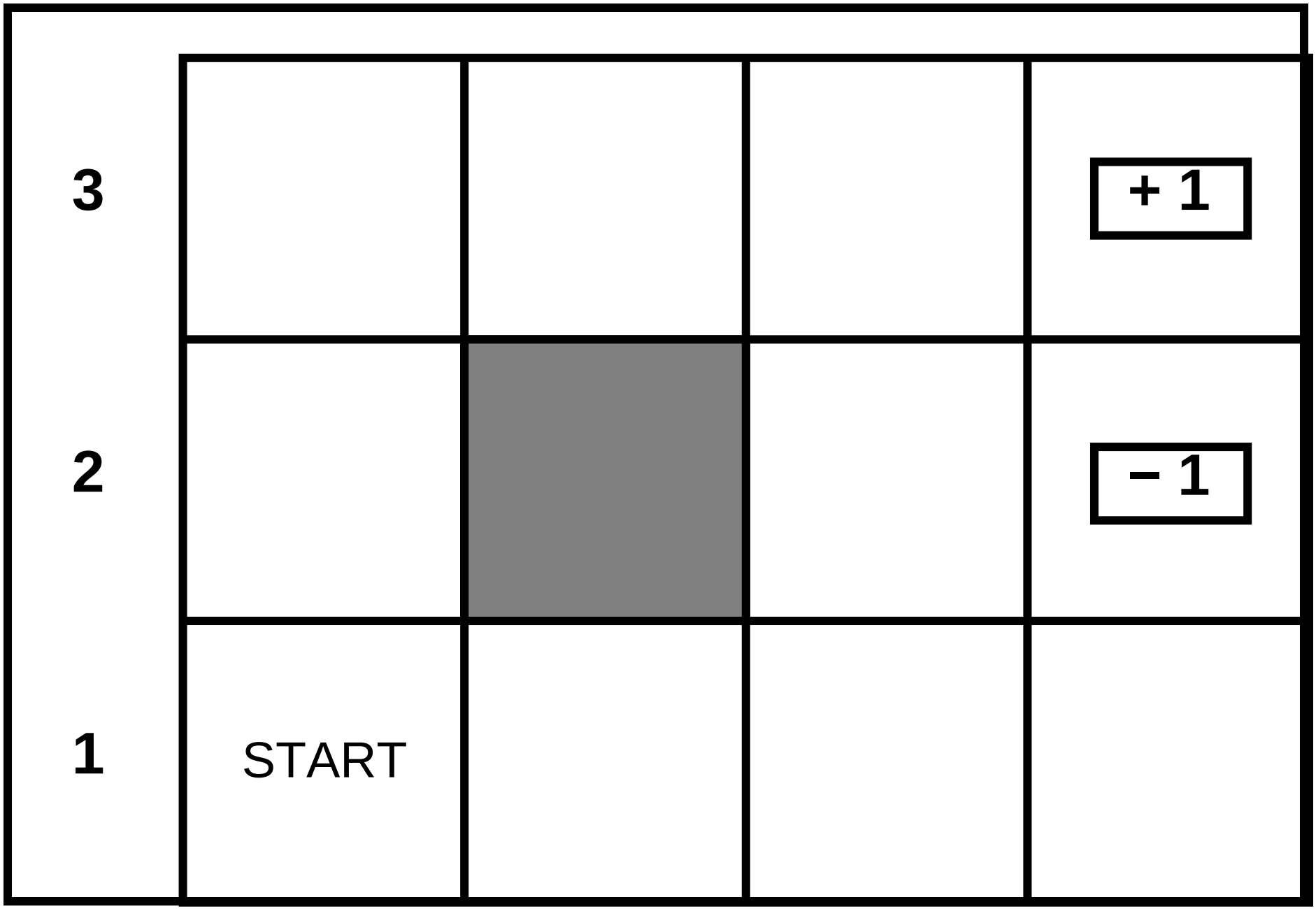
We want to know **utility** of the other states

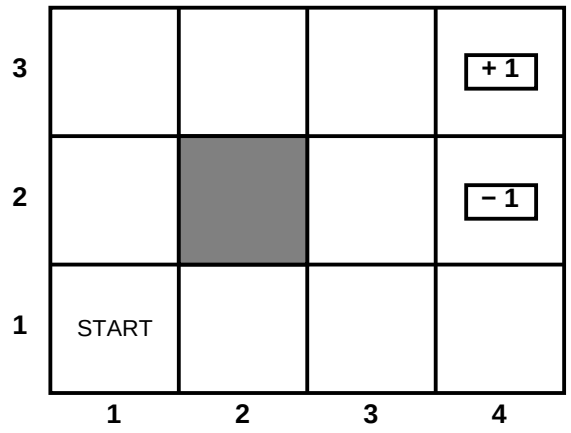
Intuitively: utility = chance of winning.

At first, we only know this for the end states.

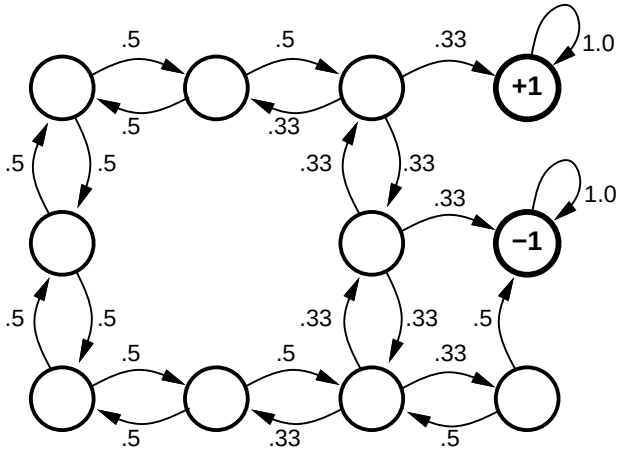
Reinforcement learning: computes for intermediate states. Play by moving to maximum utility states!

back to simplified world ...


3
2
1
START
+ 1
– 1
1
2
3
4



(a)



(b)

3	−0.0380	0.0886	0.2152	+ 1
2	−0.1646		−0.4430	− 1
1	−0.2911	−0.0380	−0.5443	−0.7722
	1	2	3	4

(c)

Example of **passive learning** in a

known environment.

Agent just wanders from state to state.

Each transition is made with a fixed probability.

Initially: only two known reward positions:

State (4,2) — a loss / poison / reward -1 (utility)
State (4,3) — a win / food / reward $+1$ (utility)

How does the agent learn about the utility, i.e.,

expected value, of the other states?

Three strategies:

(a) "Sampling" (Naive updating)

(b) "Calculation" / "Equation solving"
(Adaptive dynamic programming)

(c) "in between (a) and (b)"
(Temporal Difference Learning — TD learning)
used for backgammon

Naive updating

(a) "Sampling" — agent makes random runs through environment; collect statistics on final payoff for each state (e.g. when at (2,3), how often do you reach +1 vs. −1?)

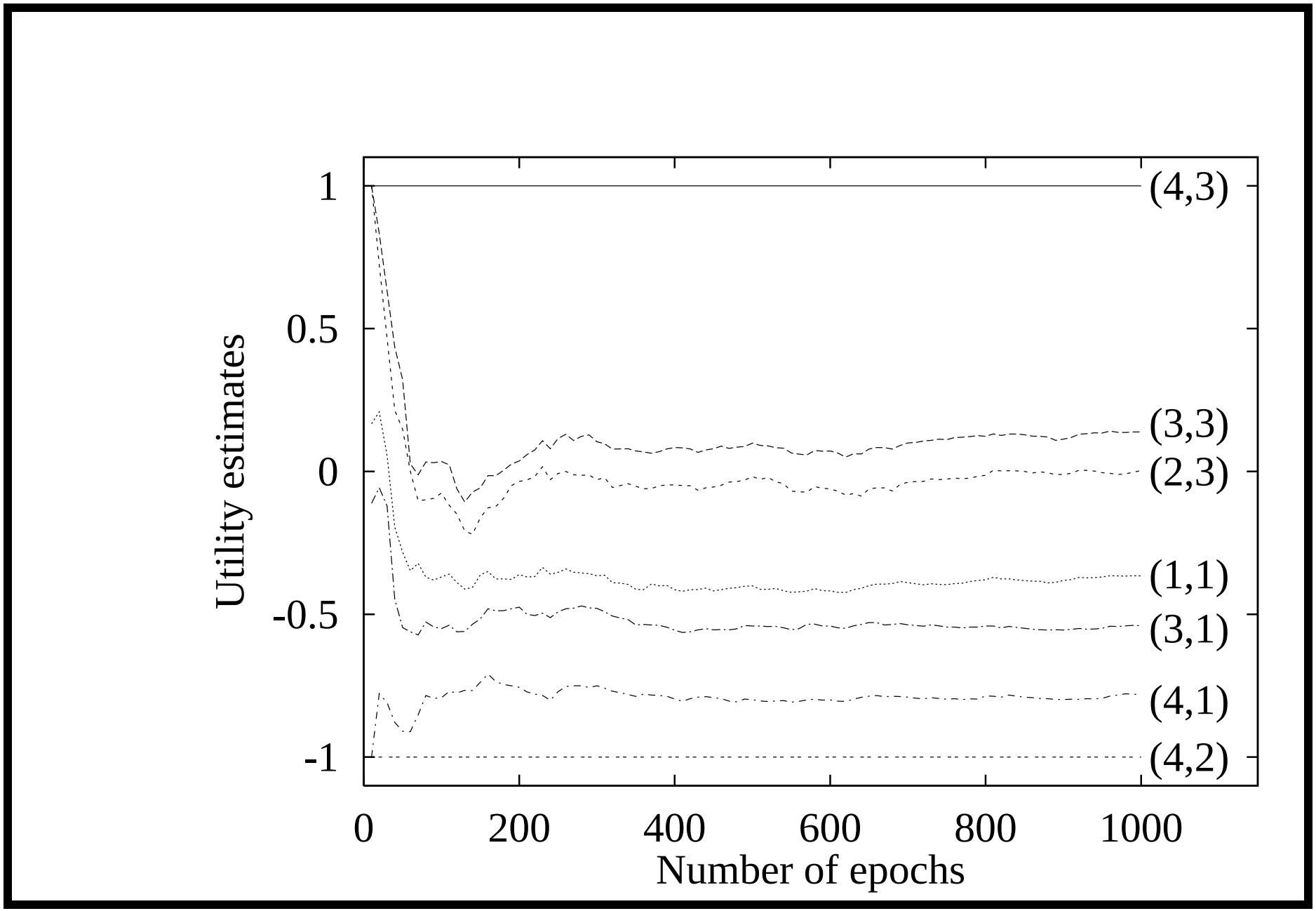
Learning algorithm keeps a running average for each state. Provably converges to true expected values (utilities).

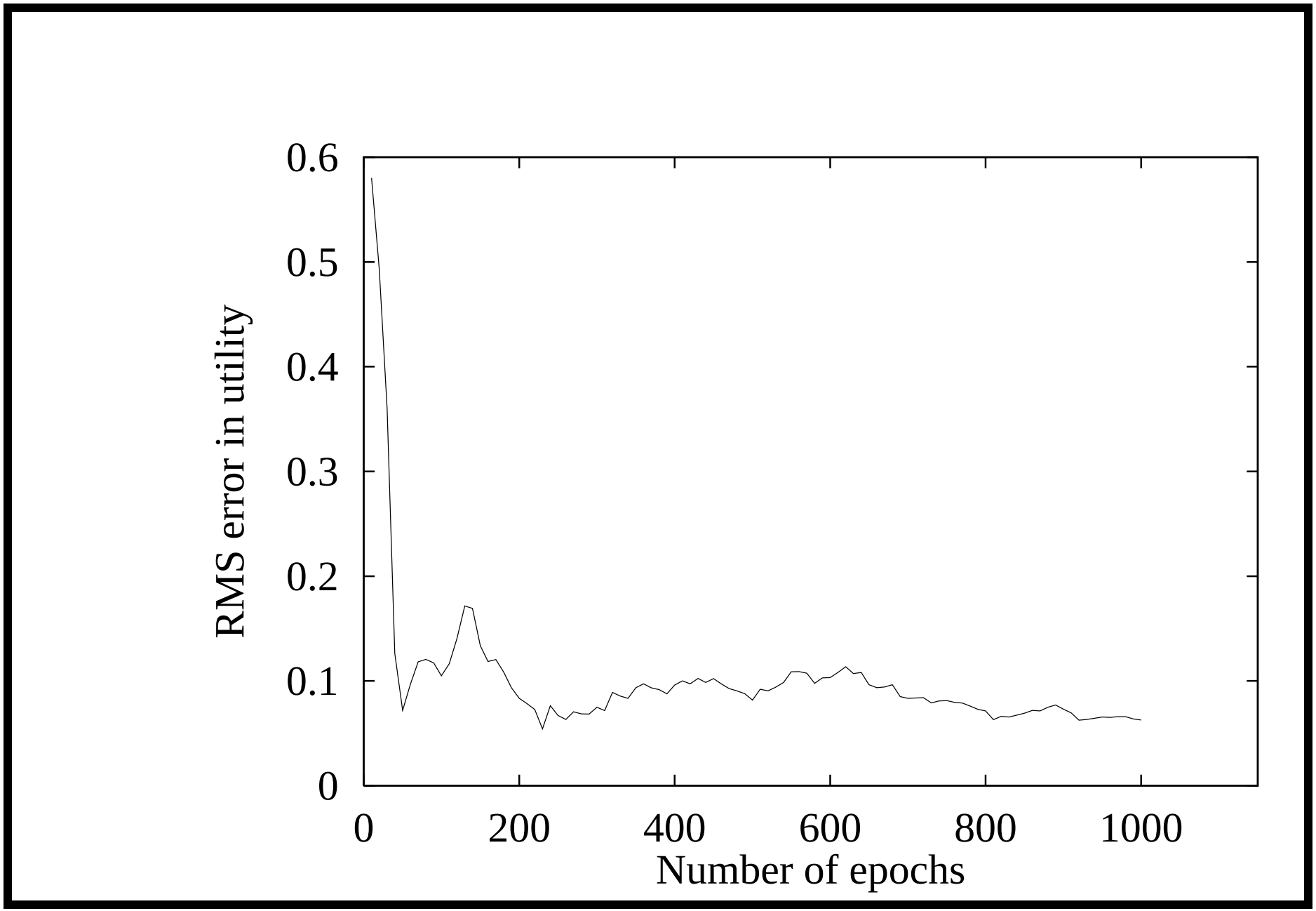
Main drawback: **slow convergence**.

See next figure.

In relatively small world takes agent

over 1000 sequences to get a reasonably small (< 0.1) **root-mean-square** error compared with true expected values.


Utility estimates
1
0.5
0
-0.5
-1
0
200
400
600
800
1000
Number of epochs
(4,3)
(3,3)
(2,3)
(1,1)
(3,1)
(4,1)
(4,2)


0.6
0.5
0.4
0.3
0.2
0.1
0
RMS error in utility
0
200
400
600
800
1000
Number of epochs

Question: Is sampling necessary?

Can we do something completely different?

Note: agent knows the environment (i.e. probability of state transitions) and final rewards.

Upon, refection we note that the utilities

must be completely defined by what the agent

already knows about the environment.

Adaptive dynamic programming

Some intuition first.

Consider $U(3,3)$

From figure we see that

$$U(3,3) = 0.33 \times U(4,3) + 0.33 \times U(2,3) + 0.33 \times U(3,2)$$

$$= 0.33 \times 1.0 + 0.33 \times 0.0886 + 0.33 \times -0.4430$$

$$= 0.2152$$

Check e.g. $U(3,1)$ yourself.

Utilities follow basic laws of probabilities:

write down equations; solve for unknowns.

Utilities follow from:

$$U(i) = R(i) + \sum_j M_{i,j} U(j) \qquad (\star)$$

(note: i, j over states.)

$R(i)$ is the reward associated with being in state i.

(often non-zero for only a few end states)

$M_{i,j}$ is the probability of transition from state i to j.

Dynamic programming style methods can be used to solve the set of equations.

Major drawback: number of equations and number of unknowns.

E.g. for backgammon: roughly 10^{50} equations with 10^{50} unknowns. Infeasibly large.

Temporal difference learning

combine "sampling" with "calculation"

or stated differently: it's using a sampling approach to solve the set of equations.

Consider the transitions, observed by a wandering agent.

Use an observed transition to adjust the utilities of the observed states to bring them closer to the constraint equations.

Temporal difference learning

When observing a transition from i to j,

bring $U(i)$ value closer to that of $U(j)$

Use update rule:

$$U(i) \leftarrow U(i) + \alpha(R(i) + U(j) - U(i)) \qquad (\star\star)$$

α is the **learning rate** parameter

rule is called the **temporal-difference** or **TD**

equation. (note we take the difference between successive states.)

At first blush, the rule:

$$U(i) \leftarrow U(i) + \alpha(R(i) + U(j) - U(i)) \ (\star\star)$$

may appear to be a bad way to solve/approximate:

$$U(i) = R(i) + \sum_j M_{i,j} U(j) \ (\star)$$

Note that $(\star\star)$ brings $U(i)$ closer to $U(j)$ but

in $(\star)$ we really want the **weighted** average

over the neighboring states!

Issue resolves itself, because over time, we **sample**

from the transitions out of i. So, successive applications

of $(\star\star)$ average over neighboring states.

(keep α appropriately small)

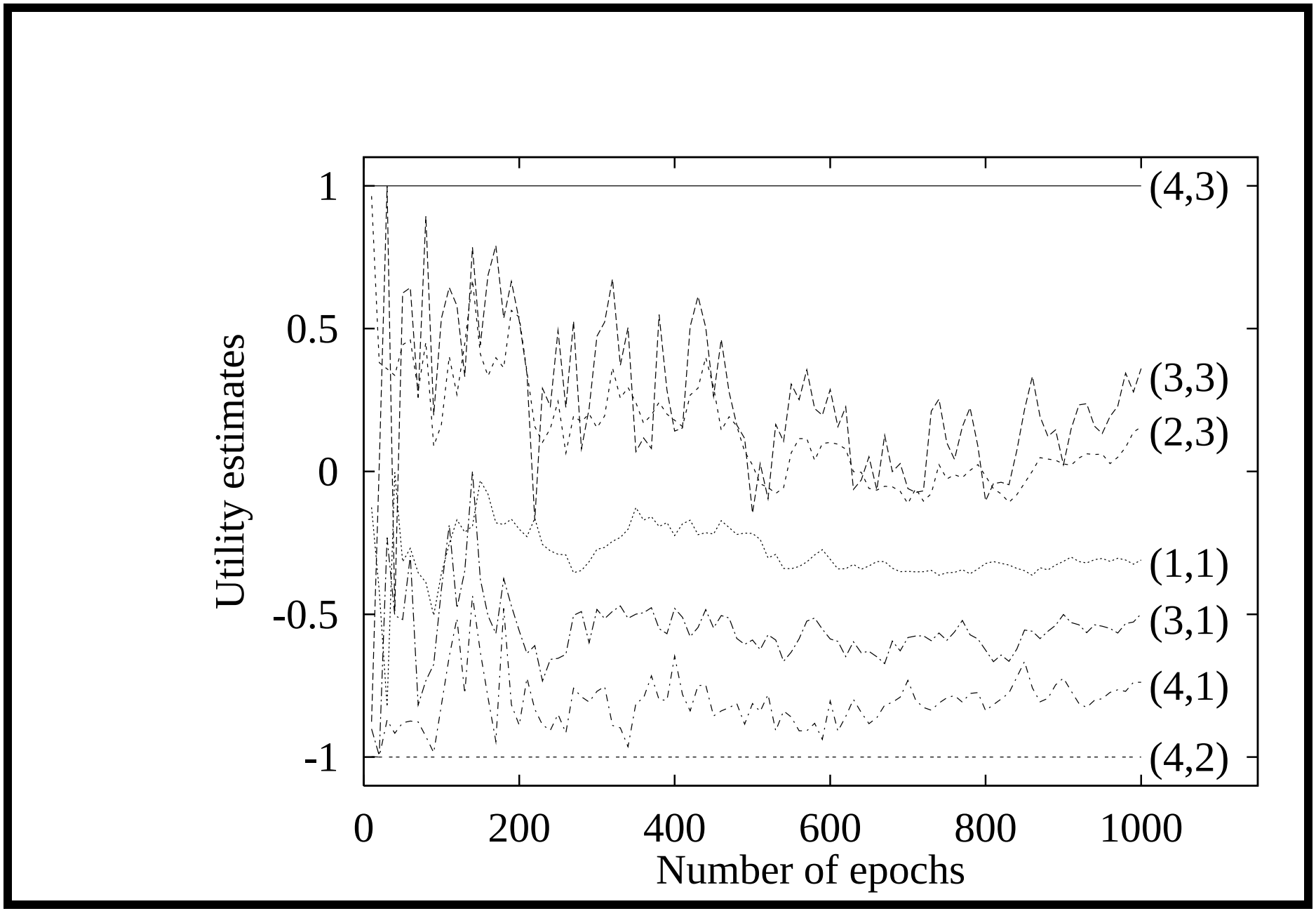
Performance

Runs noisier than Naive Updating (averaging),
but smaller error.

In our 4x3 world, we get a root-mean-square error of less than 0.07 after 1000 examples.

Also, note that compared to Adaptive Dynamic Programming we only deal with observed states during sample runs.

I.e., in backgammon consider only a few hundreds of thousands of states out of 10^{50}. Represent utility function **implicitly** (no table) in neural network.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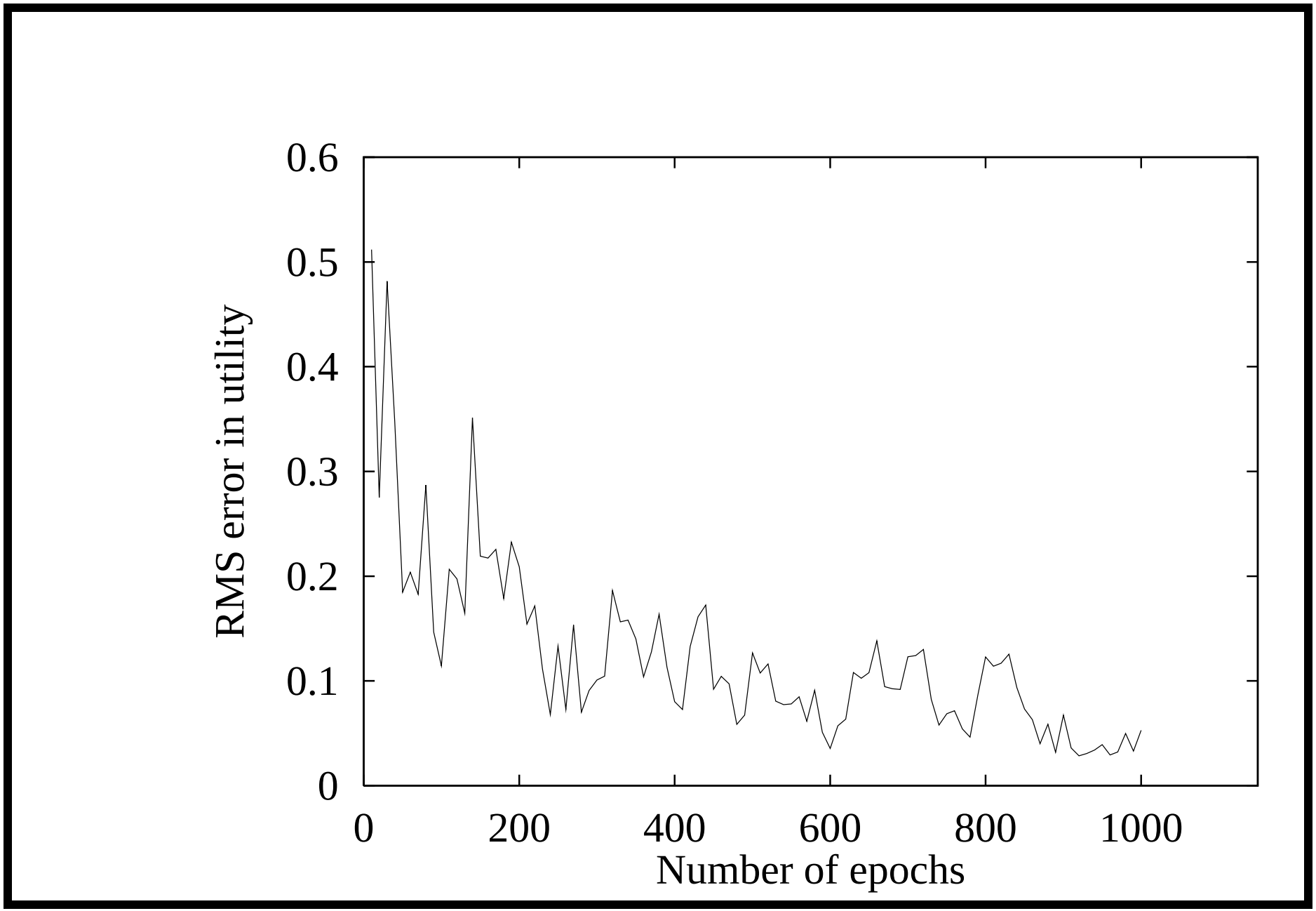
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Reinforcement learning is a very rich domain of study.

In some sense, touches on much of the core of AI.

"How does an agent learn to take the right actions in its environment"

In general, pick action that leads to state with highest utility as learned so far.

E.g. in backgammon pick legal move leading to state with highest expected payoff (chance of winning). Initially moves random. But TD rule starts learning from winning and losing games, by moving utility values backwards. (states leading to lost positions start getting low utility after a series of TD rule applications; states leading to wins see there utilities rise slowly.)

Extensions

— **Active learning** — exploration.

now and then make new (non utility optimizing move)

see n-**armed bandit** problem p. 611 R&N.

— **learning action-value functions**

$Q(a, i)$ denotes value of taking action a in state i

we have:

$$U(i) = max_a Q(a, i)$$

— **generalization in reinforcement learning**

use implicit representation of utility function

e.g. a neural network as in backgammon.

input nodes encode board position

activation of output node gives utility

— **genetic algorithms** — feedback: fitness

done in search part.