CS 4700: Foundations of Artificial Intelligence

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Module:
Informed Search

Readings R&N - Chapter 3: 3.5 and 3.6

Search

Search strategies determined by choice of node (in queue) to expand

Uninformed search:

– Distance to goal not taken into account

Informed search :

– Information about cost to goal taken into account

Aside: "Cleverness" about what option to explore next, almost seems a hallmark of intelligence. E.g., a sense of what might be a good move in chess or what step to try next in a mathematical proof. We don't do blind search…

Reminder: Perfect distance to goal info.
Eliminates search.

d = min dist. to goal

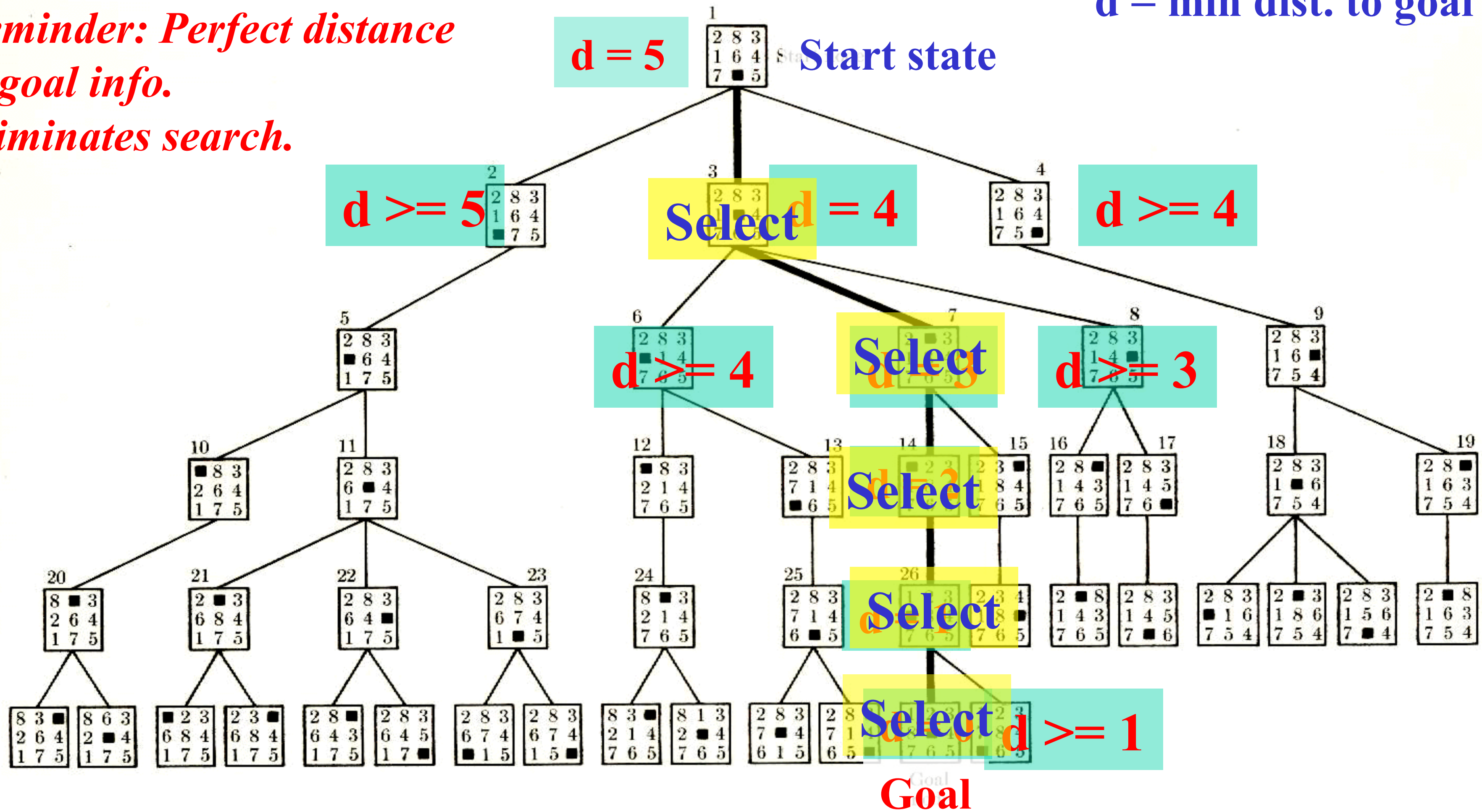


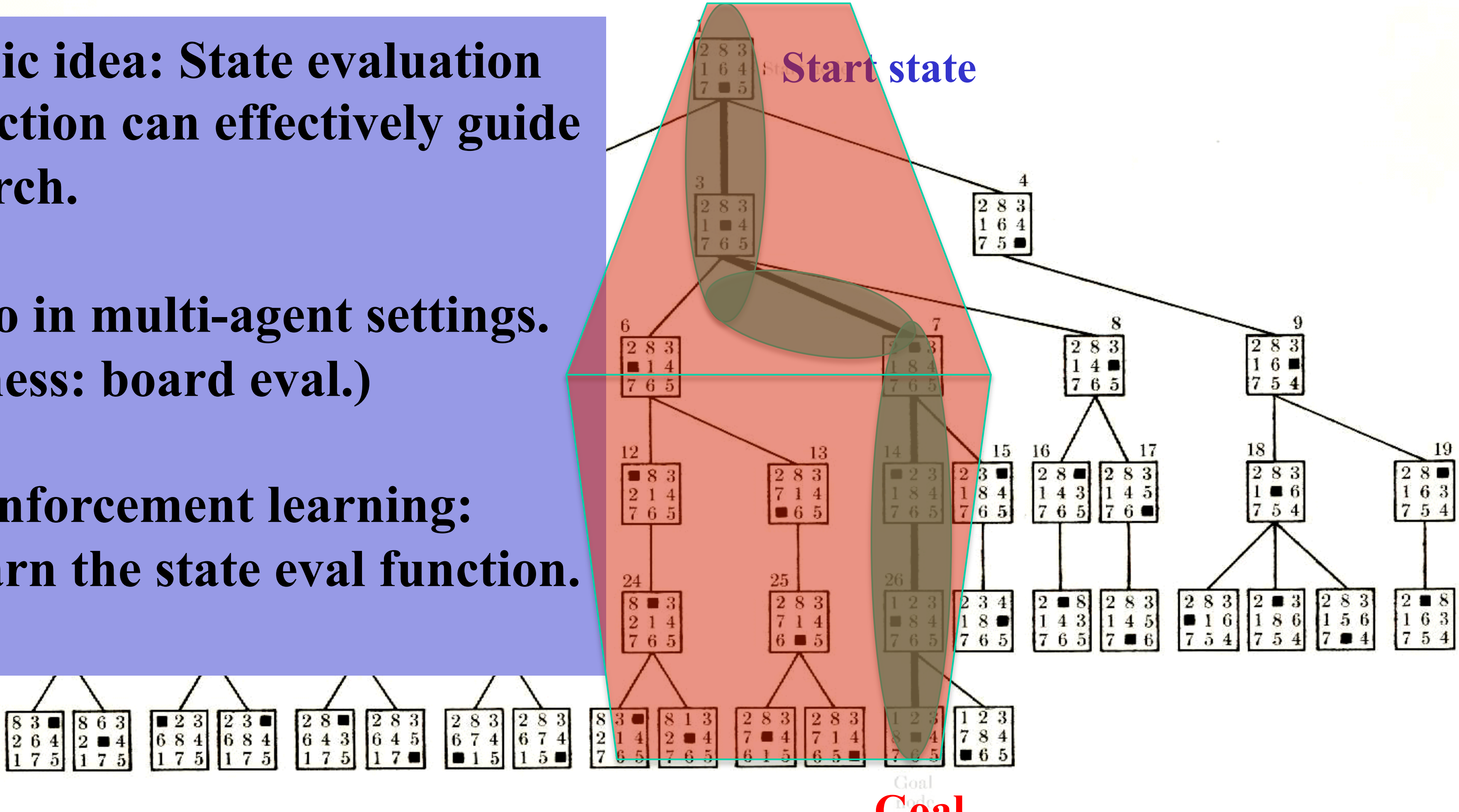
FIG. 3-2 A breadth-first search tree.

Practice:
Only have estimate of distance to goal (“heuristic information”).

Basic idea: State evaluation function can effectively guide search.

Also in multi-agent settings. (Chess: board eval.)

Reinforcement learning: Learn the state eval function.



A breadth-first search tree.

Perfect “heuristics,” eliminates search.

Approximate heuristics, significantly reduces search.
Best (provably) use of search heuristic info: Best-first / A* search.

Outline

- **Best-first search**
- **Greedy best-first search**
- **A^* search**
- **Heuristics**

How to take information into account? Best-first search.

Idea : use an evaluation function for each node

- Estimate of "desirability" of node
- Expand most desirable unexpanded node first ("best-first search")
- Heuristic Functions :
  - *f*: States → Numbers
  - *f(n)*: expresses the quality of the state *n*
    - Allows us to express problem-specific knowledge,
    - Can be imported in a generic way in the algorithms.
- Use uniform-cost search. See Figure 3.14 but use f(n) instead of path cost g(n).
- Queuing based on f(n):

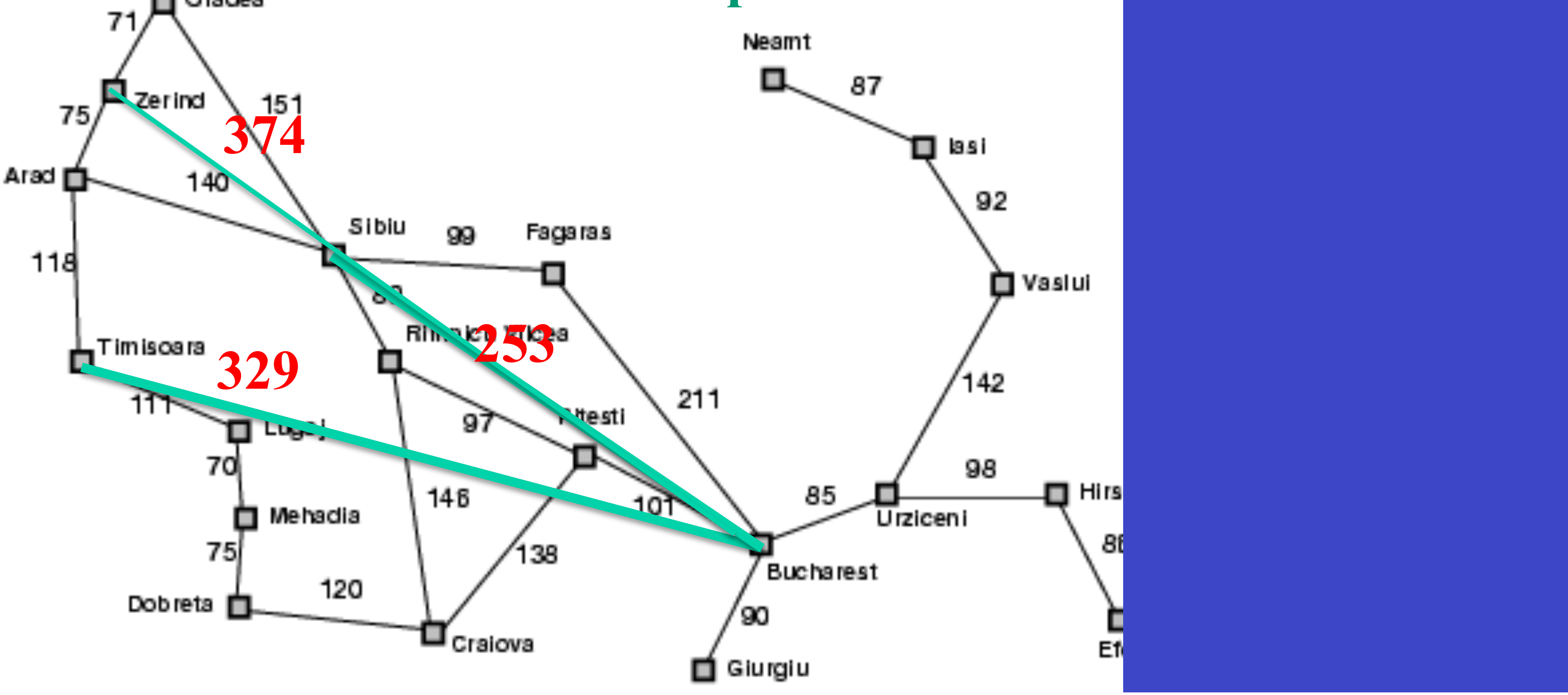
  Order the nodes in fringe in decreasing order of desirability

Special cases:

- **greedy best-first search**
- **A^* search**

Romanian path finding problem

Base eg on GPS info.
No map needed.



Searching for good path from Arad to Bucharest,
what is a reasonable “desirability measure” to expand nodes
on the fringe?

Greedy best-first search

Evaluation function at node *n*, $f(n) = h(n)$ (heuristic)

= *estimate* of cost from *n* to *goal*

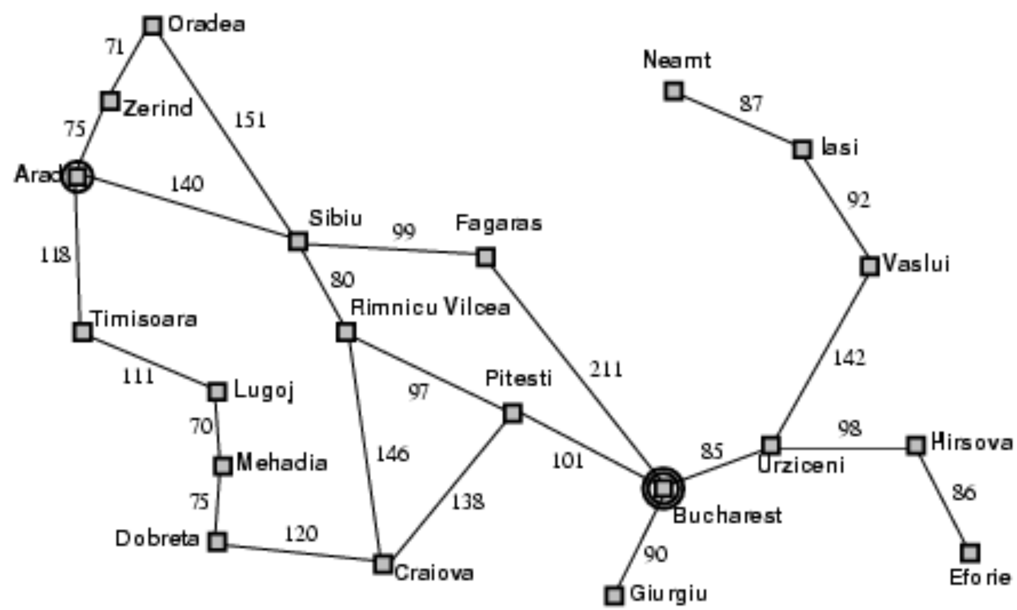
e.g., $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest

Greedy best-first search expands the node that appears to have shortest path to goal.

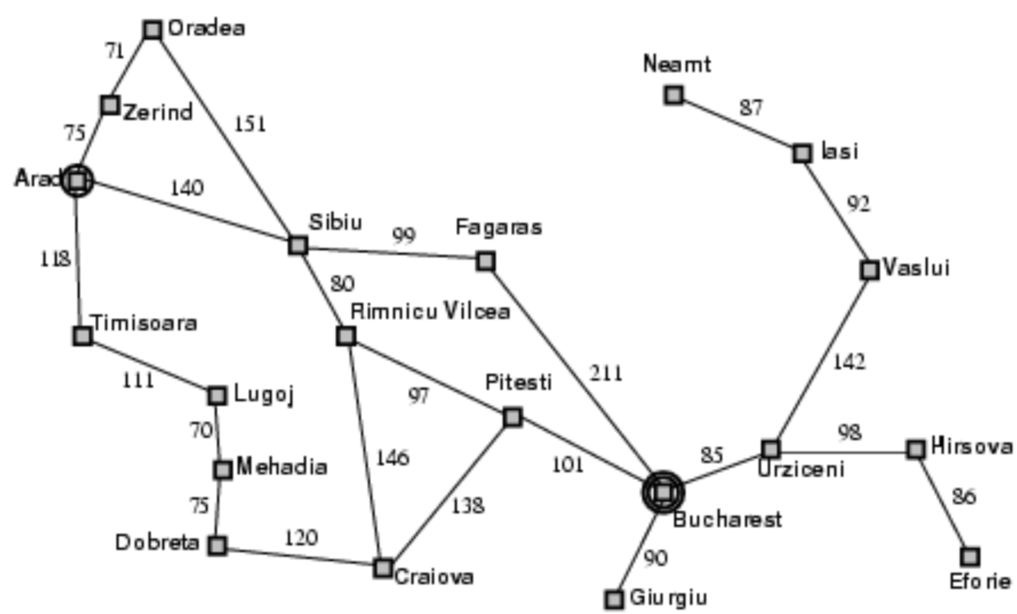
Idea: those nodes may lead to solution quickly.

Similar to depth-first search: It prefers to follow a single path to goal (guided by the heuristic), backing up when it hits a dead-end.

Greedy best-first search example



Greedy best-first search example



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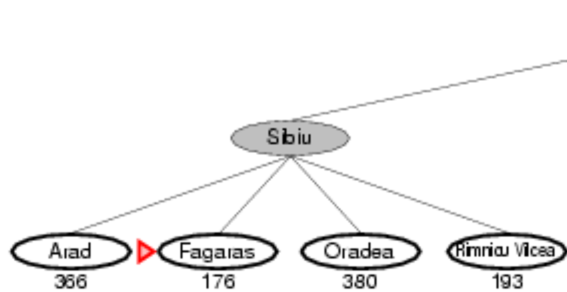
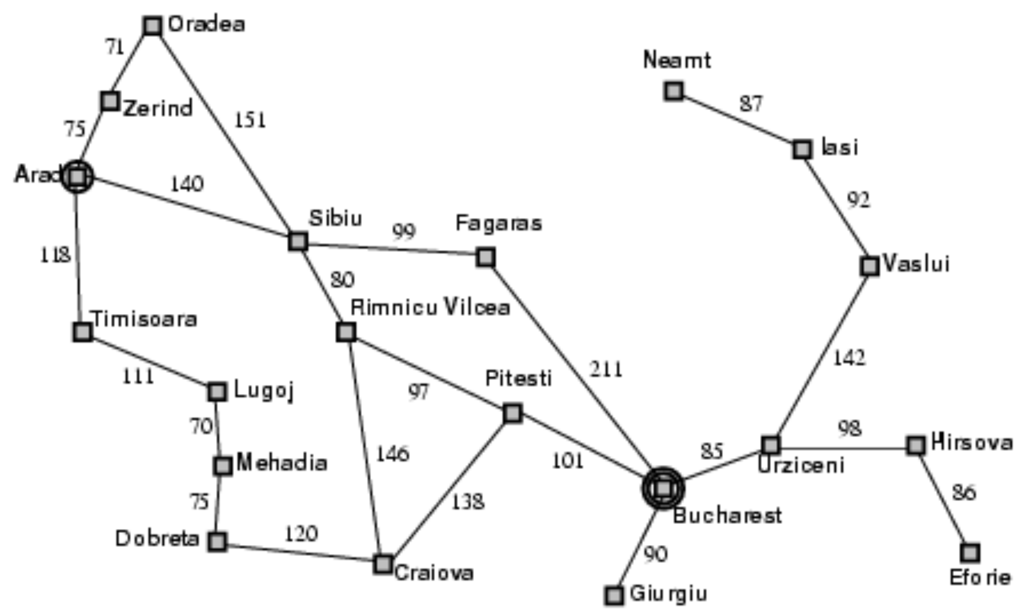
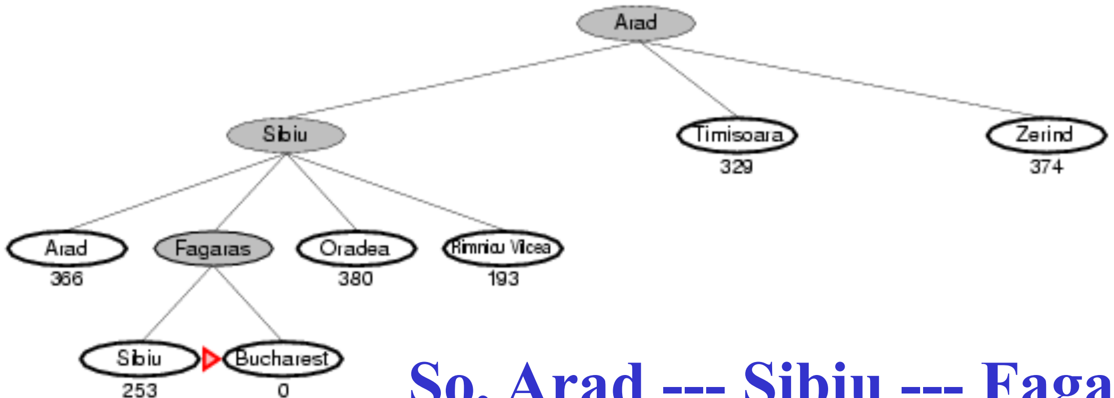




Greedy best-first search example

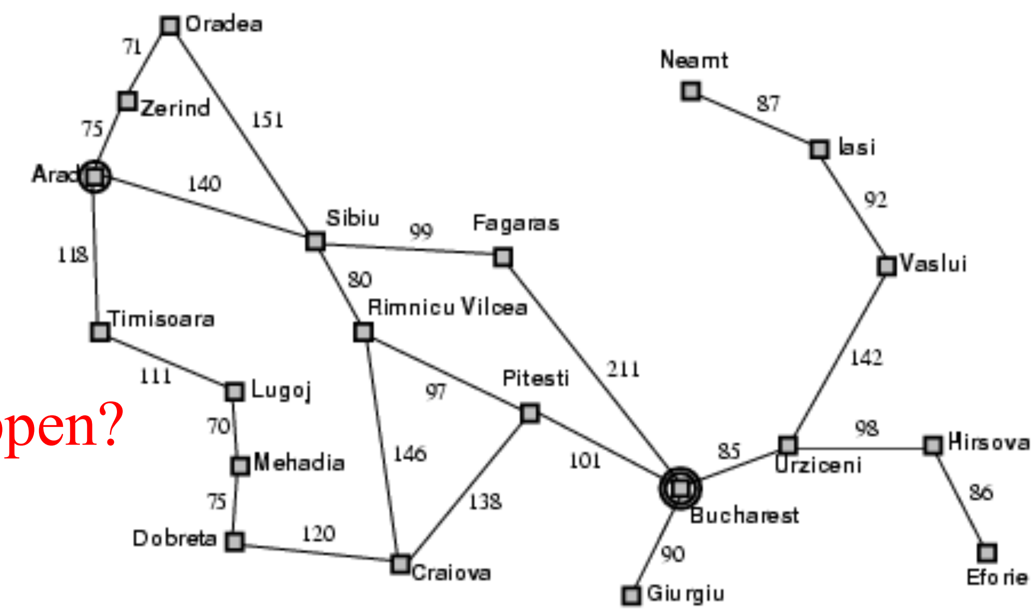


So, Arad --- Sibiu --- Fagaras --- Bucharest
140+99+211 = 450

Is it optimal?

What are we ignoring?

Also, consider going from
Iasi to Fagaras – what can happen?



Properties of greedy best-first search

Complete? No – can get stuck in loops, e.g.,

Iasi → Neamt → Iasi → Neamt…

But, complete in finite space with repeated state elimination.

Time? $O(b^m)$ **(imagine nodes all have same distance estimate to goal)**

but a good heuristic can give dramatic improvement → Becomes more similar to depth-first search, with reduced branching.

Space? $O(b^m)$ **-- keeps all nodes in memory**

Optimal? No!

How can we fix this?

b: maximum branching factor of the search tree

d: depth of the least-cost solution

m: maximum depth of the state space (may be ∞)

A^* search

Note: Greedy best-first search expands the node that *appears* to have shortest path to goal. But what about cost of getting to that node? Take it into account!

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

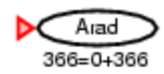
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated **total** cost of path through n to goal

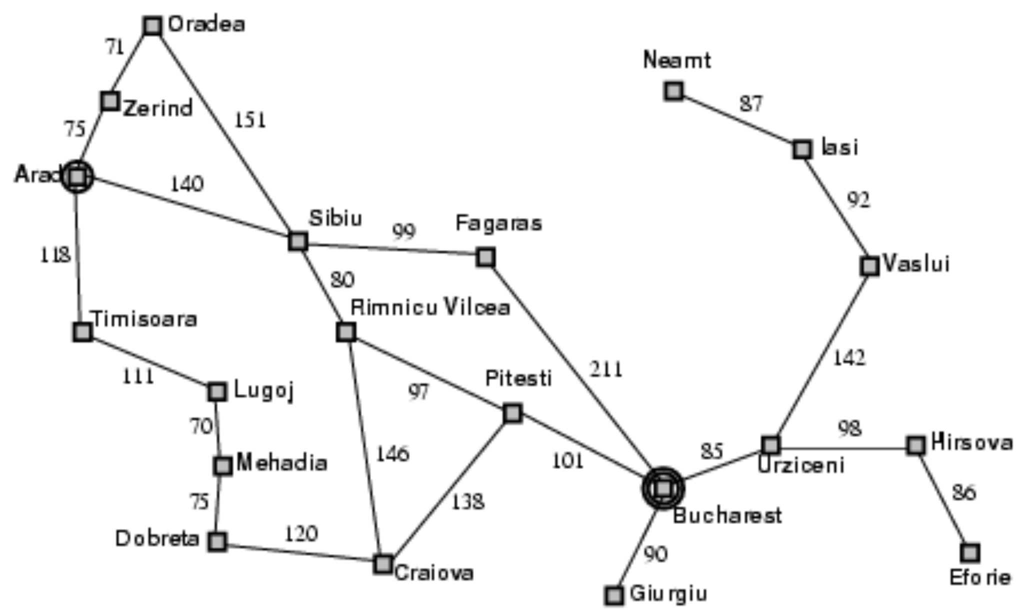
Aside: do we still have "looping problem"?
Iasi to Fagaras:
Iasi → Neamt → Iasi → Neamt…

No! We'll eventually get out of it. g(n) keeps going up.

Using: $f(n) = g(n) + h(n)$

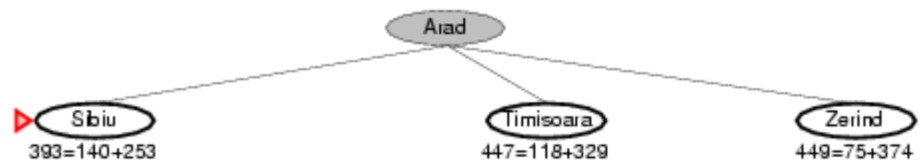
A* search example

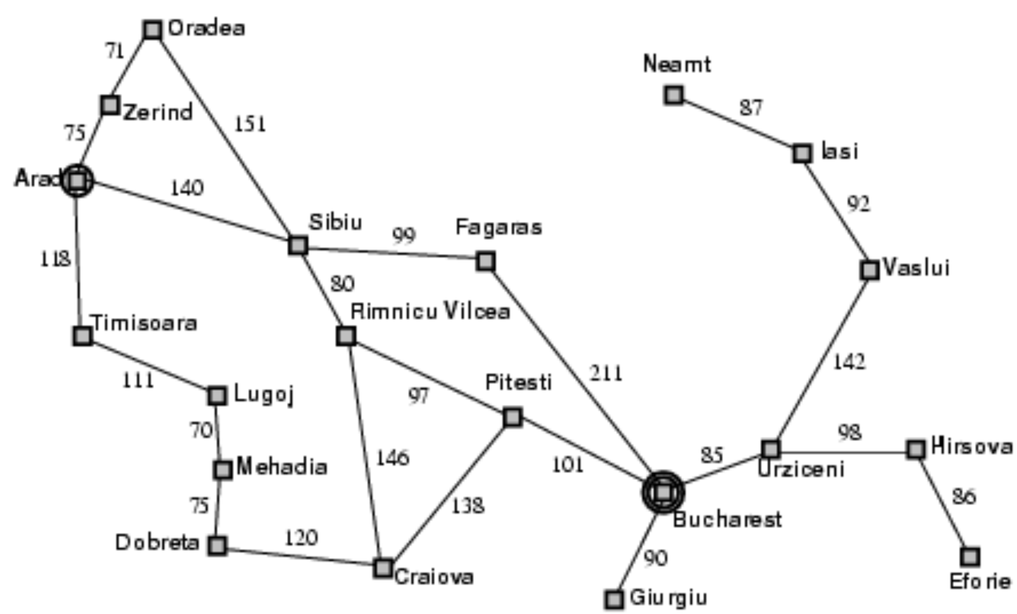




Using: f(n) = g(n) + h(n)

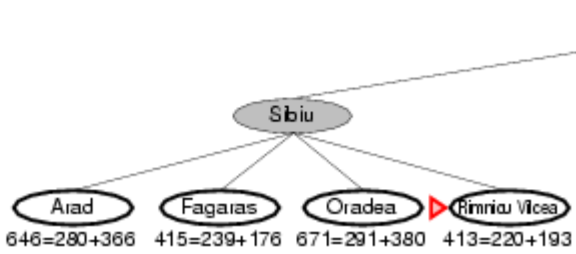
A* search example

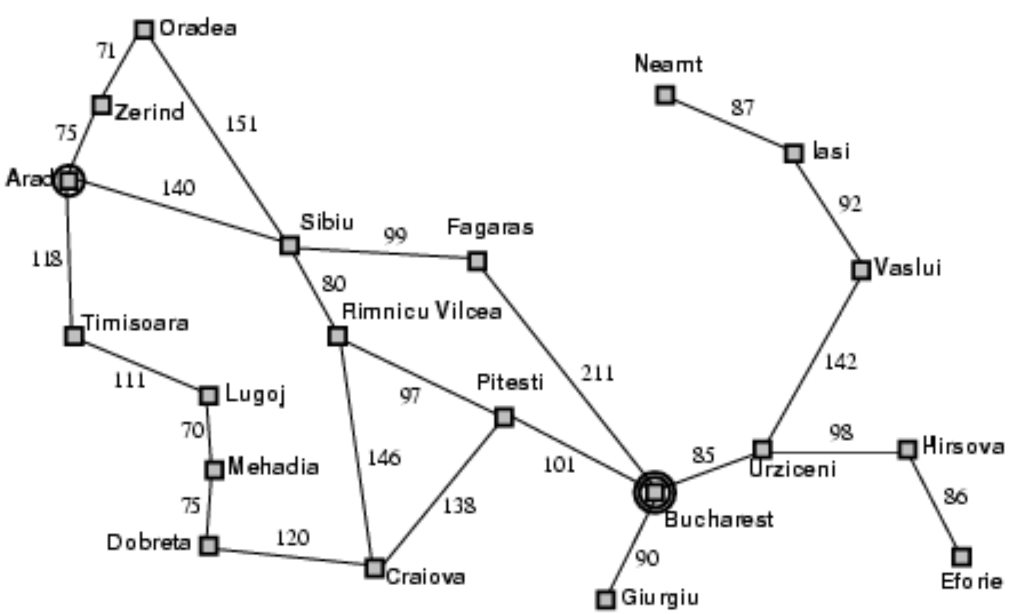




Using: $f(n) = g(n) + h(n)$

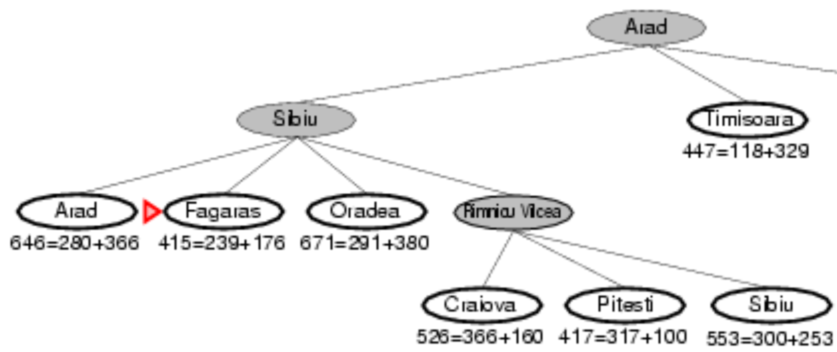
A* search example

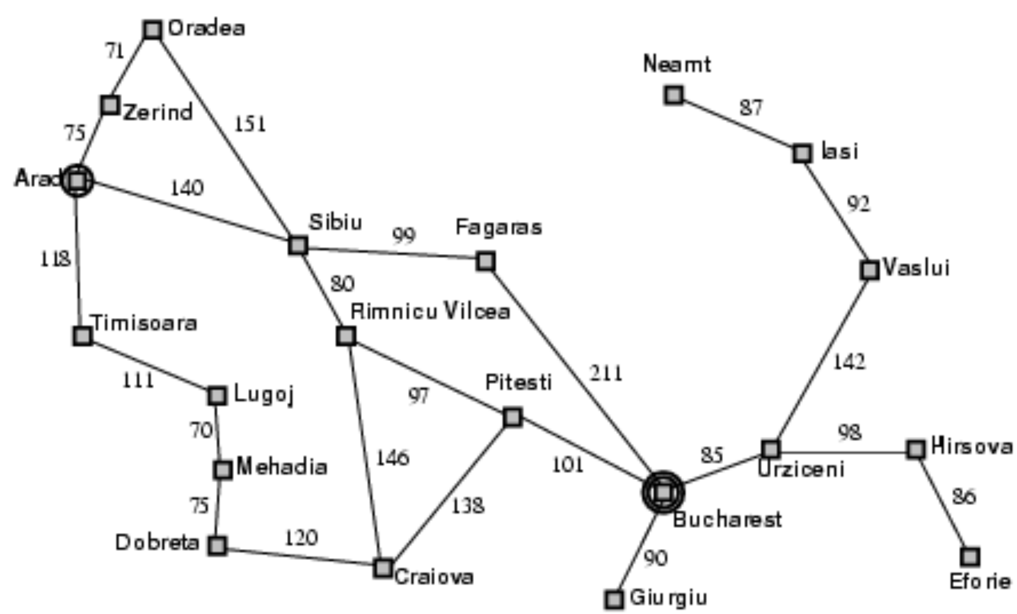




Using: $f(n) = g(n) + h(n)$

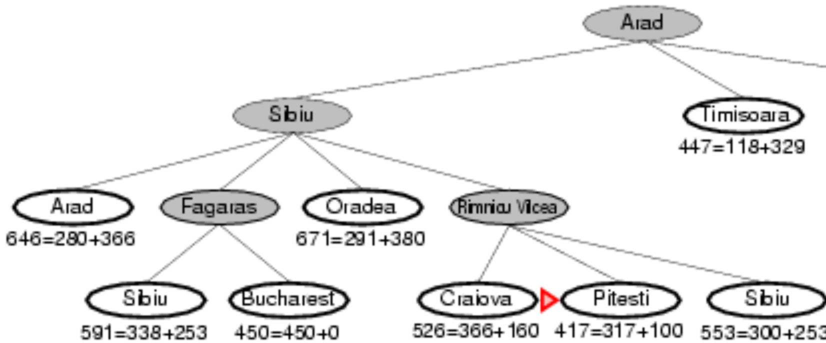
A* search example





Using: f(n) = g(n) + h(n)

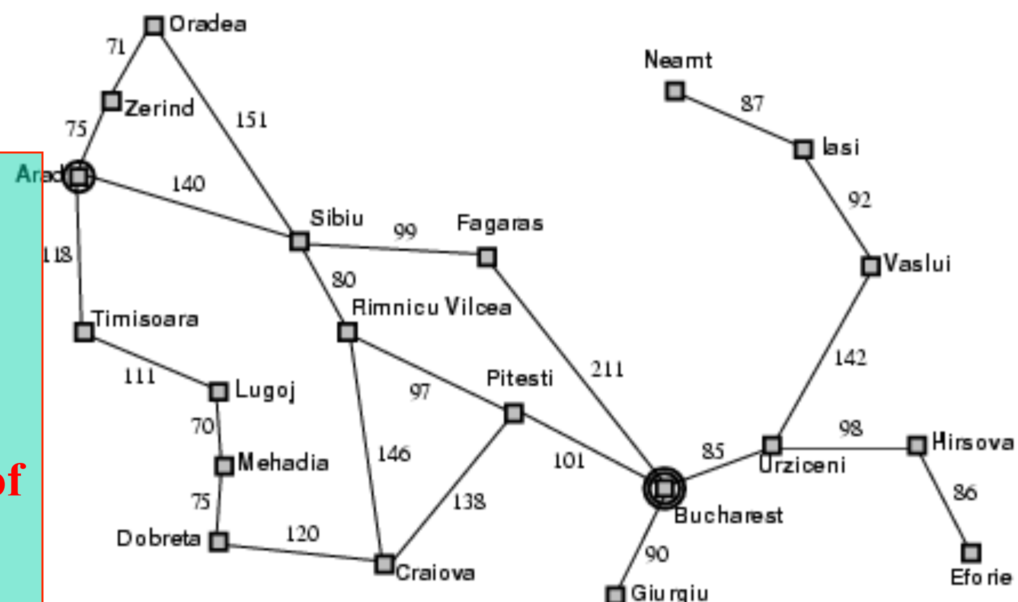
A^* search example



What happens if h(Pitesti) = 150?

Bucharest appears on the fringe but not selected for expansion since its cost (450) is higher than that of Pitesti (417).

Important to understand for the proof of optimality of A*

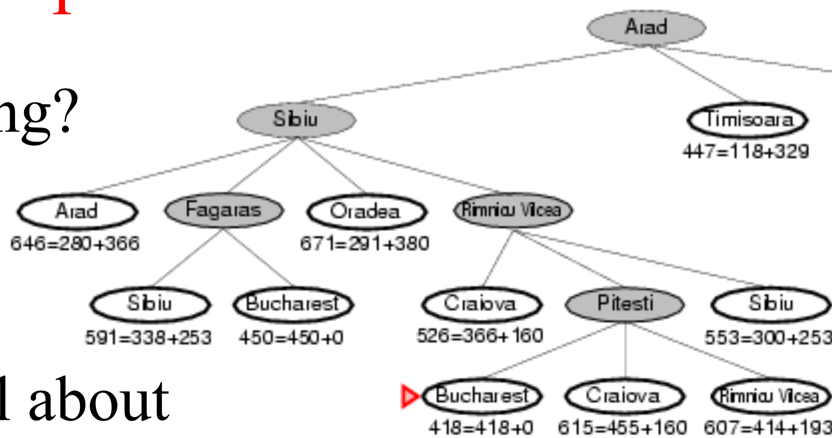


A* search example

Using: f(n) = g(n) + h(n)

Arad --- Sibiu --- Rimnicu --- Pitesti --- Bucharest

Claim: Optimal path found!



1) Can it go wrong?

Note: Best first
Arad --- Sibiu --- Fagaras --- Bucharest

2) What's special about "straight distance" to goal?

It underestimates true path distance!

Uniform cost search

3) What if all our estimates to goal are 0? Eg h(n) = 0

4) What if we overestimate?

5) What if h(n) is true distance (h*(n))? What is f(n)?

Shortest dist. through n --- perfect heuristics --- no search

A* properties

Under some reasonable conditions for the heuristics, we have:

Complete

– Yes, unless there are infinitely many nodes with f(n) < f(Goal)

Time

– Sub-exponential grow when $\left|h(n) - h^{*}(n)\right| \le O(\log h^{*}(n))$

– So, a good heuristics can bring exponential search down significantly!

Space

– Fringe nodes in memory. Often exponential. Solution: IDA*

Optimal

– Yes (under admissible heuristics; discussed next)

– Also, optimal use of heuristics information!

Widely used. After almost 40 yrs, still new applications found.

Also, optimal use of heuristic information.

Provably: Can't do better!

Heuristics: (1) Admissibility

A heuristic *$h(n)$* is admissible if for every node *n*,

$h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from *n*.

An admissible heuristic never overestimates the cost to reach the goal,

i.e., it is optimistic. (But no info of where the goal is if set to 0.)

Example: $h_{SLD}(n)$ (never overestimates the actual road distance)

Note: it follows that h(goal) = 0.

Evaluation function $f(n) = g(n) + h(n)$

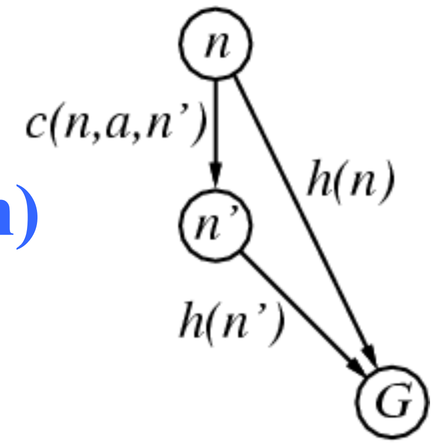
Note: less optimistic heuristic push nodes to be expanded later. Can prune a lot more.

Heuristics: (2) Consistency

A heuristic is consistent (or monotone) if for every node *n*, every successor *n'* of *n* generated by any action *a*,

$$h(n) \leq c(n,a,n') + h(n')$$

(form of the triangle inequality)



$$f(n') \geq f(n)$$

If *h* is consistent, we have

$$\begin{aligned} \mathbf{f(n')} &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\geq g(n) + h(n) \\ &= \mathbf{f(n)} \end{aligned}$$

→ sequence of nodes expanded by A* is in nondecreasing order of f(n)

→ the first goal selected for expansion must be an optimal goal.

i.e., *f(n)* is non-decreasing along any path.

Note: Monotonicity is a stronger condition than admissibility.
Any consistent heuristic is also admissible.
(Exercise 3.29)

A*: Tree Search vs. Graph Search

TREE SEARCH (See Fig. 3.7; used in earlier examples):

If *h(n)* is admissible, A* using tree search is optimal.

GRAPH SEARCH (See Fig. 3.7) A modification of tree search that includes an "explored set" (or "closed list"; list of expanded nodes to avoid re-visiting the same state); if the current node matches a node on the closed list, it is discarded instead of being expanded. In order to guarantee optimality of A*, we need to make sure that the optimal path to any repeated state is always the first one followed:

If *h(n)* is monotonic, A* using graph search is optimal.
(proof next)

(see details page 95 R&N)

Reminder: Bit of "sloppiness" in fig. 3.7. Need to be careful with nodes on frontier; allow repetitions or as in Fig. 3.14.

Intuition: Contours of A*

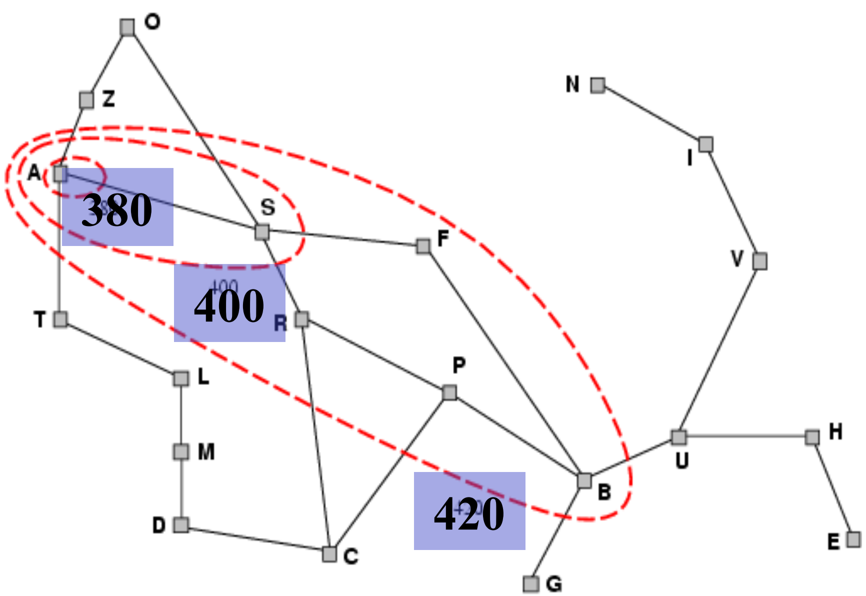
A* expands nodes in order of increasing *f* value.

Gradually adds "*f*-contours" of nodes.

Contour *i* has all nodes with $f \le f_i$, where $f_i < f_{i+1}$

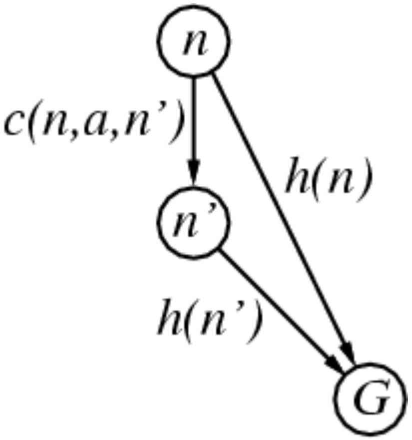
A* expands all nodes with f(n)<C*
Uniform-cost (h(n)=0) expands in circles.

Note: with uniform cost (h(n)=0) the bands will be circular around the start state.



Completeness (intuition)
As we add bands of increasing f, we must eventually reach a band where f is equal to the cost of the path to a goal state. (assuming b finite and step cost exceed some positive finite ε).

Optimality (intuition)
1st solution found (goal node expanded) must be an optimal one since <u>goal nodes in subsequent contours will have higher f-cost and therefore higher g-cost</u> (since h(goal)=0)



A* Search: Optimality

Theorem:

A* used with a *consistent* heuristic ensures optimality with graph search.

Proof:

(1) If h(n) is consistent, then the values of f(n) along any path are non-decreasing. See consistent heuristics slide.

(2) Whenever A* selects a node n for expansion, the optimal path to that node has been found. Why?
Assume *not*. Then, the optimal path, P, must have some not yet expanded nodes. (*) Thus, on P, there must be an **unexpanded** node n` on the current frontier (because of graph separation; fig. 3.9; frontier separates explored region from unexplored region). But, because f is nondecreasing along any path, n` would have a lower f-cost than n and would have been selected first for expansion before n. Contradiction.

From (1) and (2), it follows that the sequence of nodes expanded by A* using Graph-Search is in non-decreasing order of f(n). Thus, the first goal node selected must have the optimal path, because f(n) is the true path cost for goal nodes (h(Goal) = 0), and all later goal nodes have paths that are are at least as expensive. QED

(*) requires a bit of thought. Must argue that there cannot be a shorter path going only through expanded nodes (by contradiction).

Note: Termination / Completeness

Termination is guaranteed when the number of nodes with $f(n) \leq f^*$ is finite.

None-termination can only happen when

- **There is a node with an infinite branching factor, or**
- **There is a path with a finite cost but an infinite number of nodes along it.**
  - **Can be avoided by assuming that the cost of each action is larger than a positive constant *d***

A* Optimal in Another Way

It has also been shown that A* makes optimal use of the heuristics in the sense that there is no search algorithm that could expand fewer nodes using the heuristic information (and still find the optimal / least cost solution.

So, A* is "the best we can get."

Note: We're assuming a search based approach with states/nodes, actions on them leading to other states/nodes, start and goal states/nodes.

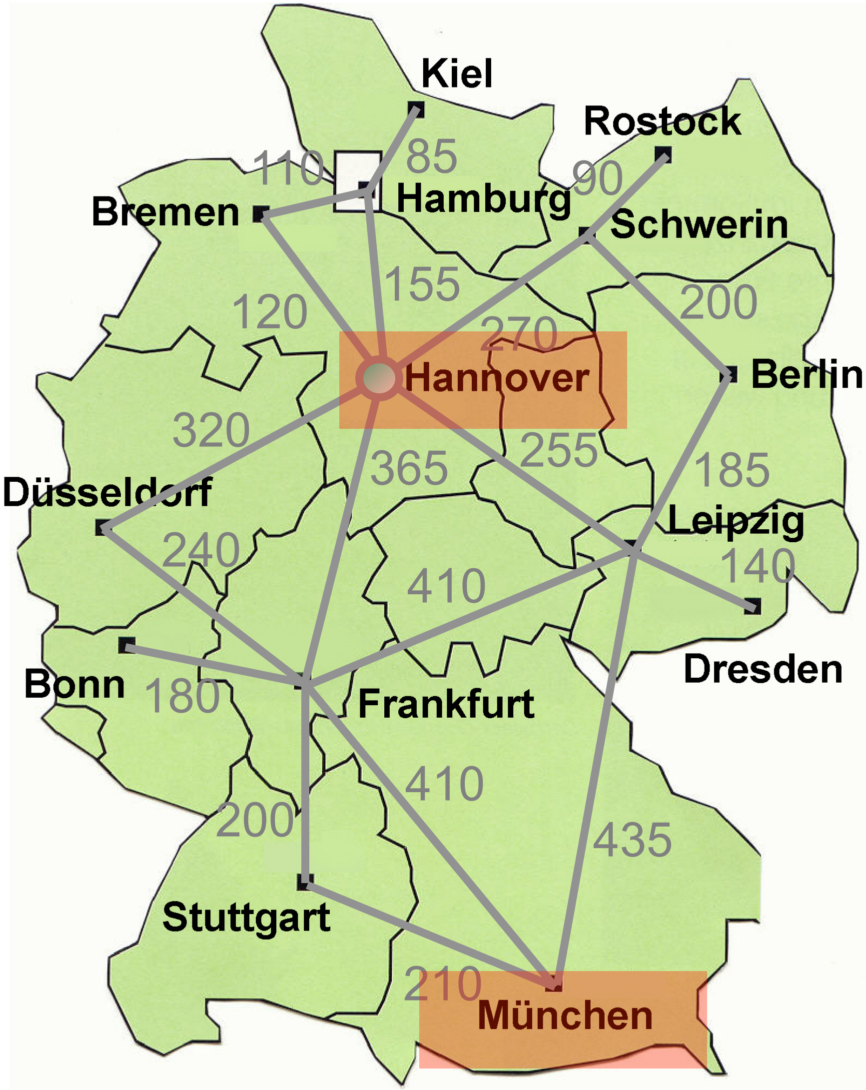
Example: Contrasting A* with Uniform Cost (Dijkstra's algorithm)

Example: The shortest route from Hannover to Munich

1) Dijkstra's alg., i.e., A* with $h(n)=0$ (Uniform cost search)
2) A* search

Example thanks to Meinolf Sellmann

Shortest Paths in Germany



Hannover	0
Bremen	∞
Hamburg	∞
Kiel	∞
Leipzig	∞
Schwerin	∞
Duesseldorf	∞
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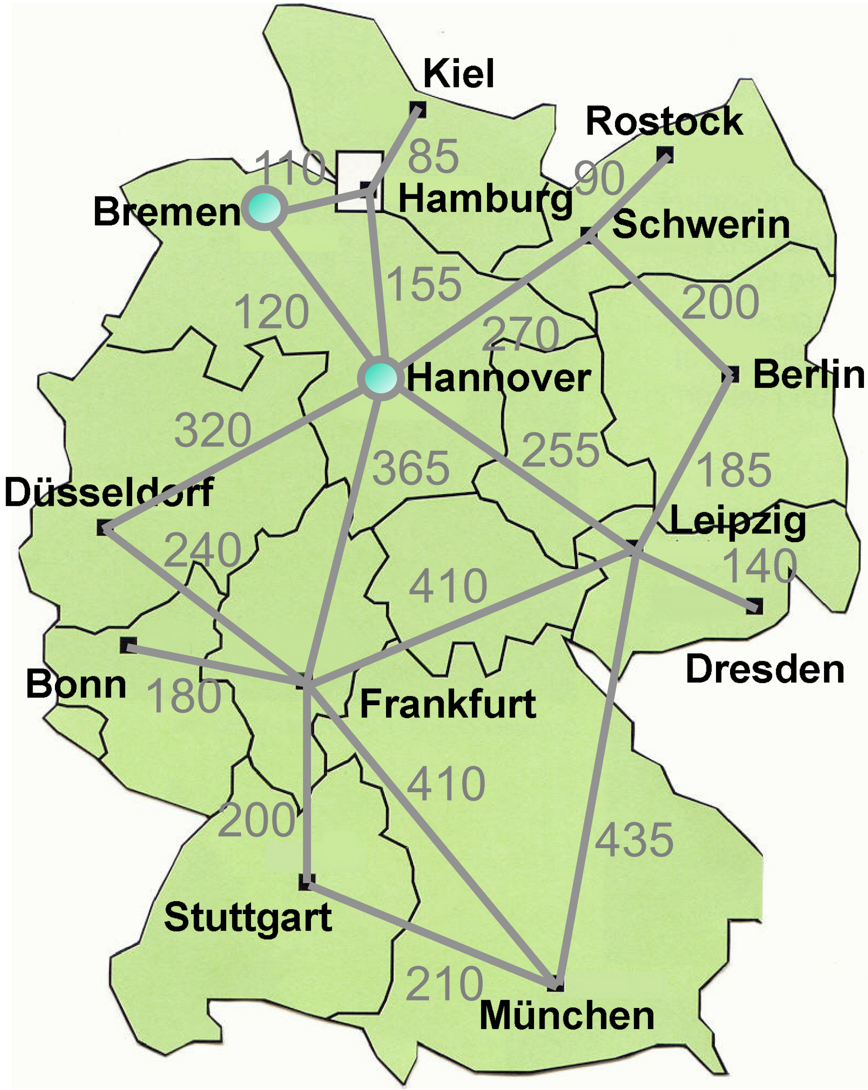

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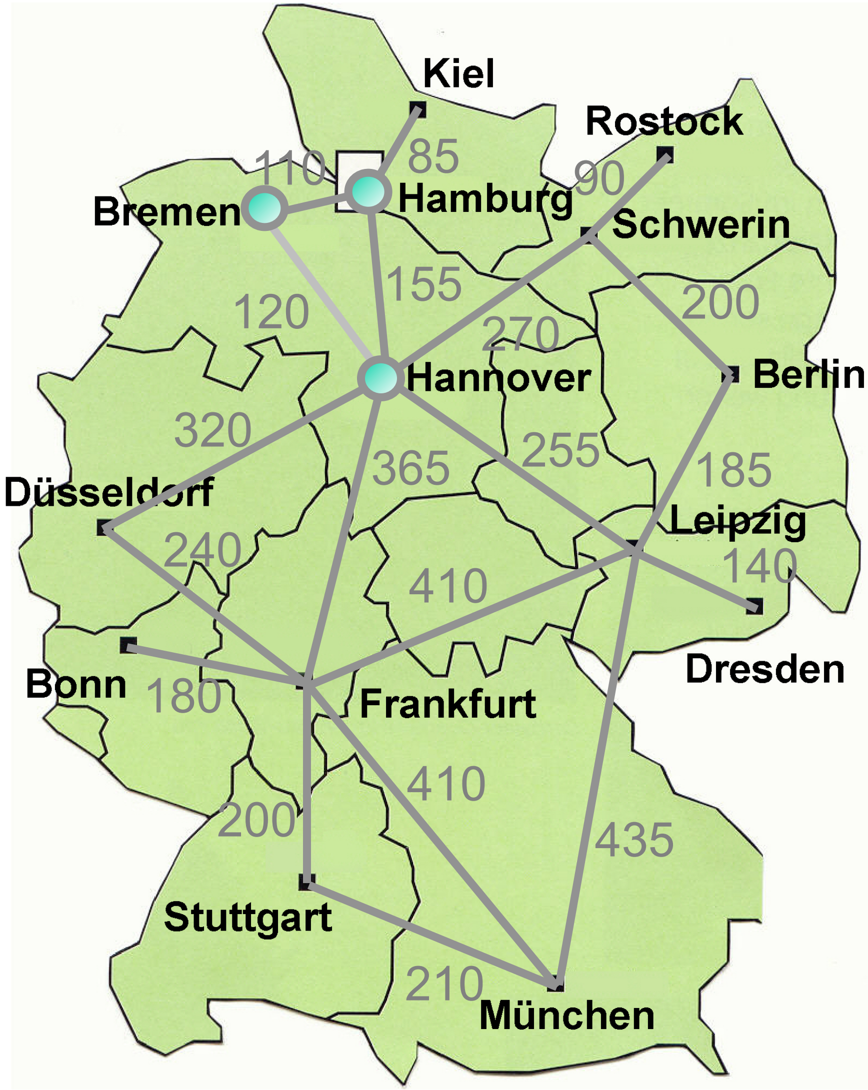

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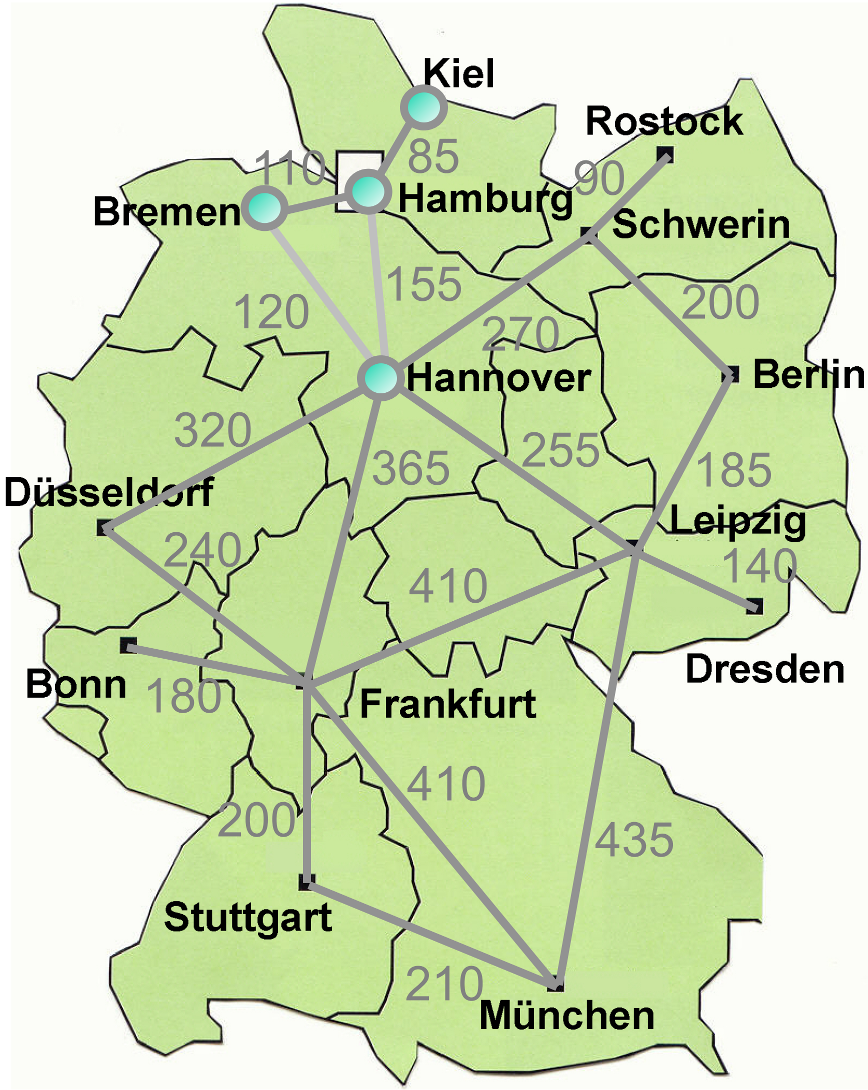

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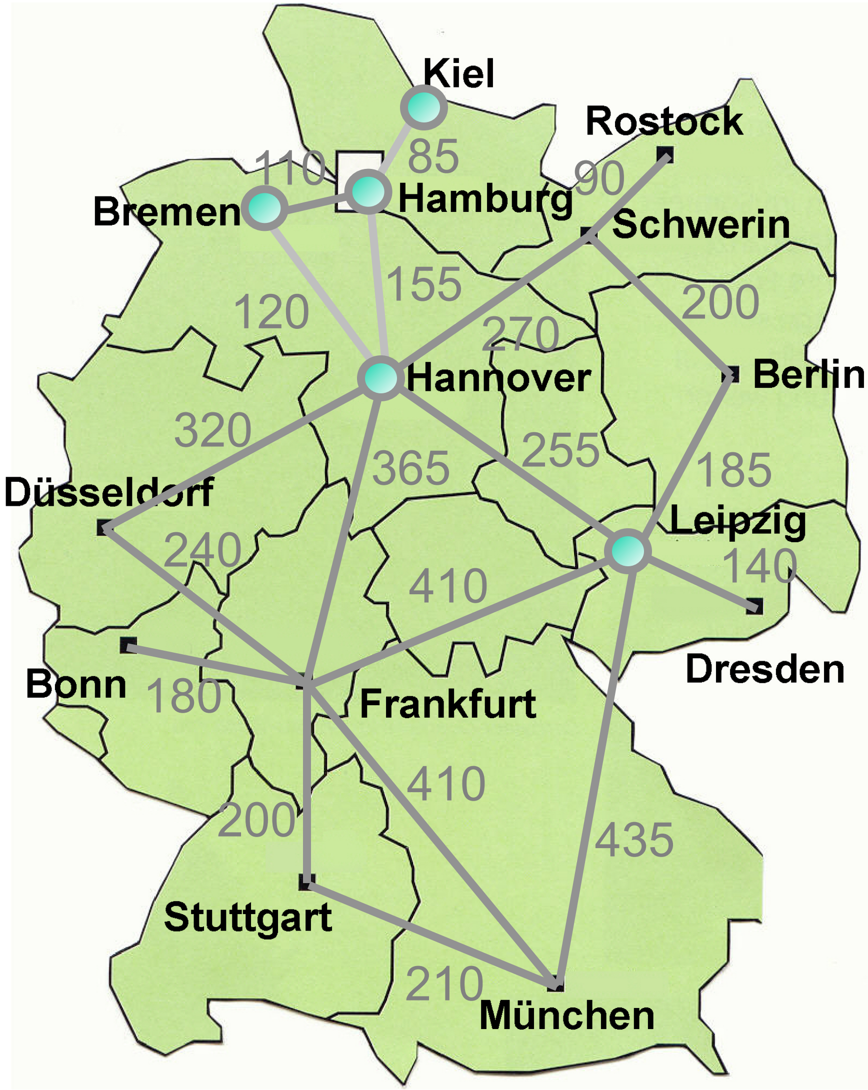

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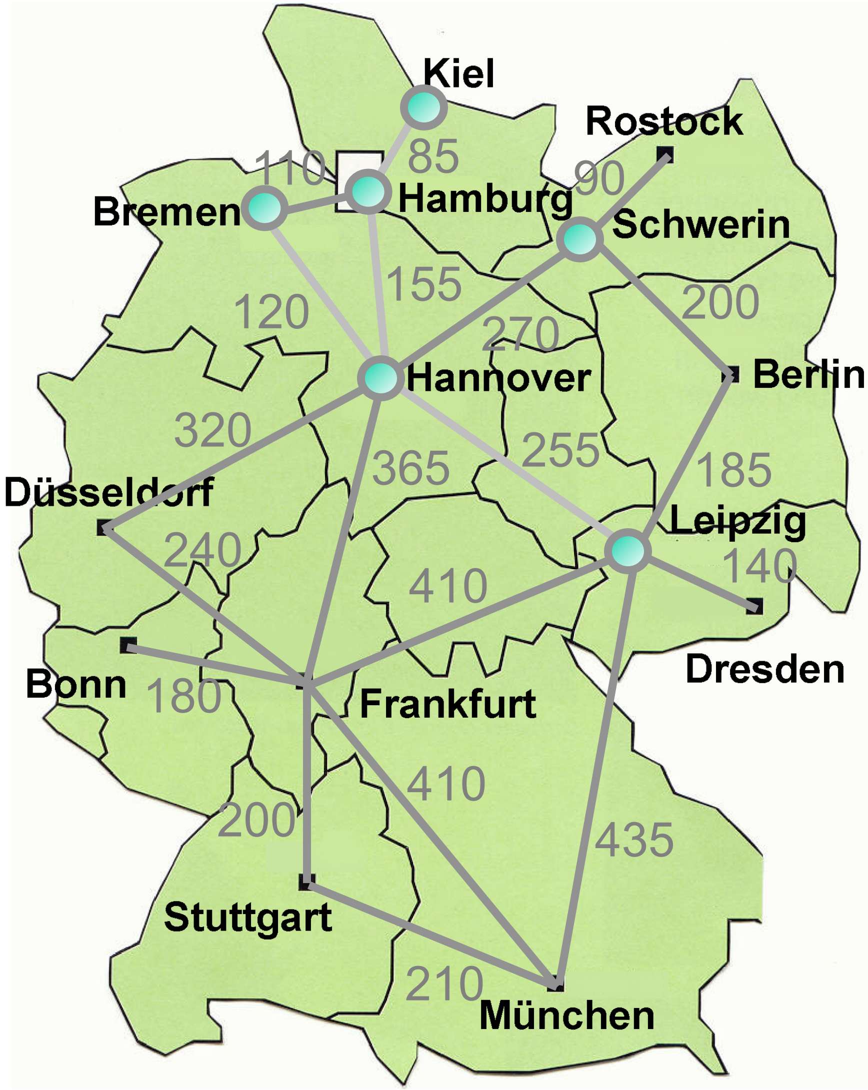

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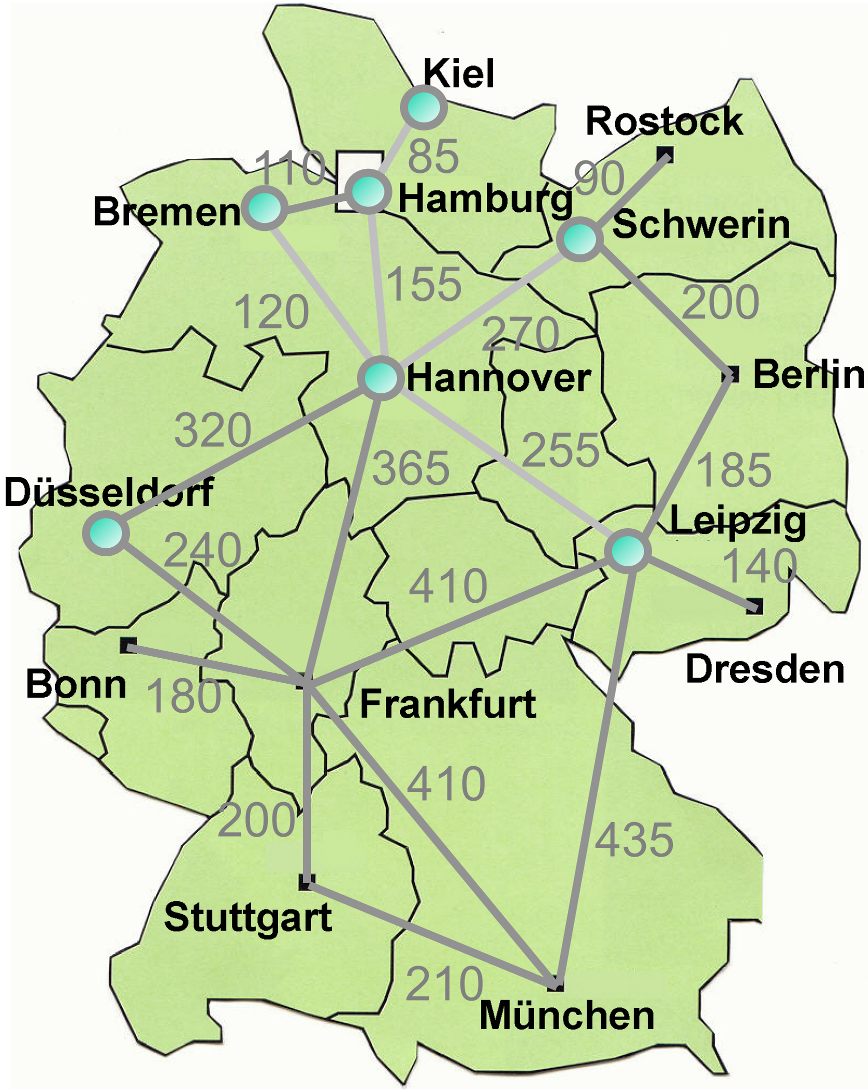
Shortest Paths in Germany



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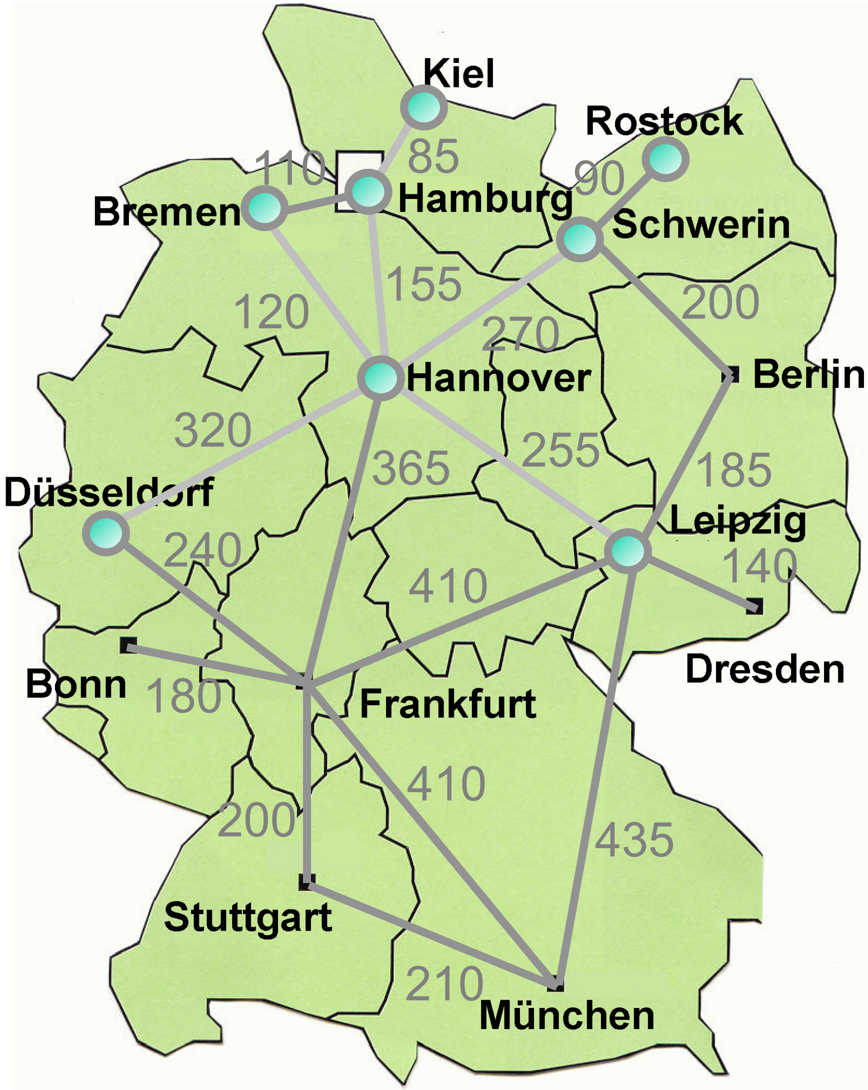
Shortest Paths in Germany



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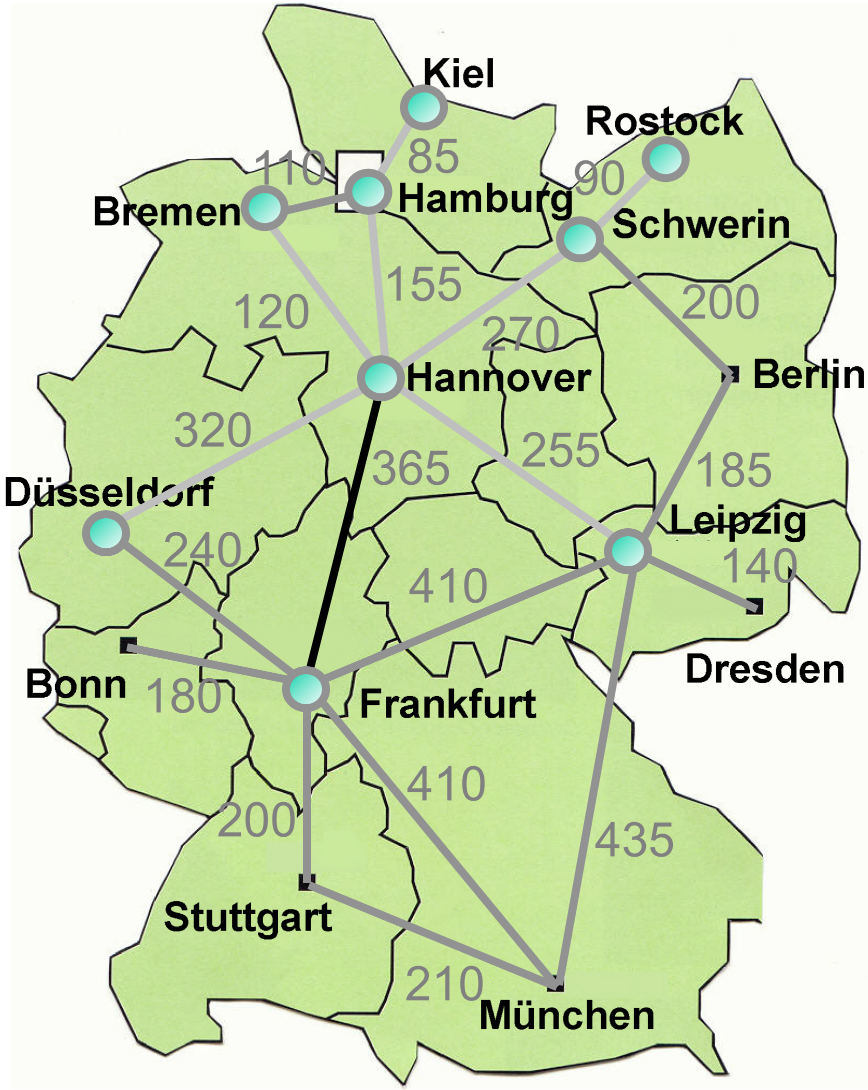

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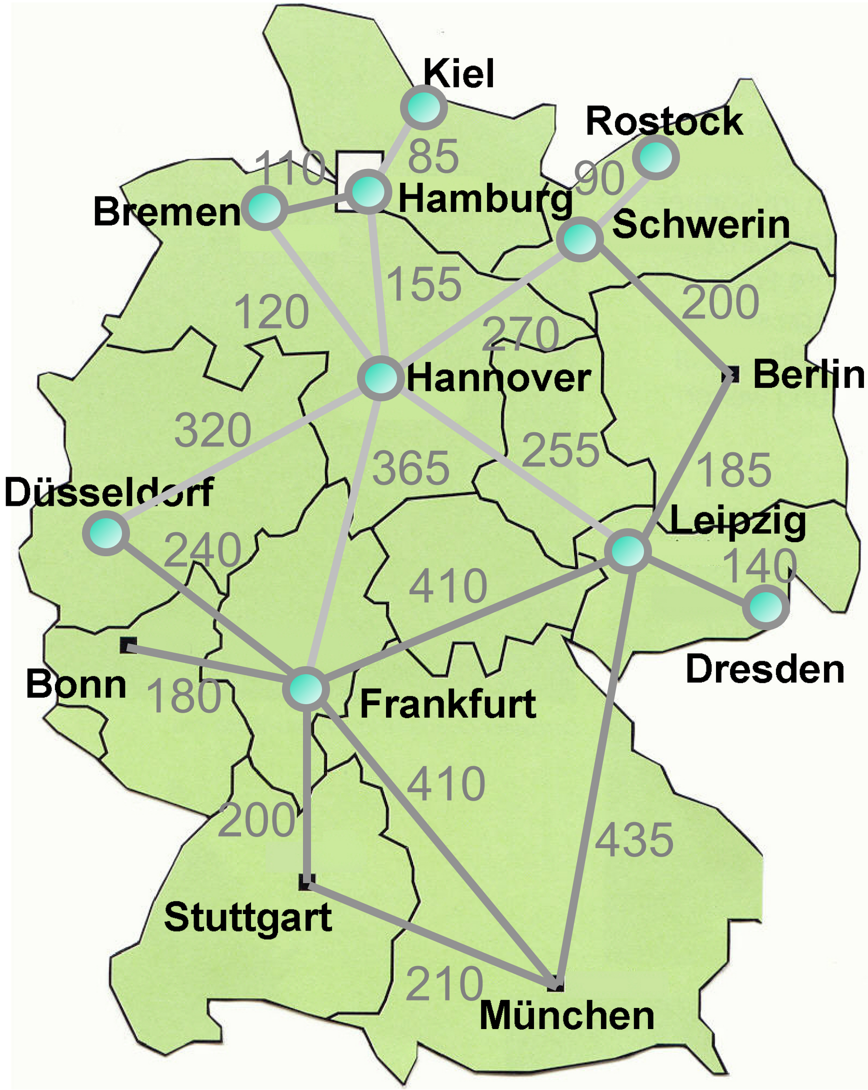

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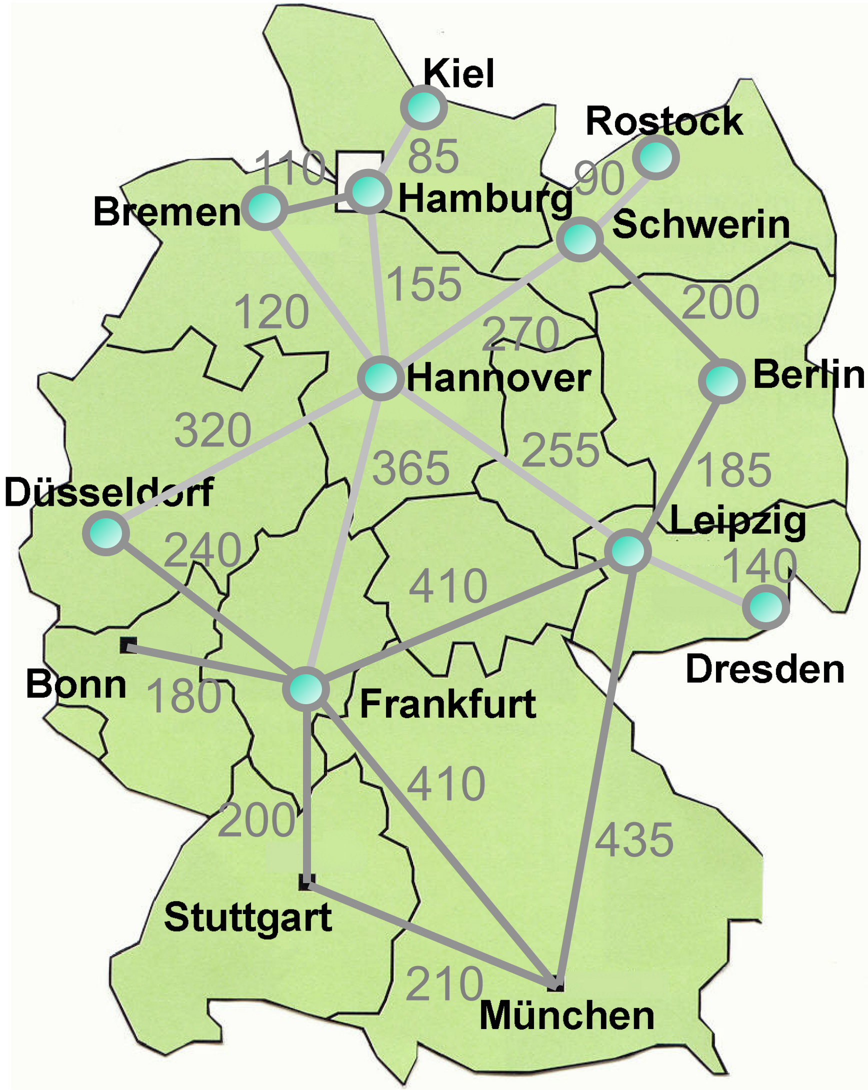
Note: route via Frankfurt longer than current one.

Shortest Paths in Germany



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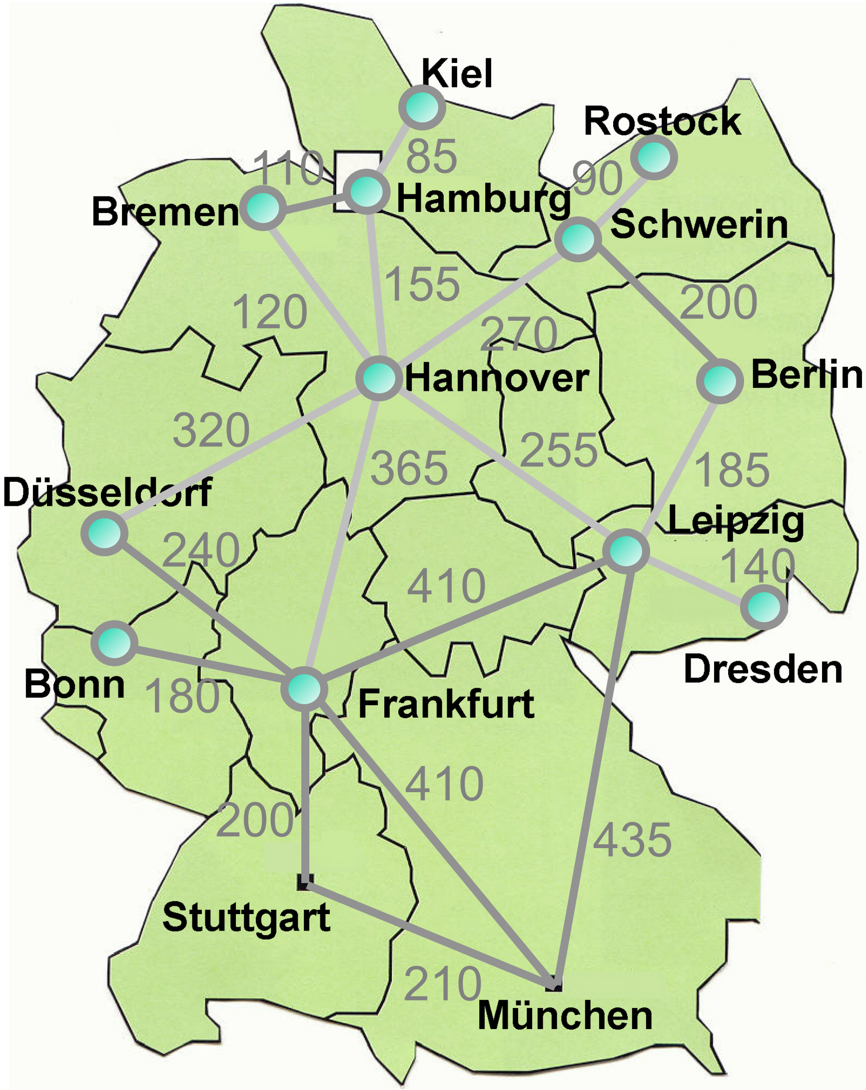
Shortest Paths in Germany



Hannover	0
Bremen	120
Hamburg	155
Kiel	240
Leipzig	255
Schwerin	270
Duesseldorf	320
Rostock	360
Frankfurt	365
Dresden	395
Berlin	440
Bonn	**545**
Stuttgart	**565**
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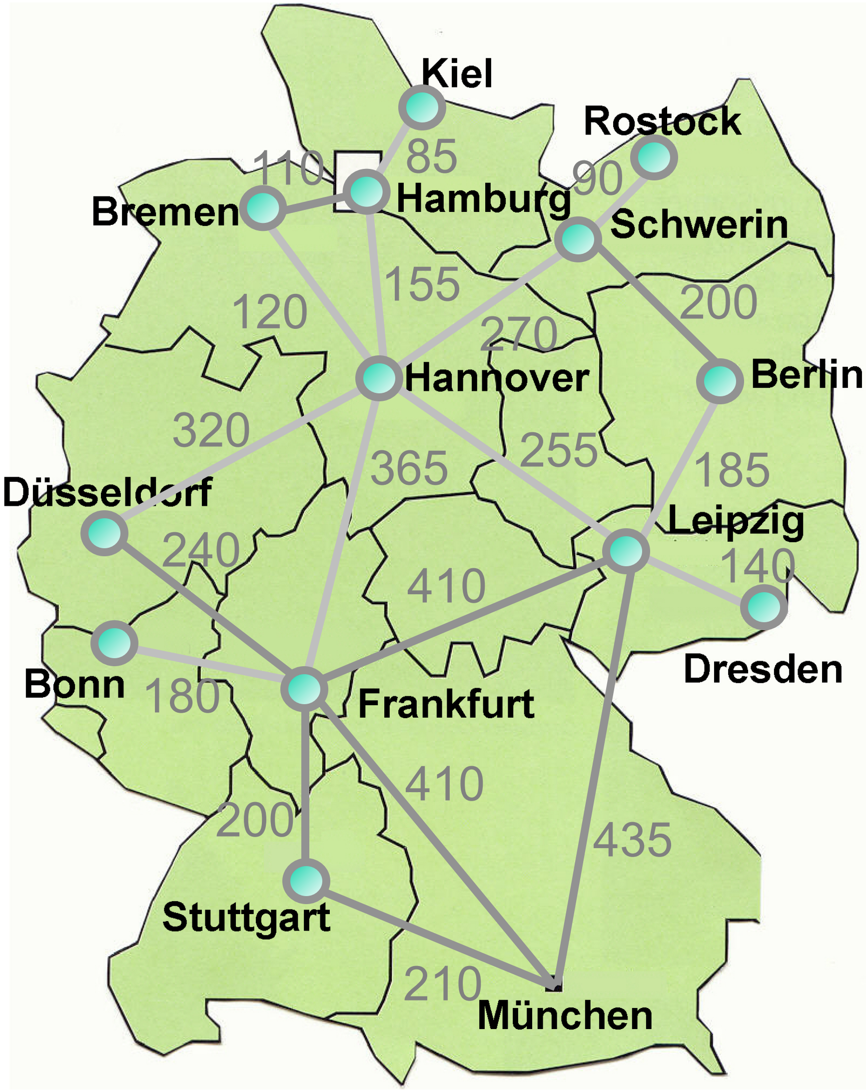
Shortest Paths in Germany



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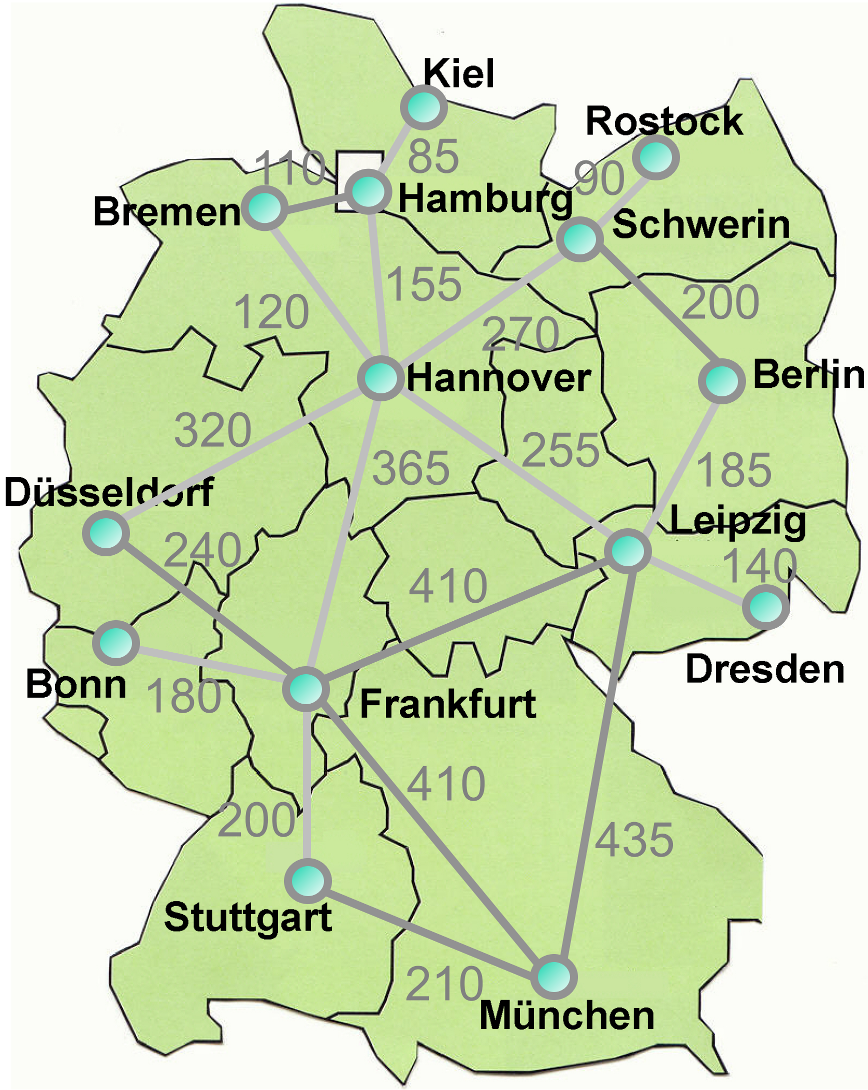
Shortest Paths in Germany



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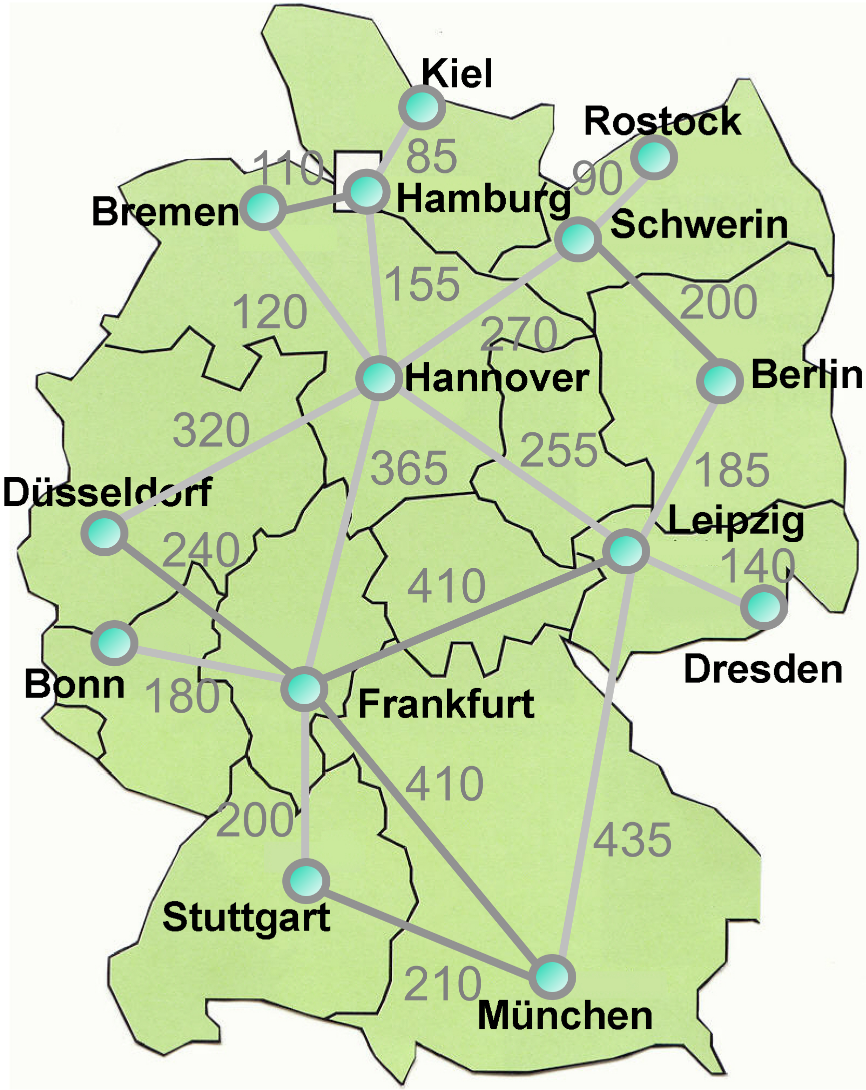

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Shortest Paths in Germany

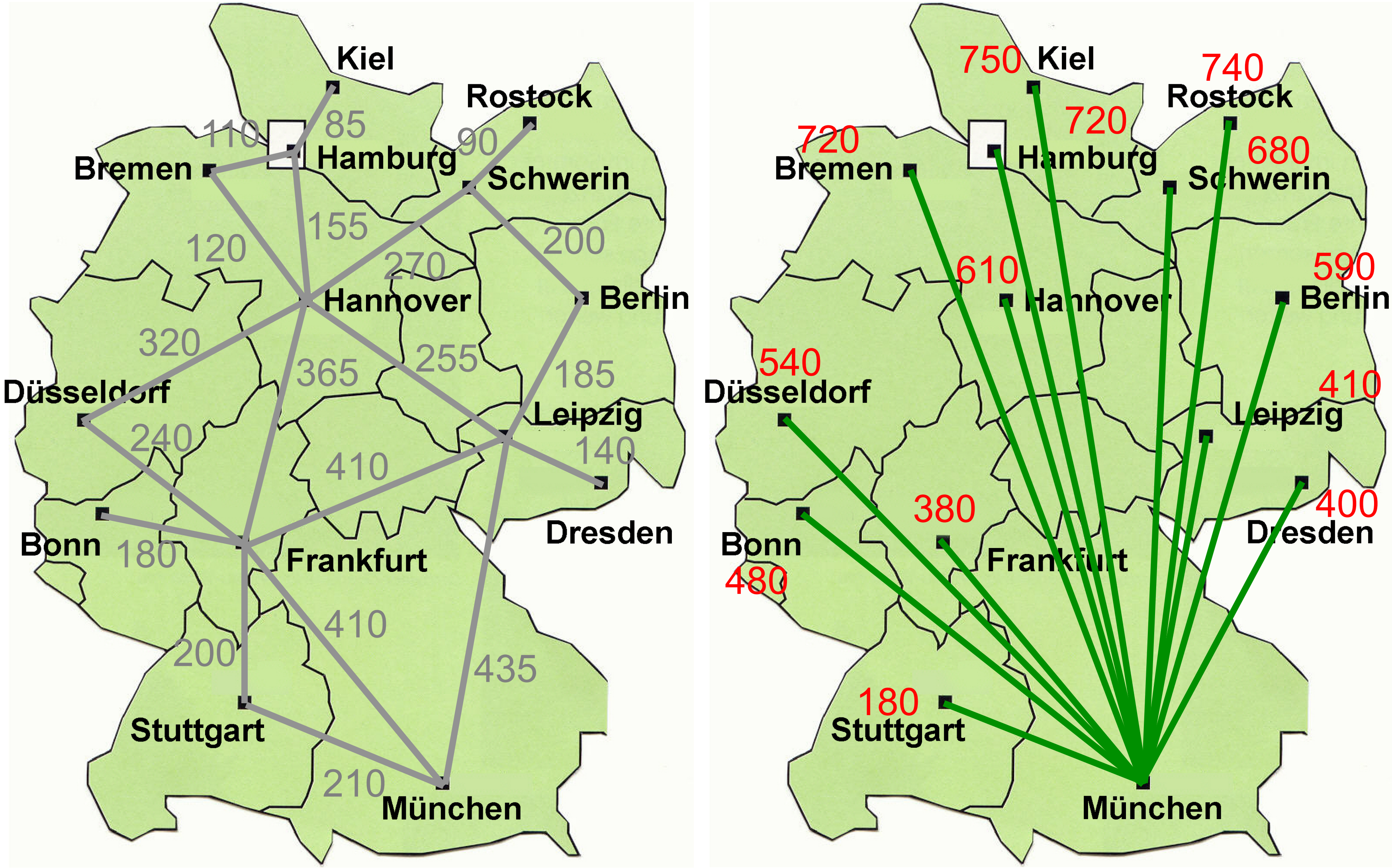
We just solved a shortest path problem by means of the algorithm from Dijkstra.

If we denote the cost to reach a state n by g(n), then Dijkstra chooses the state n from the fringe that has minimal cost g(n). (I.e., uniform cost search.)

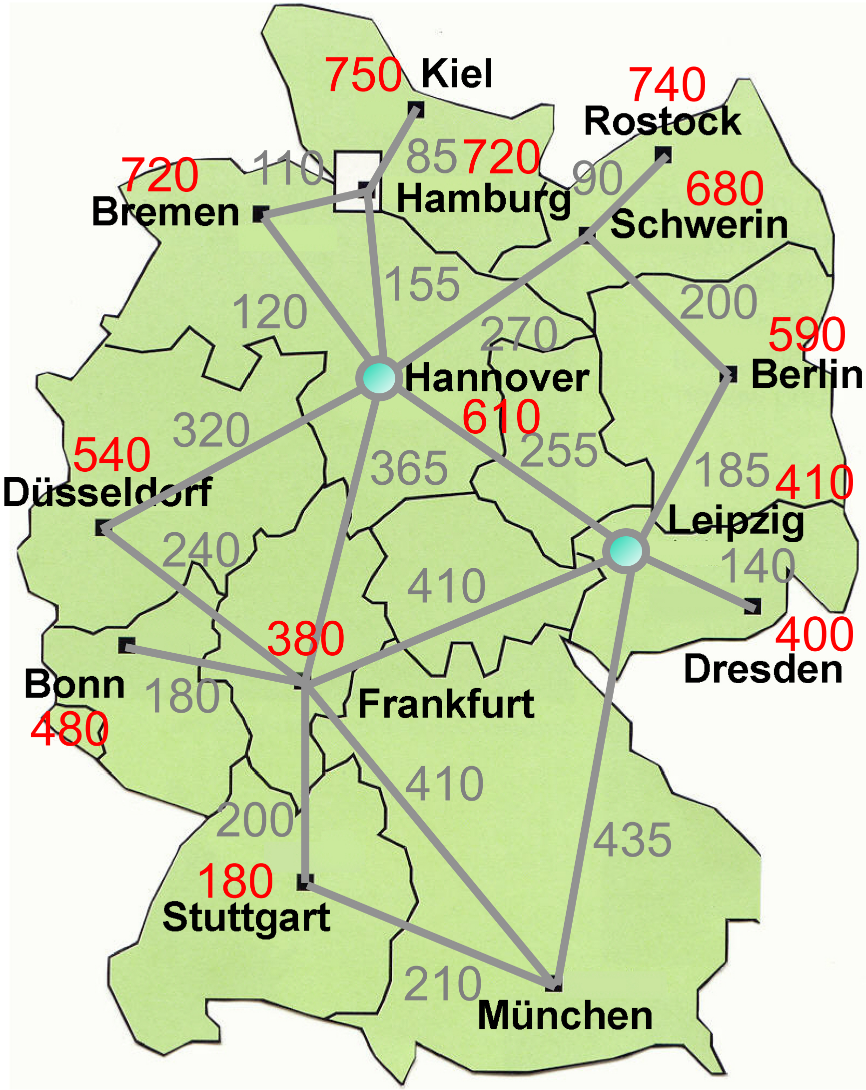
The algorithm can be implemented to run in time O(n log n + m) where n is the number of nodes, and m is the number of edges in the graph. (As noted before, in most settings n (number of world states) and m (number of possible transitions between world states) grow exponentially with problem size. E.g. (N^2-1)-puzzle.)

Approach is rather wasteful. Moves in circles around start city. Let's try A* with non-zero heuristics (i.e., straight distance).

Shortest Paths in Germany

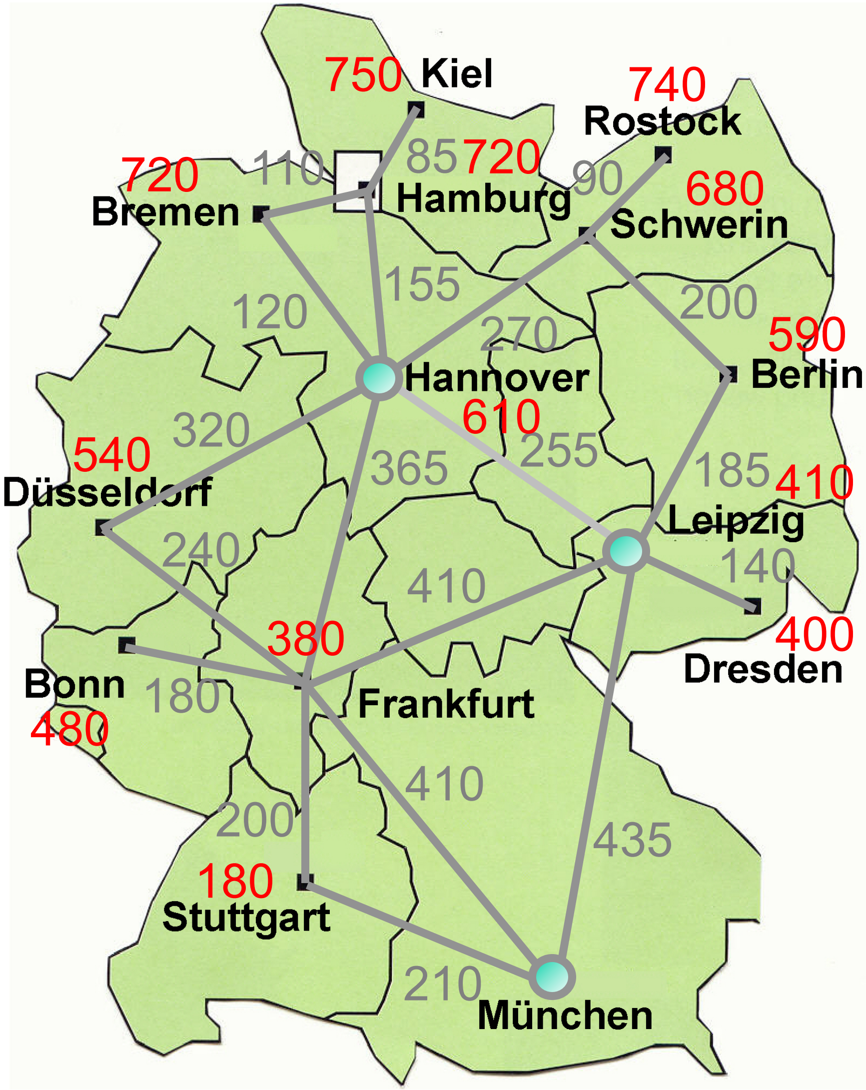


Shortest Paths in Germany



Hannover	0 + 610 = 610
Bremen	120 + 720 = 840
Hamburg	155 + 720 = 875
Kiel	∞ + 750 = ∞
Leipzig	255 + 410 = 665
Schwerin	270 + 680 = 950
Duesseldorf	320 + 540 = 860
Rostock	∞ + 740 = ∞
Frankfurt	365 + 380 = 745
Dresden	∞ + 400 = ∞
Berlin	∞ + 590 = ∞
Bonn	∞ + 480 = ∞
Stuttgart	∞ + 180 = ∞
Muenchen	∞ + 0 = ∞

Shortest Paths in Germany



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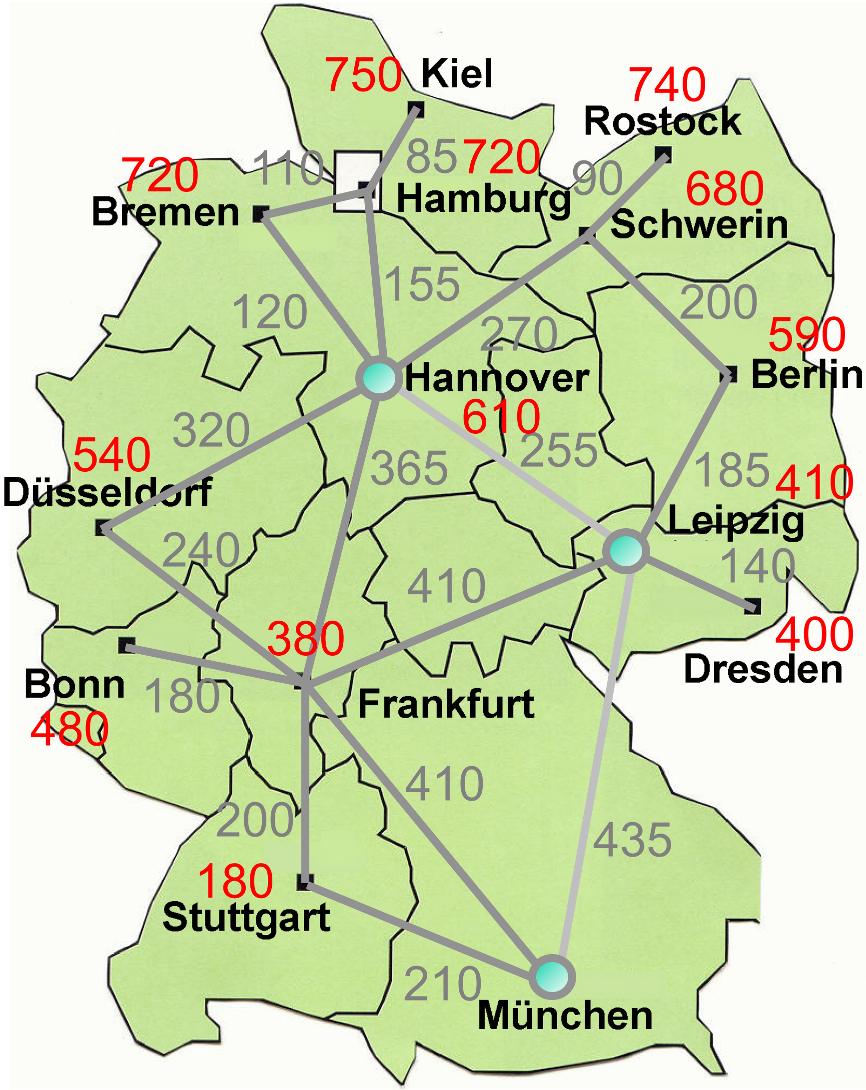
Shortest Paths in Germany



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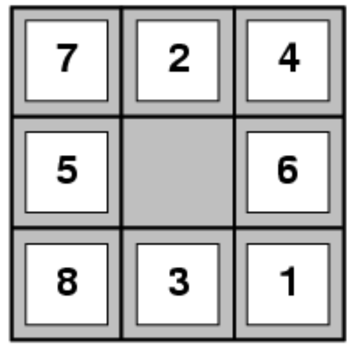
Heuristics

8-Puzzle

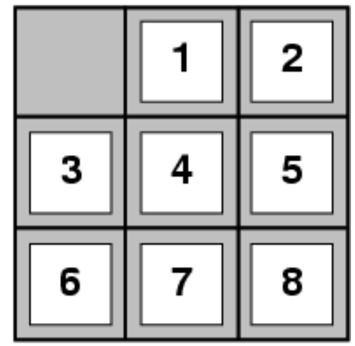
Slide the tiles horizontally or vertically into the empty space until the configuration matches the goal configuration

What's the branching factor?
(slide "empty space")

About 3, depending on location of empty tile:
middle → 4; corner → 2; edge → 3



Start State



Goal State

The average solution cost for a randomly generated 8-puzzle instance → about 22 steps
So, search space to depth 22 is about $3^{22} \approx 3.1 \times 10^{10}$ states.
→Reduced to by a factor of about 170,000 by keeping track of repeated states
(9!/2 = 181,440 distinct states) note: 2 sets of disjoint states. See exercise 3.4

But: 15-puzzle → 10^{13} distinct states!

We'd better find a good heuristic to speed up search! Can you suggest one?

Note: "Clever" heuristics now allow us to solve the 15-puzzle in a few milliseconds!

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of steps from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

$h_1(\text{Start}) = ?$ 8

$h_2(\text{Start}) = ?$ $3+1+2+2+2+3+3+2 = 18$

True cost = 26

Why are heuristics admissible?

Which is better?

How can we get the optimal heuristics? (Given H_opt(Start) = 26. How would we find the next board on the optimal path to the goal?)

Desired properties heuristics:
(1) consistent (admissible)
(2) As close to opt as we can get (sometimes go a bit over…)
(3) Easy to compute! We want to explore many nodes.

Note: each empty-square-move = 1 step tile move.

Comparing heuristics

Effective Branching Factor, b*

- If A* generates N nodes to find the goal at depth d

  b* = branching factor such that a uniform tree of depth d contains N+1 nodes (we add one for the root node that wasn't included in N)

  $N+1 = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$

  E.g., if A* finds solution at depth 5 using 52 nodes, then the effective branching factor is 1.92.

- b* close to 1 is ideal
  - because this means the heuristic guided the A* search is closer to ideal (linear).
  - If b* were 100, on average, the heuristic had to consider 100 children for each node
  - Compare heuristics based on their b*

Comparison of heuristics

	Search Cost			Effective Branching Factor		
d	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	–	539	113	–	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Figure 4.8 Comparison of the search costs and effective branching factors for the ITERATIVE-DEEPENING-SEARCH and A^* algorithms with h_1, h_2. Data are averaged over 100 instances of the 8-puzzle, for various solution lengths.

h2 indeed significantly better than h1

d – depth of goal node

Dominating heuristics

h_2 is always better than h_1

- **Because for any node, n, $h_2(n) >= h_1(n)$. (Why?)**

We say h_2 dominates h_1

It follows that h1 will expand at least as many nodes as h2.

Because:

Recall all nodes with $f(n) < C^*$ will be expanded.

This means all nodes, $h(n) + g(n) < C^*$, will be expanded.

So, all nodes n where $h(n) < C^* - g(n)$ will be expanded

All nodes h_2 expands will also be expanded by h_1 and because h_1 is smaller, others may be expanded as well

Inventing admissible heuristics: Relaxed Problems

Can we generate h(n) automatically?

- **Simplify problem by reducing restrictions on actions**

A problem with fewer restrictions on the actions is called a relaxed problem

Examples of relaxed problems

Original: **A tile can move from square A to square B iff**
(1) A is horizontally or vertically adjacent to B *and* (2) B is blank

Relaxed versions:

- **A tile can move from A to B if A is adjacent to B (“overlap”; Manhattan distance)**
- **A tile can move from A to B if B is blank (“teleport”)**
- **A tile can move from A to B (“teleport and overlap”)**

Key: **Solutions to these relaxed problems can be computed <u>without search</u> and therefore provide a heuristic that is easy/fast to compute.**

This technique was used by ABSOLVER (1993) to invent heuristics for the 8-puzzle better than existing ones and it also found a useful heuristic for famous Rubik’s cube puzzle.

Inventing admissible heuristics: Relaxed Problems

The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem. Why?

1) The optimal solution in the original problem is also a solution to the relaxed problem (satisfying in addition all the relaxed constraints). So, the solution cost matches at most the original optimal solution.

2) The relaxed problem has fewer constraints. So, there may be other, less expensive solutions, given a lower cost (admissible) relaxed solution.

What if we have multiple heuristics available? I.e., h_1(n), h_2(n), …

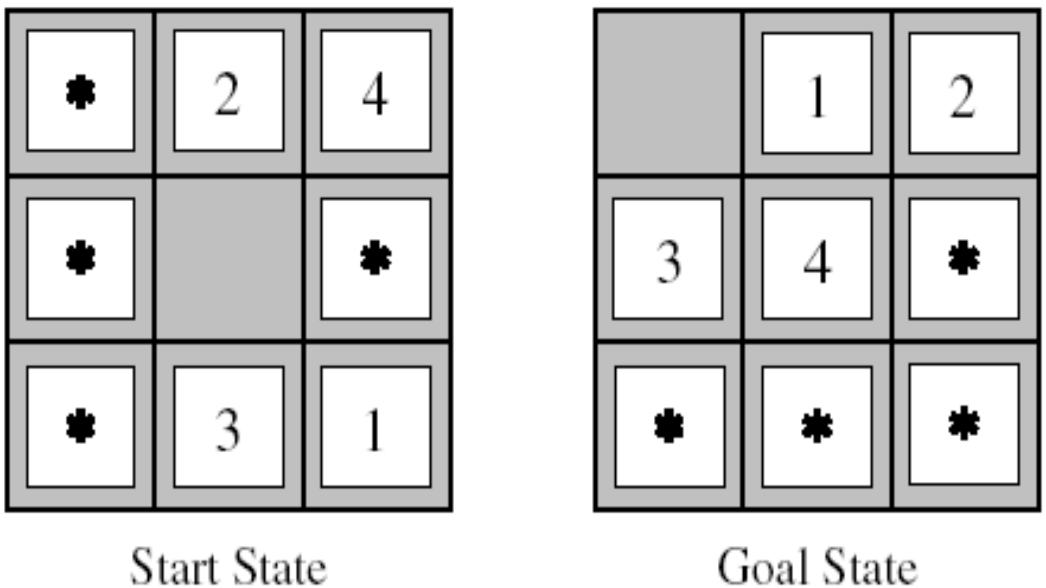
$$h(n) = \max \{h_1(n), h_2(n), \ldots, h_m(n)\}$$

If component heuristics are admissible so is the composite.

Inventing admissible heuristics: Sub-problem solutions as heuristic

What is the optimal cost of solving some portion of original problem?

- **subproblem solution is heuristic of original problem**



Pattern Databases

Store optimal solutions to subproblems in database

- We use an exhaustive search to solve every permutation of the 1,2,3,4-piece subproblem of the 8-puzzle
- During solution of 8-puzzle, look up optimal cost to solve the 1,2,3,4-piece sub-problem and use as heuristic
- Other configurations can be considered

Inventing admissible heuristics: Learning

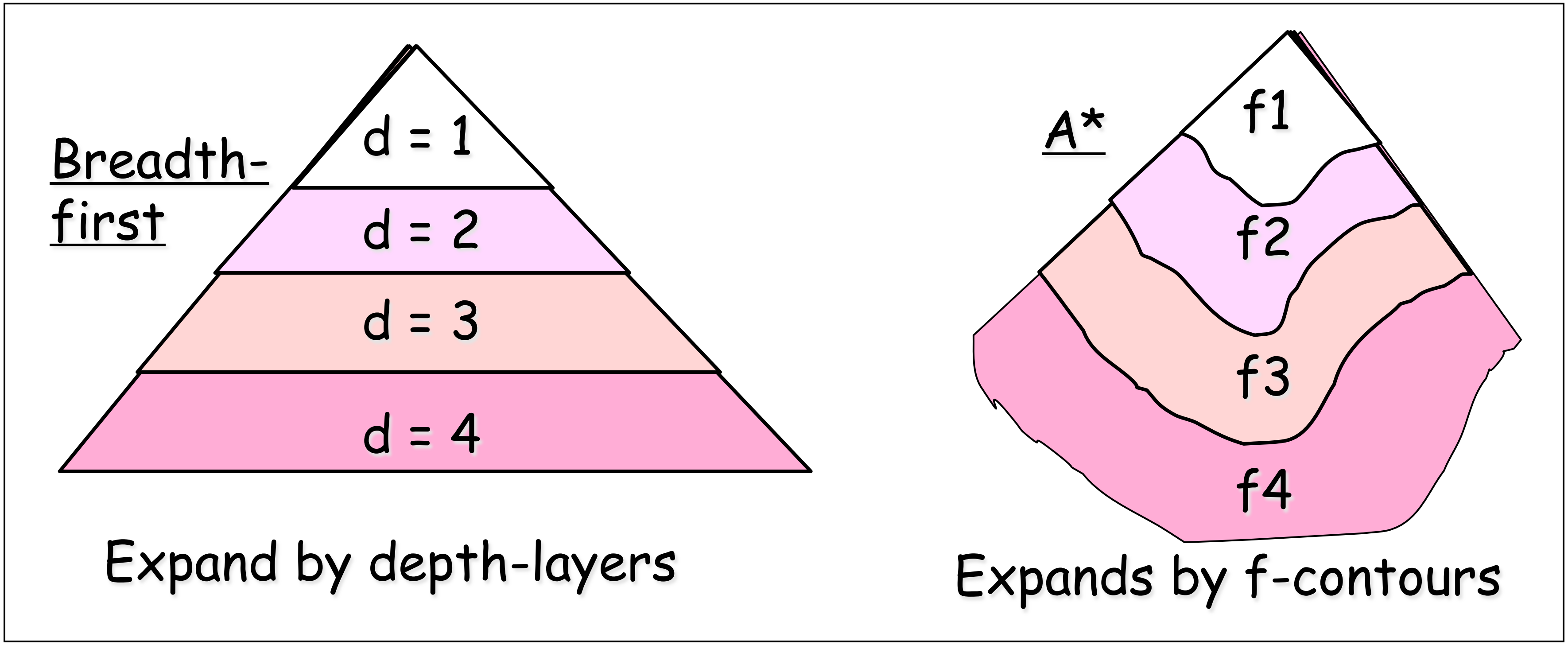
Also automatically learning admissible heuristics using machine learning techniques, e.g., inductive learning and reinforcement learning.

Generally, you try to learn a “state-evaluation” function or “board evaluation” function. (How desirable is state in terms of getting to the goal?) Key: What “features / properties” of state are most useful?

More later…

Memory problems with A*

A* is similar to breadth-first:

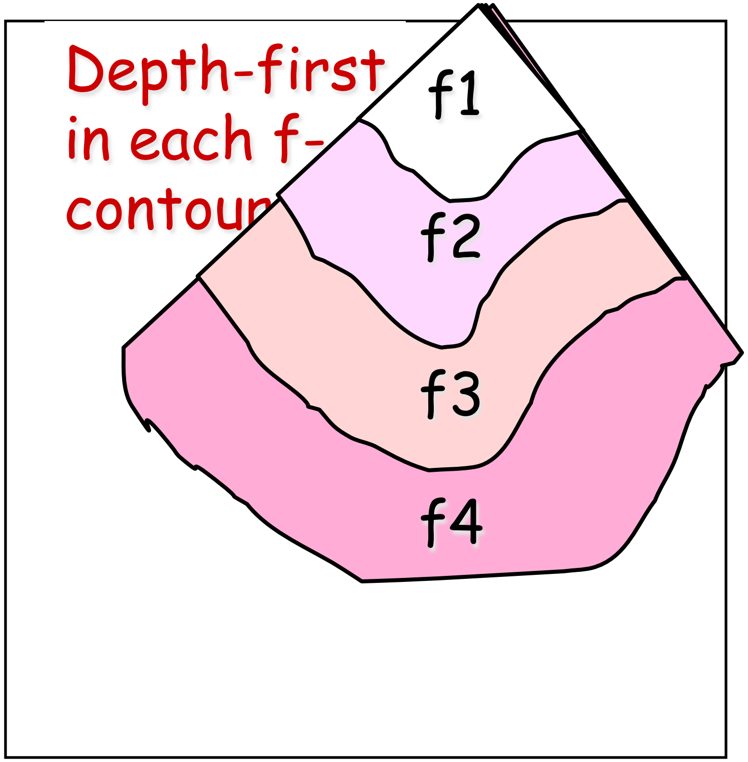


IDA* : Iterative Deepening A* (Korf 1985)

Use idea similar to Iterative Deepening DFS.

Slides adapted from Daniel de Schreye's

Iterative deepening A* (IDA*)



Perform *depth-first search* LIMITED to some f-bound.

If goal found: ok.

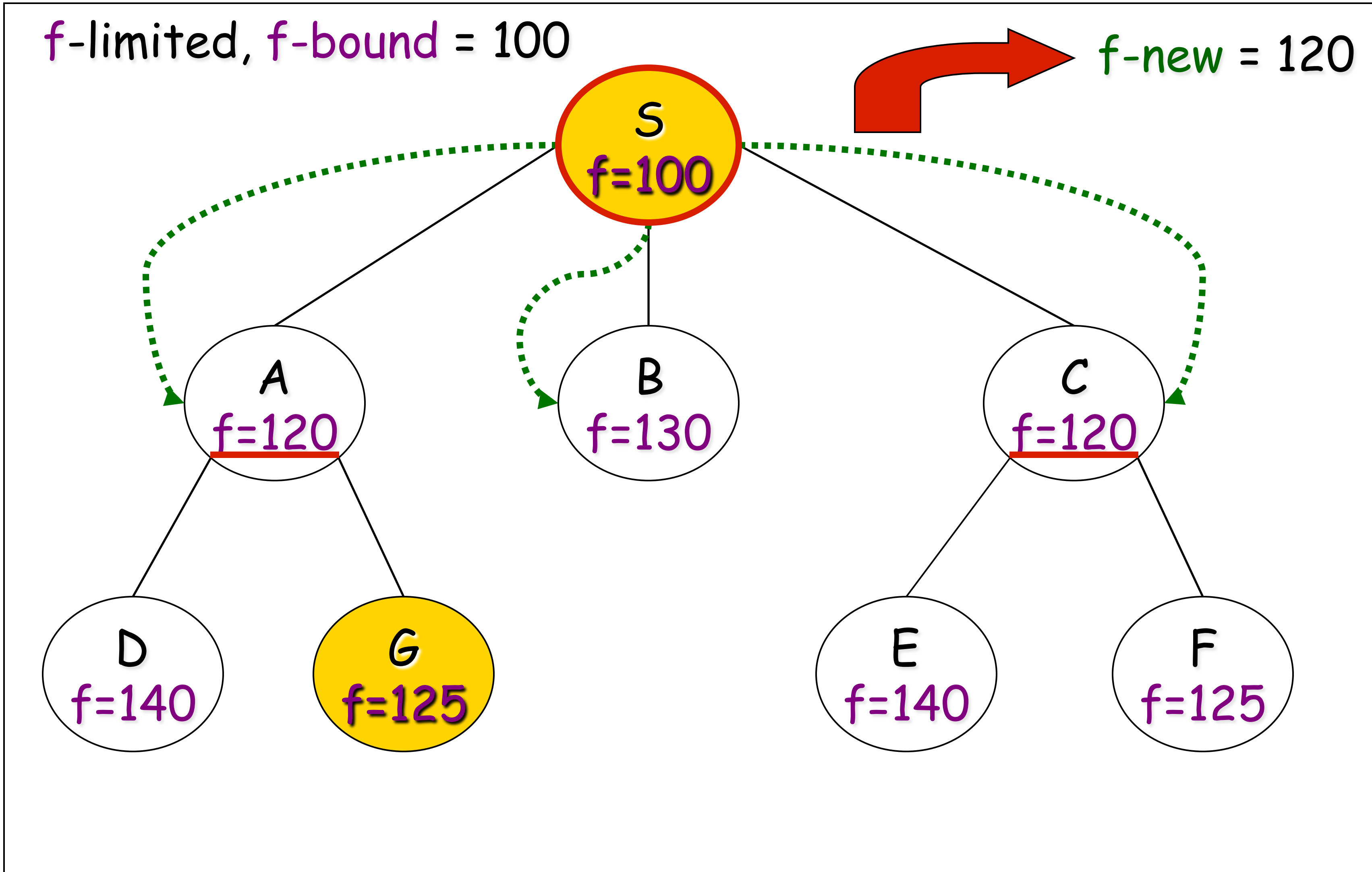
Else: increase f-bound and restart.

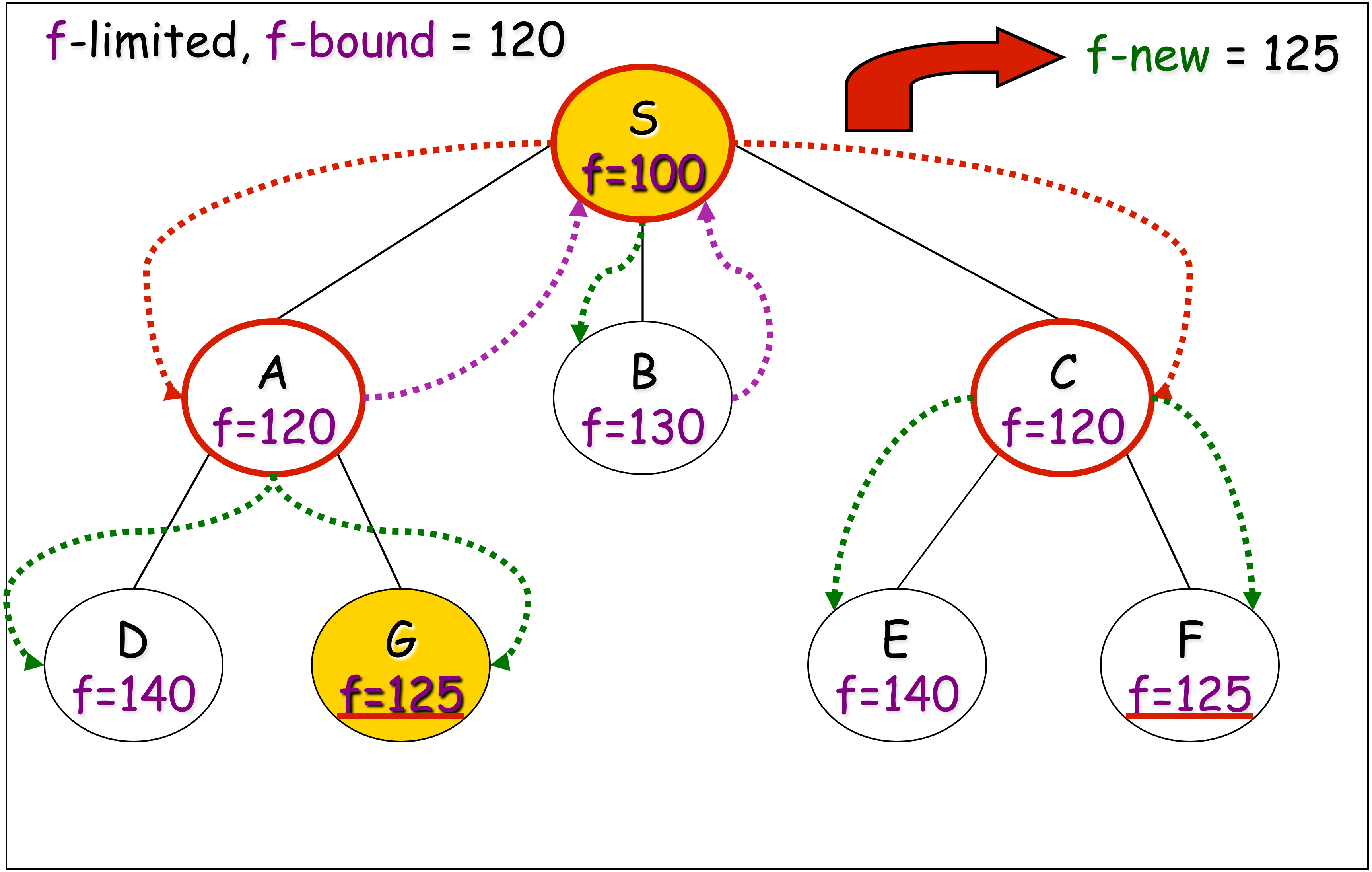
Note: DFS does not need to “sort” the frontier nodes. All at f-bound value.

How to establish the f-bounds?
- initially: f(S) (S – start node)
  - generate all successors
  - record the minimal f(succ) > f(S)

Continue with minimal f(succ) instead of f(S)

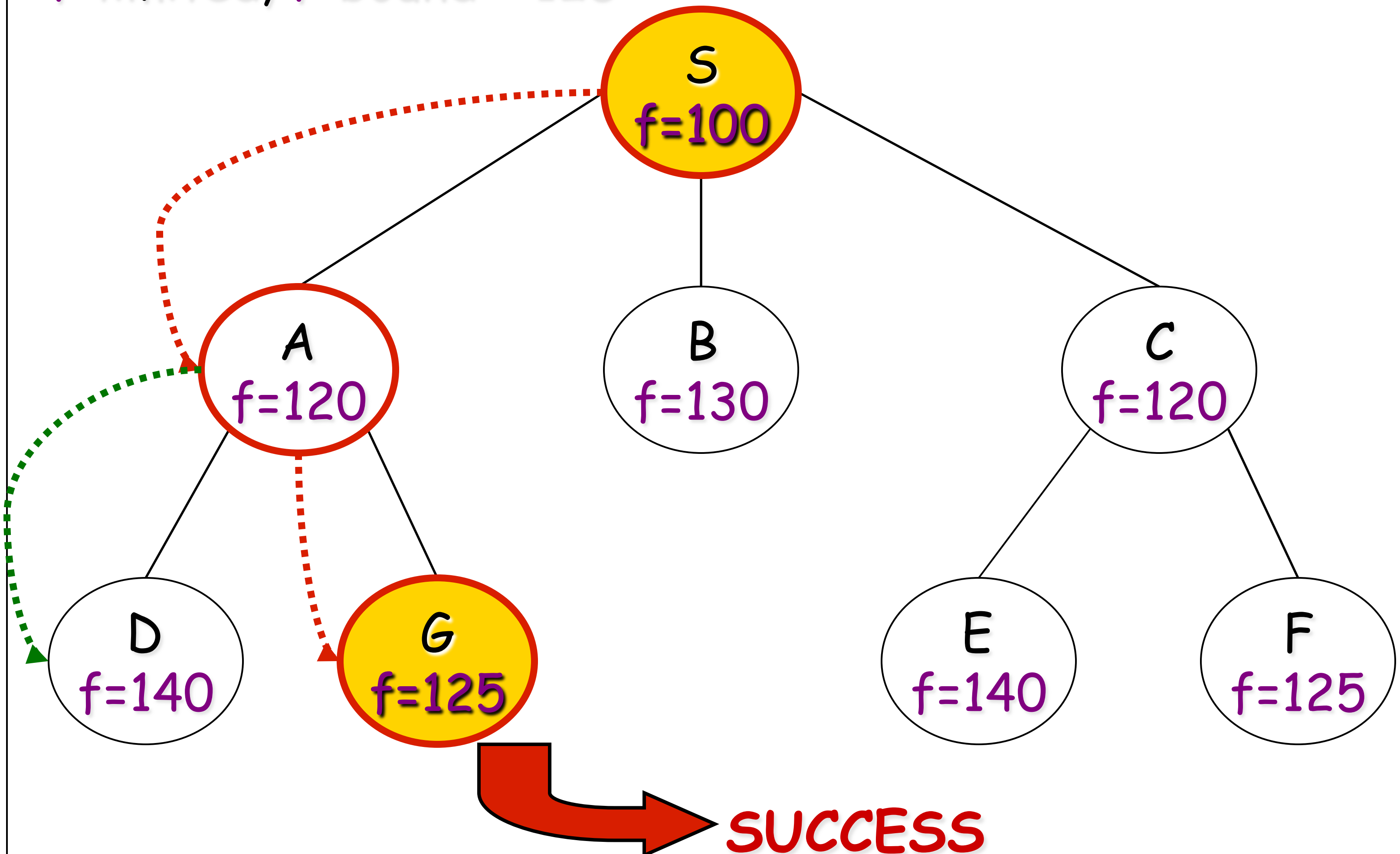
Example
[check at home]





f-limited, f-bound = 120
f-new = 125
S
f=100
A
f=120
B
f=130
C
f=120
D
f=140
G
f=125
E
f=140
F
f=125

f-limited, f-bound = 125



Properties: practical

If there are only a reduced number of different contours:

- IDA* is one of the very best optimal state-space search techniques!
  - Example: the 15-puzzle
  - Also for many other practical problems

Else, the gain of the extended f-contour is not sufficient to compensate recalculating the previous parts. Do:

- increase f-bound by a fixed number ε at each iteration:
  - effects: less re-computations, BUT: optimality is lost: obtained solution can deviate up to ε
  - Can be remedied by completing the search at this layer
    Issue: finding goal state potentially "too early" in final layer, without having the optimal path. (Consider "Bucharest" twice on frontier; but we don't keep full frontier in memory, with DFS.)

Summary

Uninformed search:

(1) Breadth-first search (2) Uniform-cost search

(3) Depth-first search (4) Depth-limited search

(5) Iterative deepening search (6) Bidirectional search

Informed search:

(1) Greedy Best-First

(2) A*

Summary, cont.

Heuristics allow us to scale up solutions dramatically!

Can now search combinatorial (exponential size) spaces with easily 10^15 states and even up to 10^100 or more states.
Especially, in modern heuristics search planners (eg FF).

Before informed search, considered totally infeasible.

Still many variations and subtleties:
There are conferences and journals dedicated solely to search.

Lots of variants of A*. Research in A* has increased dramatically since A* is the key algorithm used by map engines.

Also used in path planning algorithms (autonomous vehicles), and general (robotics) planning, problem solving, and even NLP parsing.