Ungraded quiz: camera calibration and stereo

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1. When performing camera calibration, we set up a system of equations $A\|\mathbf{p}\| = \mathbf{0}$ in the parameters $\mathbf{p}$ that define the camera projection matrix. We then tried to minimize $A\|\mathbf{p}\|$ subject to $\|\mathbf{p}\| = 1$. Here, we constrain $\|\mathbf{p}\| = 1$ because:

   (a) A camera projection matrix is valid only if its Frobenius norm is 1.

   (b) The constraint makes the optimization easier to implement.

   (c) The correspondences used to form A might be noisy.

   (d) The equations $A\|\mathbf{p}\| = \mathbf{0}$ are not sufficient to produce a unique matrix P, and will produce a family of solutions.

   (d) is correct. Because the projection equations are all expressed in homogenous coordinates, if P is a solution, so is αP. To isolate a single solution, we need to add a constraint on the Frobenius norm.

2. For a particular camera, the intrinsic camera parameters are $K = I$. Its projection matrix P is one of the following. Which is it?

   (a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 0.8 & 0.6 & 0 \\ -0.6 & 0.8 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 0.8 & 0.6 & 0 & 5 \\ -0.6 & 0.8 & 0 & 7 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 7 & 0 \end{bmatrix}$

   P is a 3×4 matrix. If K is identity, then $P = \begin{bmatrix} R & \mathbf{t} \end{bmatrix}$, where R is a rotation matrix. Thus the first 3×3 submatrix of P must be a rotation matrix. This in turn means that the first 3 columns of P must be (a) orthogonal to each other, and (b) unit norm. Only (c) satisfies this constraint.

3. Two cameras are looking at a scene. They have projection matrices $P^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $P^{(2)} = \begin{bmatrix} 0.8 & 0 & 0.6 & -4 \\ 0 & 1 & 0 & 0 \\ -0.6 & 0 & 0.8 & 3 \end{bmatrix}$. A 3D world point appears in the first image at the location $(2, 0)$, and in the second image at location $(-18, 0)$ (Each tuple is the (x,y) coordinates). What is the 3D location of this world point?

Suppose the world point is $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$. From the first camera, we have:

$$\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \equiv P^{(1)} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \tag{1}$$

$$\Rightarrow \lambda \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = P^{(1)} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \tag{2}$$

$$\Rightarrow \lambda \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{3}$$

$$\begin{aligned} \Rightarrow 2\lambda &= X \\ 0 &= Y \\ \lambda &= Z \end{aligned} \tag{4}$$

$$\Rightarrow X = 2Z \tag{5}$$

$$Y = 0 \tag{6}$$

Let us now look at the second camerra:

$$\begin{bmatrix} -18 \\ 0 \\ 1 \end{bmatrix} \equiv P^{(2)} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \tag{7}$$

$$\Rightarrow \lambda \begin{bmatrix} -18 \\ 0 \\ 1 \end{bmatrix} = P^{(2)} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \tag{8}$$

$$\begin{aligned} \Rightarrow -18\lambda &= 0.8X + 0.6Z - 4 \\ 0 &= Y \\ \lambda &= -0.6X + 0.8Z + 3 \end{aligned} \tag{9}$$

Substituting $X = 2Z$ in the first and third equations, we get:

$$-18\lambda = 2.2Z - 4 \tag{10}$$

$$\lambda = -0.4Z + 3 \tag{11}$$

$$\Rightarrow -18(-0.4Z + 3) = 2.2Z - 4 \tag{12}$$

$$\Rightarrow 7.2Z - 54 = 2.2Z - 4 \tag{13}$$

$$\Rightarrow 5Z = 50 \tag{14}$$

$$\Rightarrow Z = 10 \tag{15}$$

$$\Rightarrow X = 20 \tag{16}$$

Thus, the point is $(20, 0, 10)$.