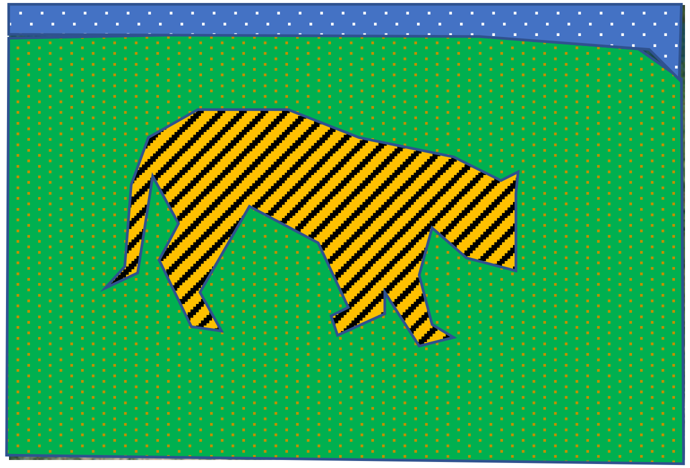
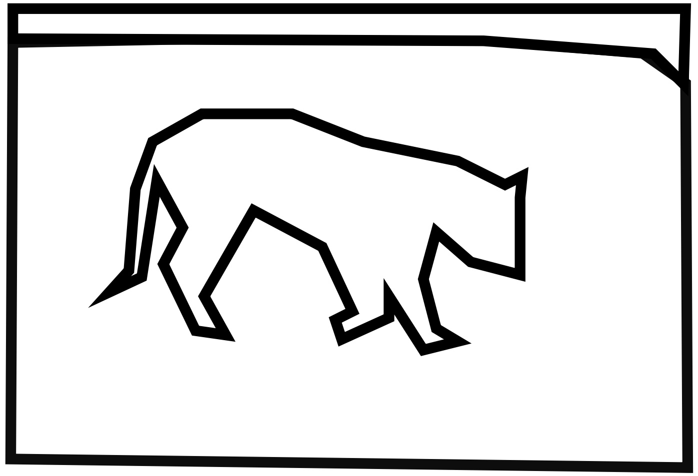
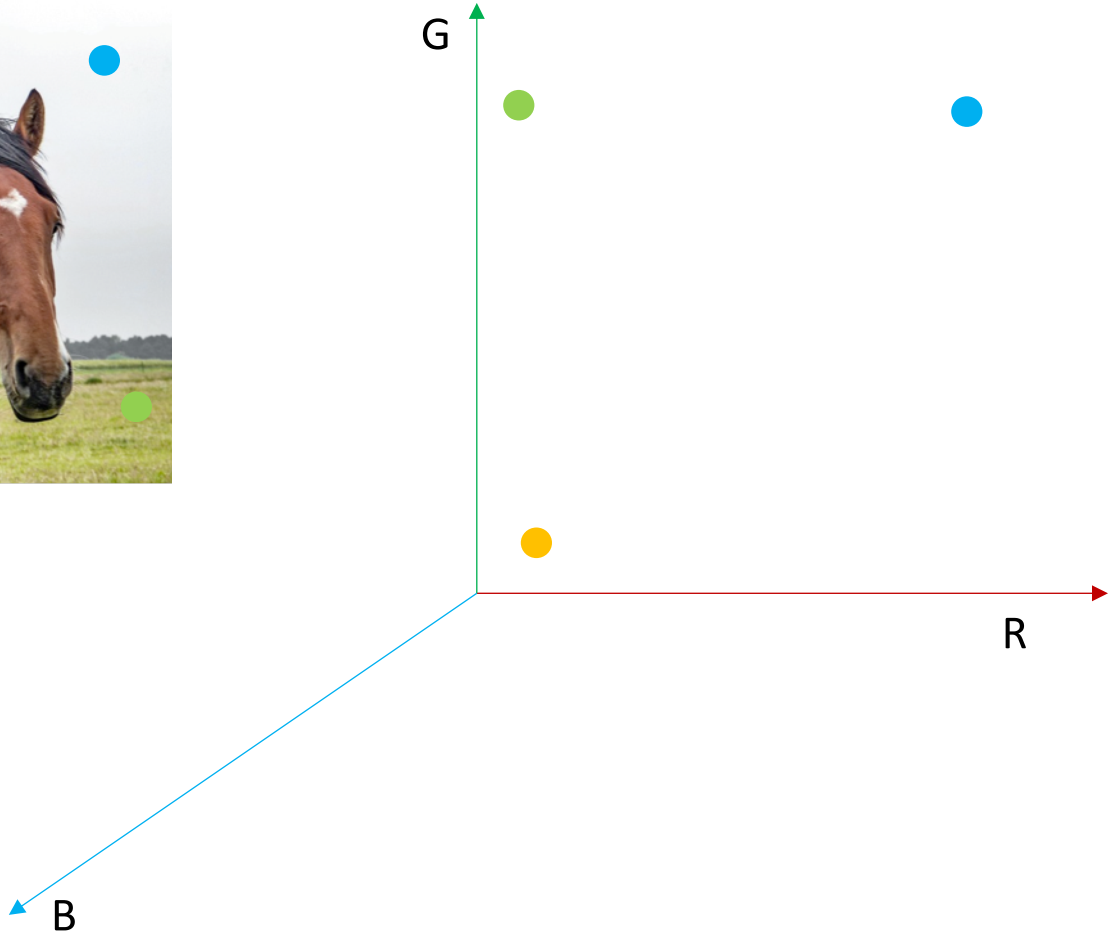
Grouping

Regions ⟷ Boundaries



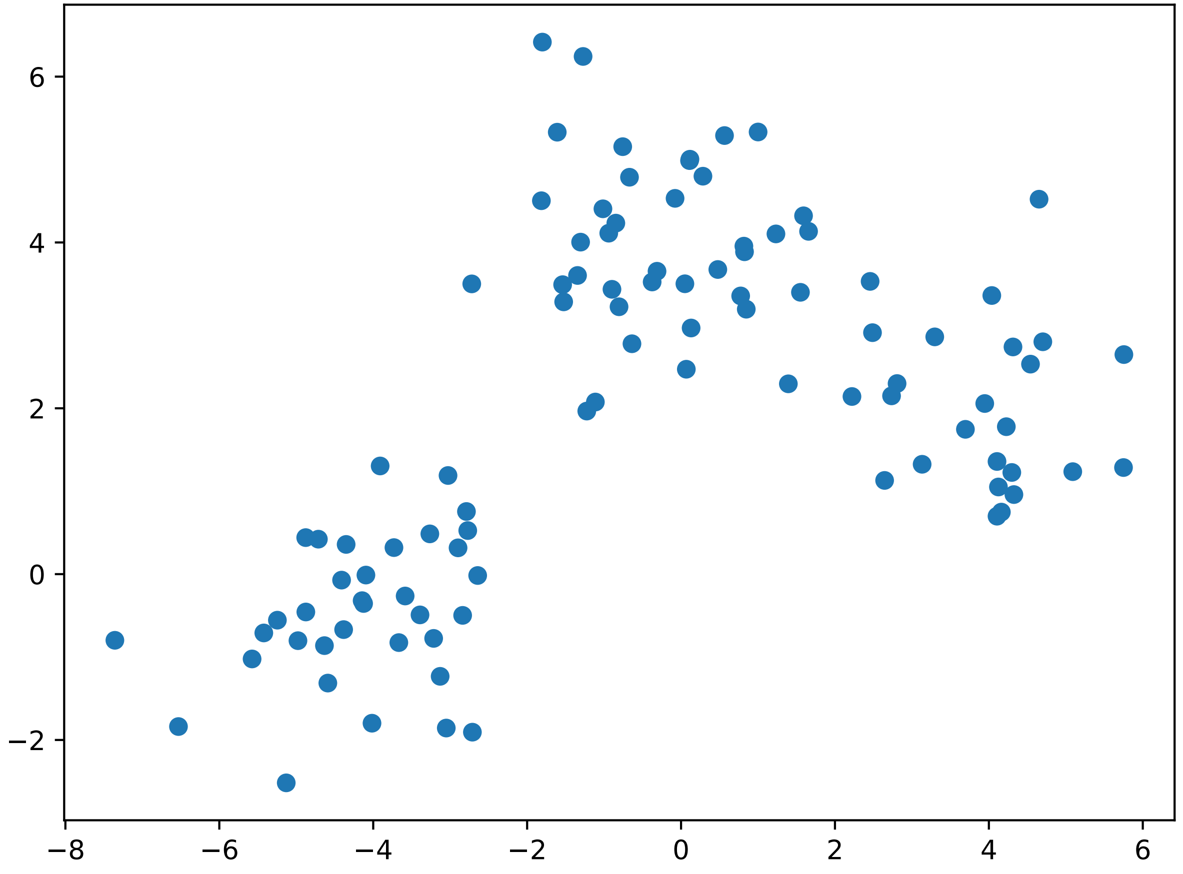


Grouping by clustering



Grouping by clustering

- Idea: embed pixels into high-dimensional space (e.g. 3-dimensions)
- Each pixel is a point
- Group together points



K-means

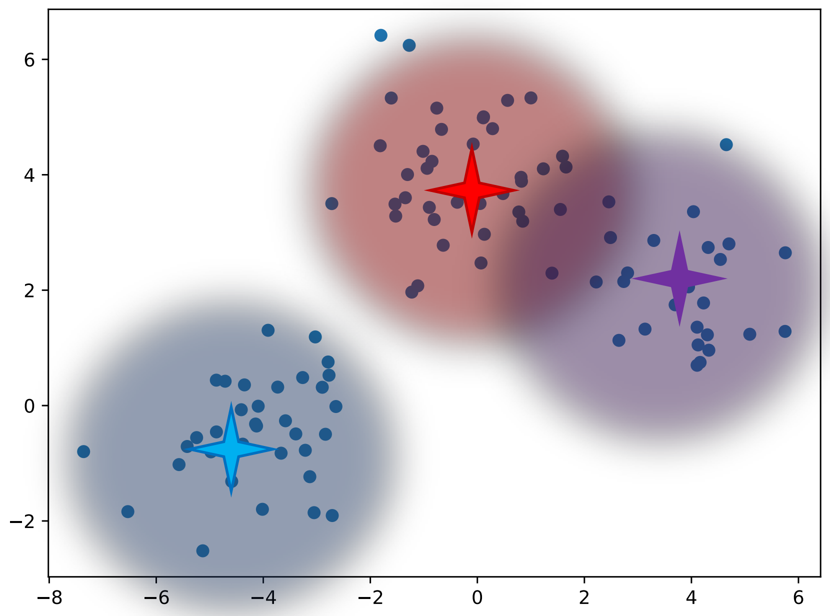
- Assumption: each group is a Gaussian with different means and same standard deviation

$$P(x_i|\mu_j) \propto e^{-\frac{1}{2\sigma^2}\|x_i-\mu_j\|^2}$$

- Suppose we know all μ_j. Which group should a point x_i belong to?
  - The j with highest $P(x_i|\mu_j)$
  - = The j with smallest $||x_i - \mu_j||^2$

K-means

- Assumption: each group = a Gaussian with different means and same standard deviation
- If means are known, then trivial assignment to groups. How?
- Assign data point to *nearest mean*!



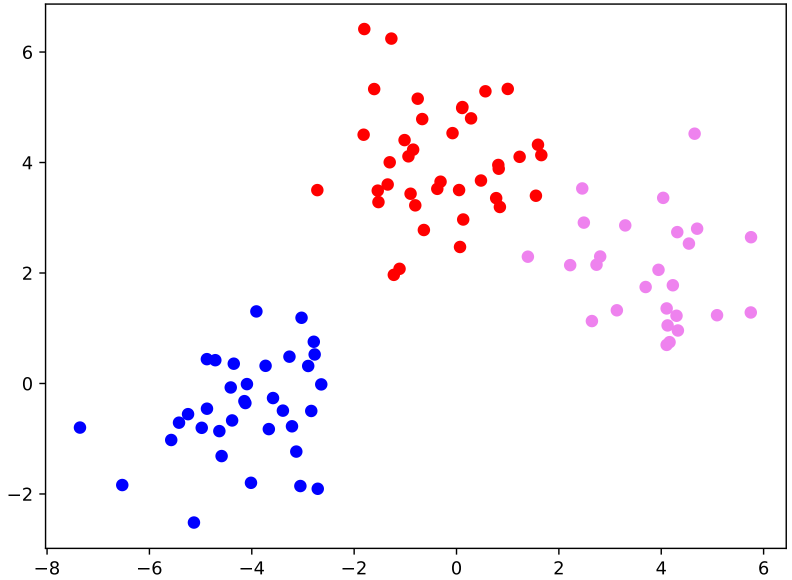
K-means

- Problem: means are not known
- What if we know a set of points from each cluster?
- $x_{k_1}, x_{k_2}, \ldots, x_{k_n}$ belong to cluster k
- What should be μ_k?

$$\mu_k = \frac{(x_{k_1} + x_{k_2} + \ldots + x_{k_n})}{n}$$

K-means

- Problem: means are not known
- If assignment of points to clusters is known, then finding means is easy
- How? Compute the mean of every cluster!

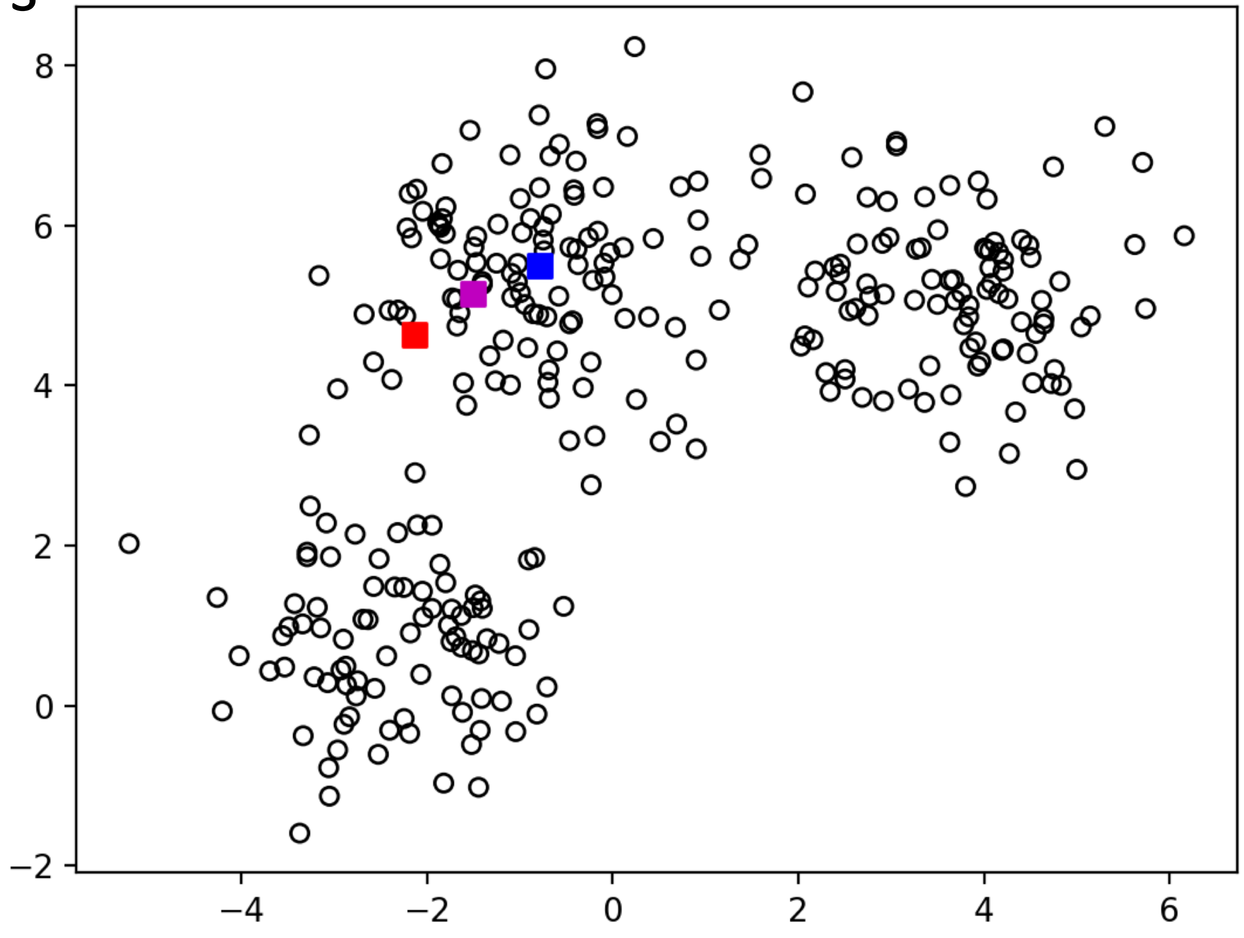


K-means

- Given means, can assign points to clusters
- Given assignments, can compute means
- Idea: iterate!

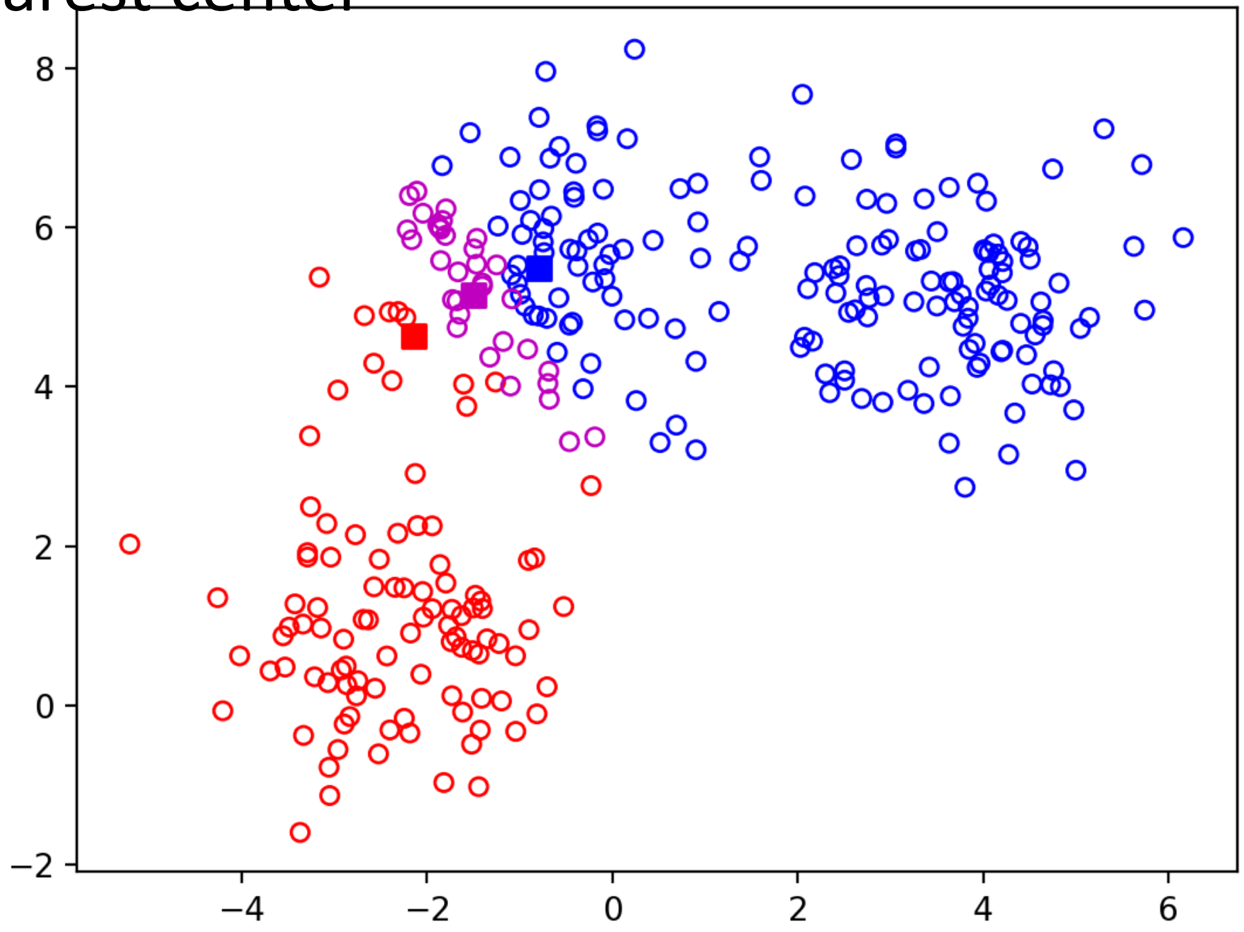
K-means

- Step-1 : randomly pick k centers



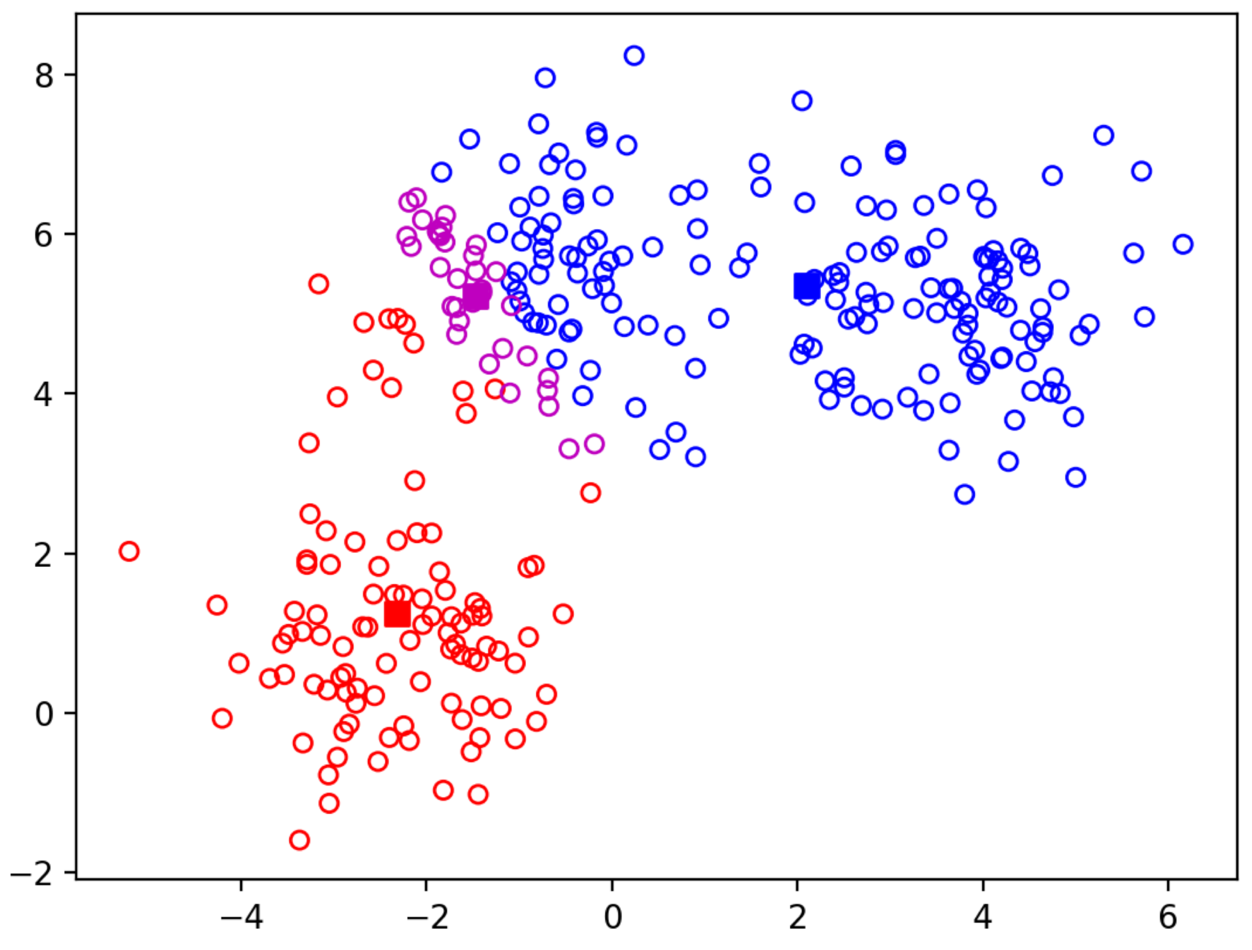
K-means

- Step 2: Assign each point to nearest center



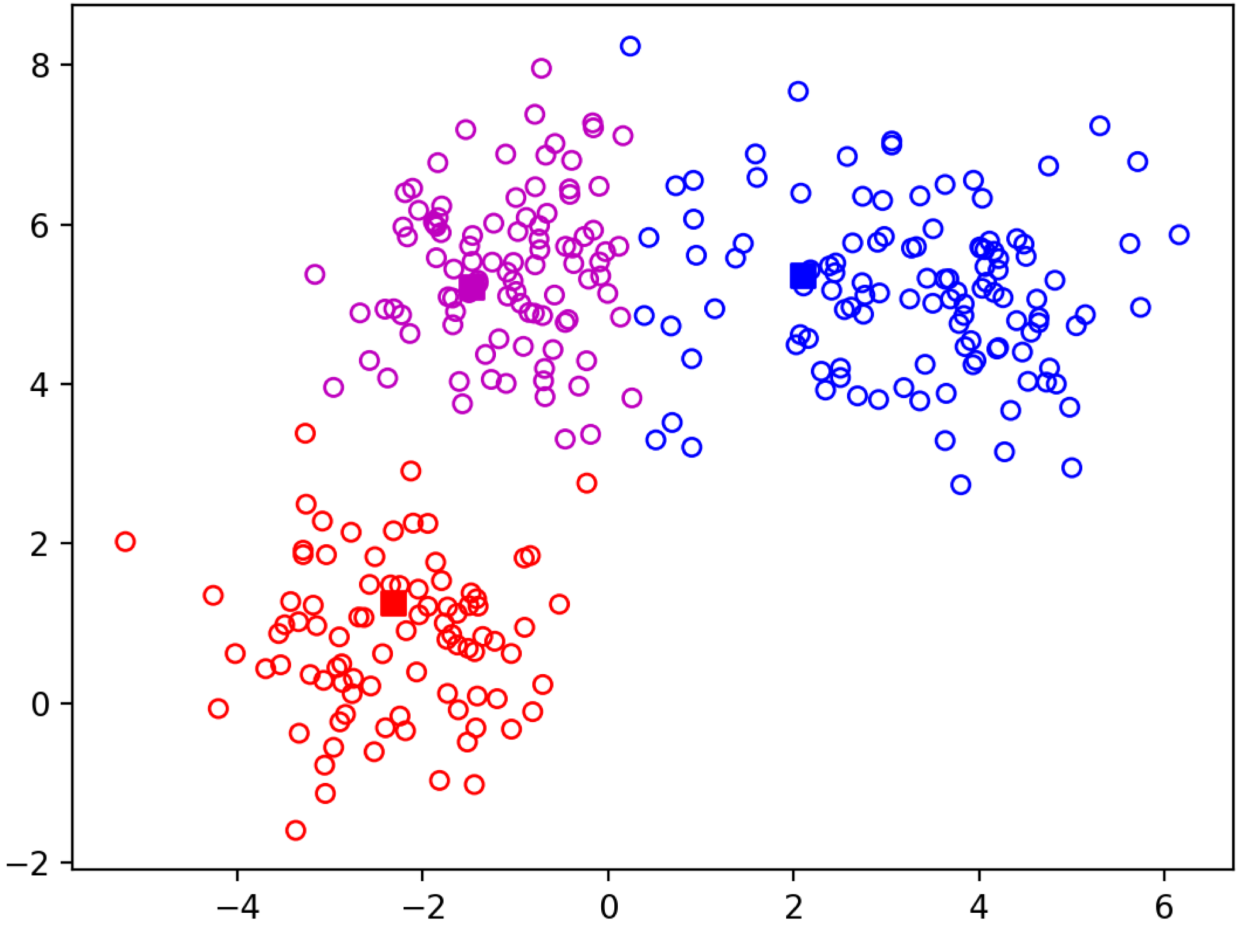
K-means

- Step 3: re-estimate centers



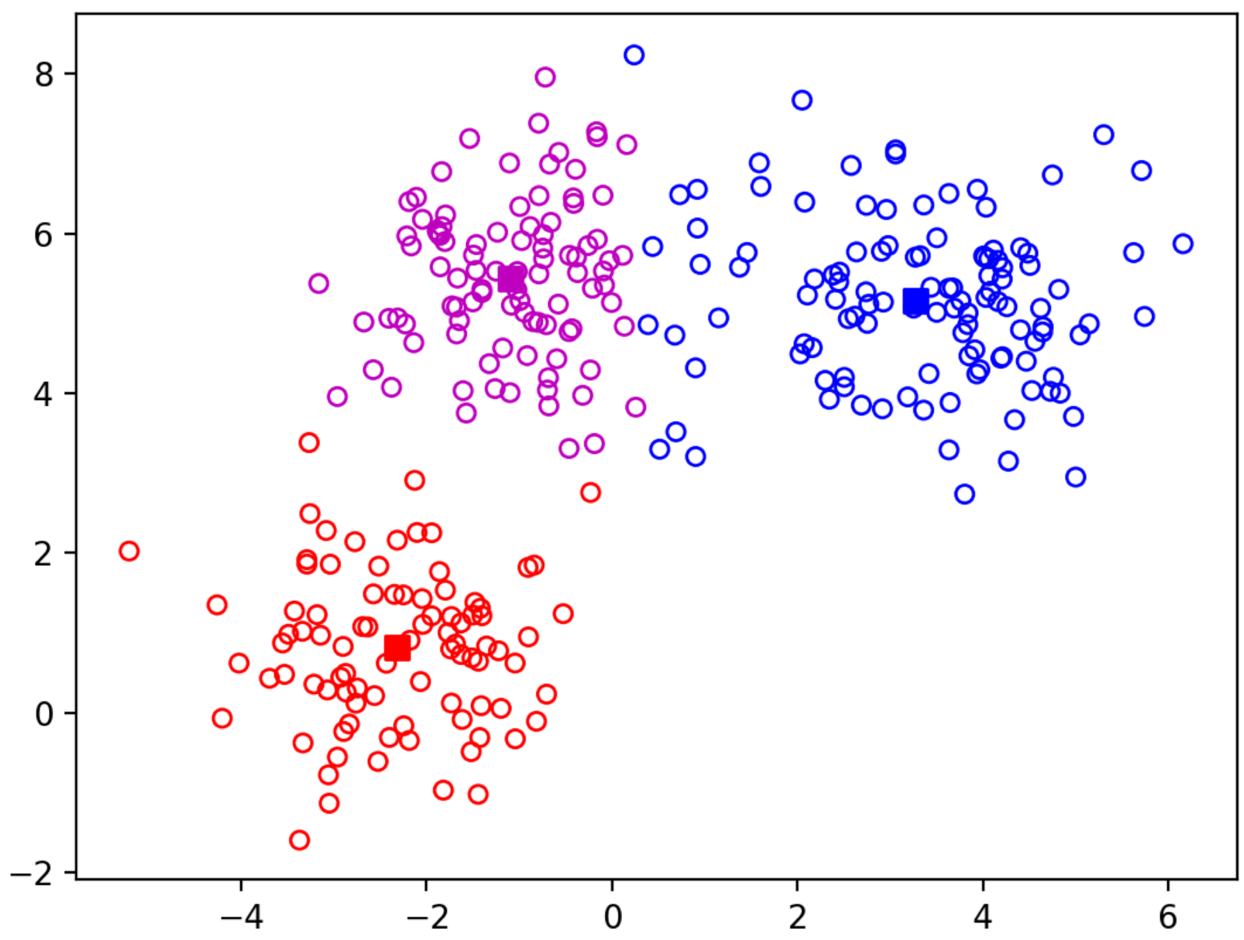
K-means

- Step 4: Repeat



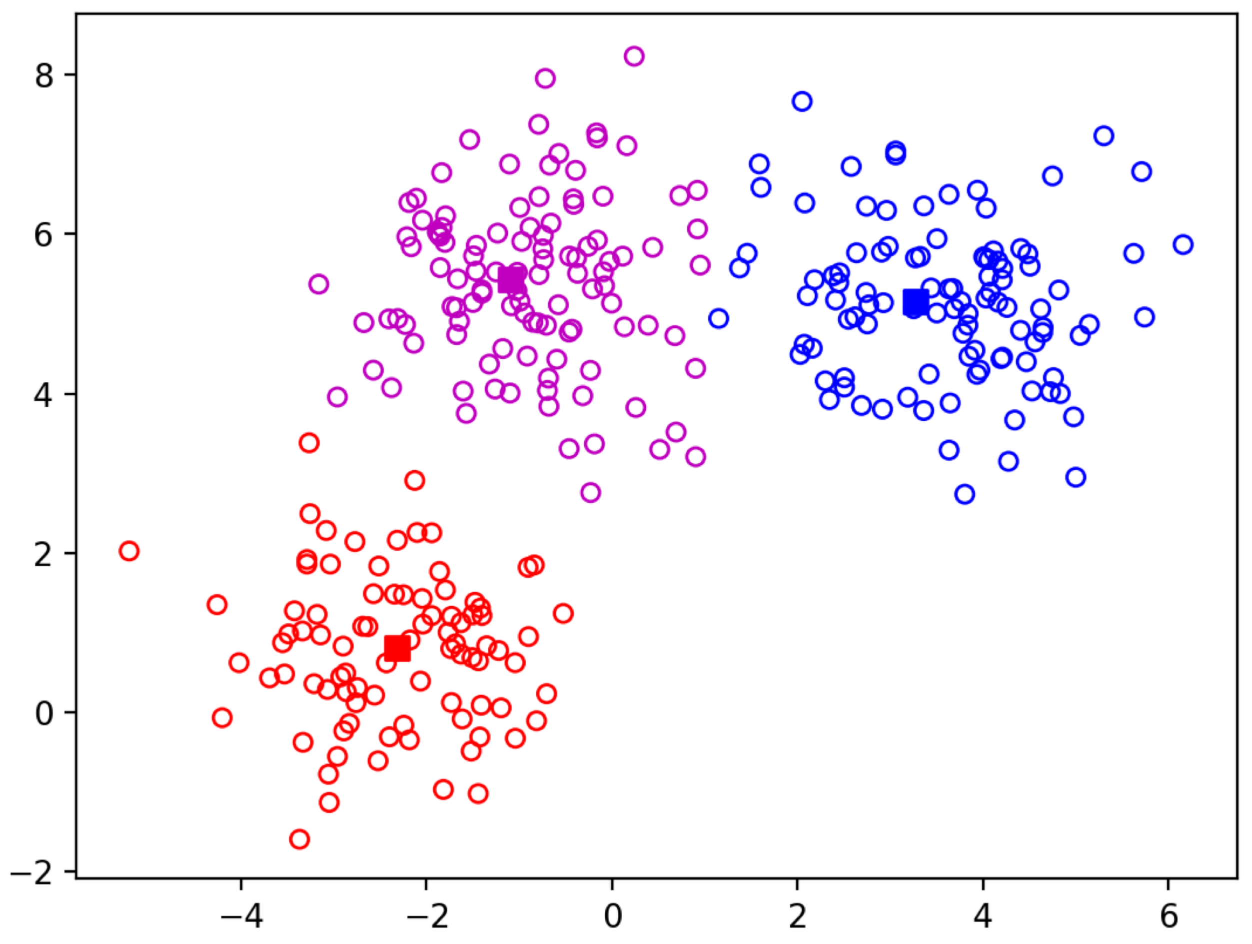
K-means

- Step 4: Repeat



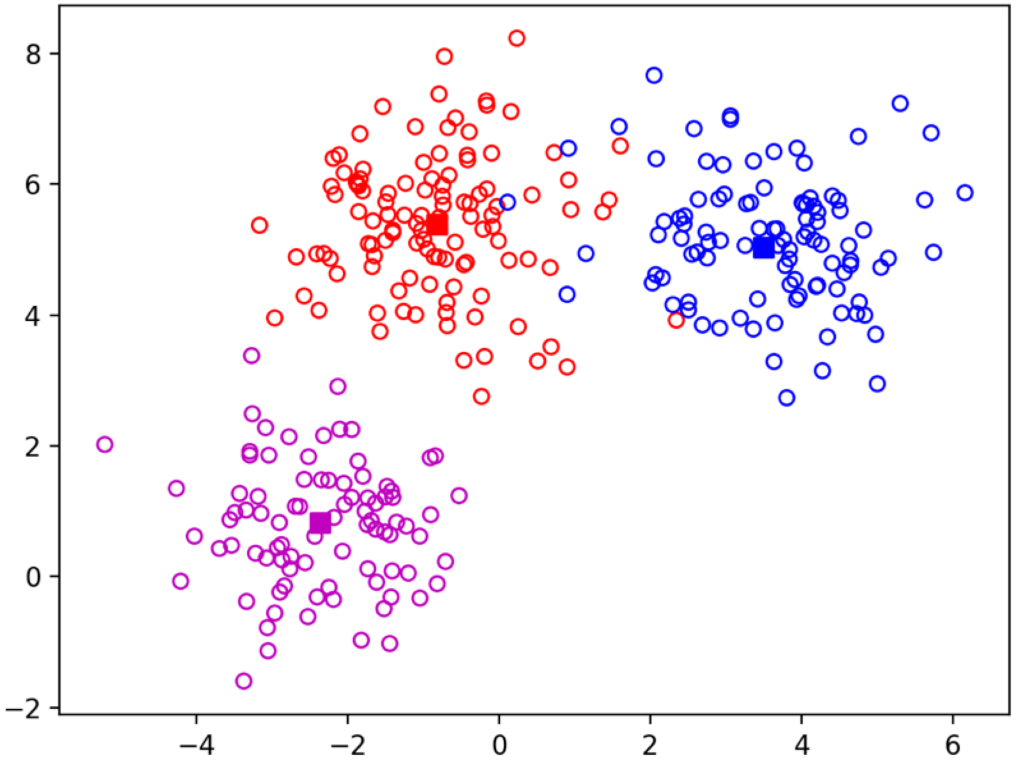
K-means

- Step 4: Repeat

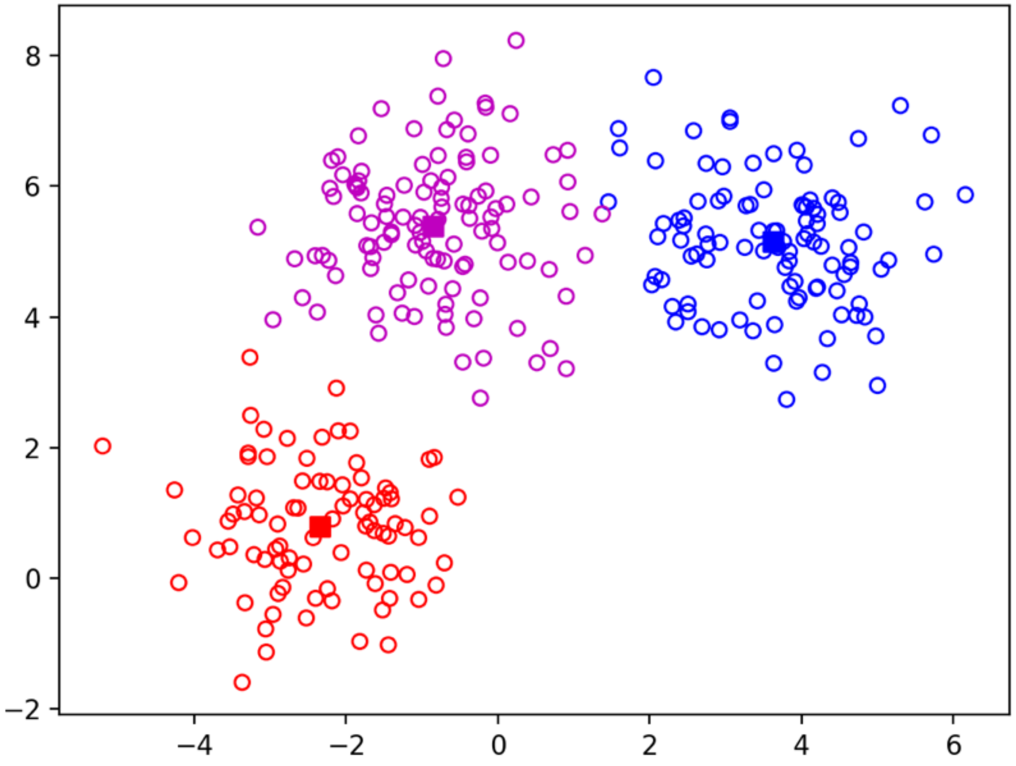


K-means

- Ground-truth vs k-means

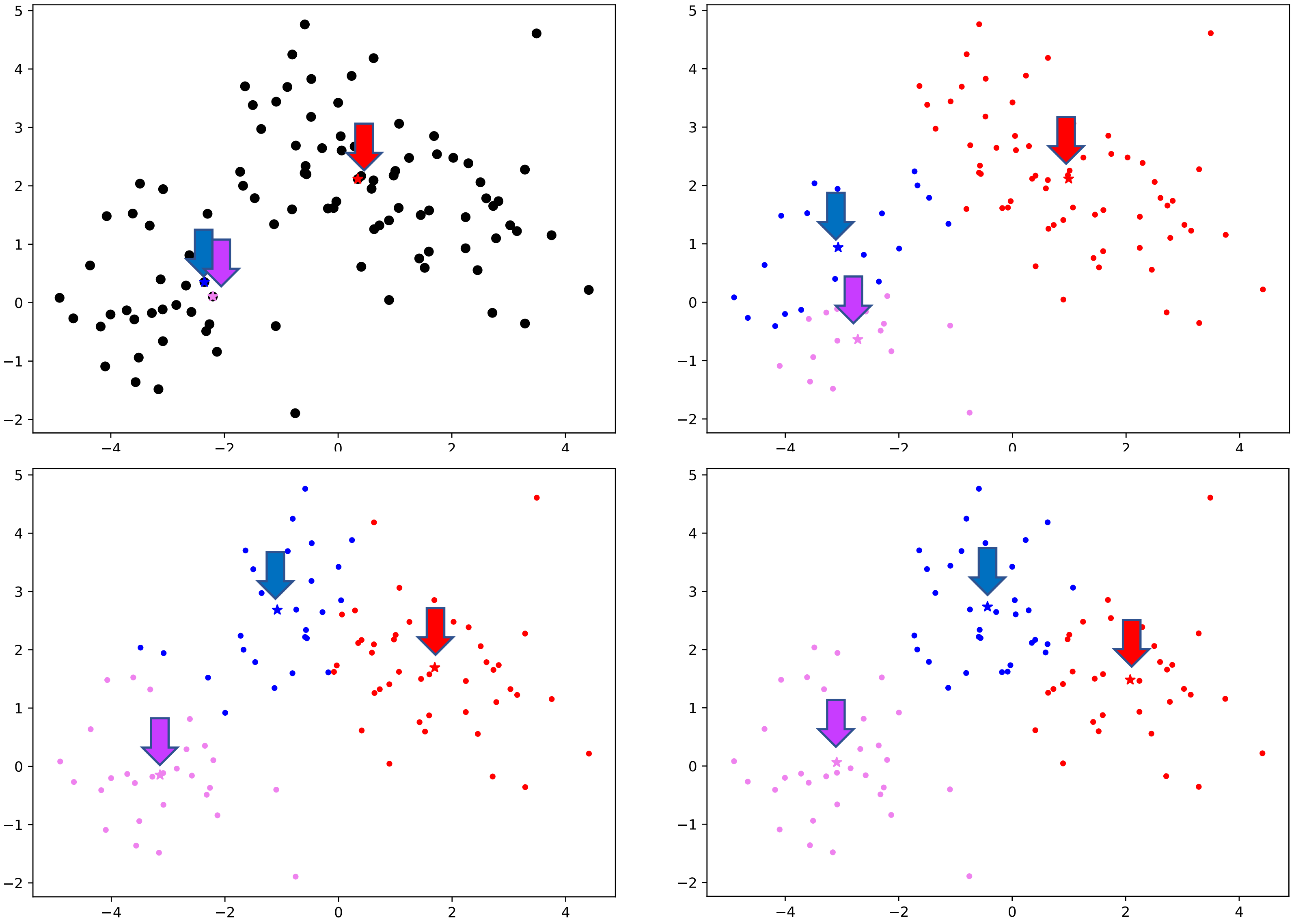


Ground truth



K-means 100 iterations

K-means - another example



K-means

Input: set of data points, k

1. Randomly pick k points as means
2. For i in [0, maxiters]:
    1. Assign each point to nearest center
    2. Re-estimate each center as mean of points assigned to it

K-means - the math

Input: set of data points X, k

1. Randomly pick k points as means $\mu_i, i = 1, \dots, k$
2. For iteration in [0, maxiters]:
    1. Assign each point to nearest center

$$y_i = \arg\min_j \|x_i - \mu_j\|^2$$

    2. Re-estimate each center as mean of points assigned to it

$$\mu_j = \frac{\sum_{i:y_i=j} x_i}{\sum_{i:y_i=j} 1}$$

K-means - the math

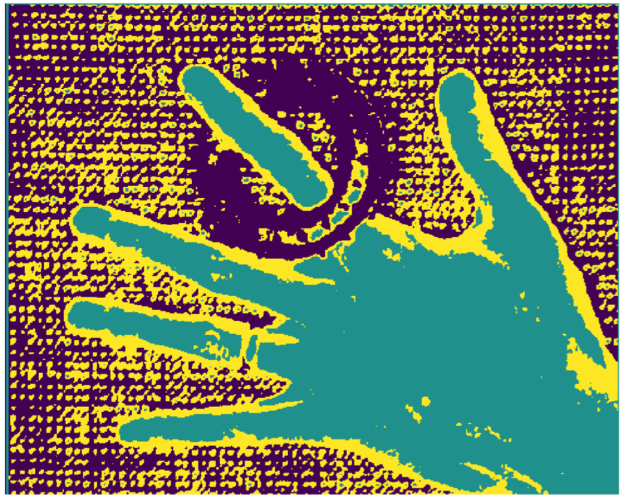
- An objective function that must be minimized:

$$\min_{\mu, y} \sum_i \|x_i - \mu_{y_i}\|^2$$

- Every iteration of k-means takes a downward step:
  - Fixes μ and sets y to minimize objective
  - Fixes y and sets μ to minimize objective

K-means on image pixels

Iteration 1



Iteration 5

Final: Iteration 17

K-means on image pixels

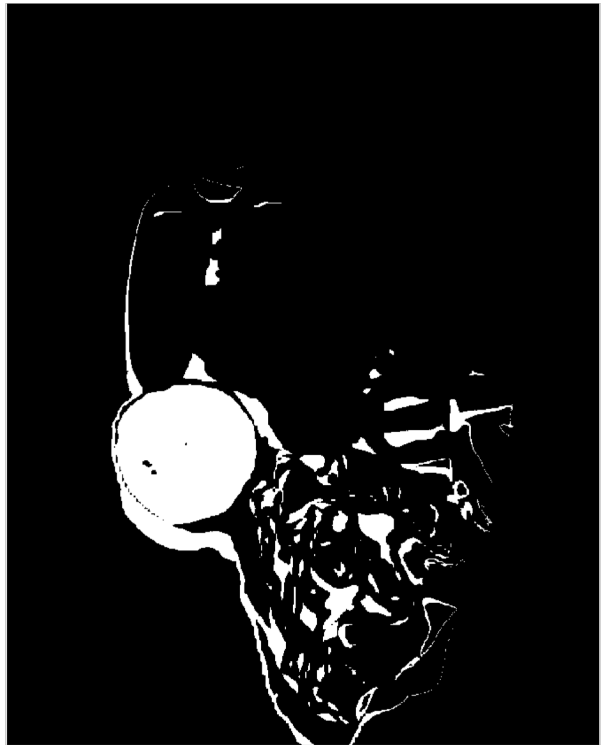
Picture courtesy David Forsyth

One of the clusters from k-means

K-means on image pixels

- What is wrong?
- Pixel position
  - Nearby pixels are likely to belong to the same object
  - Far-away pixels are likely to belong to different objects
- How do we incorporate pixel position?
  - Instead of representing each pixel as (r,g,b)
  - Represent each pixel as (r,g,b,x,y)

K-means on image pixels+position



The issues with k-means

- Captures pixel similarity but
  - Doesn’t capture continuity of contours
  - Captures near/far relationships only weakly
  - Can merge far away objects together
- Requires knowledge of k!
- Can it deal with texture?

Oversegmentation and superpixels

- We don’t know k. What is a safe choice?
- Idea: Use large k
  - Can potentially break big objects, but will hopefully not merge unrelated objects
  - Later processing can decide which groups to merge
  - Called *superpixels*