Question

- Which of the following two images when subsampled naively is likely to produce aliasing artifacts?

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Grouping

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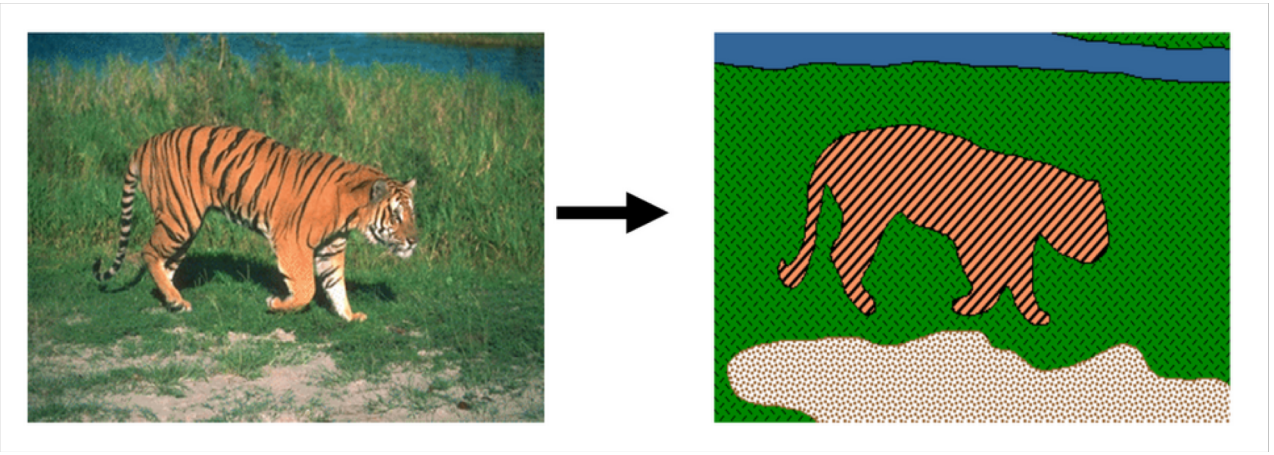
- Grouping pixels into objects (“perceptual organization”)

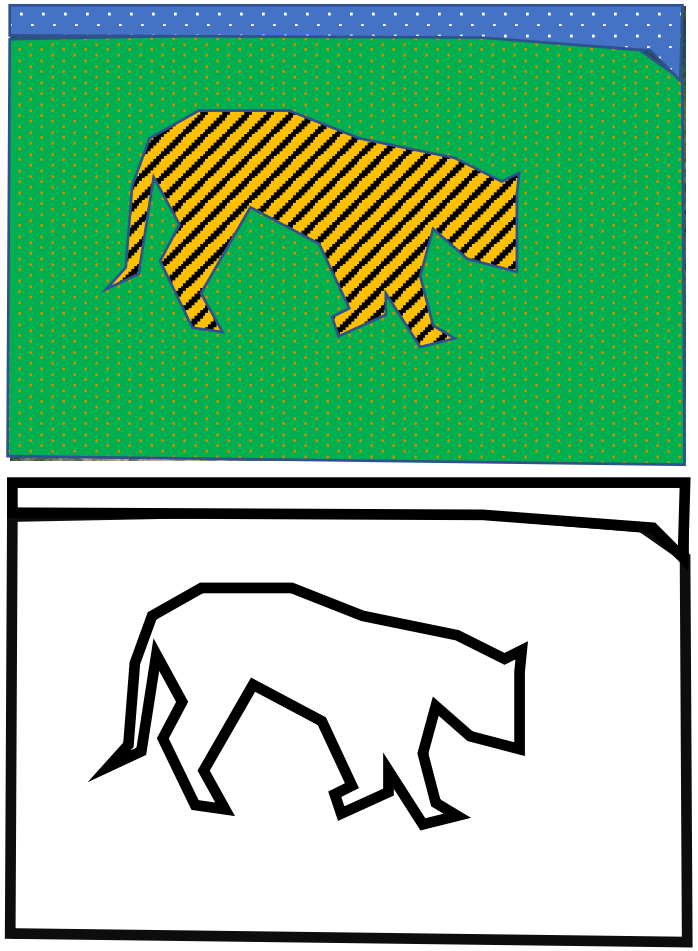


Why grouping?

- Pixels property of sensor, not world
- Reasoning at object level (might) make things easy:
  - objects at consistent depth
  - objects can be recognized
  - objects move as one

"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have "327"? No. I have sky, house, and trees."
Max Wertheimer

Regions ⟷ Boundaries

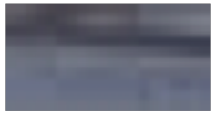


Why edges?

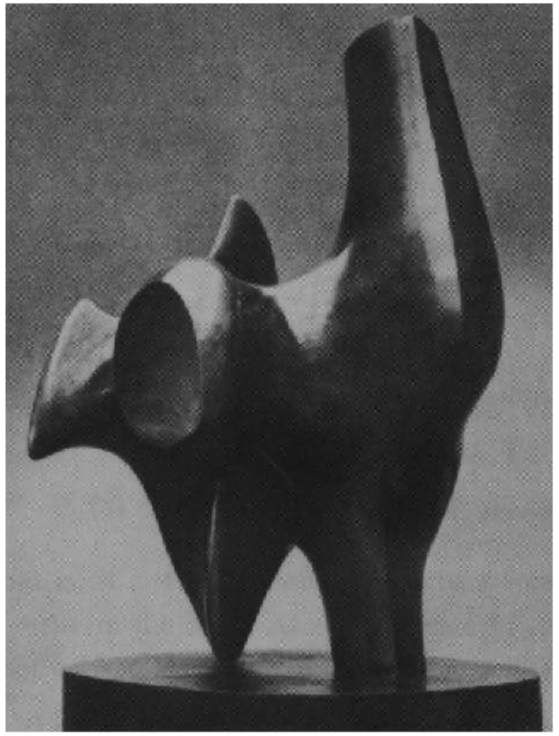
- Resilience to lighting and color
  - useful for recognition, matching patches across images

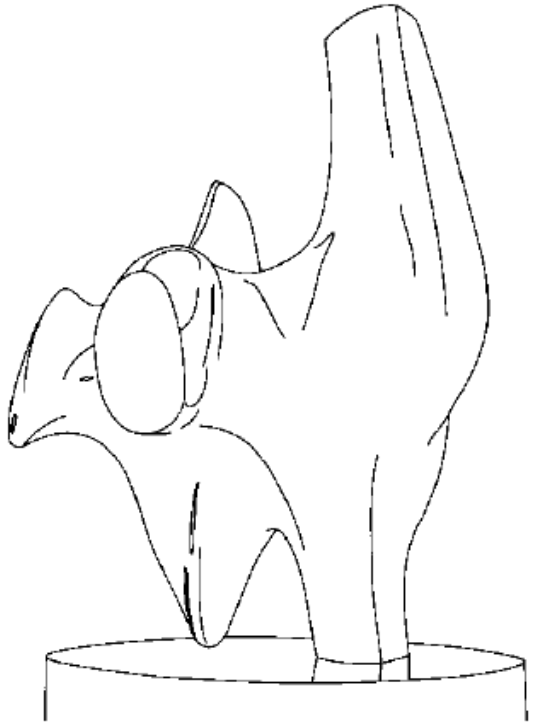







Why edges?

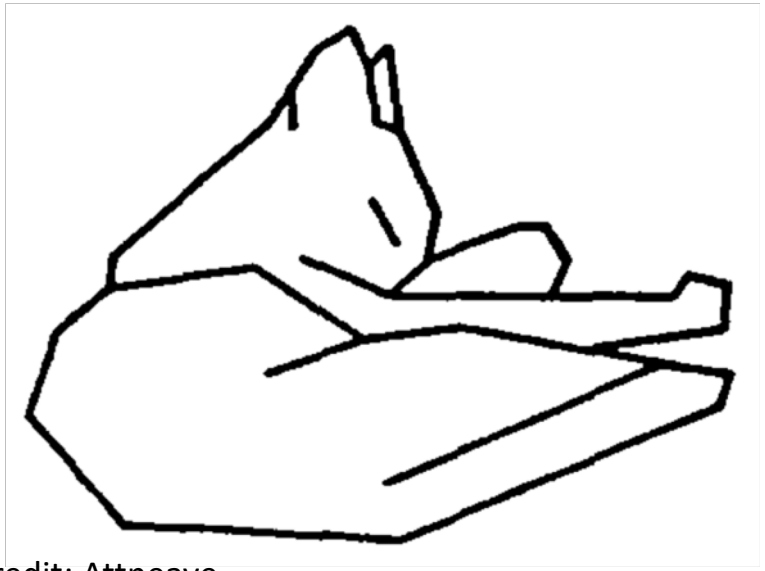




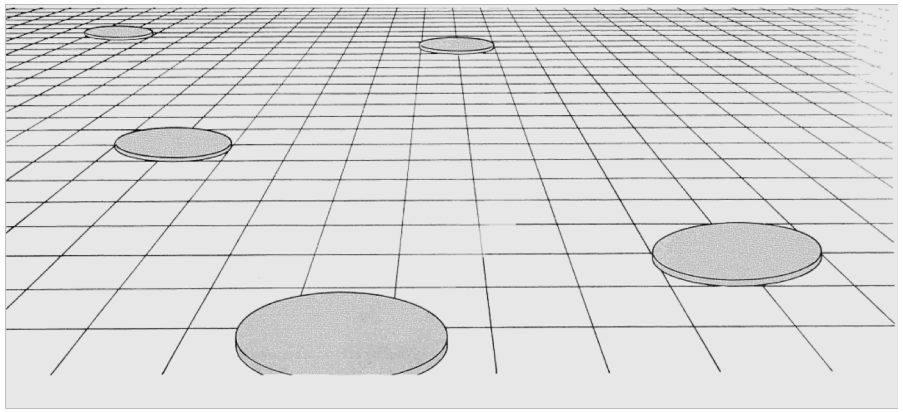
- Humans are sensitive to edges
- Convert a 2D image into a set of curves
  - Extracts salient features of the scene, more compact

Why edges?

- Cue to shape and geometry
  - useful for recognition, understanding 3D structure



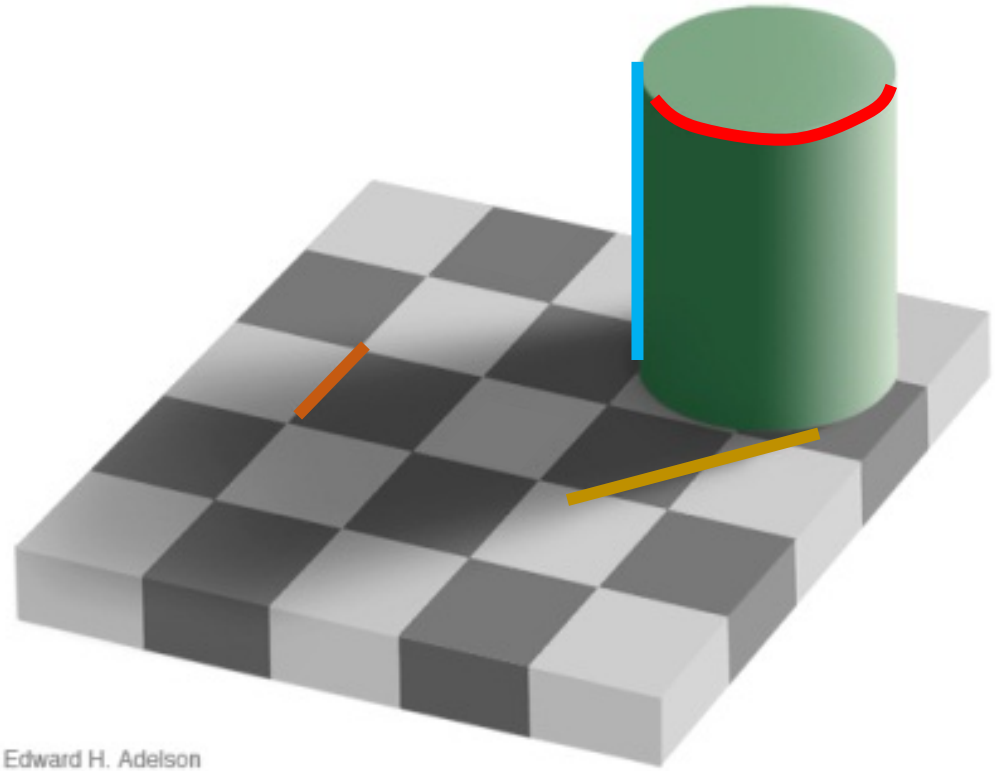
Credit: Attneave



Credit: Jitendra Malik

Edges

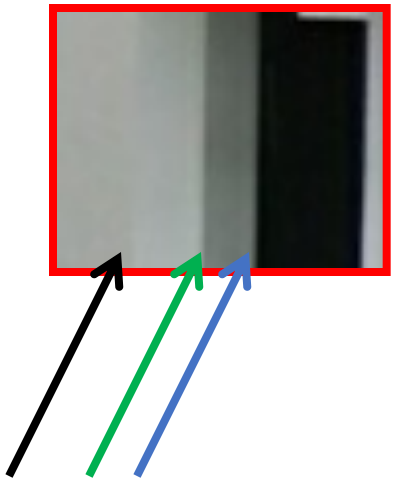
- Edges are curves in the image, across which the brightness changes “a lot”
- Corners/Junctions
- Different kinds of edges:
  - Depth discontinuities
  - Normal discontinuities
  - Discontinuities in “paint”
  - Shadows



Closeup of edges

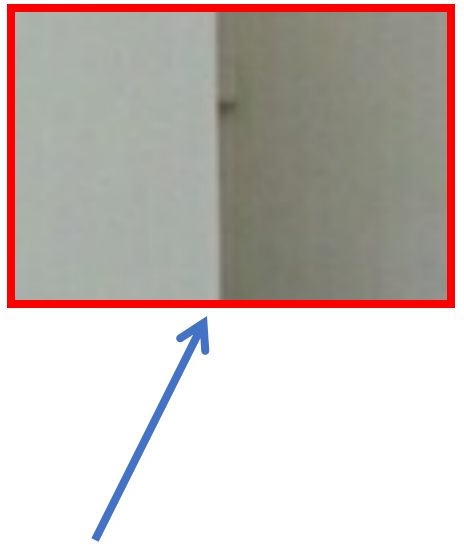
Source: D. Hoiem

Closeup of edges



Source: D. Hoiem

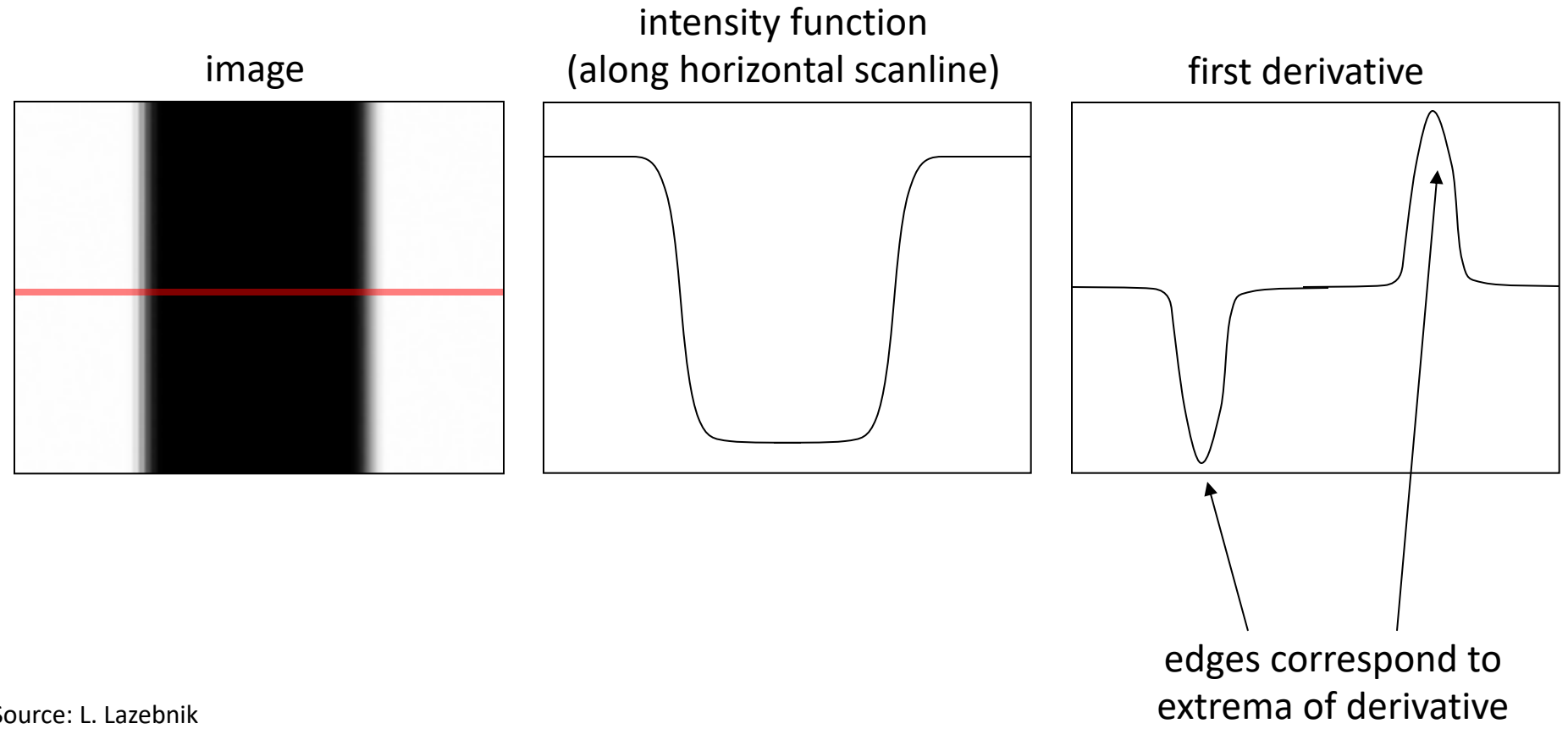
Closeup of edges



Source: D. Hoiem

Characterizing edges

- An edge is a place of *rapid change* in the image intensity function



Source: L. Lazebnik

Derivatives and convolution

- Differentiation is *linear*

$$\frac{\partial(af(x)+bg(x))}{\partial x} = a\frac{\partial f(x)}{\partial x} + b\frac{\partial g(x)}{\partial x}$$

- Differentiation is *shift-equivariant*
  - Derivative of shifted signal is shifted derivative

- Hence, differentiation can be represented as convolution!

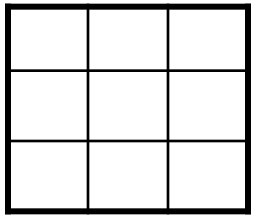
Image derivatives

- How can we differentiate a *discrete* image F[x,y]?
  - Option 1: reconstruct a continuous image, *f,* then compute the derivative
  - Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x, y] \approx F[x+1, y] - F[x, y]$$

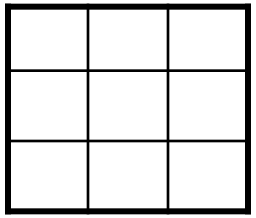
How would you implement this as a linear filter?

$\frac{\partial f}{\partial x}$:



H_x

$\frac{\partial f}{\partial y}$:



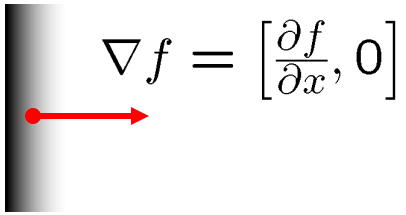
H_y

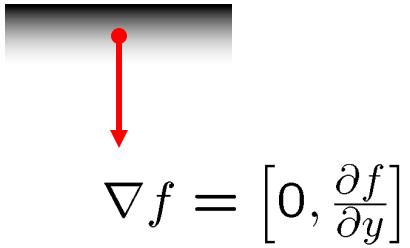
Source: S. Seitz

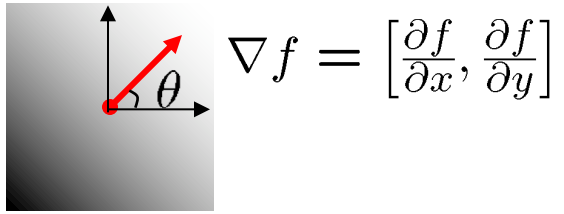
Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

  The gradient points in the direction of most rapid increase in intensity







The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

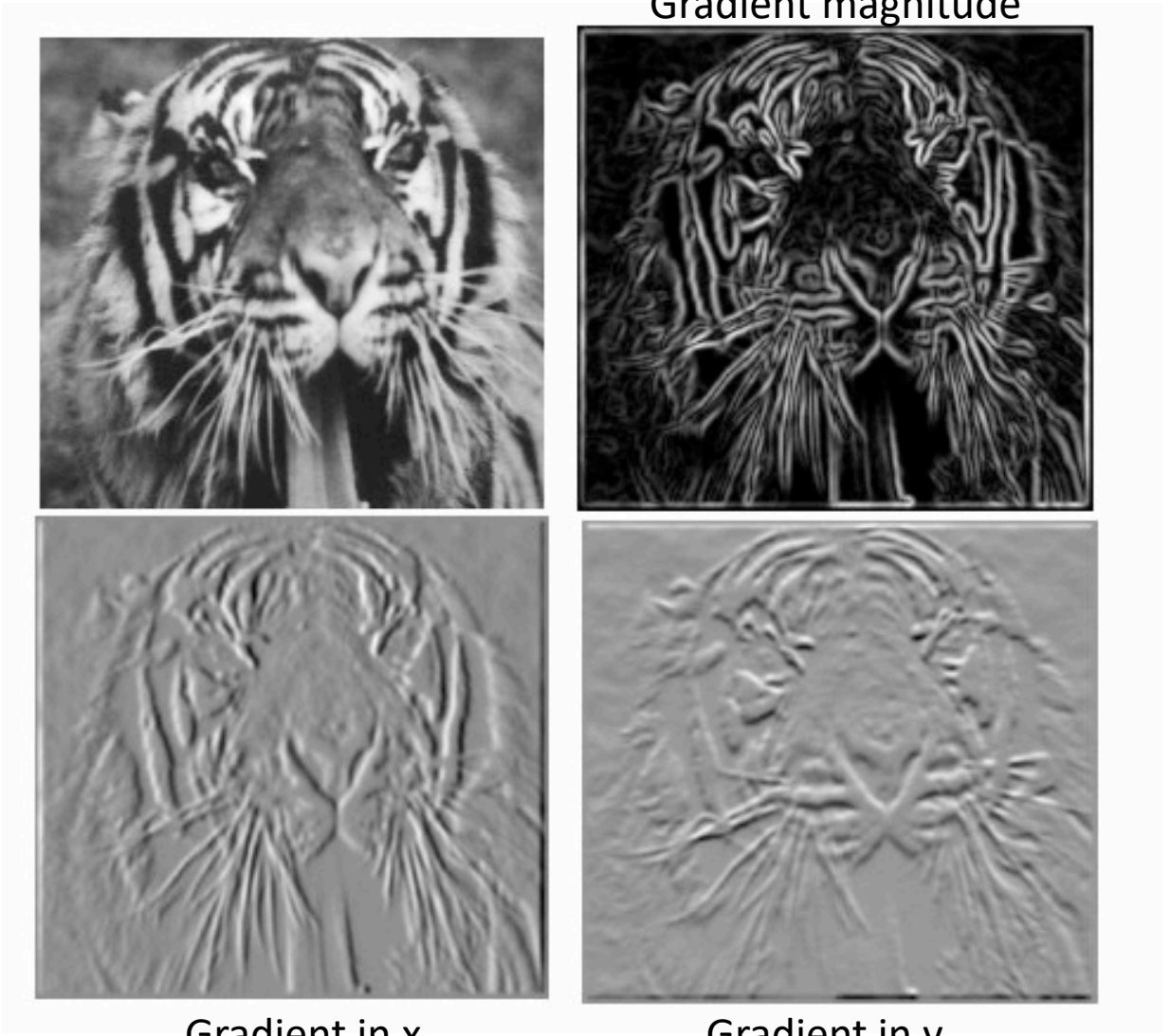
The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

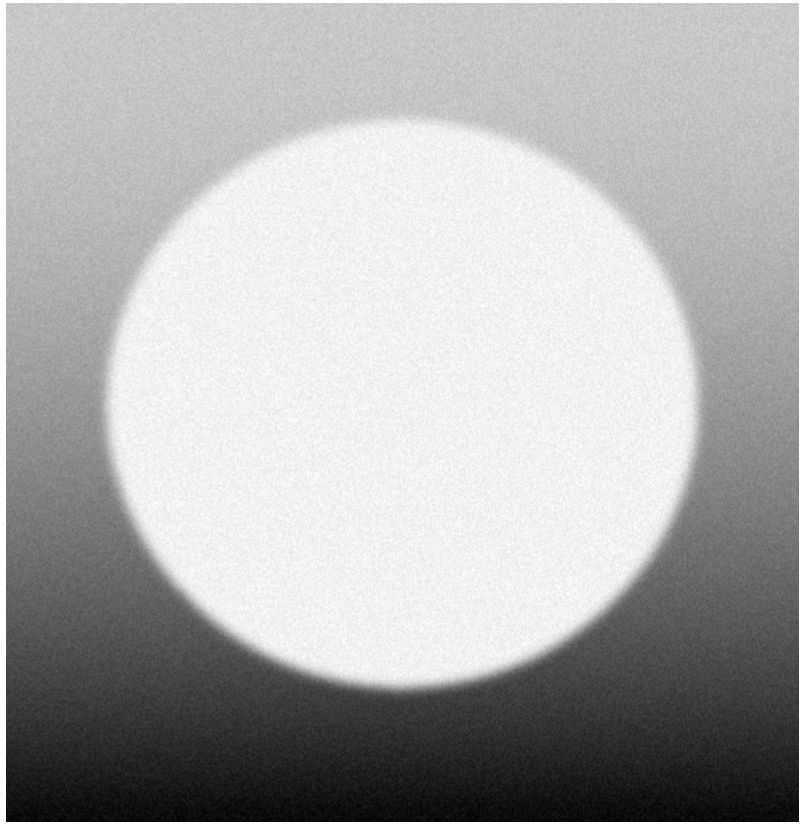
Source: Steve Seitz

Image gradient



Source: L. Lazebnik

Effects of noise



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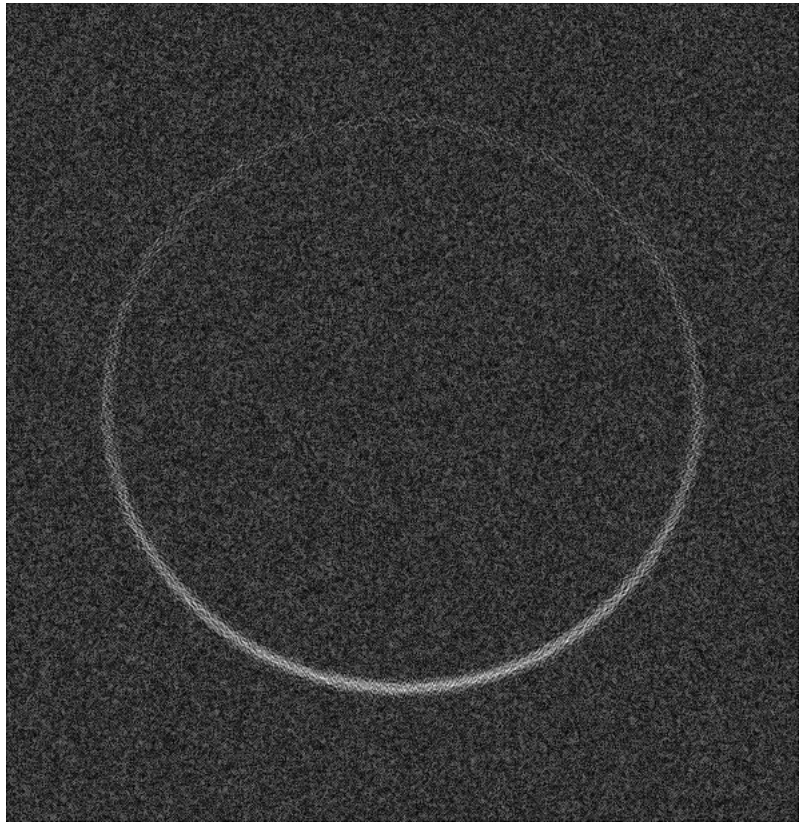
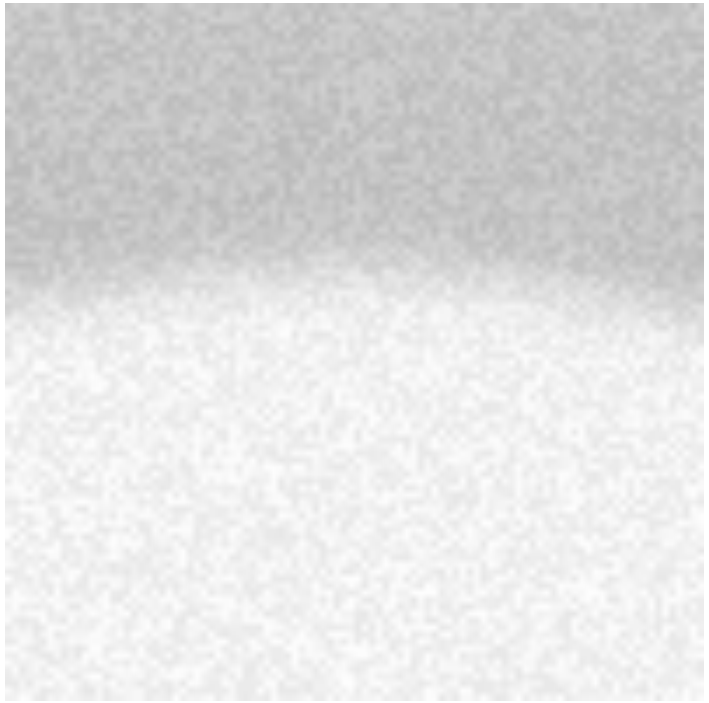


Gradient magnitude

Effects of noise



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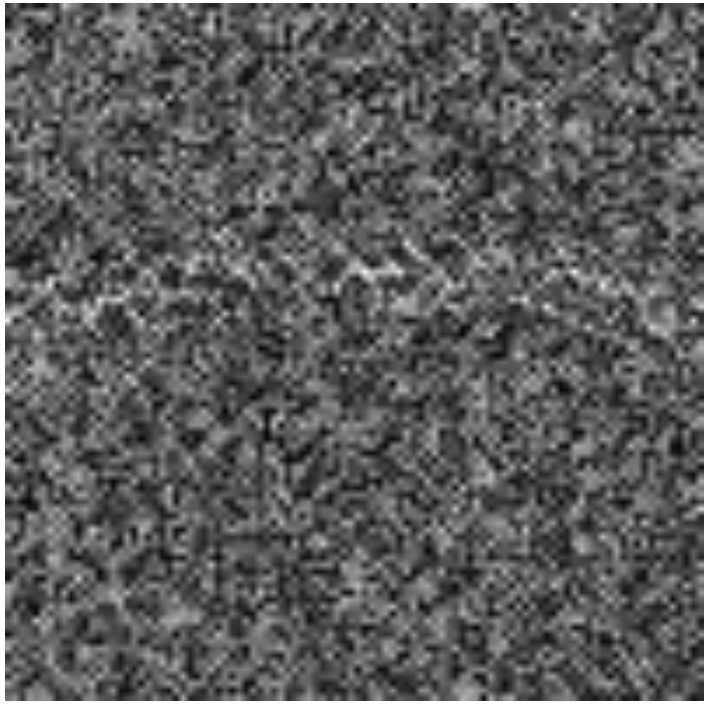
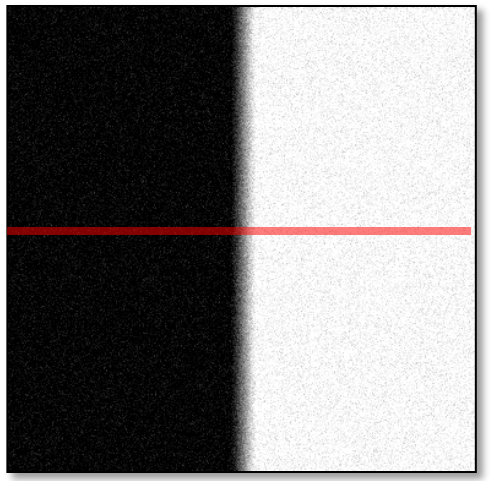


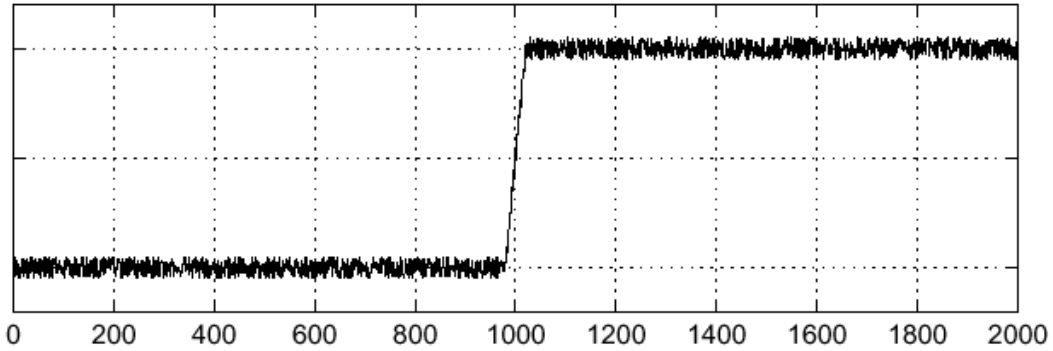
Gradient magnitude

Effects of noise

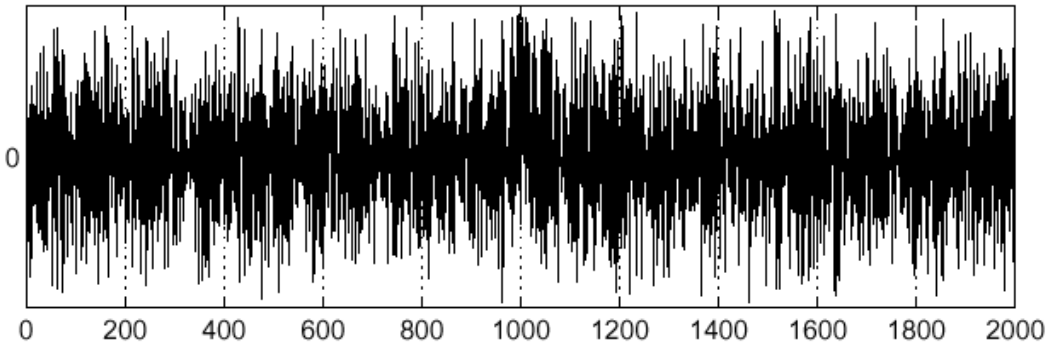


Noisy input image

$f(x)$



$\frac{d}{dx}f(x)$



Where is the edge?

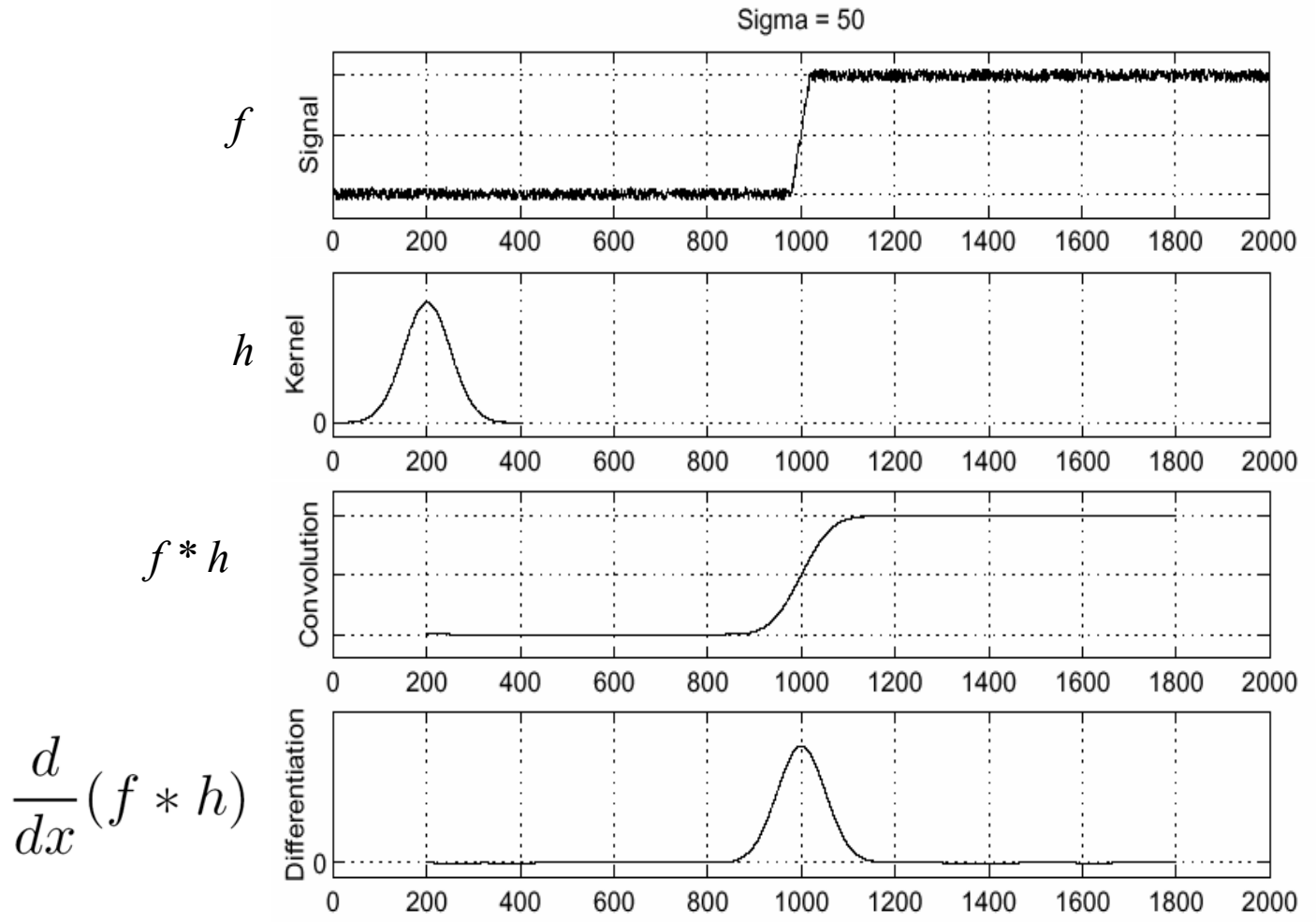
Source: S. Seitz

Effects of noise

- Noise is high frequency
- Differentiation accentuates noise

$$\frac{d \sin \omega x}{dx} = \omega \cos \omega x$$

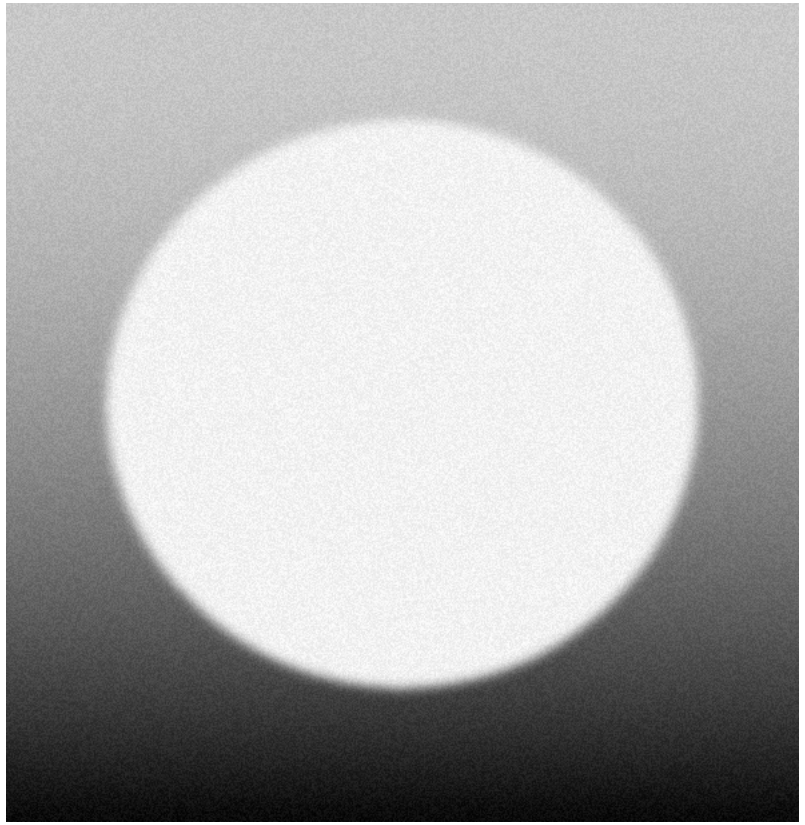
Solution: smooth first



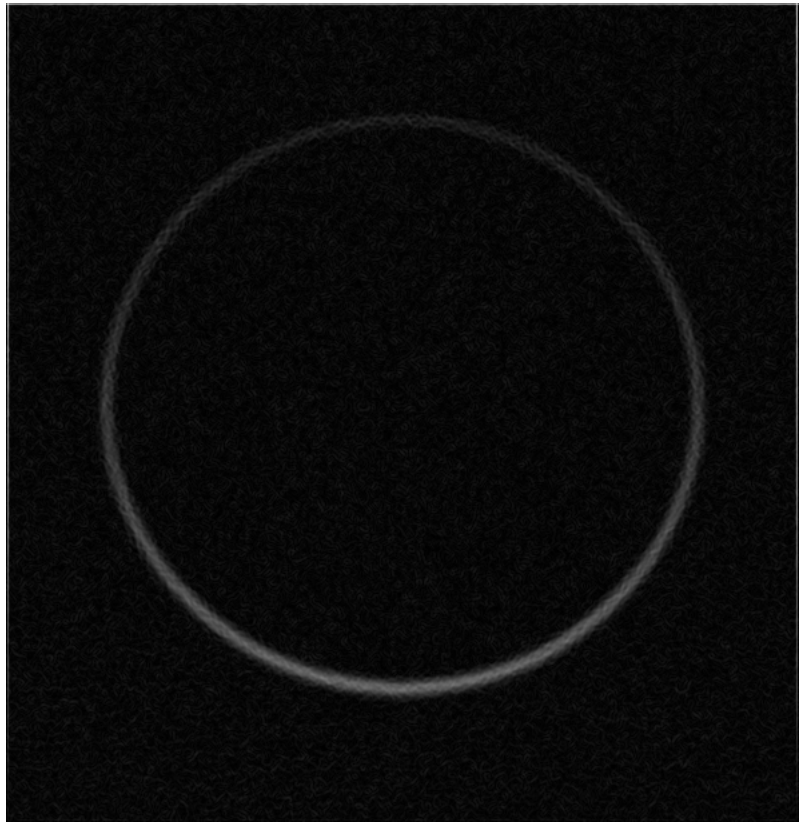
To find edges, look for peaks in $\frac{d}{dx}(f * h)$

Source: S. Seitz

Solution: smooth first

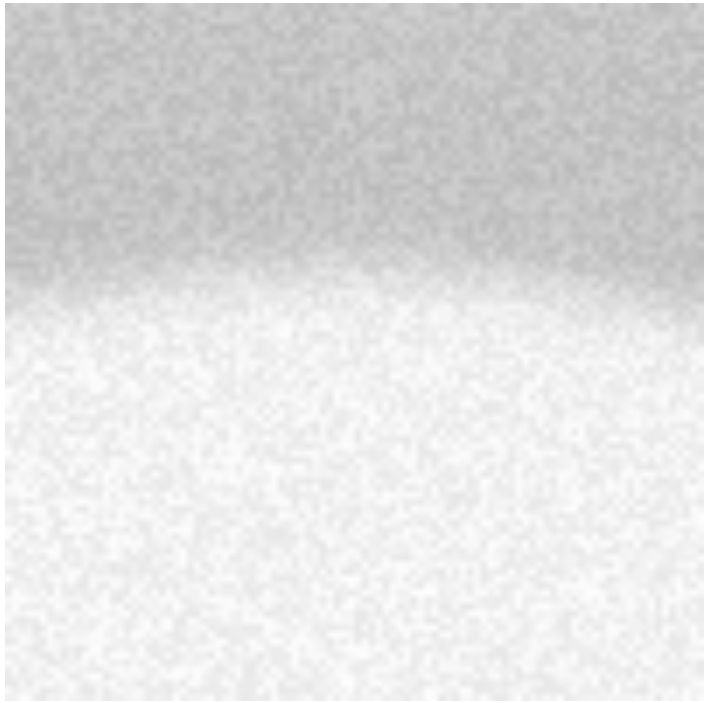


Input

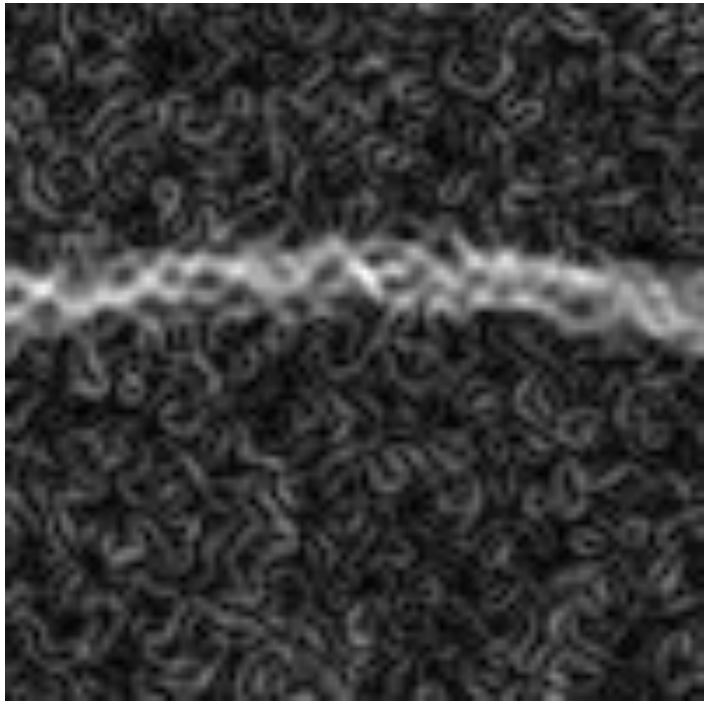


Gradient magnitude

Solution: smooth first



Input



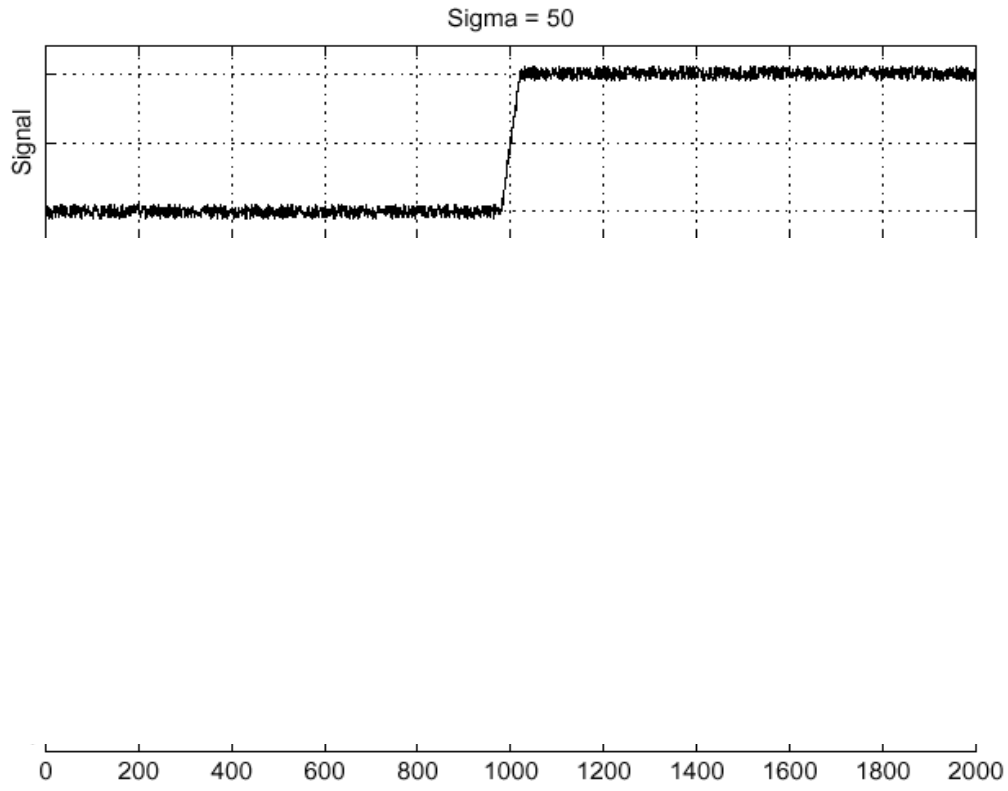
Gradient magnitude

Associative property of convolution

- Differentiation is a convolution
- Convolution is associative:
- This saves us one operation:

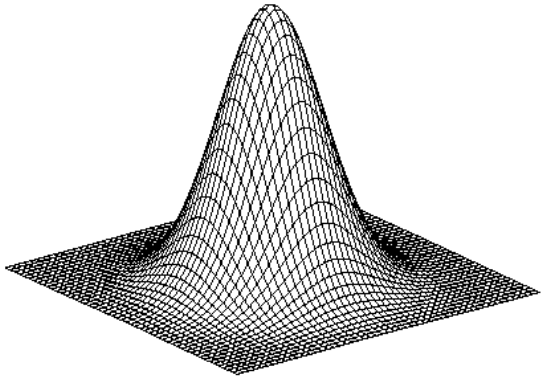
$$\frac{d}{dx}(f * h) = f * \frac{d}{dx}h$$

f



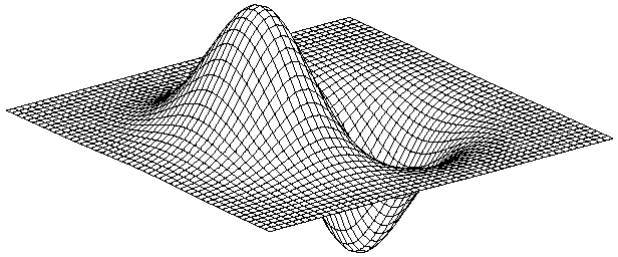
Source: S. Seitz

2D edge detection filters



Gaussian

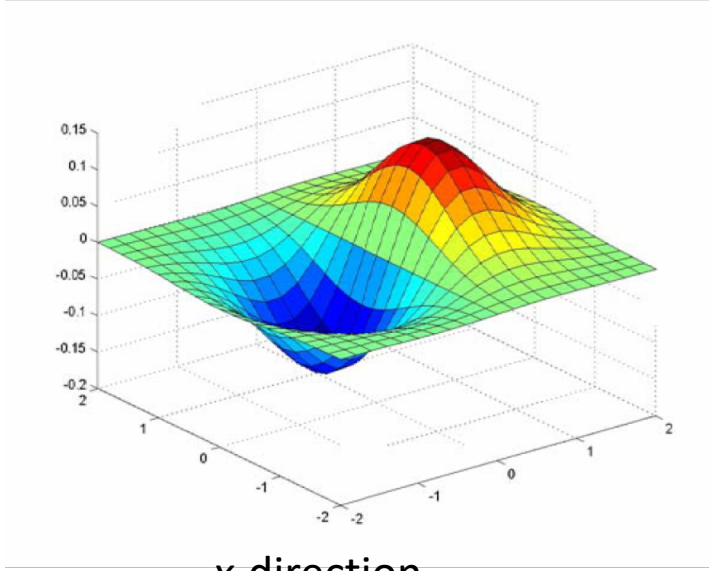
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



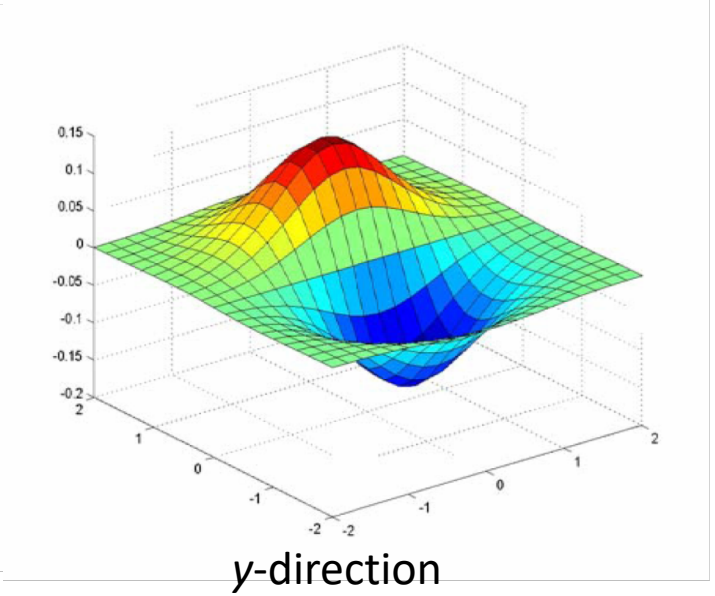
derivative of Gaussian (x)

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

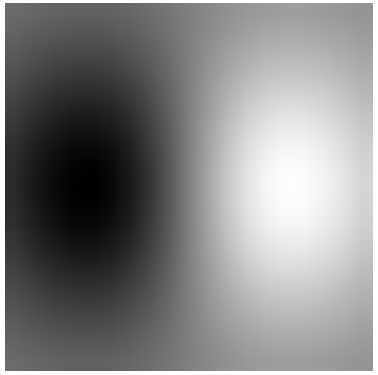
Derivative of Gaussian filter

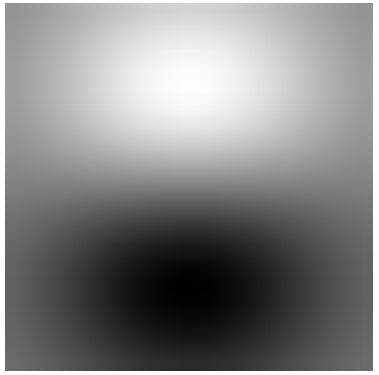


x-direction



y-direction





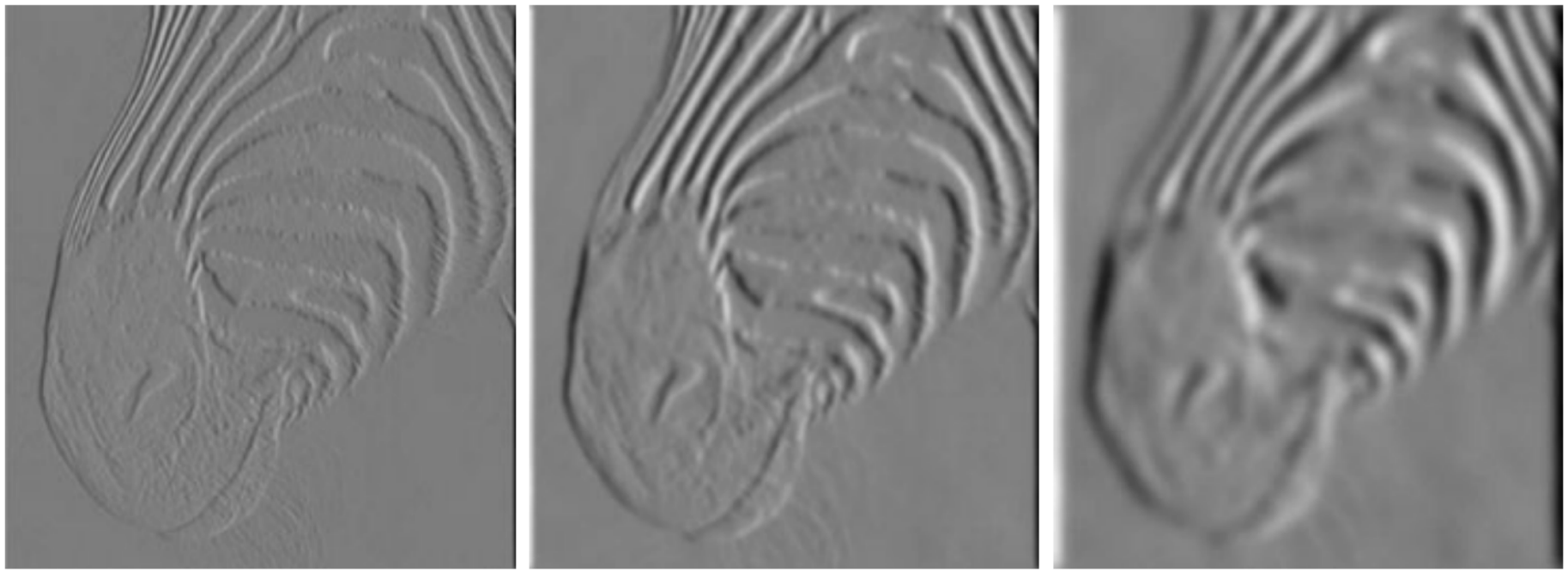
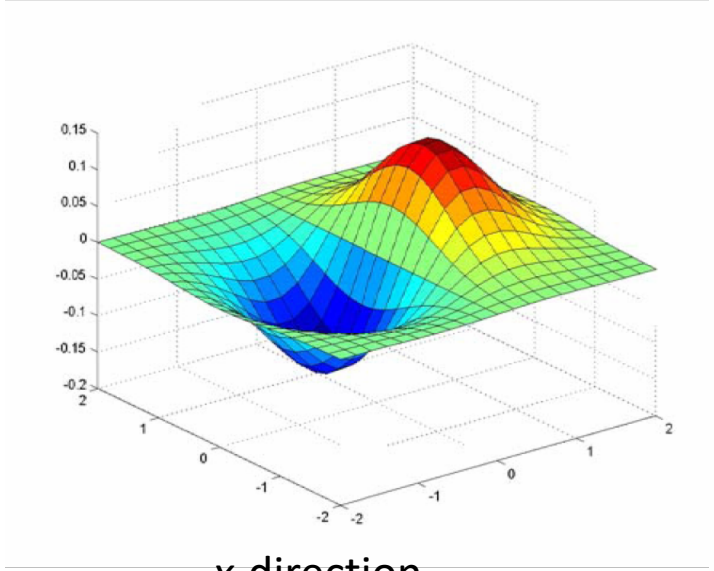
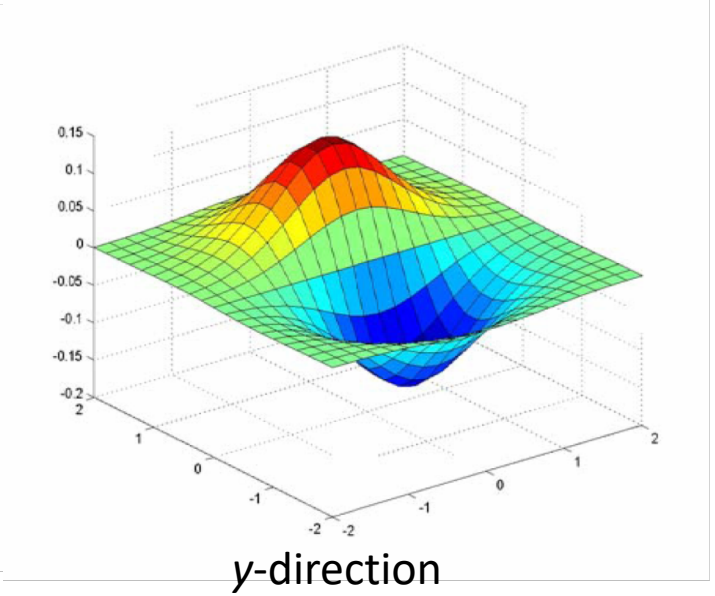


FIGURE 5.3: The scale (i.e., σ) of the Gaussian used in a derivative of Gaussian filter has significant effects on the results. The three images show estimates of the derivative in the x direction of an image of the head of a zebra obtained using a derivative of Gaussian filter with σ one pixel, three pixels, and seven pixels (**left** to **right**). Note how images at a finer scale show some hair, the animal's whiskers disappear at a medium scale, and the fine stripes at the top of the muzzle disappear at the coarser scale.

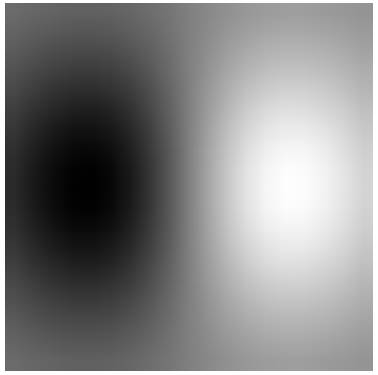
Derivative of Gaussian filter

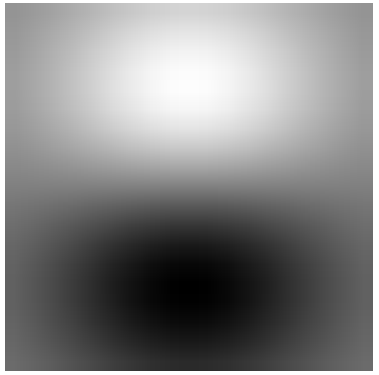


x-direction



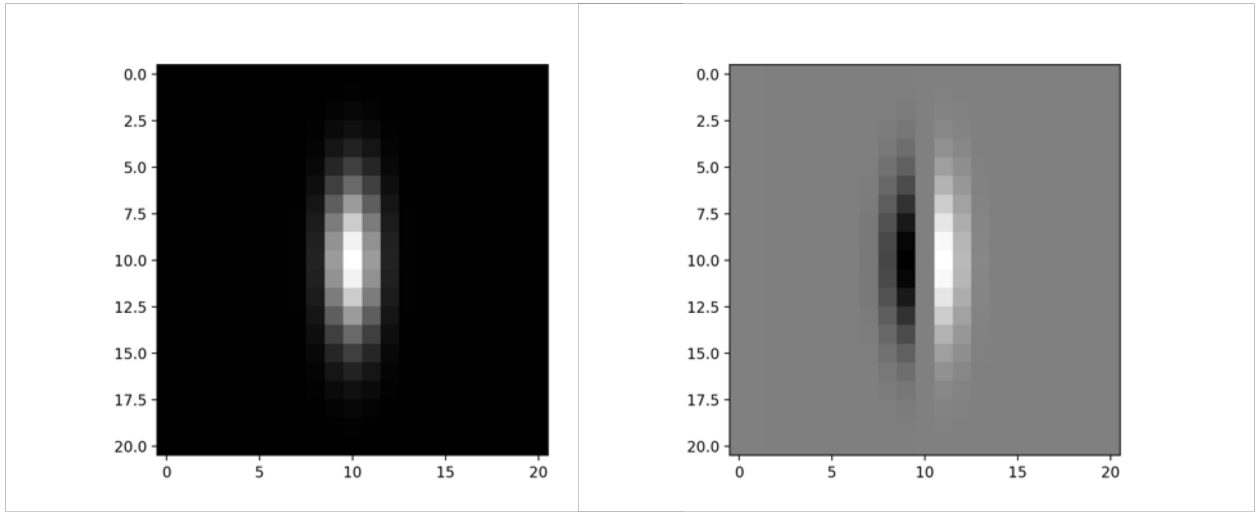
y-direction





Anisotropic Gaussian

- Should smoothing along the edge be the same as smoothing across it?
- Smooth more along the edge than across it
- Use a lower standard deviation along one direction
- $G(x, y) = e^{-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}}$
- Can also do for derivative



The Sobel filter

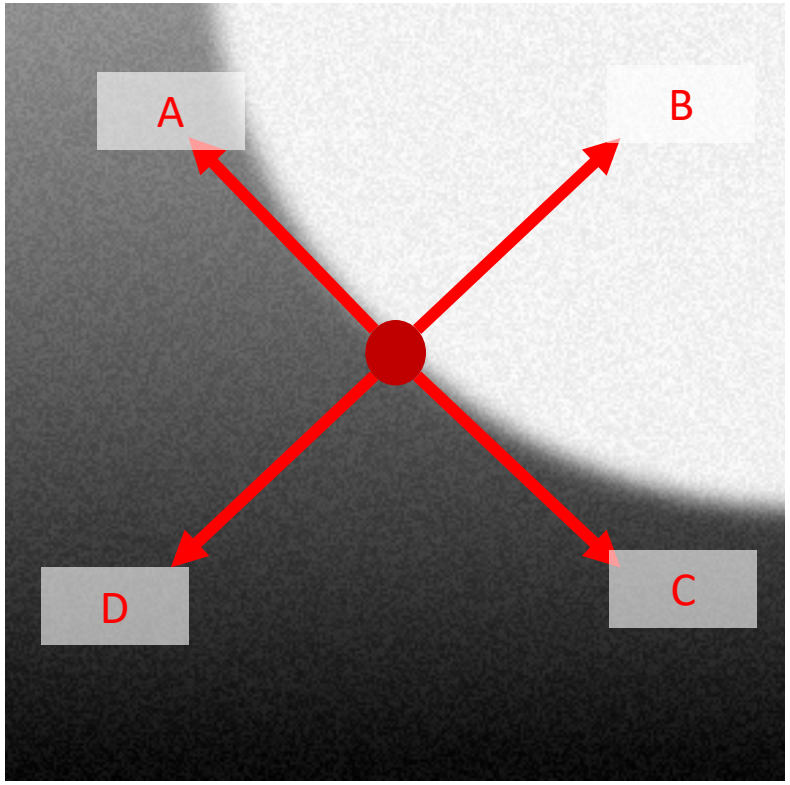
- A variant of the Anisotropic Gaussian
- Smooth *only* along the edge
- X derivative filter:

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

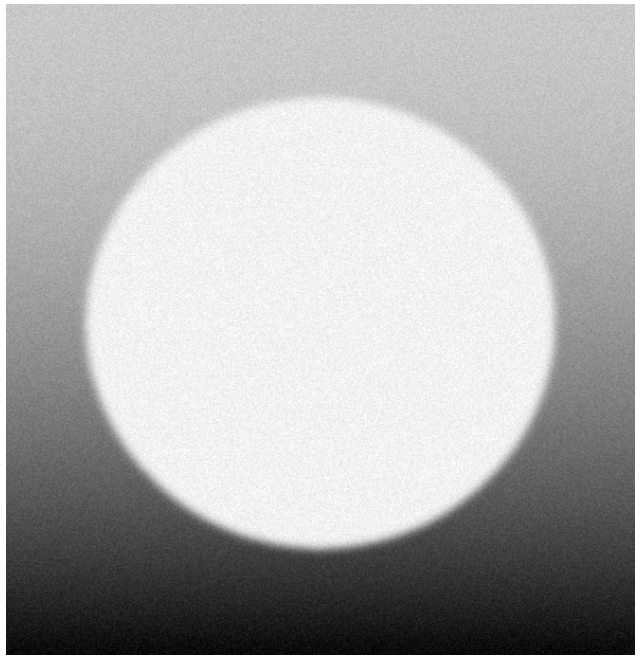
Oriented Gaussian Derivatives

- Instead of just derivative along x and y, can also rotate filter to get derivatives along different directions

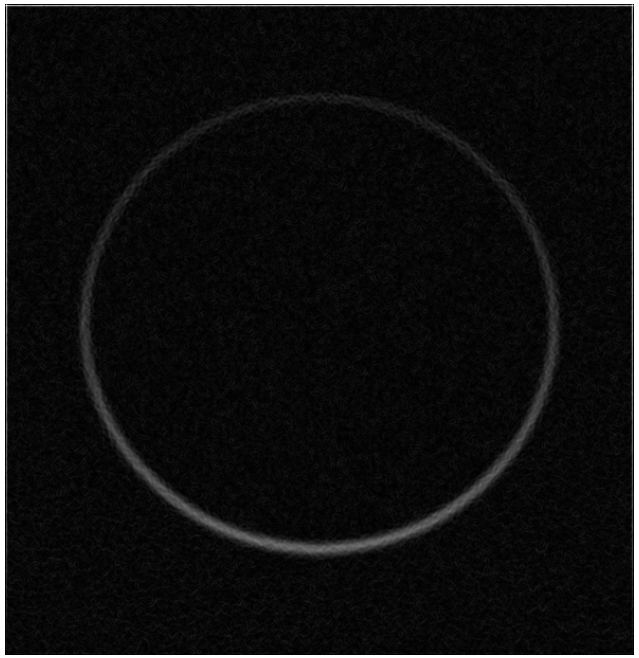
Which way does the gradient point?



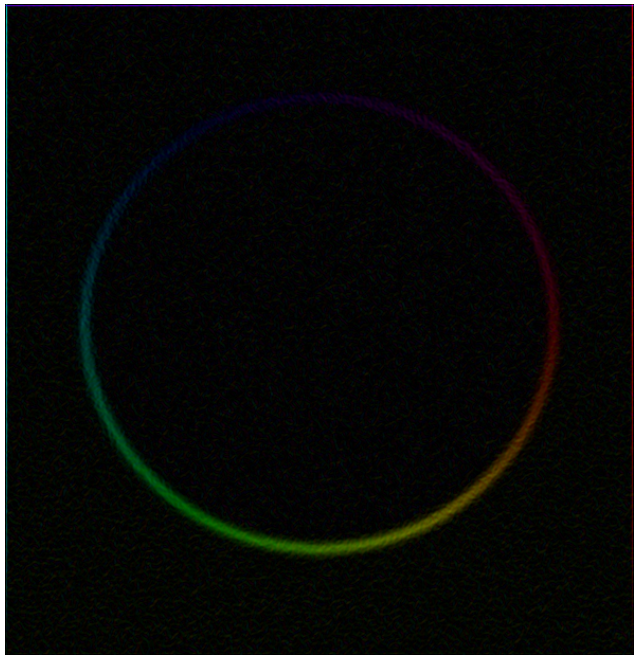
Gradient magnitude and orientation



Input



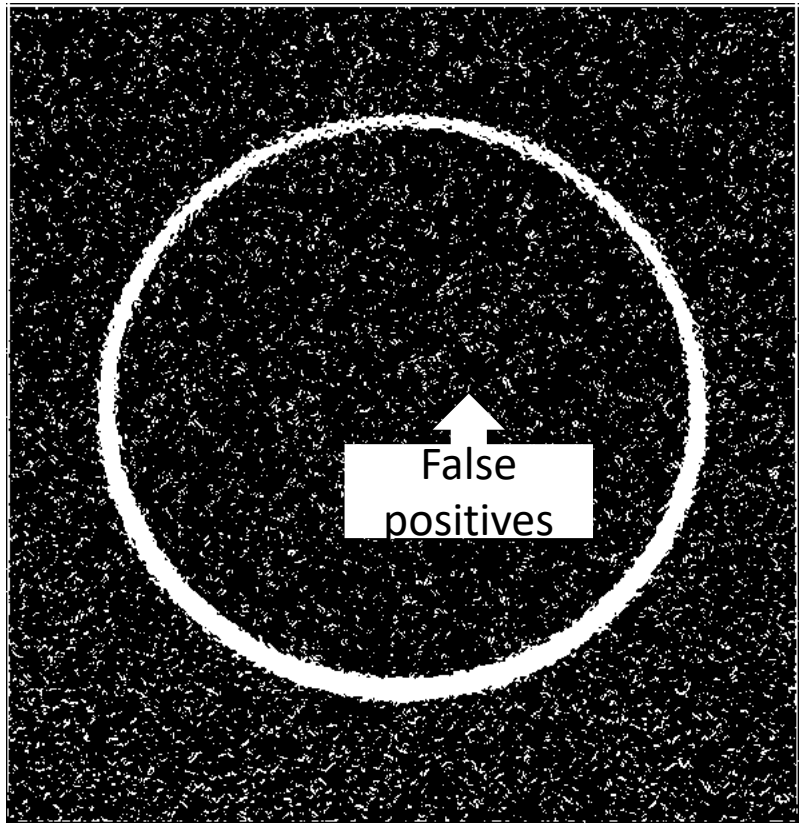
Gradient Magnitude

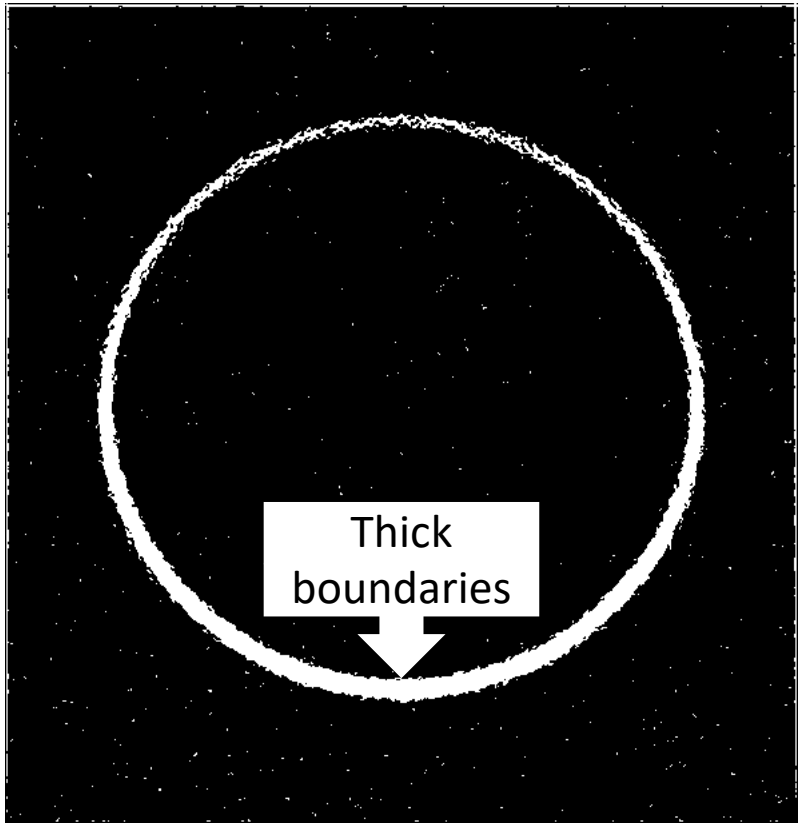


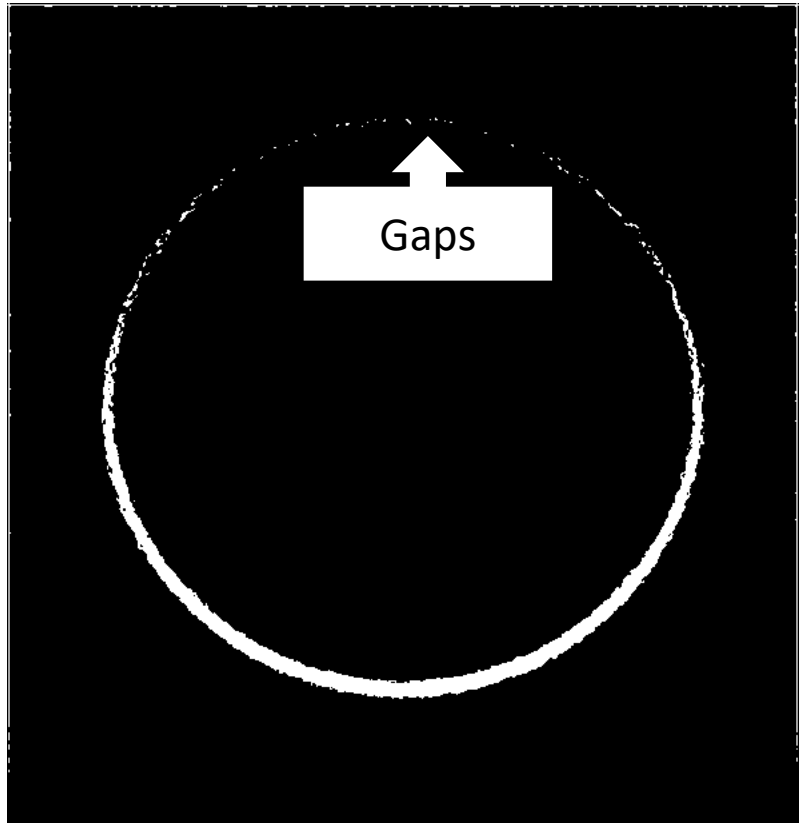
Gradient Orientation

From gradients to boundaries?

- Can we threshold the gradient magnitude to get object boundaries?
- Three different thresholds:

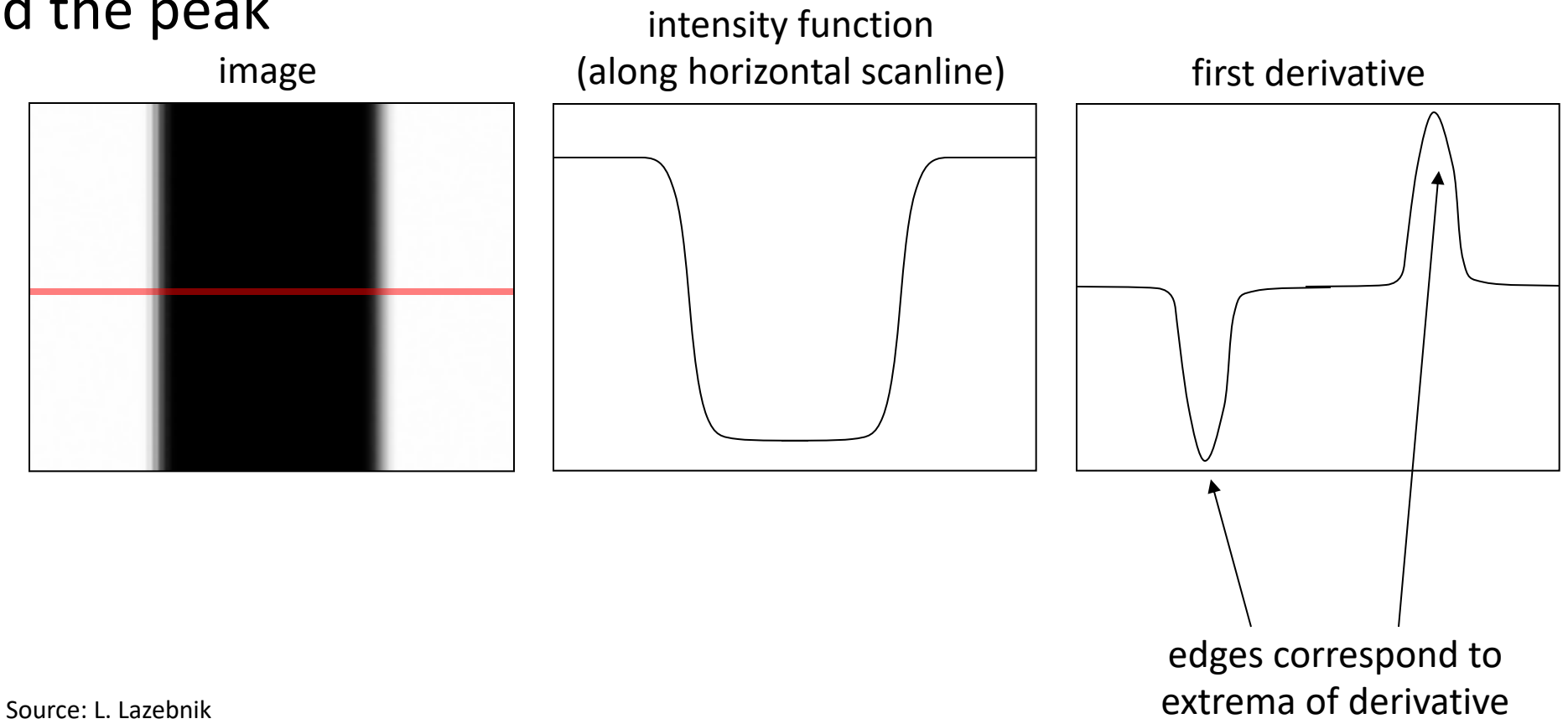






Thick boundaries

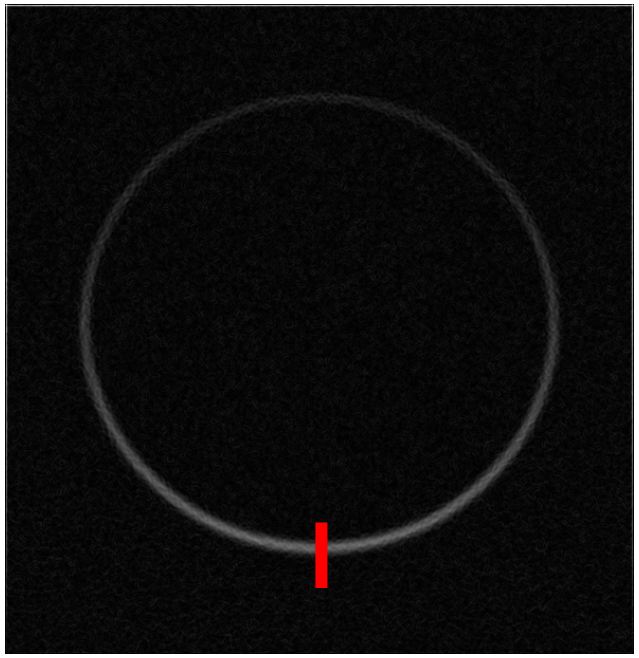
- Want to find *peaks* in the gradient magnitude
- As one moves across edge, gradient magnitude rises and then falls
- Find the peak

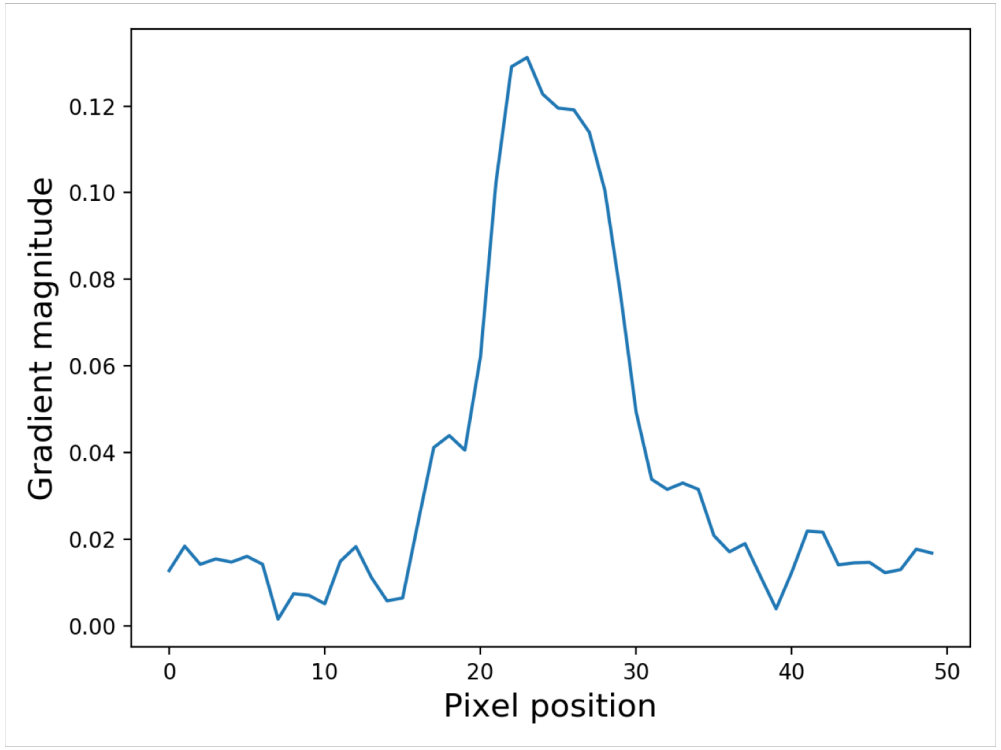


Source: L. Lazebnik

Thick boundaries

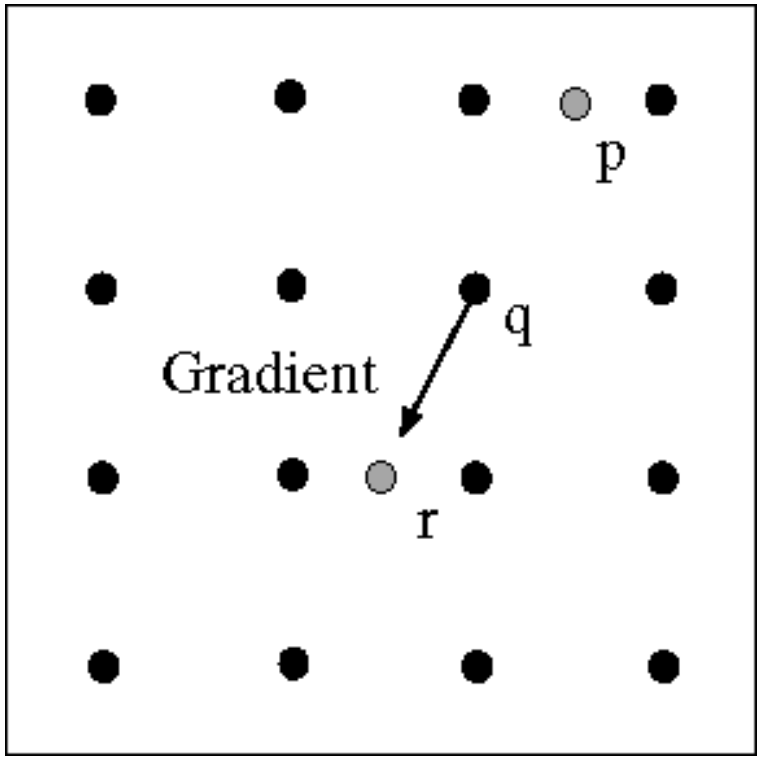
- Want to find *peaks* in the gradient magnitude
- As one moves across edge, gradient magnitude rises and then falls
- Find the peak

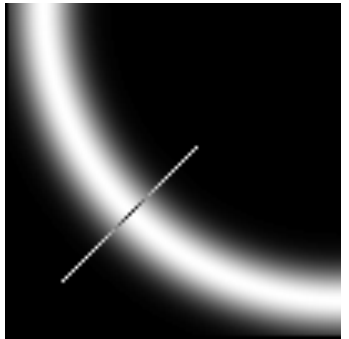




Non-maximum suppression for each orientation

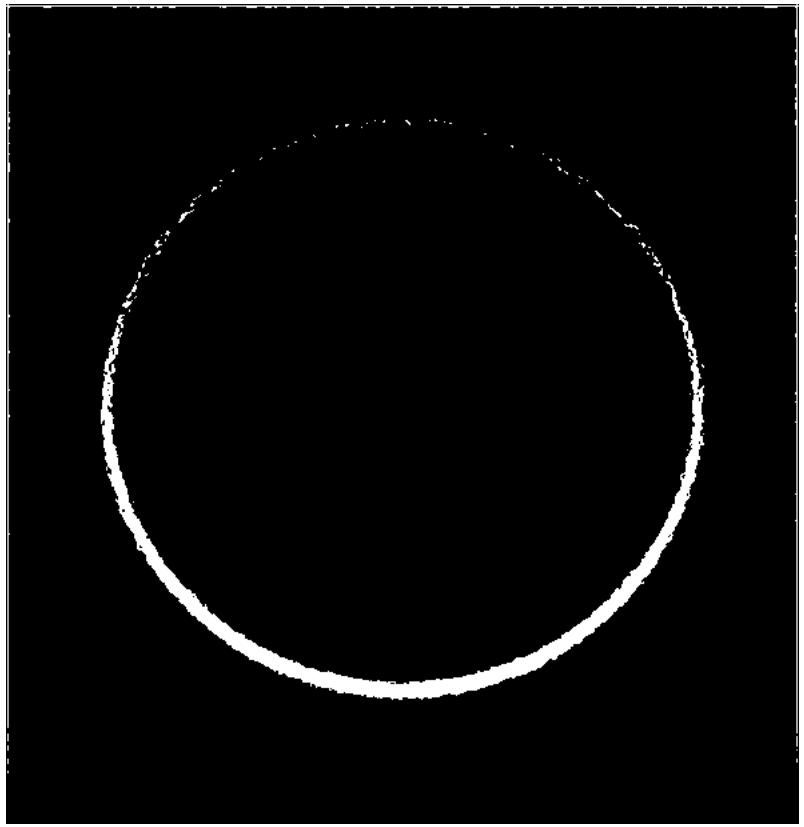
- Let $\boldsymbol{g}(q)$ be the image gradient at pixel q
  - A vector
- Take a step forwards and backwards along gradient
  - $r = q + \frac{\boldsymbol{g}(q)}{\|\boldsymbol{g}(q)\|}$
  - $p = q - \frac{\boldsymbol{g}(q)}{\|\boldsymbol{g}(q)\|}$
- Find gradient magnitude at p and r
  - Interpolate into gradient magnitude
- If magnitude at q > at p and r, ***q*** **is a local peak of gradient magnitude!**



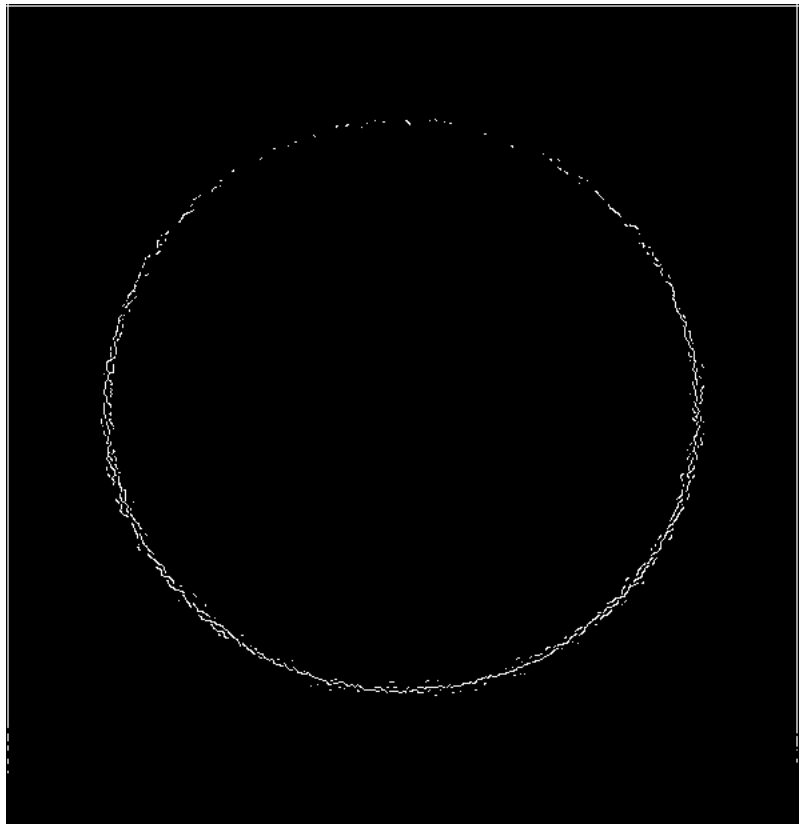


Source: D. Forsyth

Non-maximum suppression

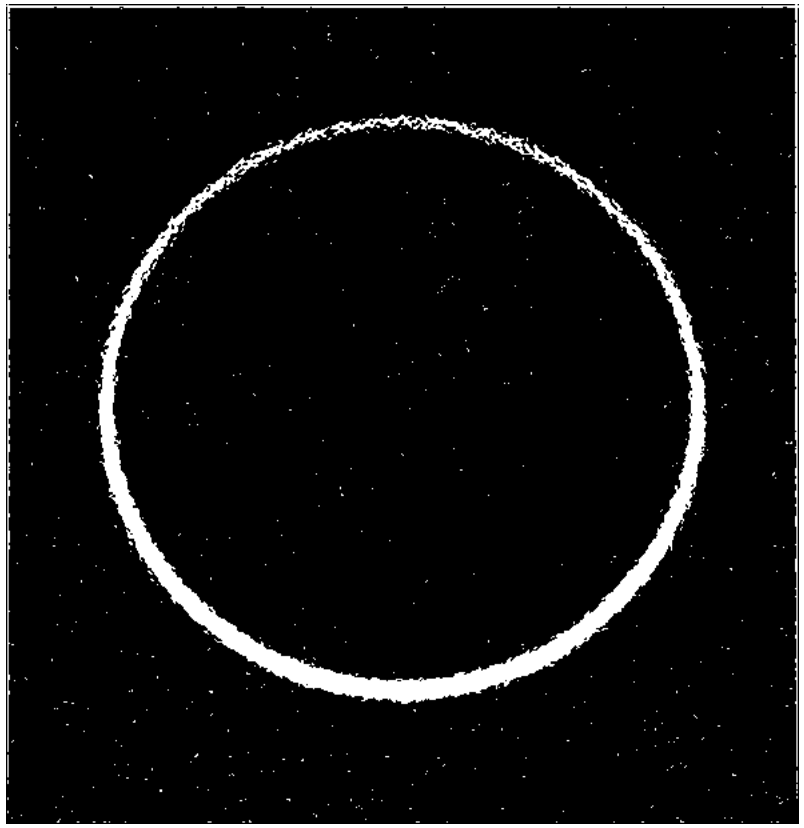


Before

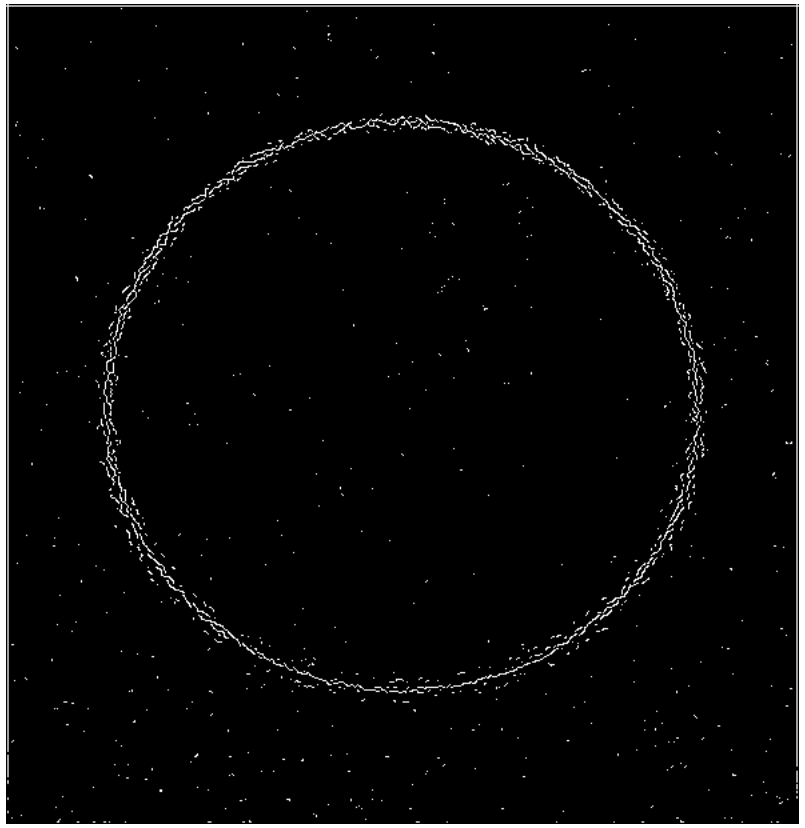


After

Non-maximum suppression

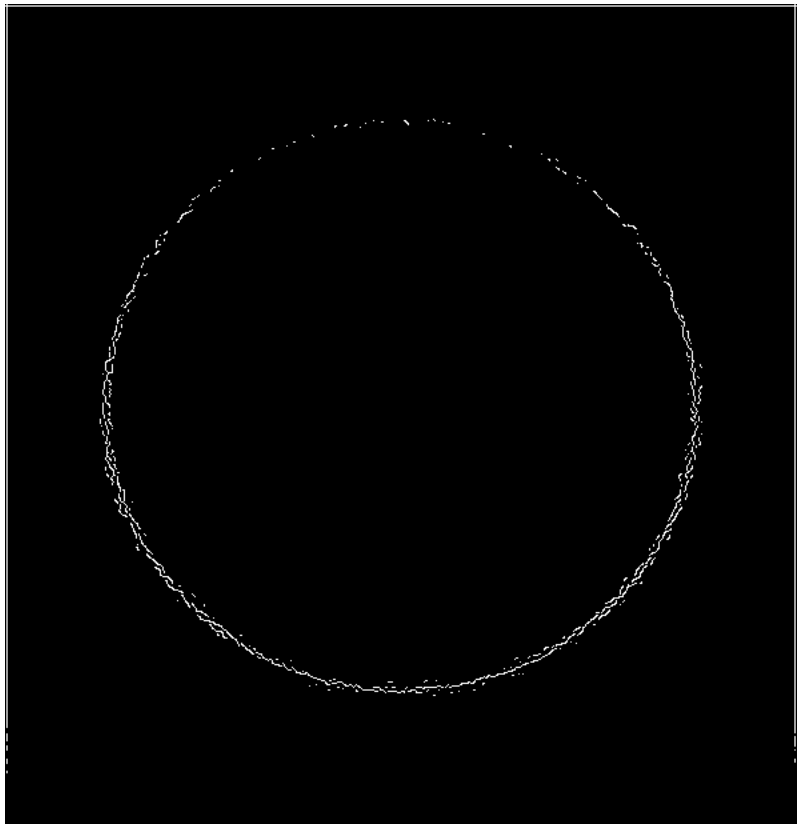


Before

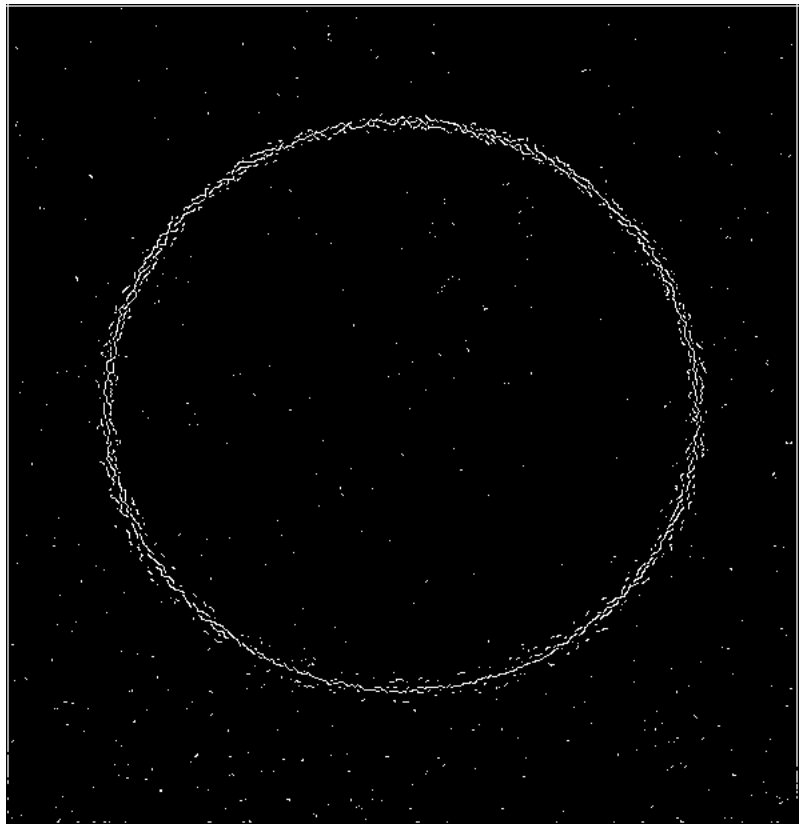


After

Non-max suppression with various thresholds



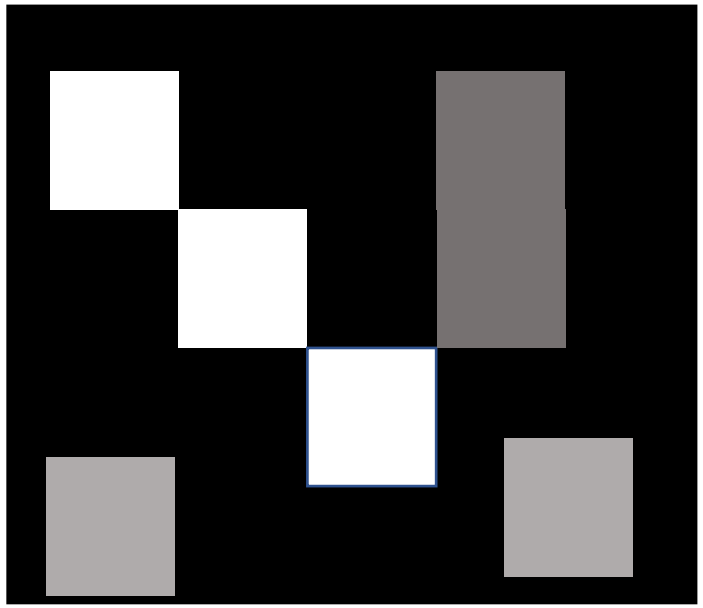
High threshold



Low threshold

Hysteresis thresholding

- Start with a high threshold
- Then grow high threshold edges by looking for neighbors that clear a lower threshold



- White: Clears high threshold
- Gray: clears low threshold but not high threshold

Hysteresis thresholding

- Start with a high threshold
- Then grow high threshold edges by looking for neighbors that clear a lower threshold

- Mark pixels that clear high threshold as boundary

Hysteresis thresholding

- Start with a high threshold
- Then grow high threshold edges by looking for neighbors that clear a lower threshold

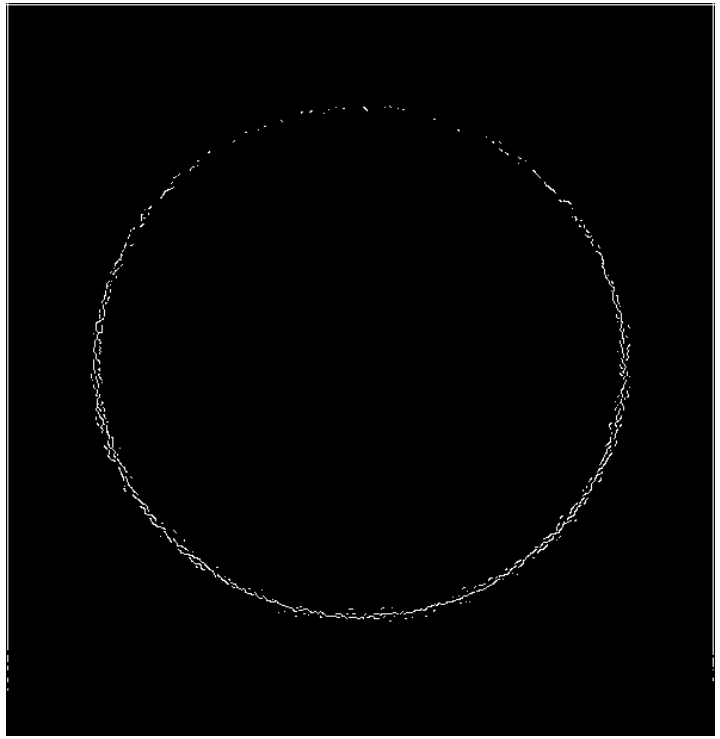
- Mark pixels that clear high threshold as boundary
- If there are pixels that clear low threshold and are connected to a boundary, mark them as boundary too

Hysteresis thresholding

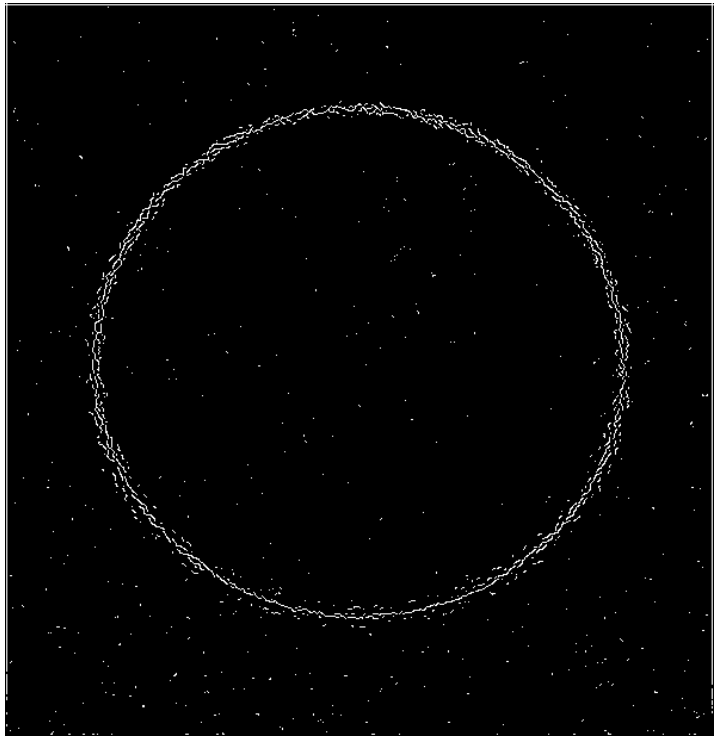
- Start with a high threshold
- Then grow high threshold edges by looking for neighbors that clear a lower threshold

- Mark pixels that clear high threshold as boundary
- If there are pixels that clear low threshold and are connected to a boundary, mark them as boundary too

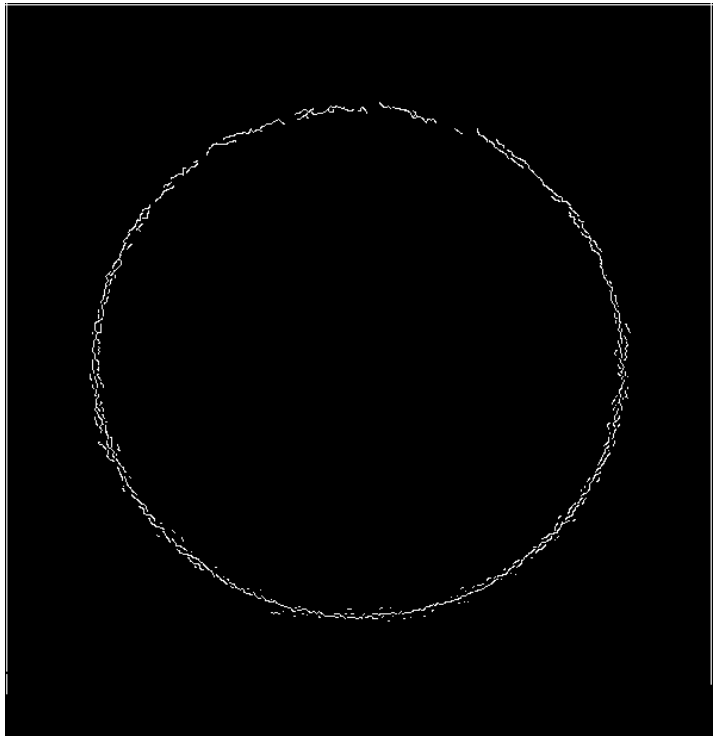
Hysteresis thresholding



High threshold



Low threshold



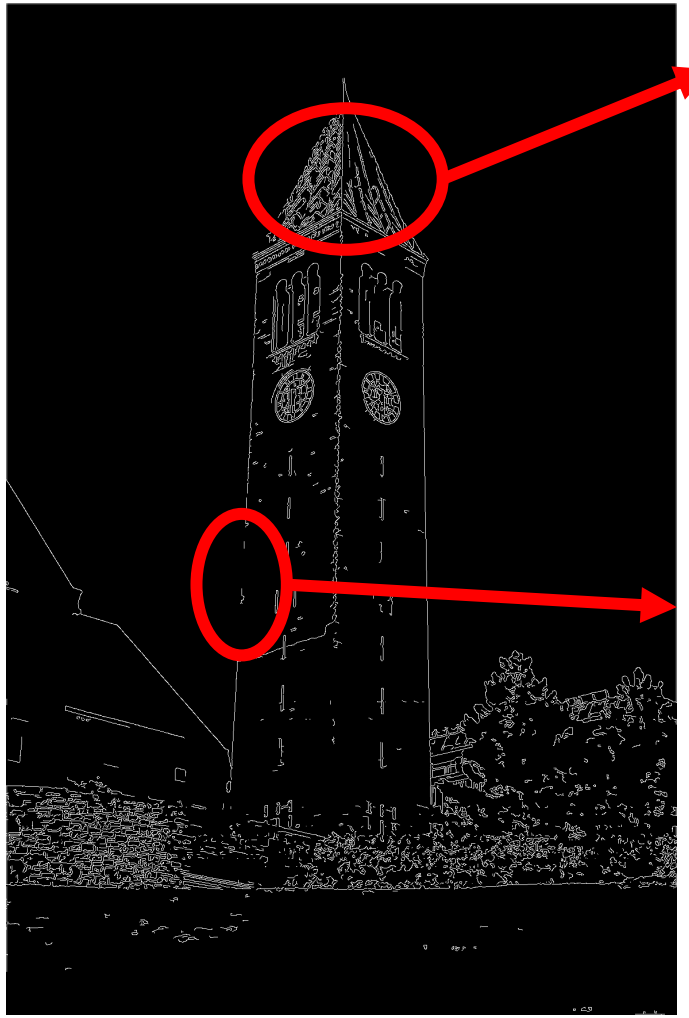
Hysteresis threshold

Canny edge detector

1. Filter image with x, y derivatives of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   - Thin multi-pixel wide “ridges” down to single pixel width
4. Thresholding and linking (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

Source: D. Lowe, L. Fei-Fei

Canny edge detection



Challenge #1: Texture

Challenge #2: Low-contrast edges