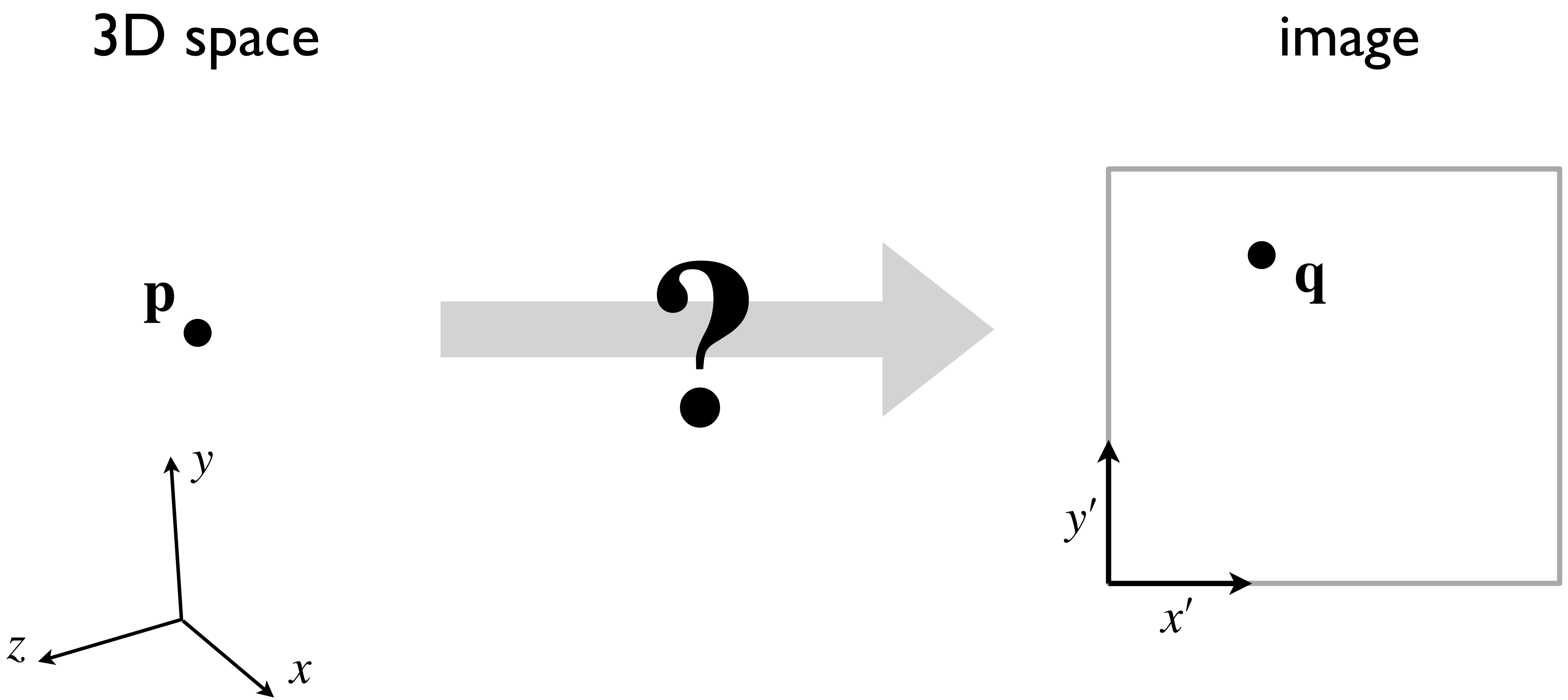
Perspective and image formation

CS 4670, 17 Feb 2020

Image formation / Viewing

- **Images that are taken by cameras are pretty special**
  - different from abstract paintings
  - different from scans of paper documents
  - different from charts, graphs, etc.
- **They are projections of objects in the 3D world**
  - properties of camera create simple geometric relationships
  - cameras are quite similar to human eye
  - these projections are relevant in both vision and graphics
  - in vision, topic tends to be called "image formation"
  - in graphics, topic tends to be called "viewing"
  - notation is a bit different but the geometry is the same

Basic question



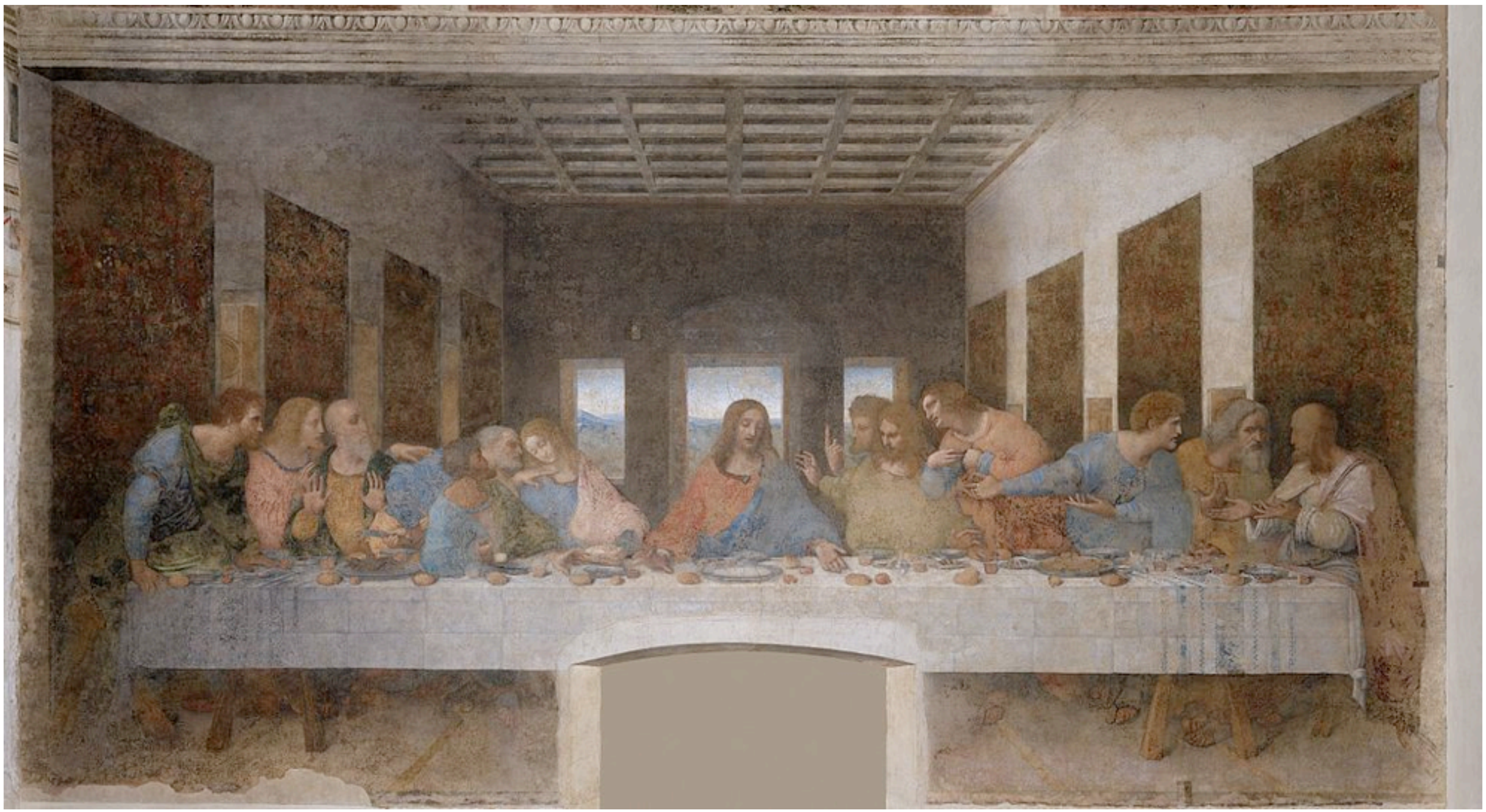
History of projection

- **Ancient times: Greeks wrote about laws of perspective**
- **Renaissance: perspective is adopted by artists**

Duccio c. 1308

History of projection

- **Later Renaissance: perspective formalized precisely**



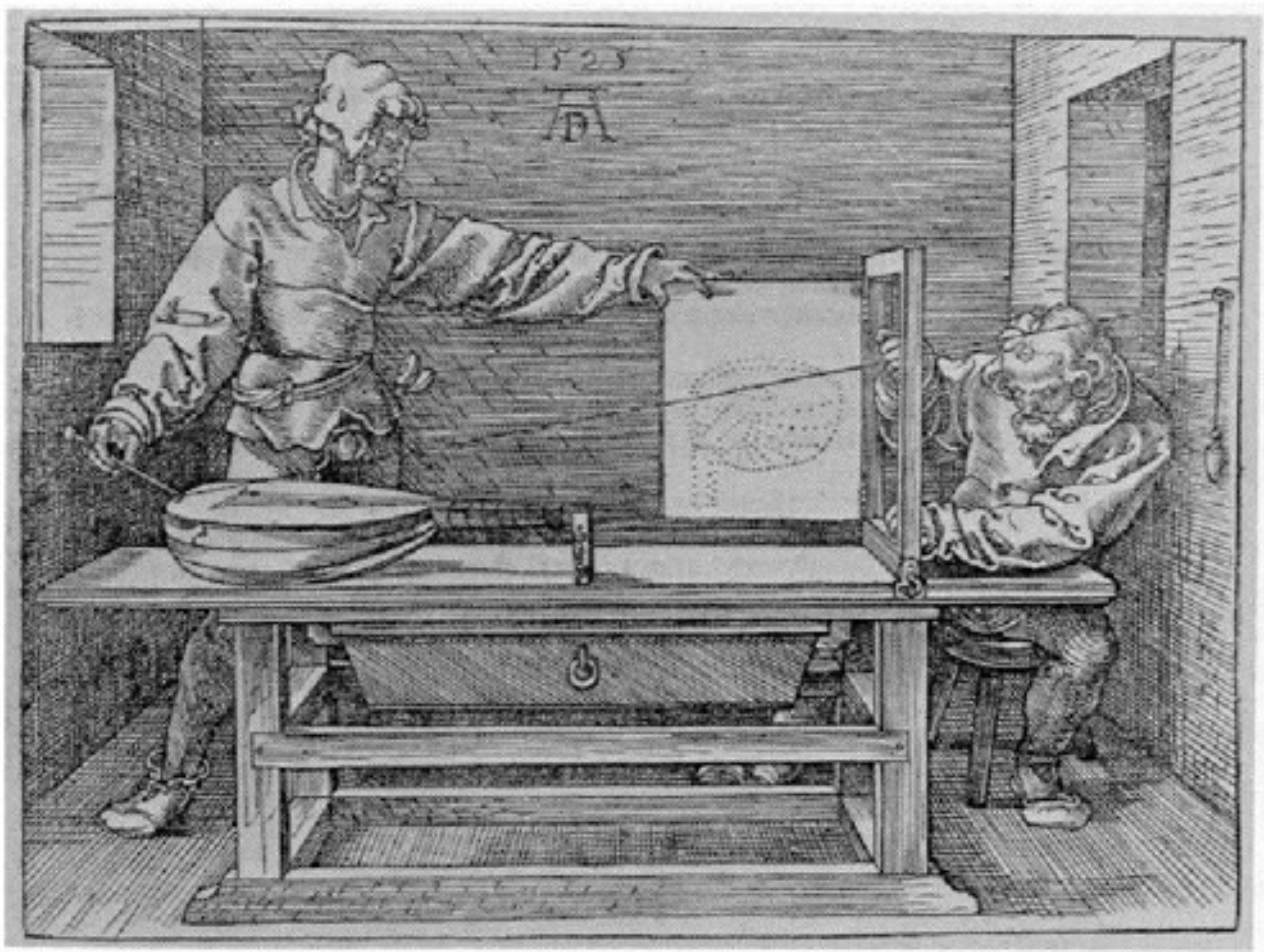
da Vinci c. 1498

Basic perspective facts

- **Objects get smaller as they get farther away**
- **Parallel lines converge at *vanishing points***

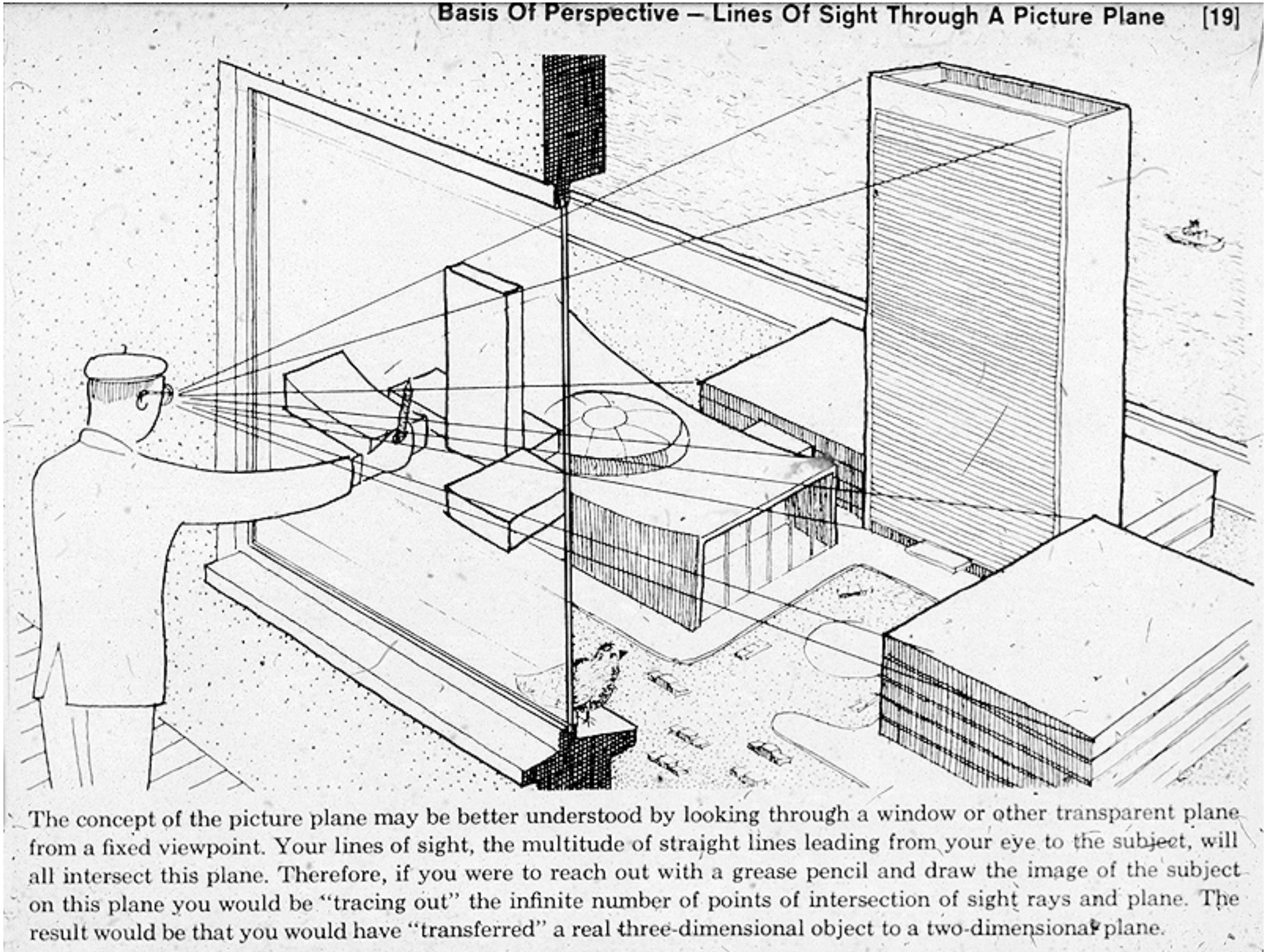

Ray Verrier

Plane projection in drawing



Albrecht Dürer

Plane projection in drawing



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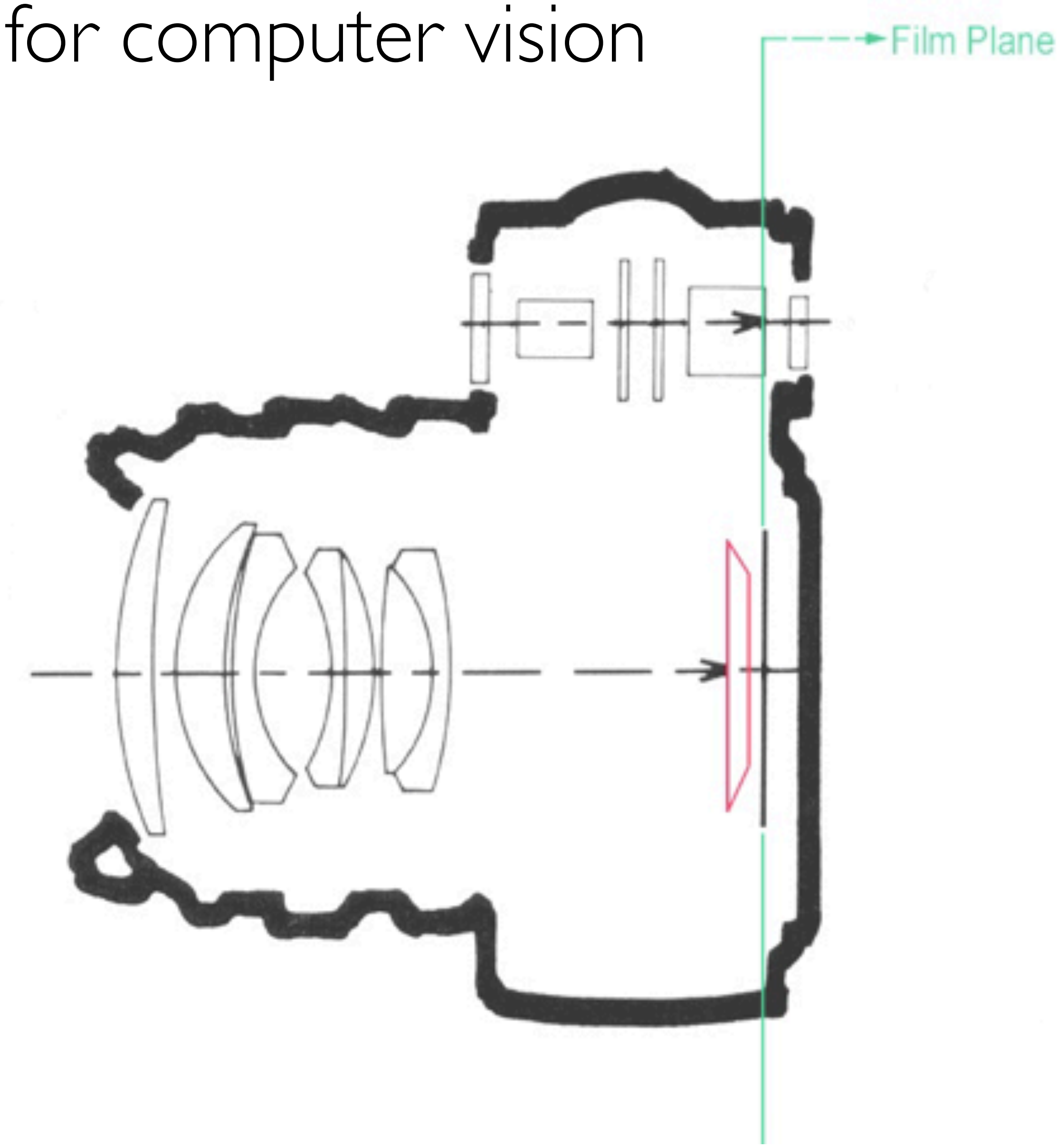
Plane projection in cameras

- **Projection through a point is a good model for many cameras**
  – this makes it a good model for computer vision



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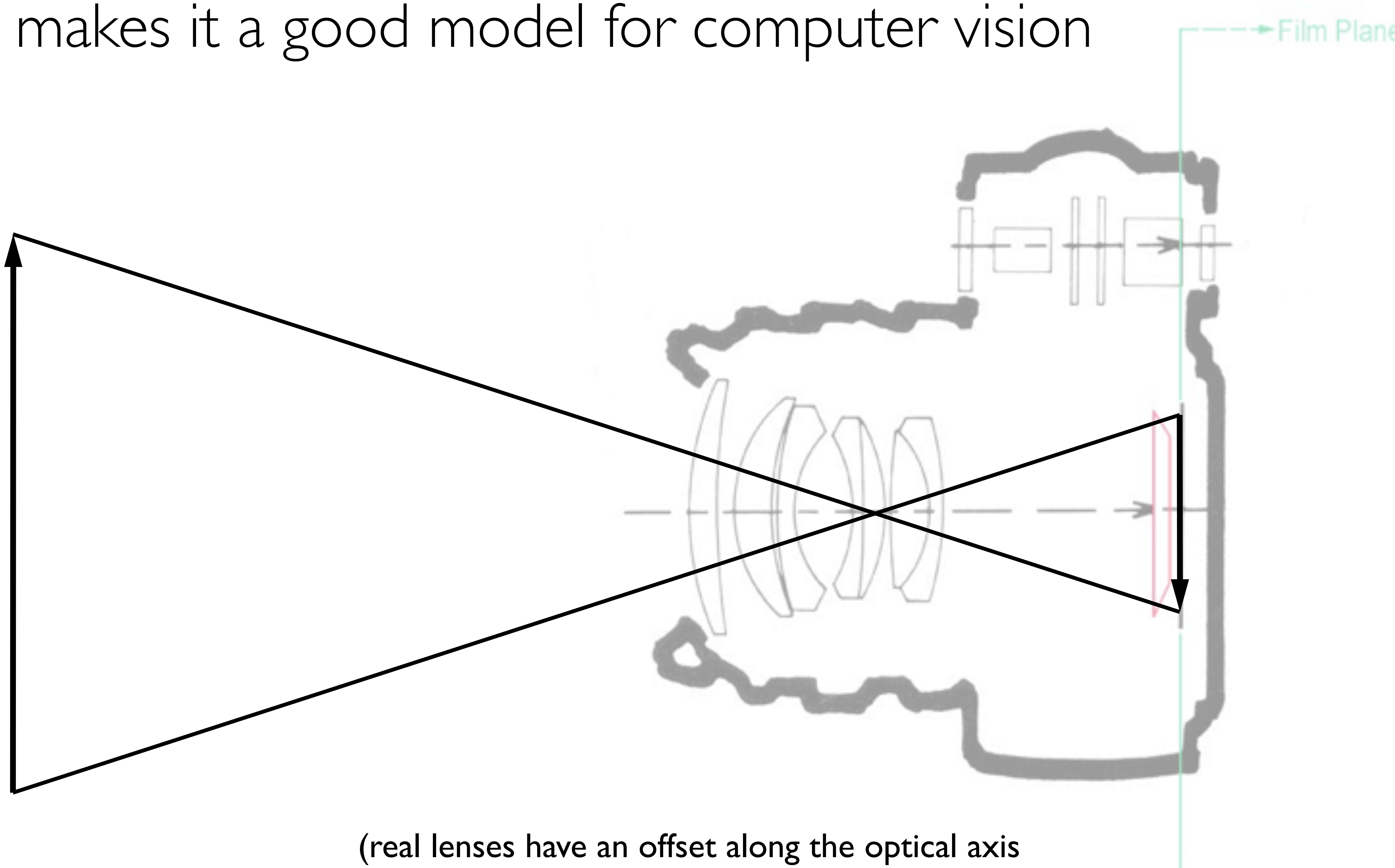


[Source unknown]

(real lenses have an offset along the optical axis between the point where the rays converge on the object side and the point from which they diverge on the image side — but this does not often matter for graphics or vision.)

Pinhole camera

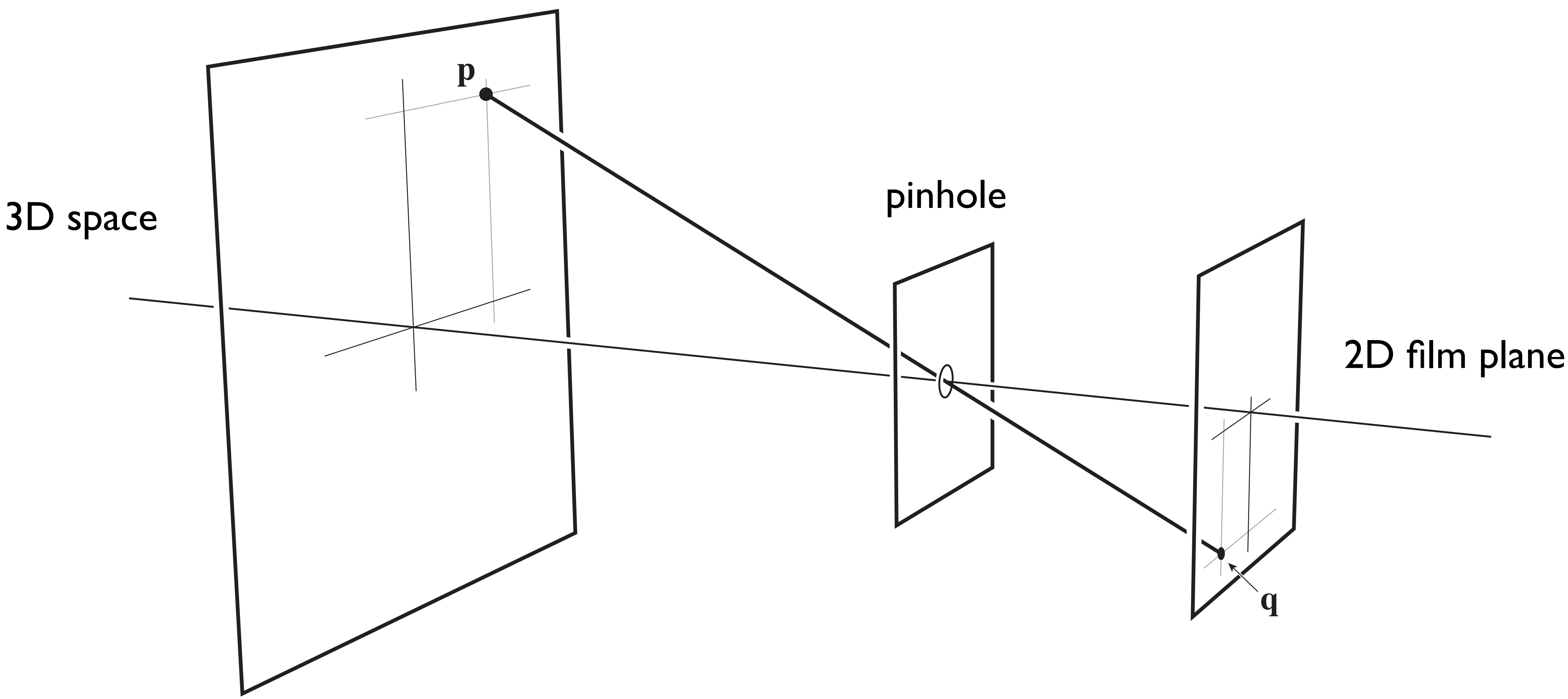
B&H Photo-Video

- **Simple model for the projection geometry of a camera with a lens**
  – but you can actually make a camera like this!
  – just is very inefficient and produces low image quality

intstructables.com | user 343GUILTYSPARK]

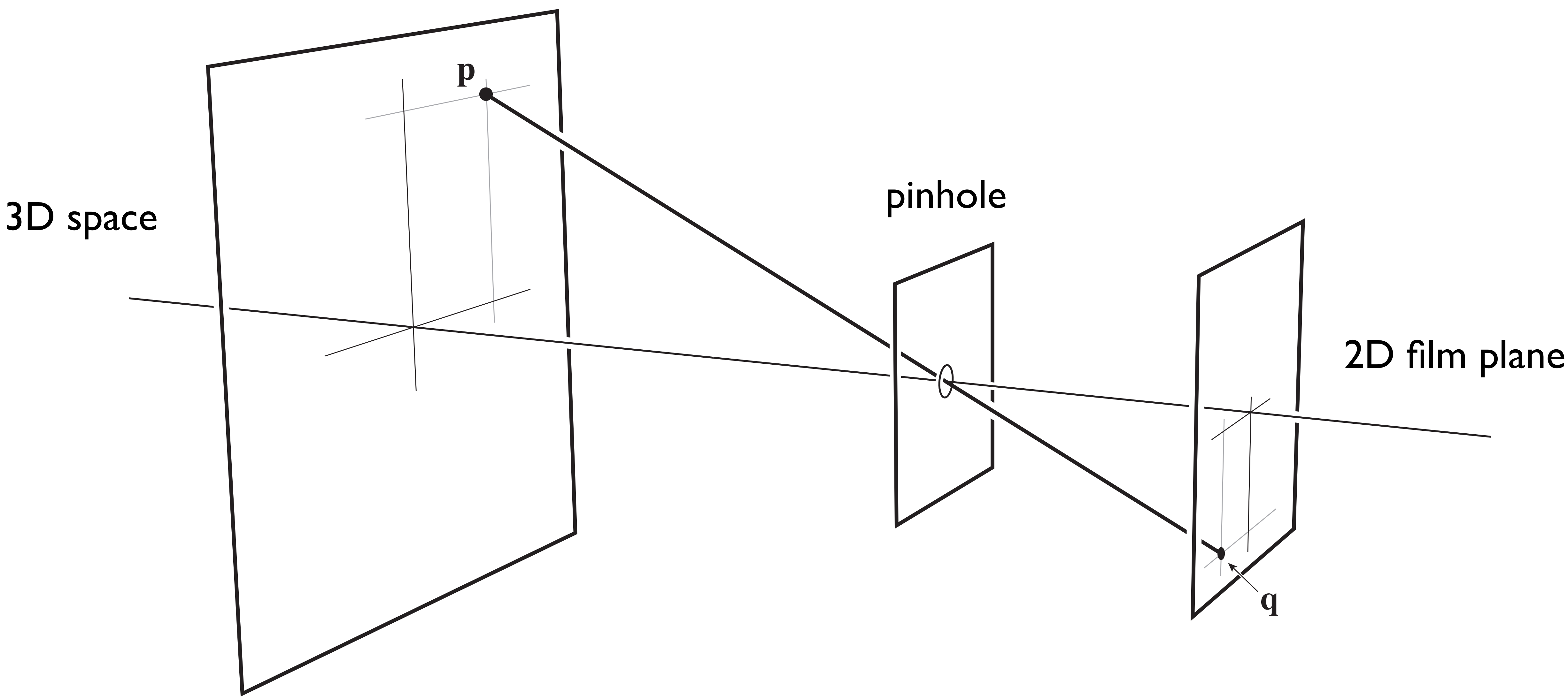
Pinhole projection

- **A 3D point p projects along the line through the pinhole to the intersection of that line with the film plane**



More convenient image plane

- **A 3D point p projects along the line through the camera center to the intersection of that line with the image plane**



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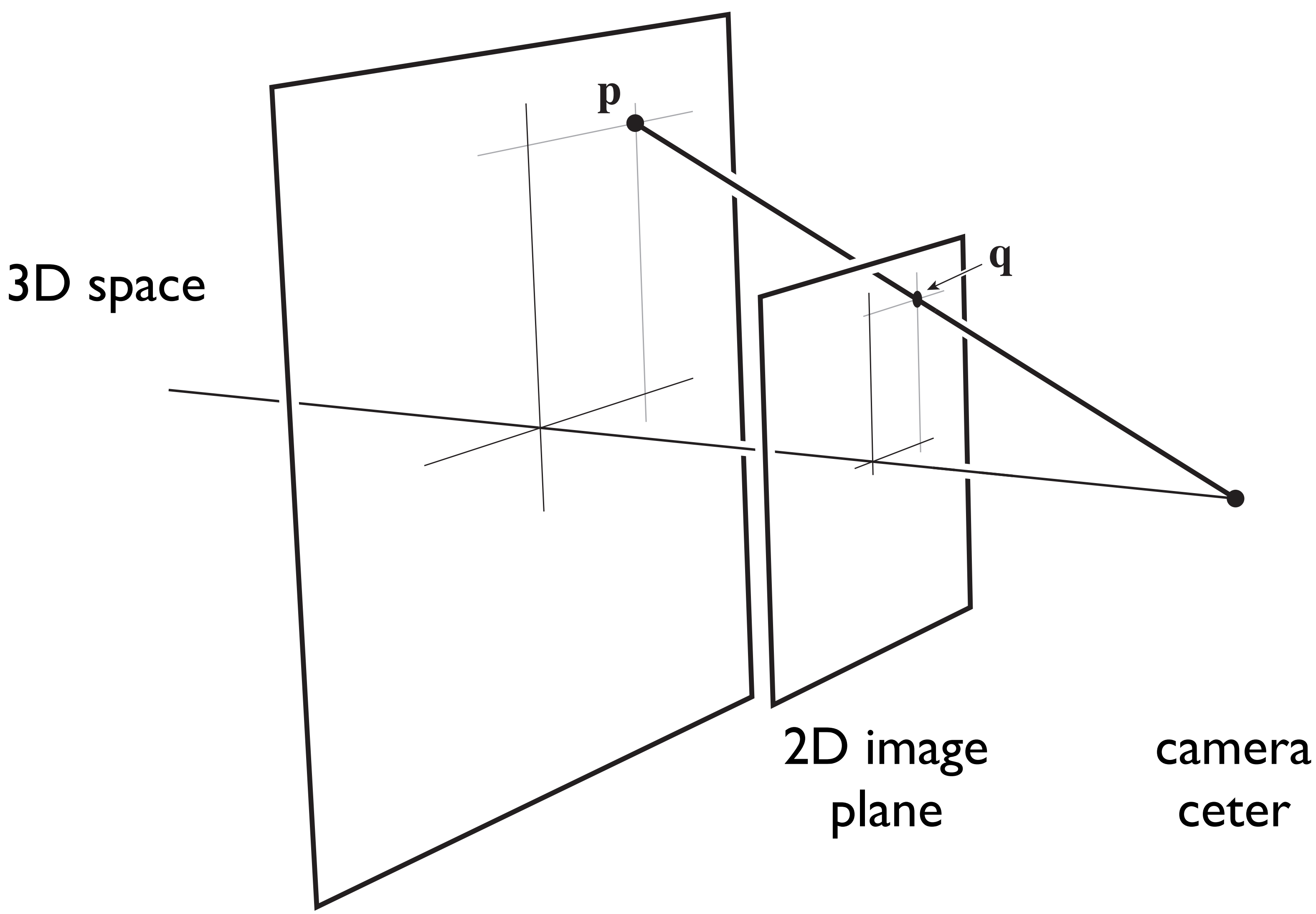
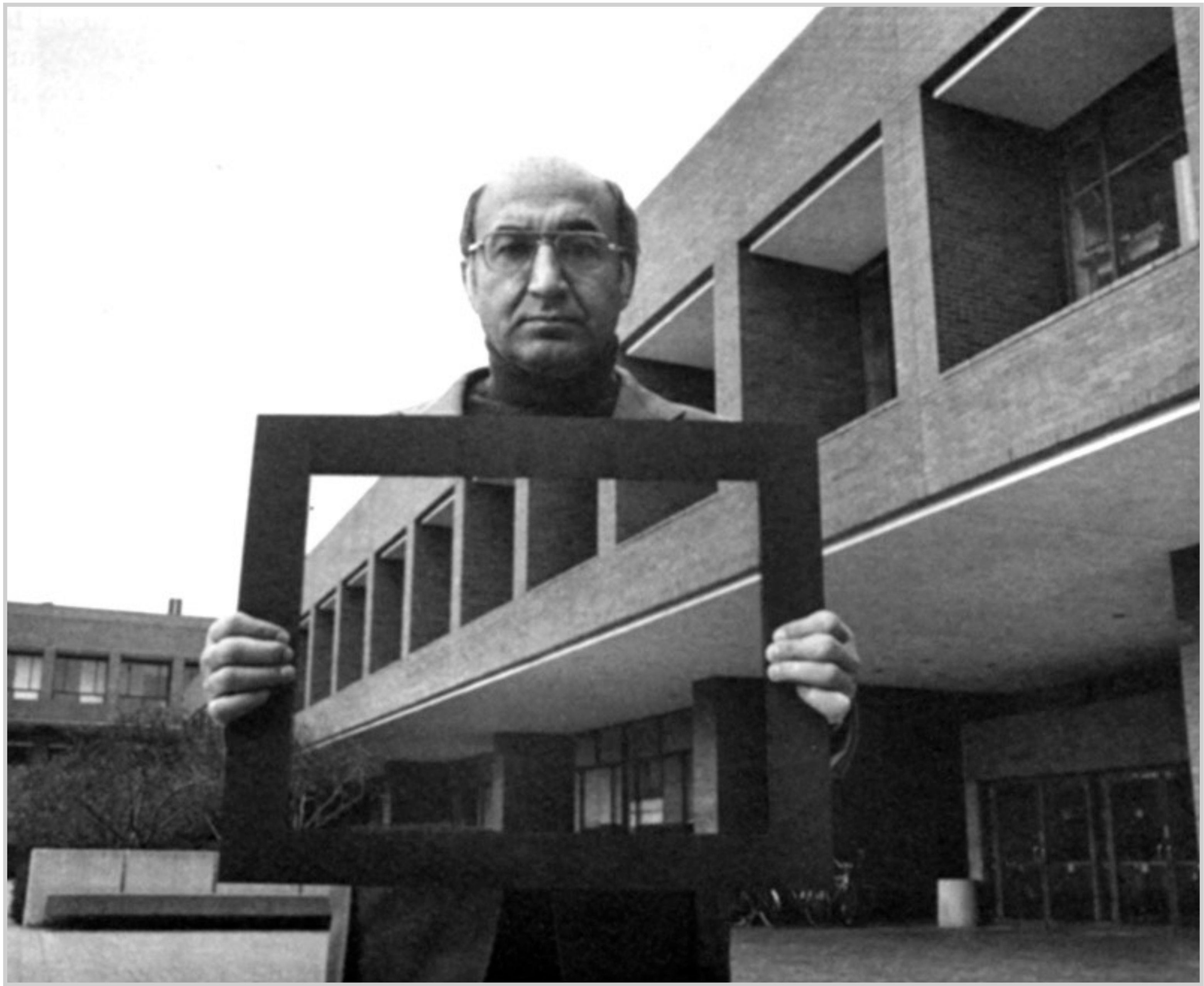


Plane projection in photography



Richard Zakia

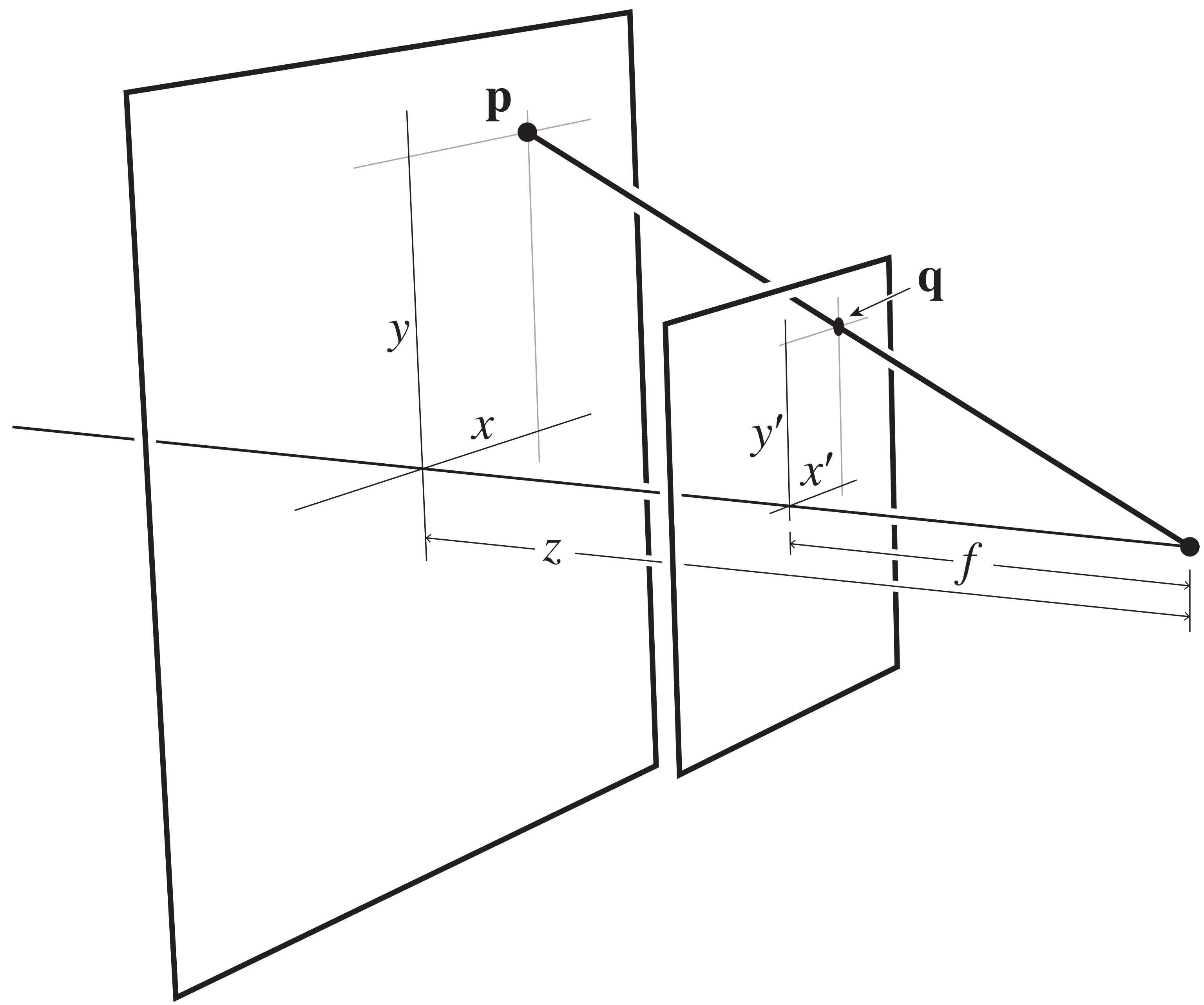
Plane projection math

- **Similar triangles in the *x-z* and *y-z* planes lead to:**

$$x' = fx/z$$

$$y' = fy/z$$

- **Distance from image plane to camera center is called the *focal length***
  - this is close to the actual focal length of the lens, when not focused very close



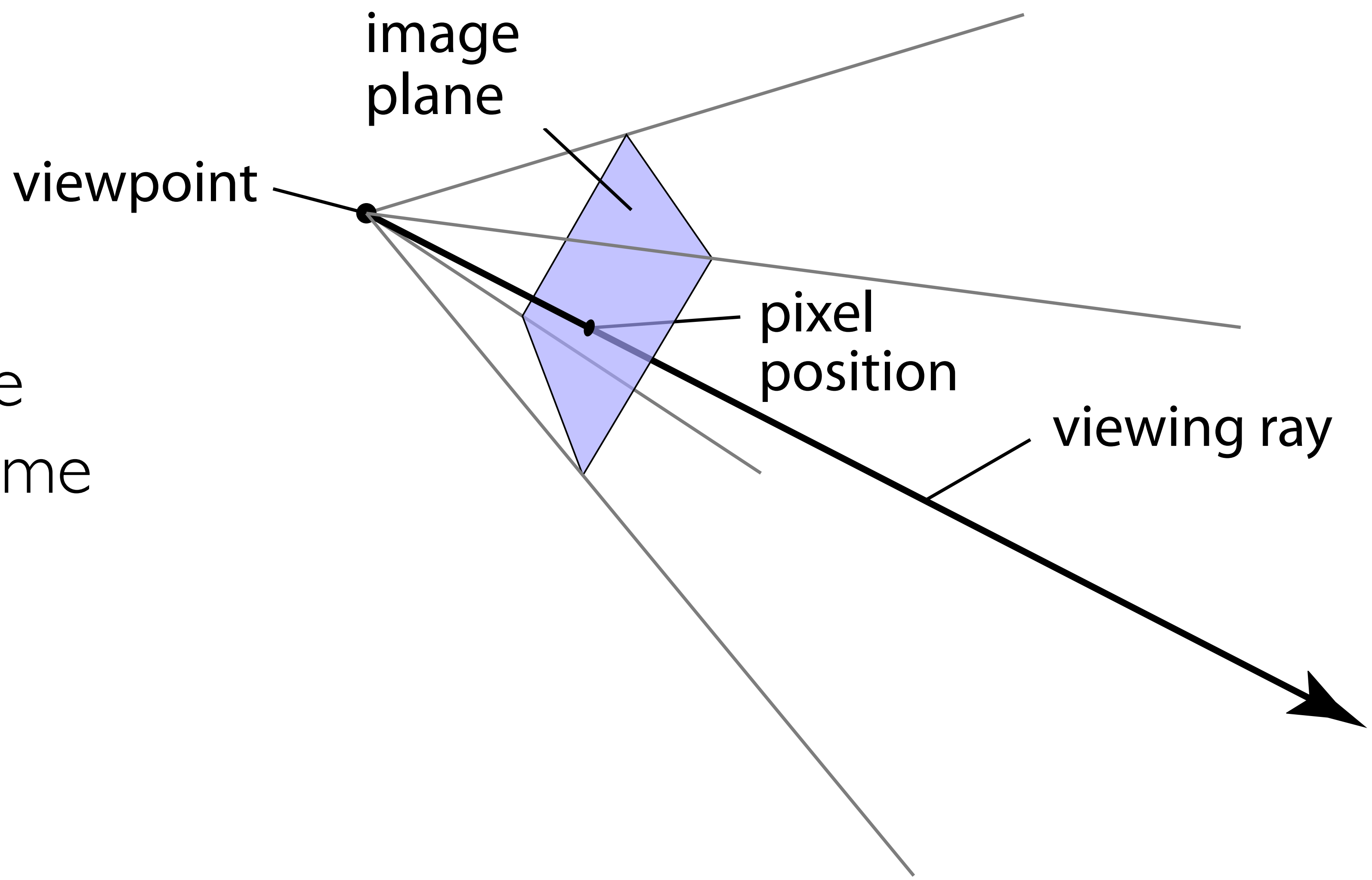
Modeling projection

- **These equations assume some things**
  - 3D points are in a coordinate system where the *z* axis is perpendicular to the image plane
  - 2D points are in a coordinate system with the same units and with the origin on the *z* axis
- **The basic phenomena:**
  - planar objects parallel to the image plane are simply magnified in projection
  - magnification is inversely proportional to *z* (distance from the camera measured along the *z* axis, a.k.a. "depth")

$$x' = fx/z$$
$$y' = fy/z$$

Forward and inverse projection

- **Perspective projection maps a 3D point to a single 2D point**
- **Thus it is not invertible!**
  - there are many points in the world that project to the same point in the image
  - the set of points projecting to a point is a ray (in graphics, a *viewing ray*—not sure there's a standard name in vision!)



View volume: perspective

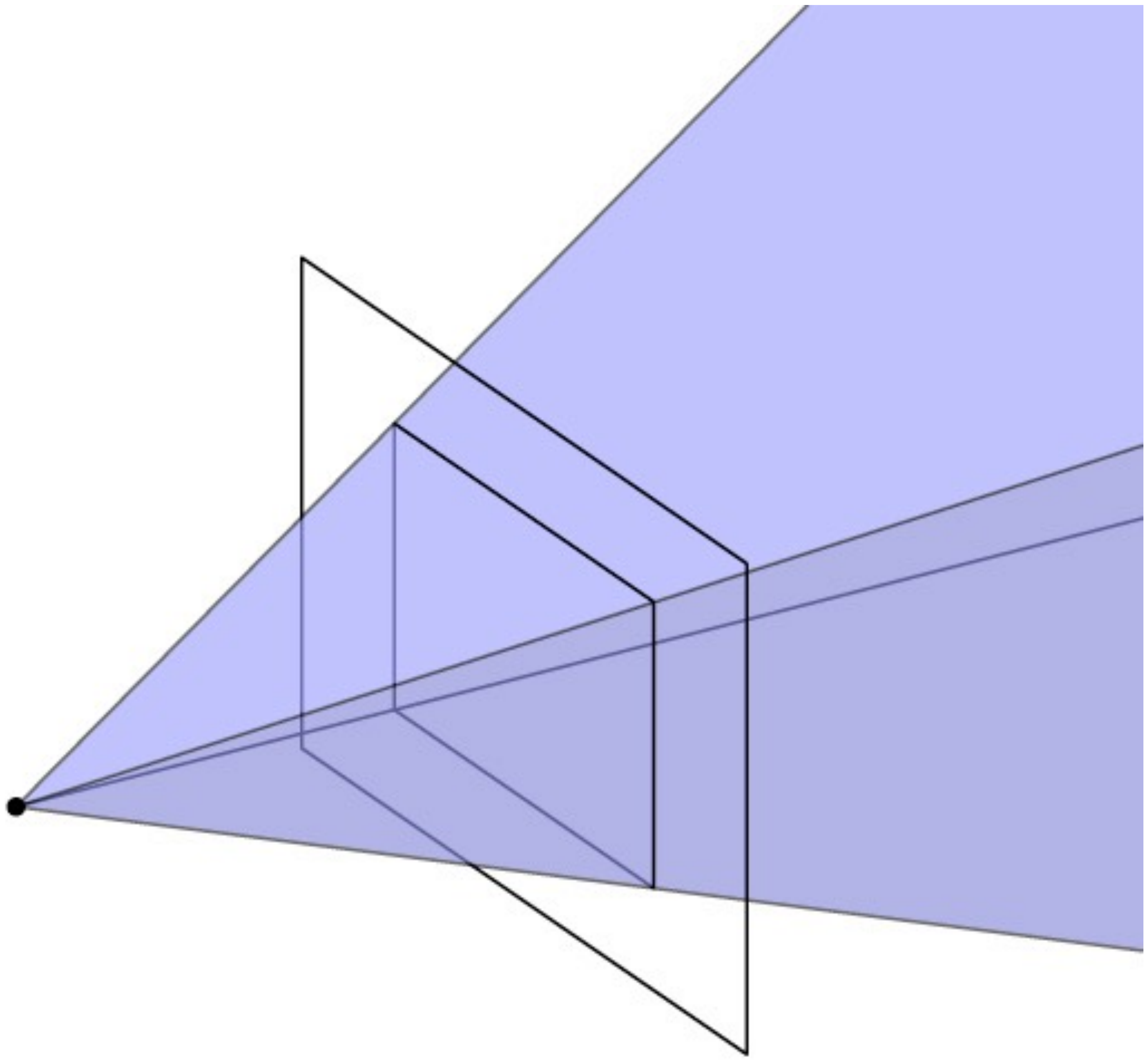


Image coordinates

- **One remaining issue is the units on the image side**
  - we probably didn't want to have to measure points in the image in terms of actual distances on the sensor
  - for many (most) images we encounter we won't know the actual details of the sensor size, etc.
  - normally in vision we measure image distances in terms of pixels
- **First fix: convert from world units to pixels**
  - suppose everything was measured in meters; then:

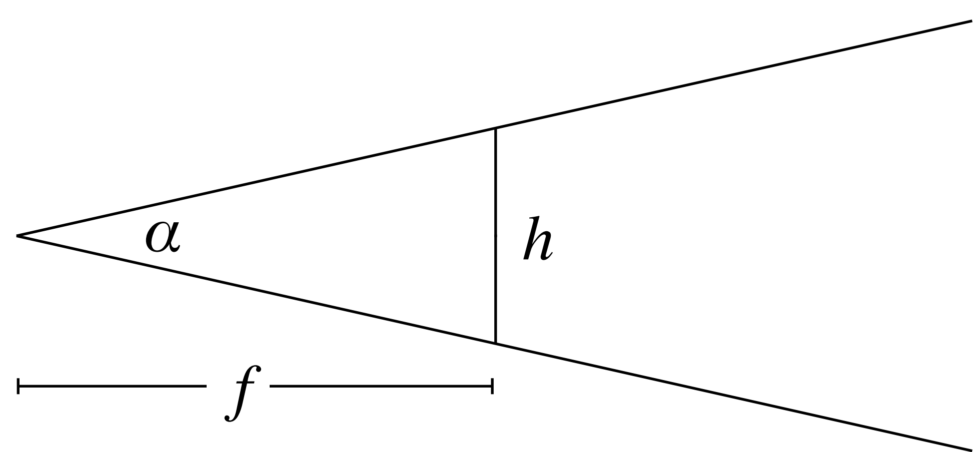
$$x' = \left(\frac{\text{pixels}}{\text{meter}}\right) fx/z$$

  - but this just amounts to converting *f* to units of pixels (pixels · meters / meters = pixels)

Focal length and field of view

- **Focal length gets measured in pixels**
  - roughly, the number written on the lens dividid by the size of a pixel (typical pixel sizes are single-digit microns)
  - e.g. many popular smartphones have ~4000x3000 sensors with 1.4μm pixels (therefore about 5.6 x 4.2 mm in size)
  - mine has a focal length of 4.55mm*, so measured in pixels this is (4.55mm / 1.4μm) = 3250 pixels
- **Focal length (in pixels) plus image size determines the field of view**
  - formula for vertical f.o.v. with image *h* pixels high:

$$\alpha = 2\tan^{-1}\frac{h}{2f}$$



* Spec sheet for my phone says "26mm equivalent," which means the same f.o.v. as a 36x24mm film frame with a 26mm focal length. Assume matching vertical f.o.v., then (4.2 / 24) 26mm = 4.55mm is the focal length for this camera

Field of view (or f.o.v.)

- **The angle between the rays corresponding to opposite edges of a perspective image**
  – have to decide to measure vert., horiz., or diag.
- **In cameras, determined by focal length**
  – confusing because of many image sizes
  – for 35mm format (36mm by 24mm image)
    – 18mm = 67° v.f.o.v. — super-wide angle
    – 28mm = 46° v.f.o.v. — wide angle
    – 50mm = 27° v.f.o.v. — "normal"
    – 100mm = 14° v.f.o.v. — narrow angle ("telephoto")
  – cameras with other size sensors often still use these same numbers, because they are familiar/traditional.

Field of view

- **Determines "strength" of perspective effects**

close viewpoint
wide angle
large scale differences

far viewpoint
narrow angle
small scale differences

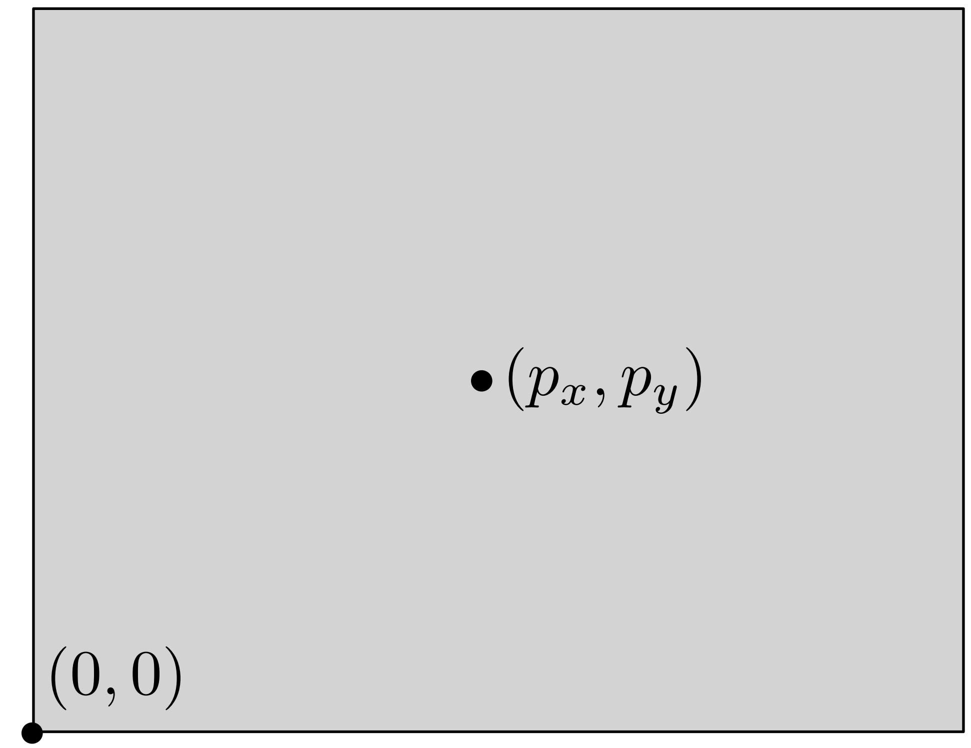
[Ansel Adams]

Principal point

- **Second fix to img. coords: measure from image corner**
  - with our simple projection formulas, image coordinates are measured from wherever the *z* axis hits the image plane
  - this is usually near the center of the image, but we usually measure pixels with the origin at a corner of the image
  - this mismatch is fixed by adding an offset

$$x' = fx/z + p_x$$
$$y' = fy/z + p_y$$

  - the point (p_x, p_y) is called the *principal point*
  - the exact center of the image is a good first guess for this



Projection redux

focal length
(pixels)

$$x' = fx/z + p_x$$
$$y' = fy/z + p_y$$

2D image
coordinates
(pixels)

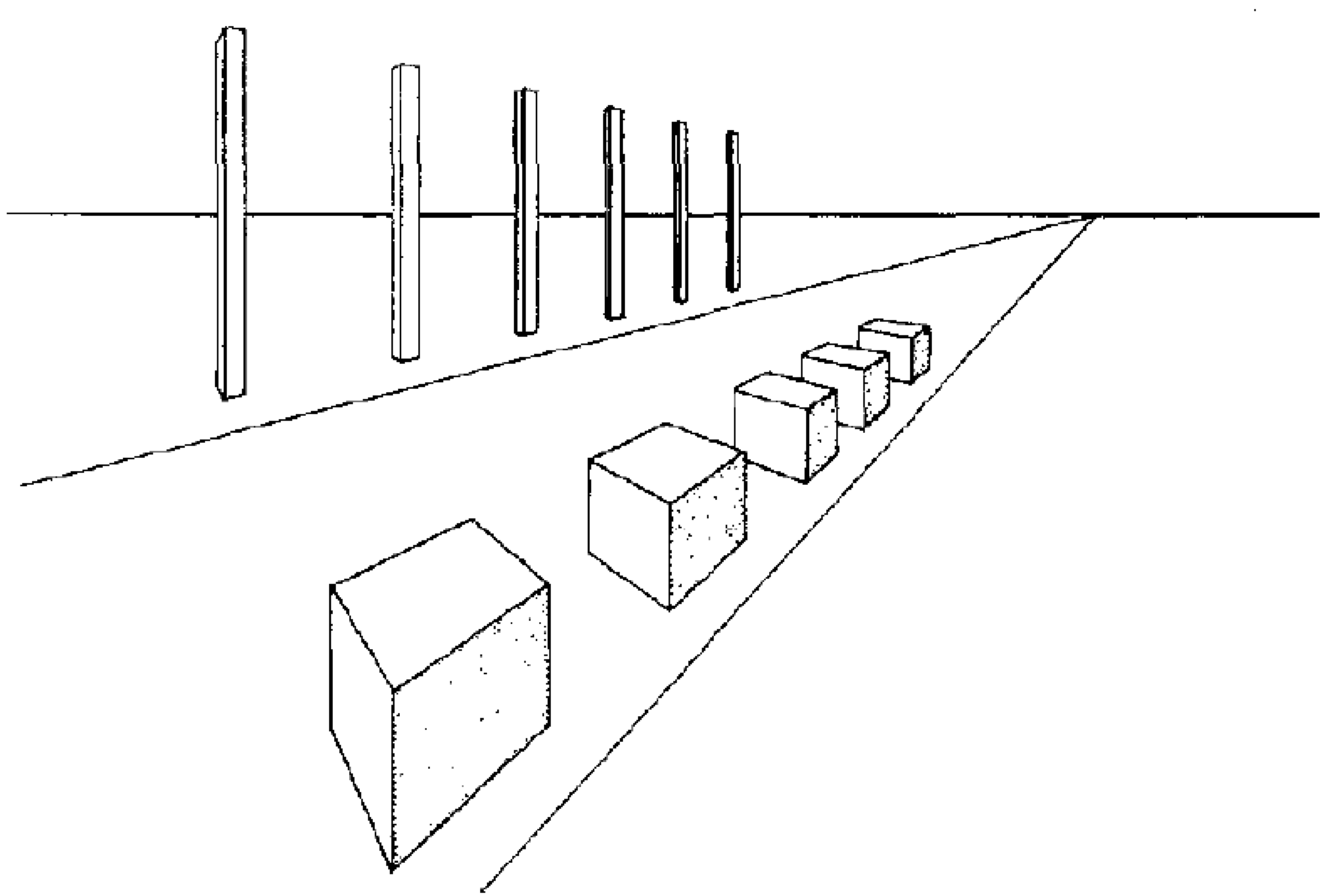
3D space
coordinates
(meters)

principal
point
(pixels)

Assumptions:
- pixels are square
- camera center is at (0, 0, 0)
- image sensor is aligned with (x, y) axes in 3D space

Perspective "distortions"

- **Lengths, length ratios**



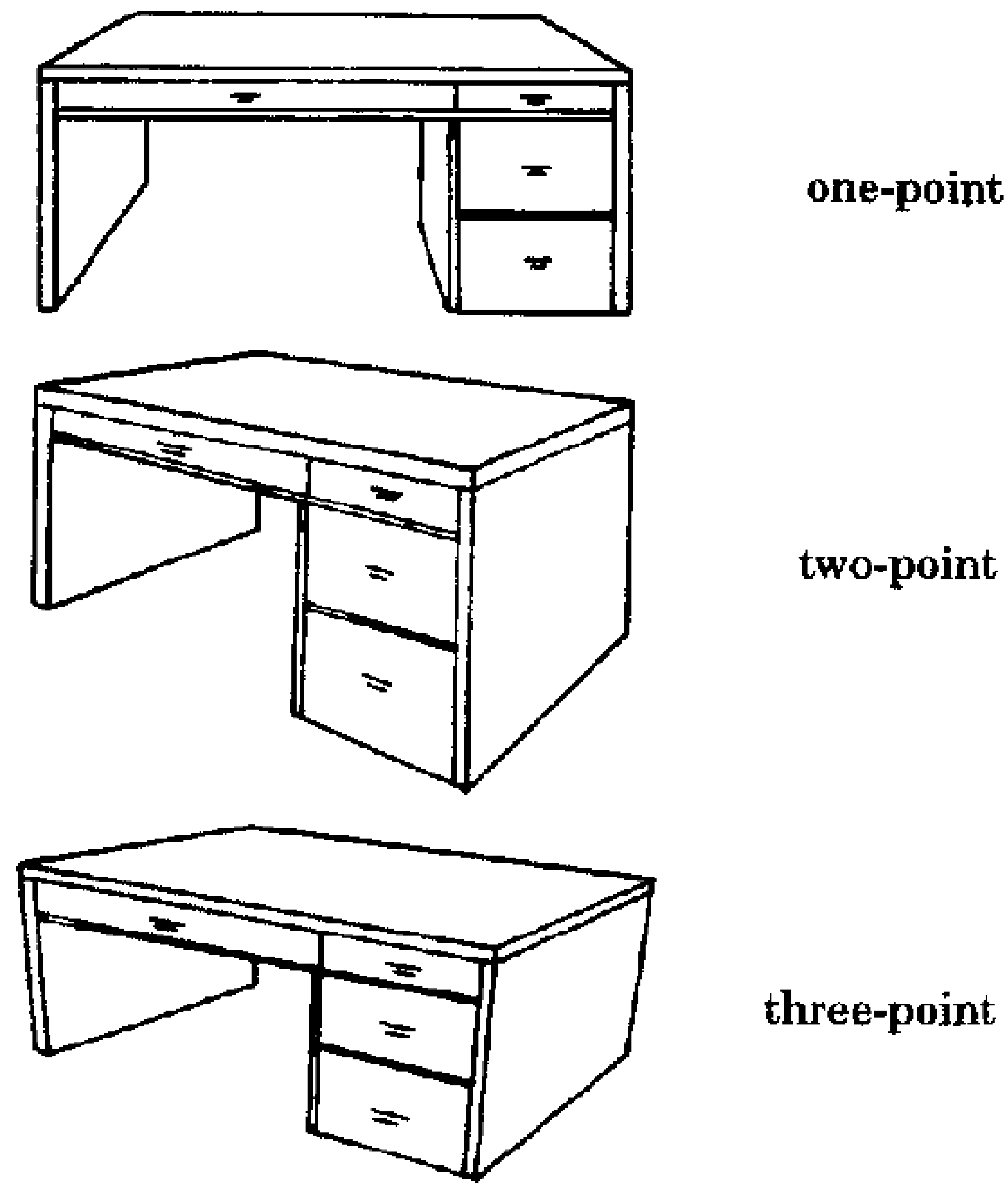
[Carlbom & Paciorek 78]

Perspective

one-point: projection plane parallel to a coordinate plane (to two coordinate axes)

two-point: projection plane parallel to one coordinate axis

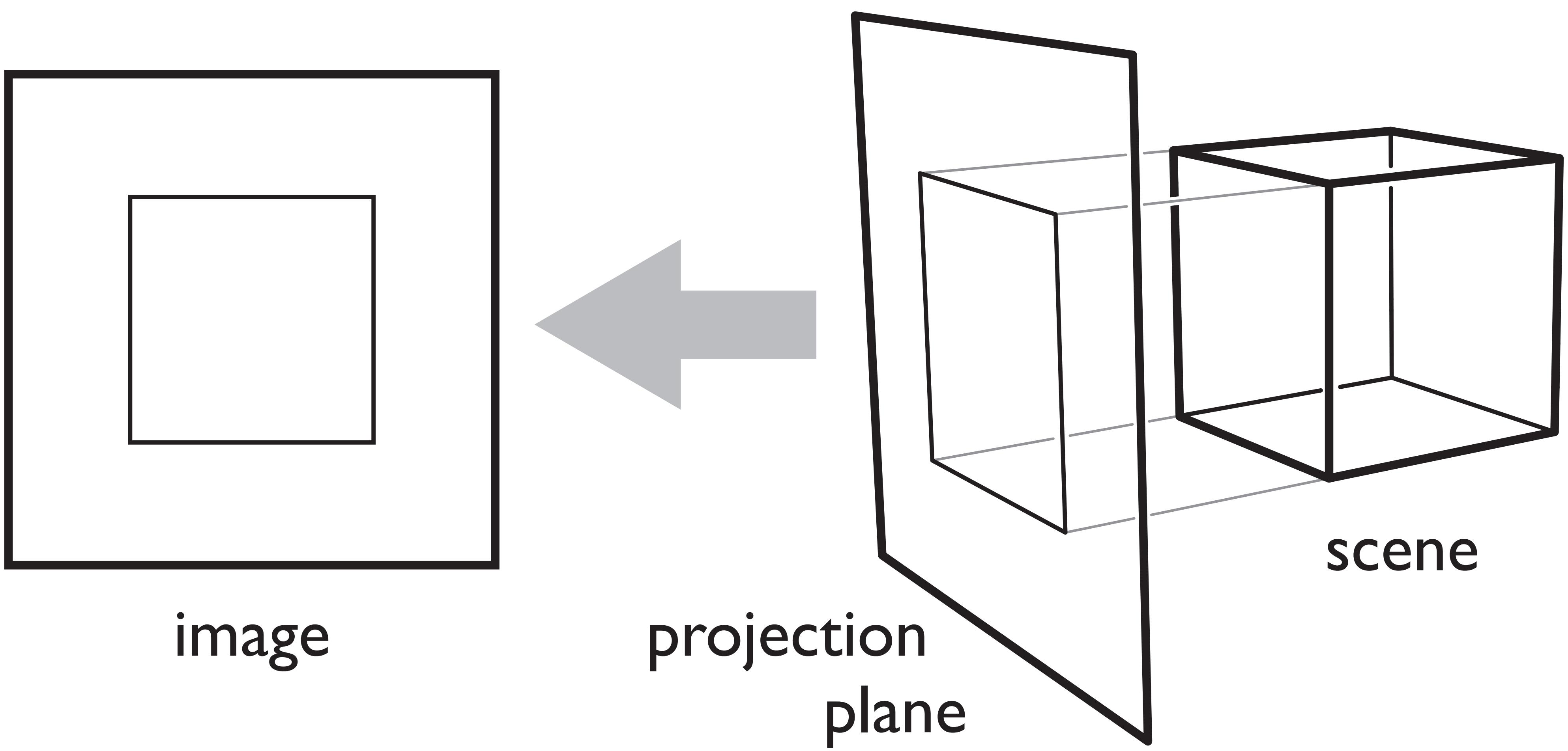
three-point: projection plane not parallel to a coordinate axis



[Carlbom & Paciorek 78]

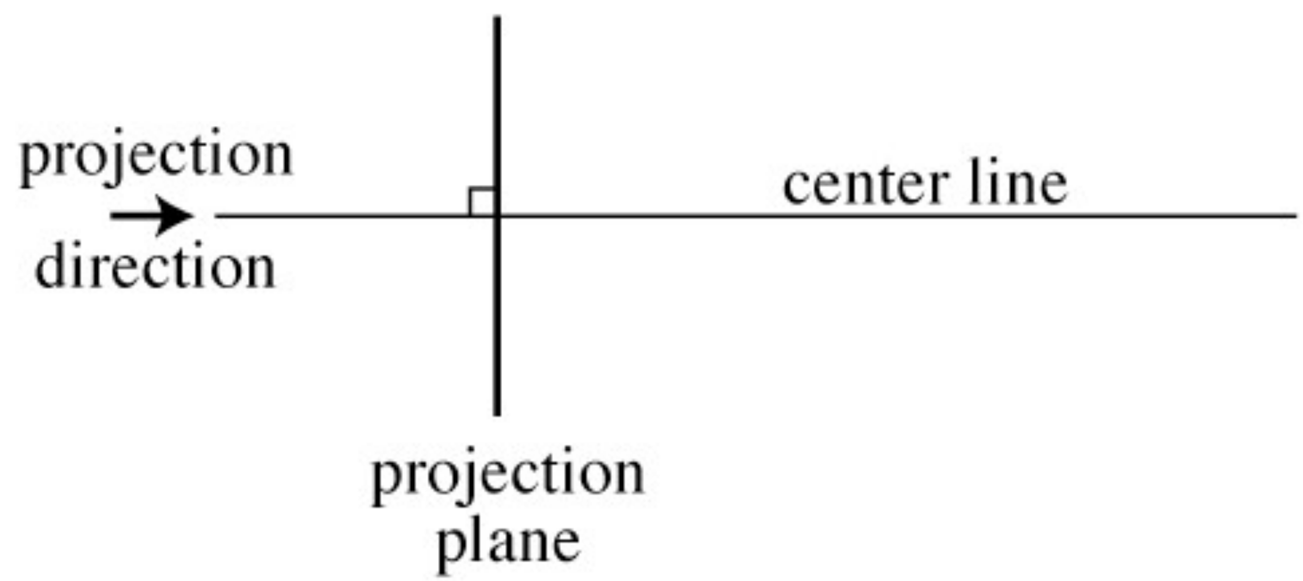
Parallel projection

- **A simpler kind of projection than perspective is *parallel projection***



Orthographic projection

- **In vision and graphics, usually we call all parallel projections orthographic**
  - projection plane perpendicular to projection direction
  - image height determines size of objects in image independent of depth!



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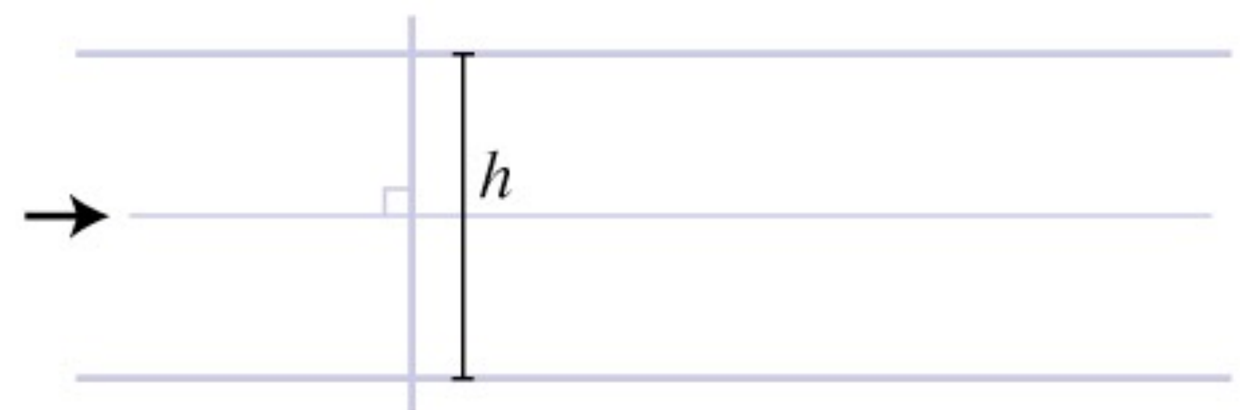

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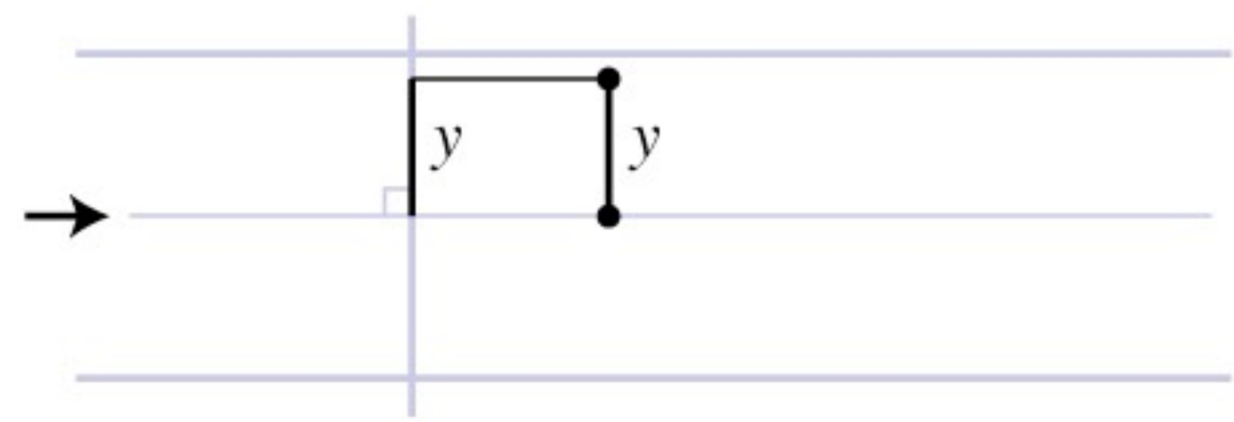

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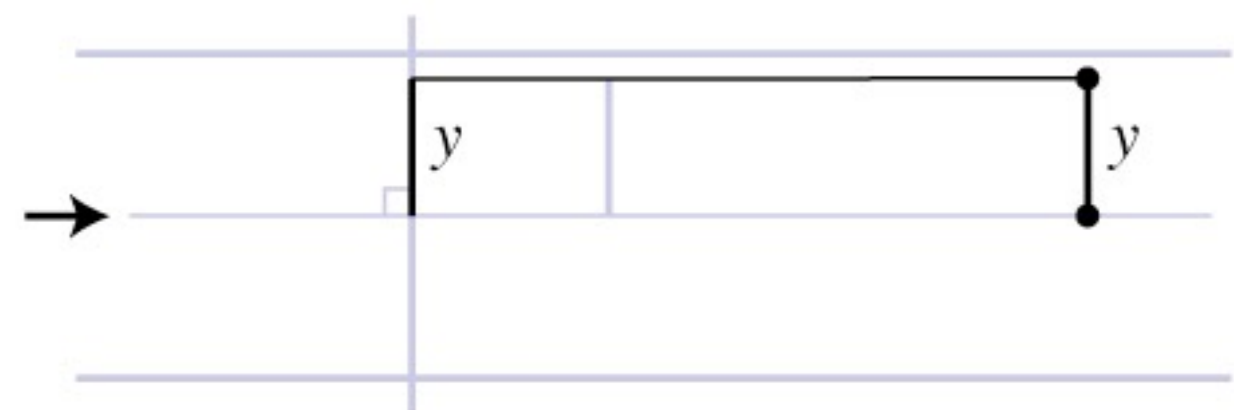
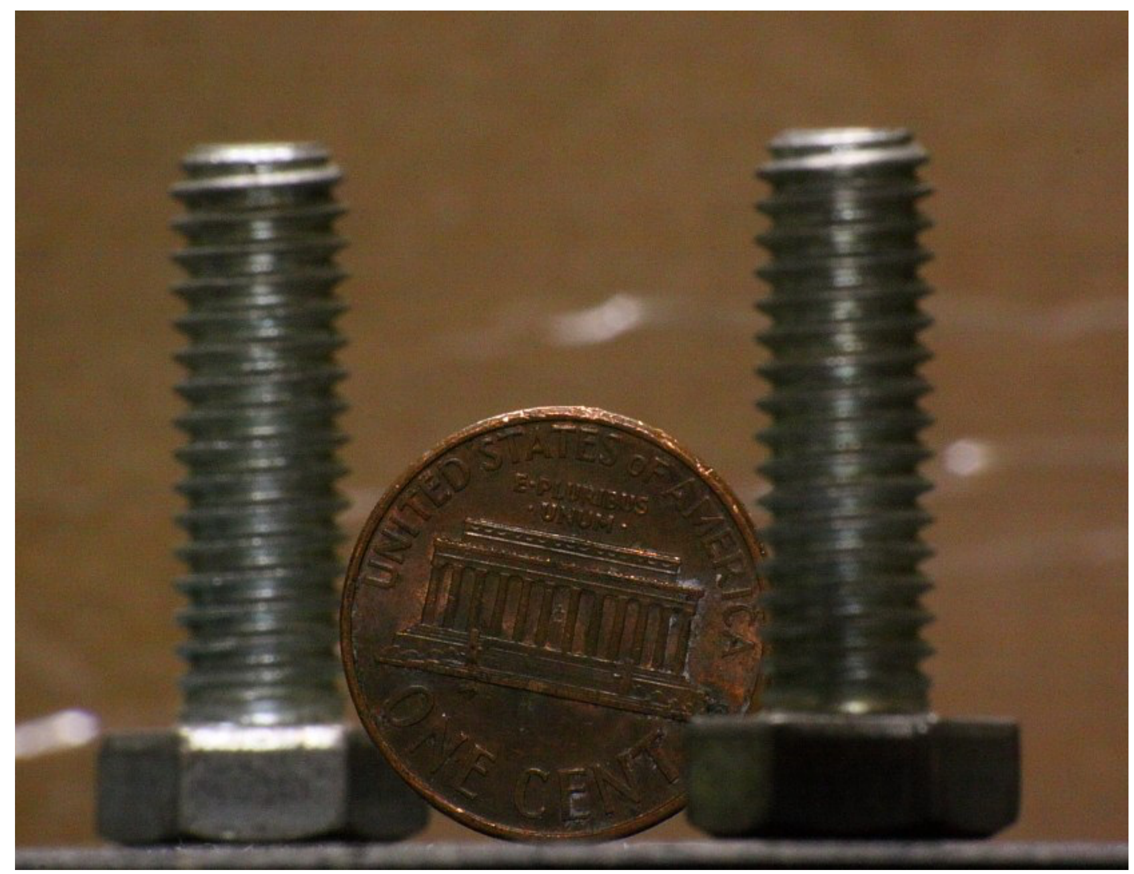


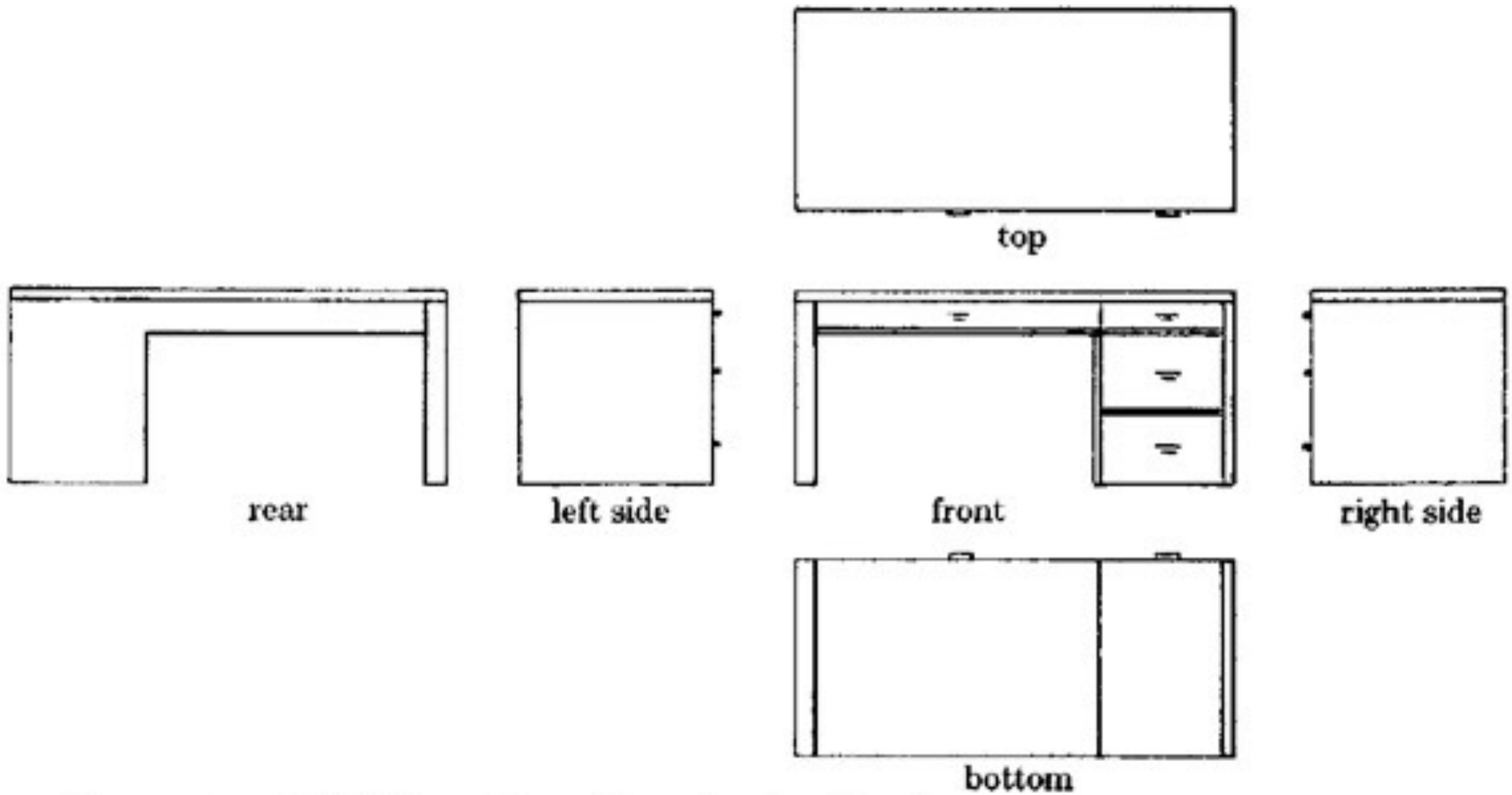
Orthographic cameras?

- **Orthographic projection is a handy approximation**
  - when objects are far from the camera compared to their size
  - provides simpler math when approximation is OK
- **Special *telecentric* lenses are exactly orthographic**
  - often used in inspection/ measurement applications
  - can measure object dimensions in image directly



photomacrography.net | user dbur

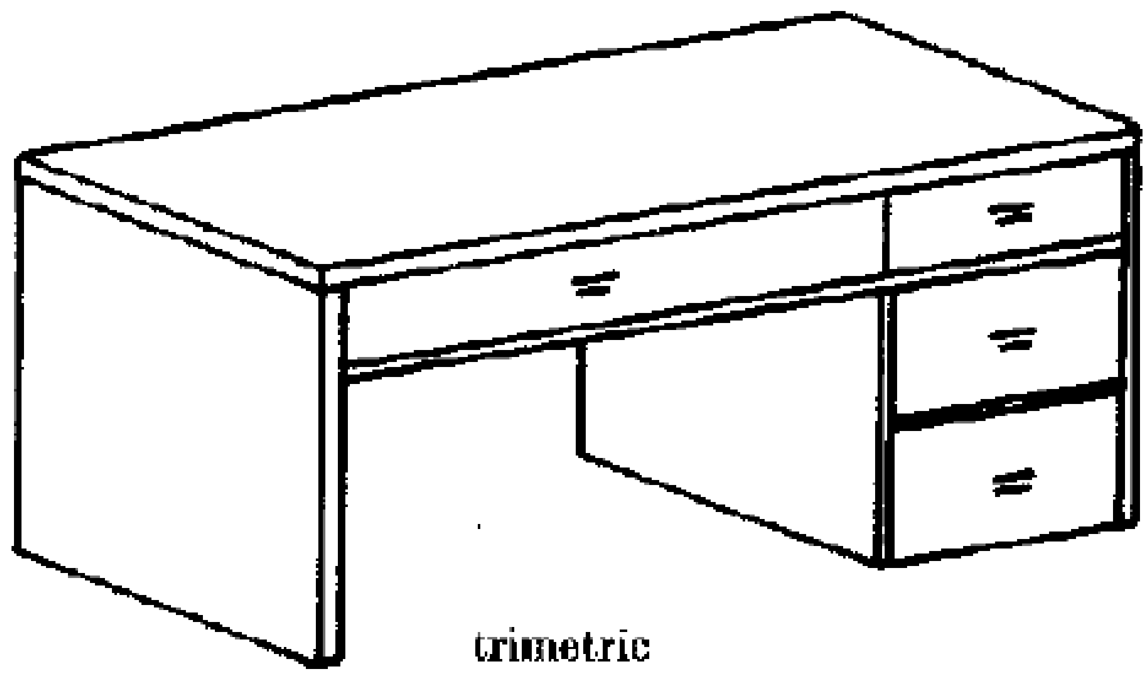
Orthographic projection in drafting



[Carlbom & Paciorek 78]

– projection plane parallel to a coordinate plane
– projection direction perpendicular to projection plane

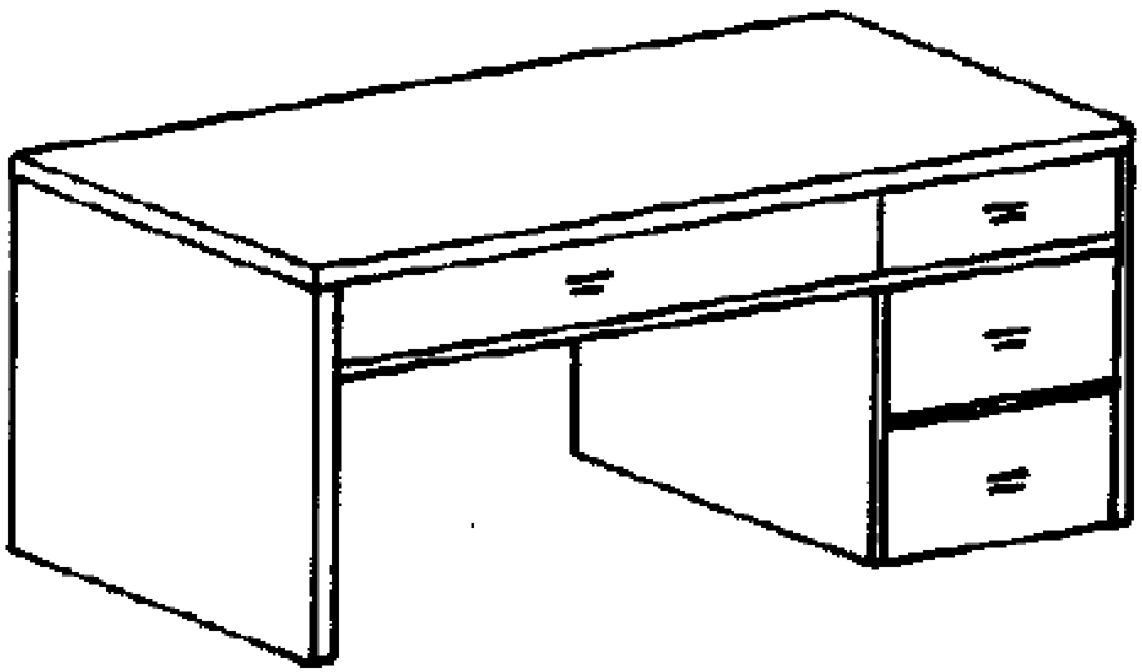
Orthographic projection in drafting

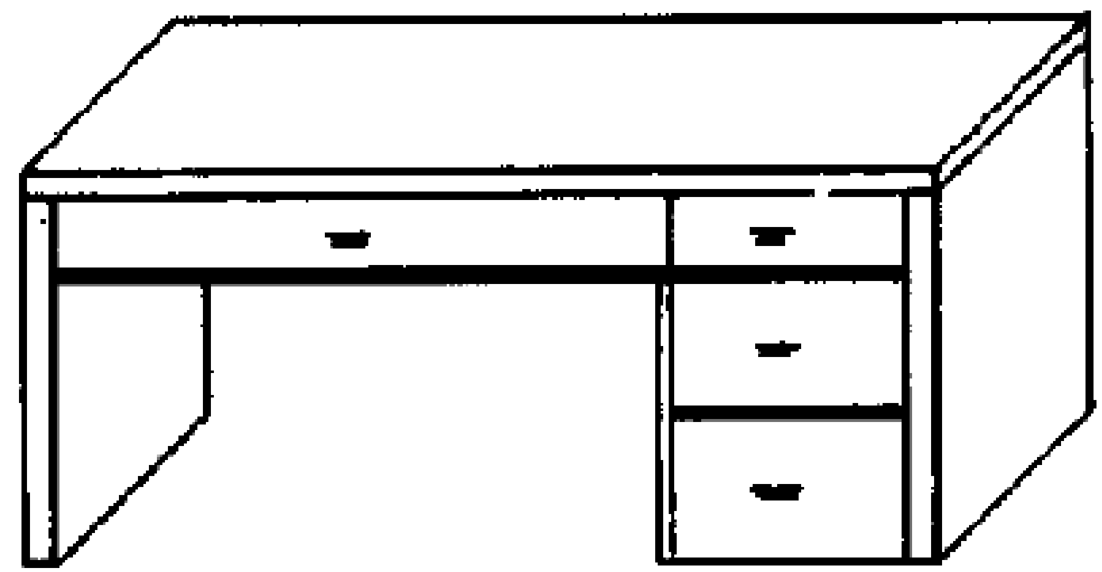


[Carlbom & Paciorek 78]

axonometric: projection plane perpendicular to projection direction but not parallel to coordinate planes

Off-axis parallel projections





[Carlbom & Paciorek 78]

axonometric: projection plane perpendicular to projection direction but not parallel to coordinate planes

oblique: projection plane parallel to a coordinate plane but not perpendicular to projection direction.

