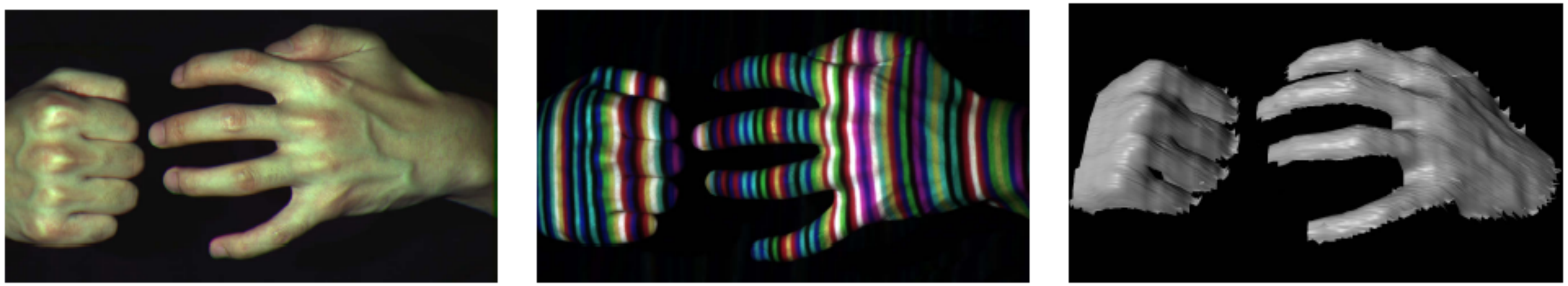
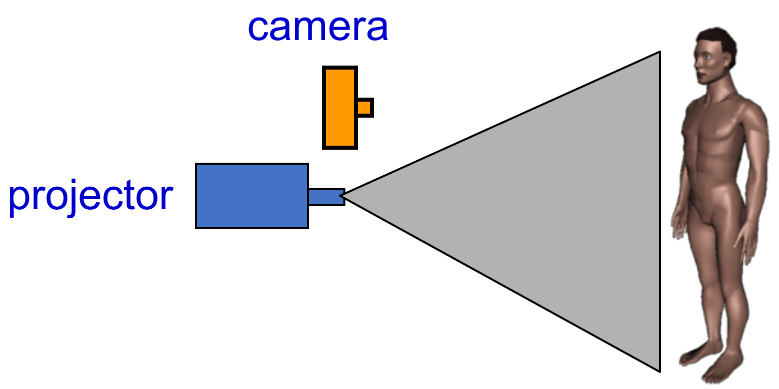
Other approaches to obtaining 3D structure

Active stereo with structured light

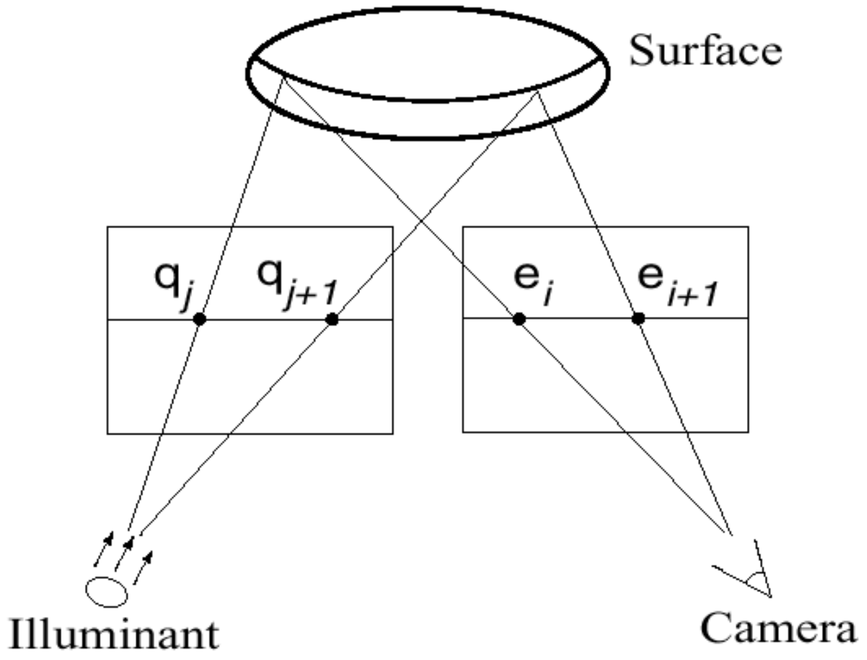


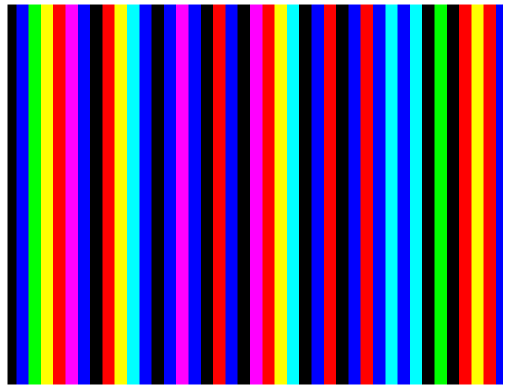
- Project “structured” light patterns onto the object
  - simplifies the correspondence problem
  - Allows us to use only one camera

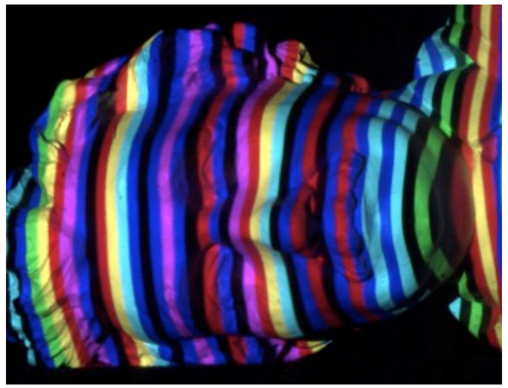


L. Zhang, B. Curless, and S. M. Seitz. Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming. *3DPVT* 2002

Active stereo with structured light

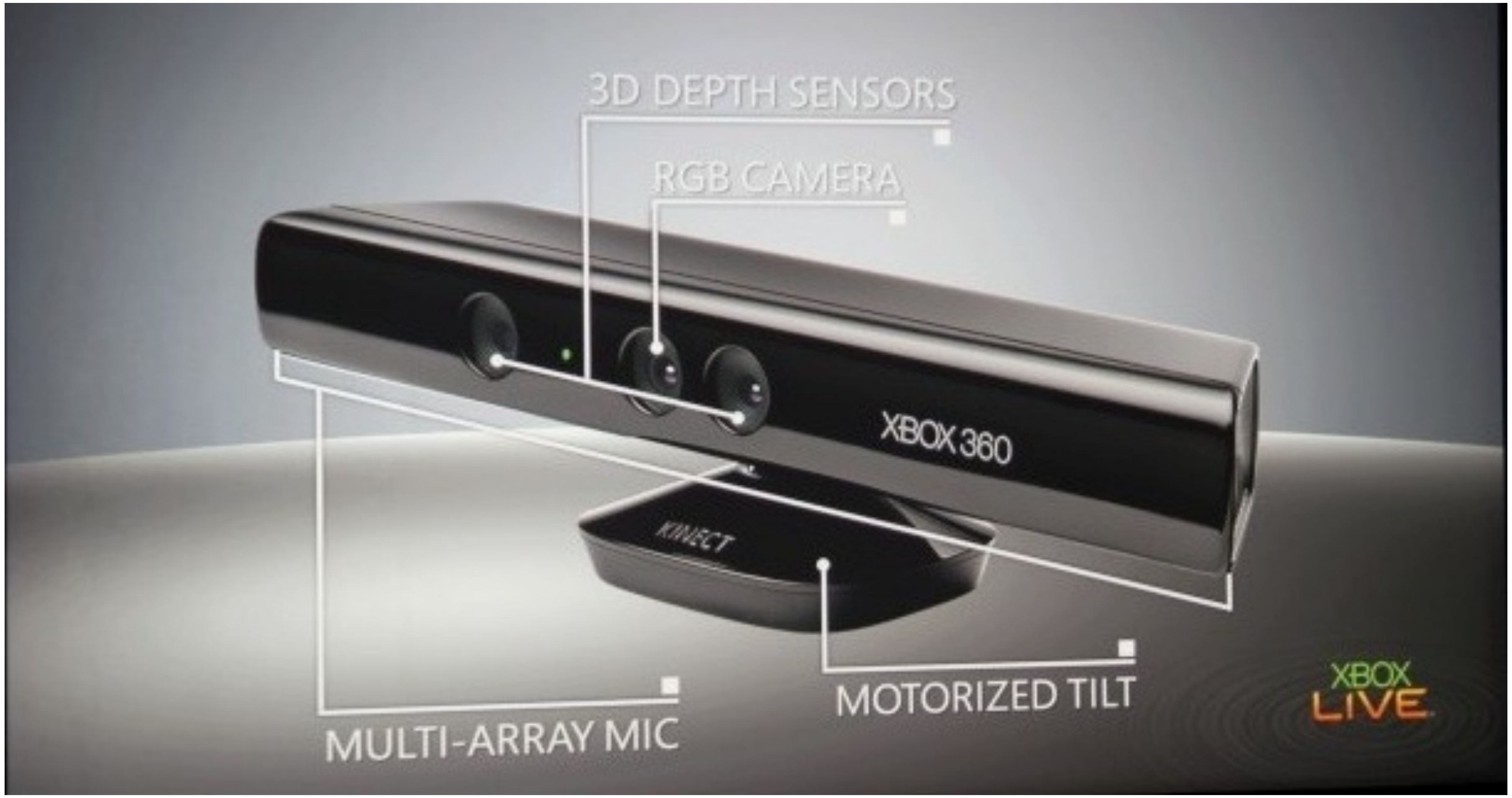






L. Zhang, B. Curless, and S. M. Seitz. Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming. *3DPVT* 2002

Microsoft Kinect



Light and geometry

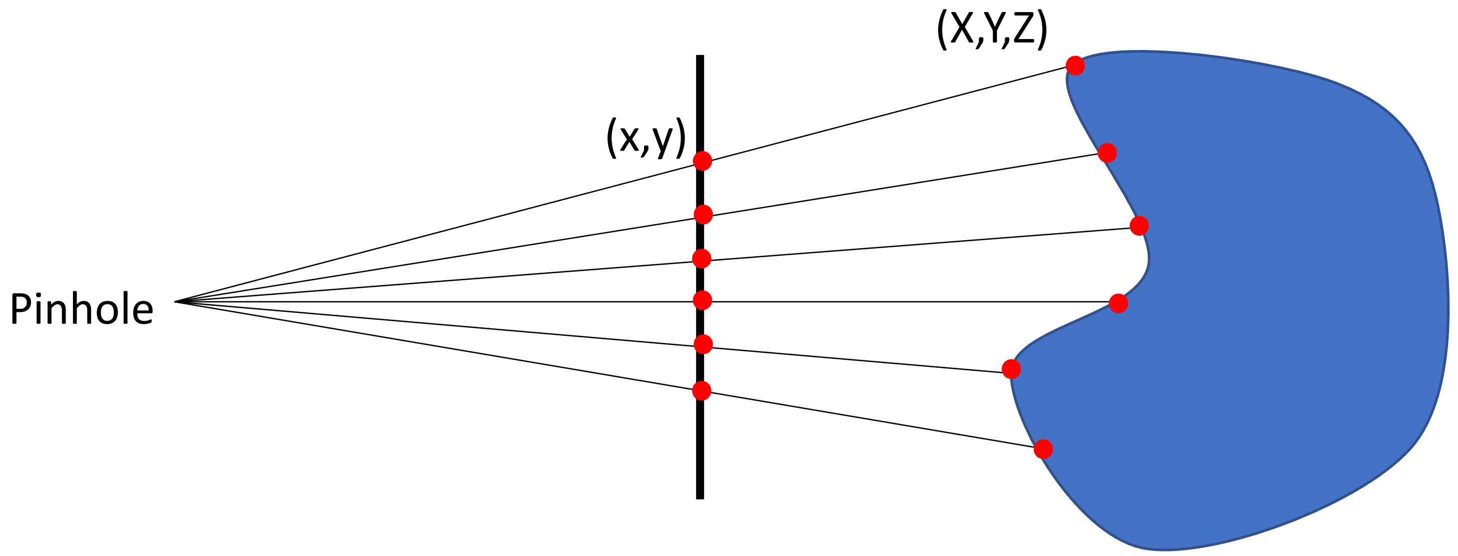
Till now: 3D structure from multiple cameras

- Problems:
  - requires calibrated cameras
  - requires correspondence
- Other cues to 3D structure?

Key Idea: use feature motion to understand shape

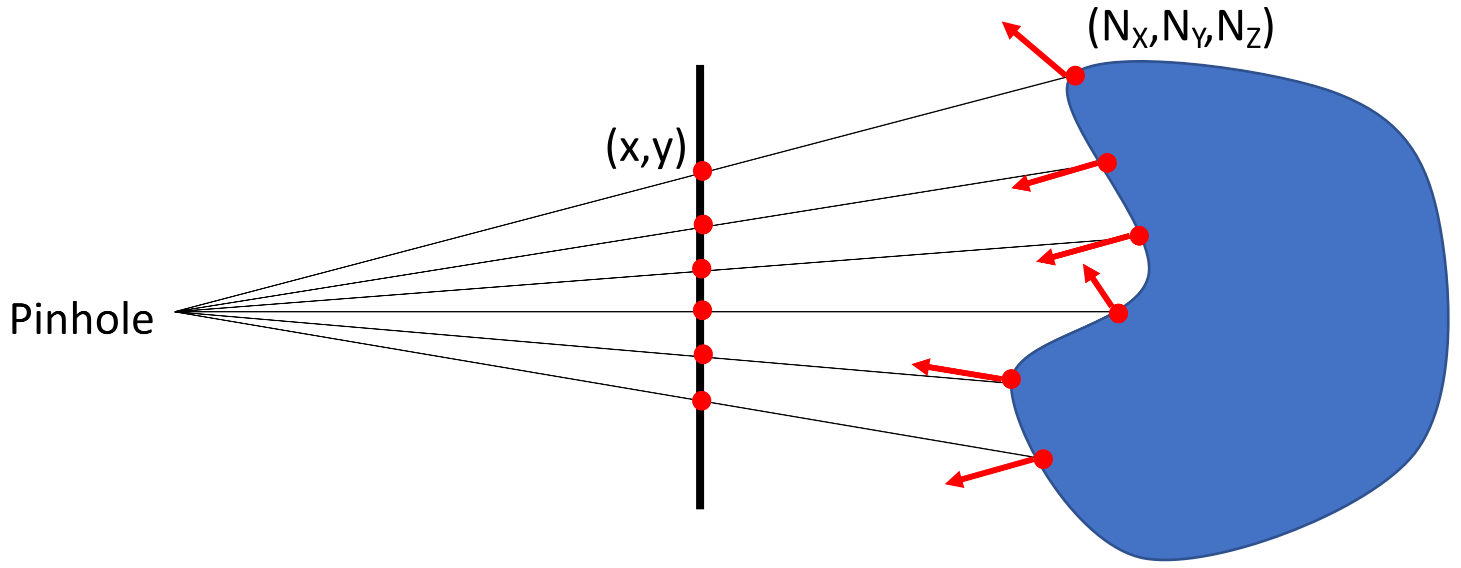
What does 3D structure mean?

- We have been talking about the depth of a pixel



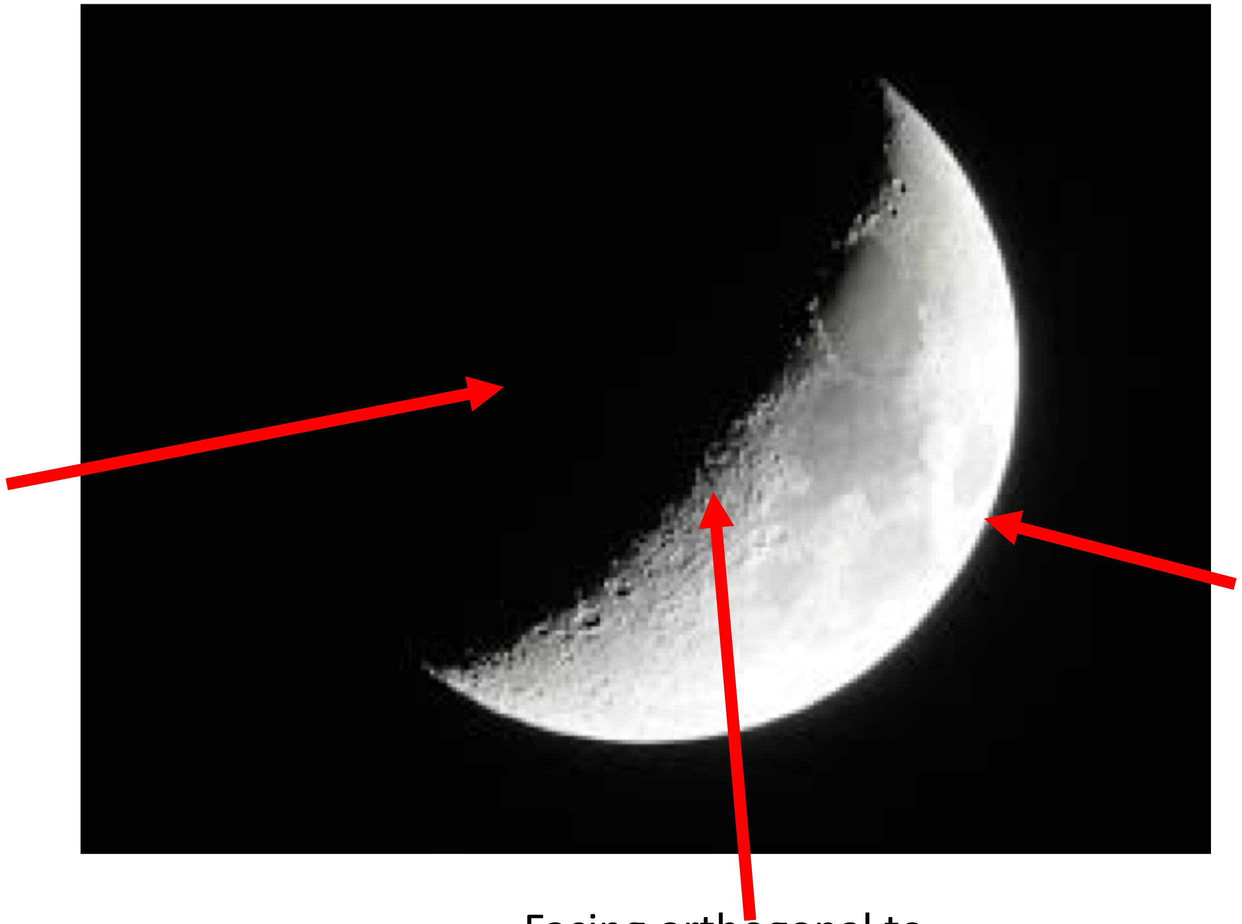
What does 3D structure mean?

- But we can also look at the orientation of the surface at each pixel: *surface normal*



Not enough by itself to reveal absolute locations, but gives enough of a clue to object shape

Shading is a cue to surface orientation


Facing away from the sun, hence dark – "shadow"
Facing the sun, hence bright
Facing orthogonal to the sun, hence dark

Shading is a cue to surface orientation

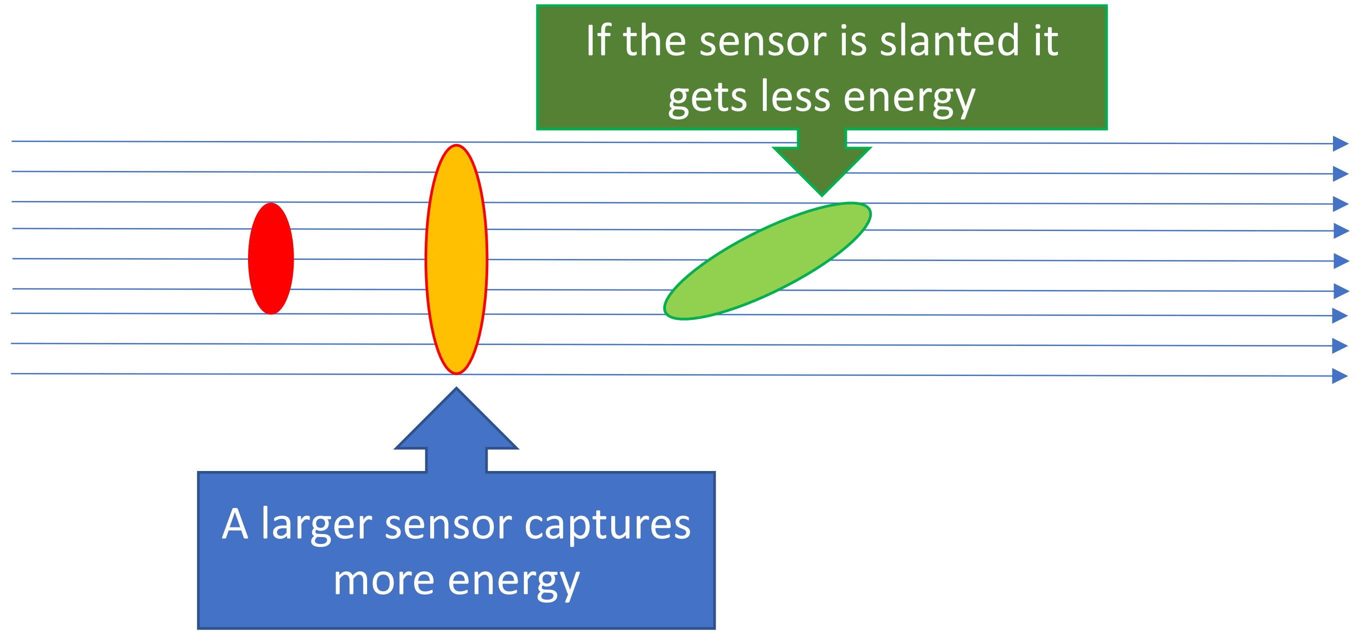
- Till now we have looked at *where a pixel comes from*
- Now: *what is its color?*
- Depends on:
  - Color and amount of lighting
  - Orientation of surface relative to lighting
  - Paint on the surface

How does light interact with the scene?

- Light is a bunch of photons
- Photons are energy packets
- Light starts from the light source, is reflected / absorbed by surfaces and lands on the camera
- Two key questions:
  - What property of light does a camera pixel record? *Radiance*
  - How does the radiance of a pixel depend on lighting, shape and paint?

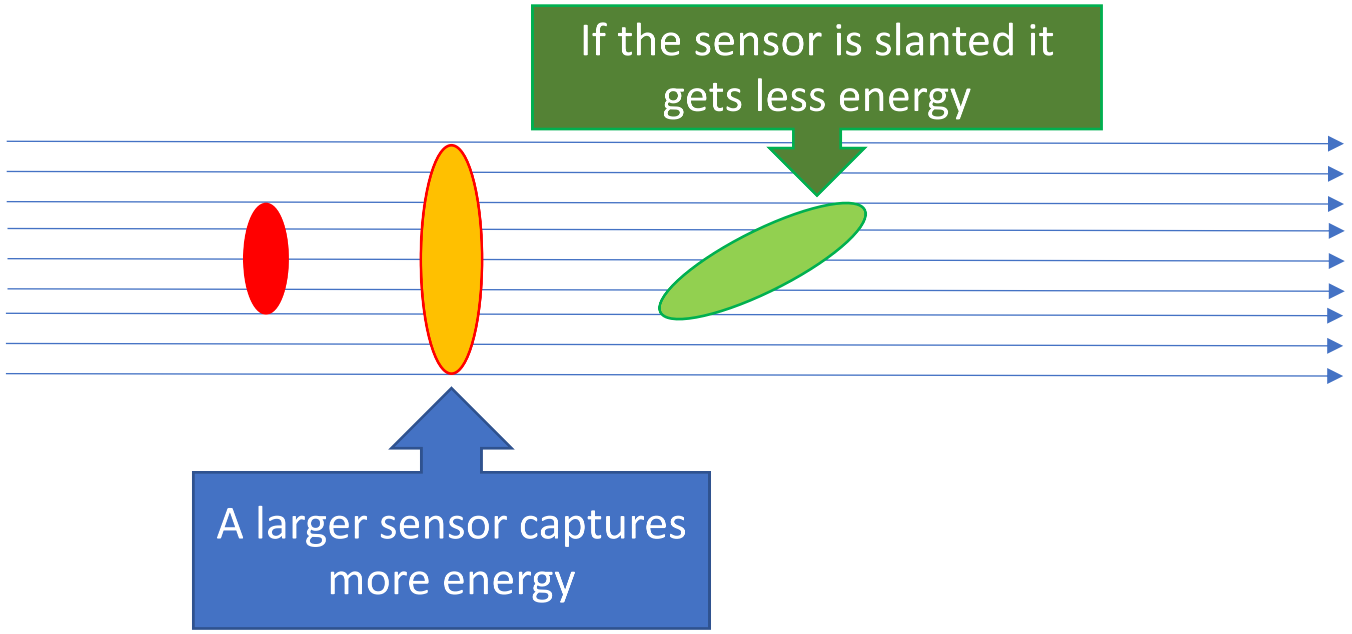
Radiance

- How do we measure the “strength” of a beam of light?
- Idea: put a sensor and see how much energy it gets


If the sensor is slanted it gets less energy
A larger sensor captures more energy

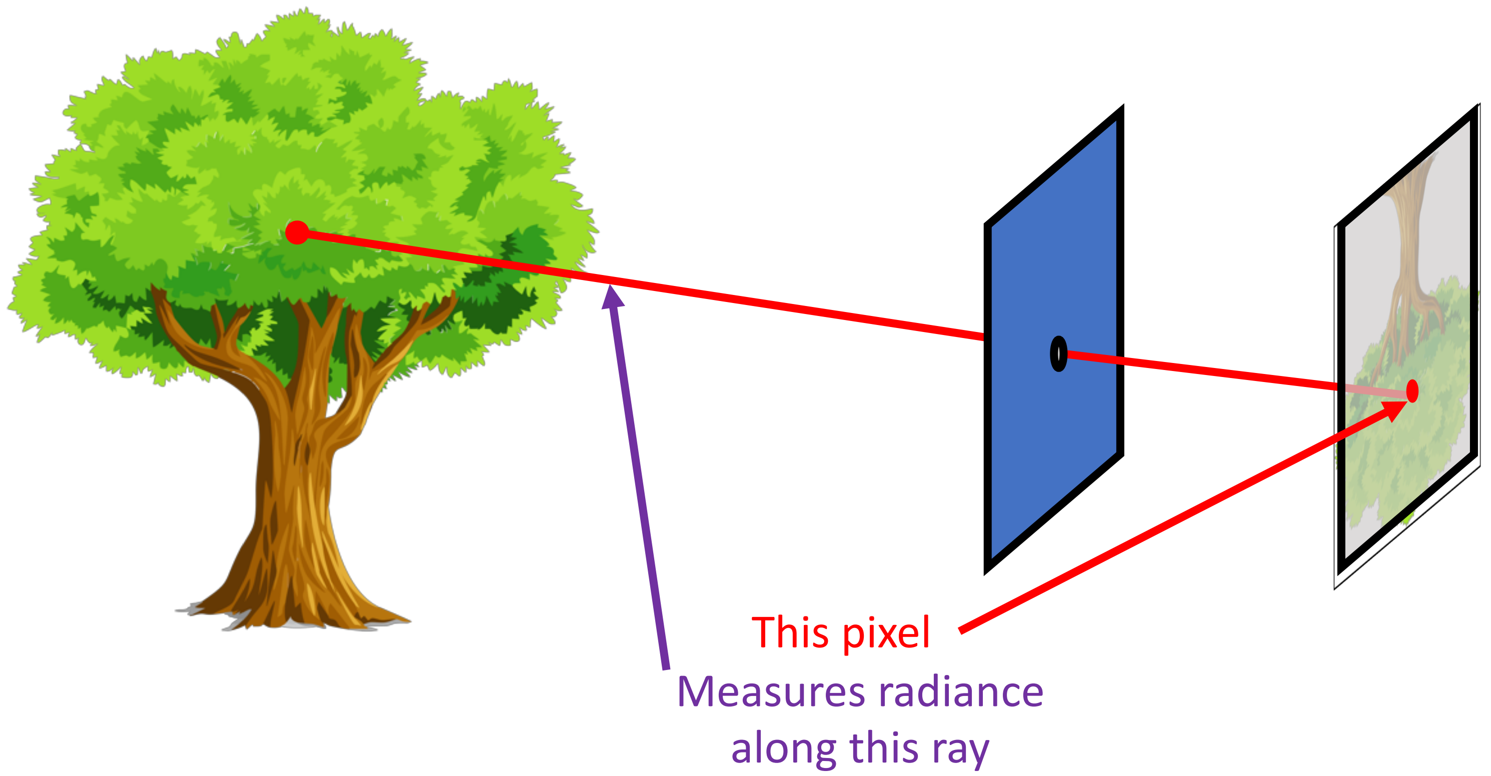
Radiance

- How do we measure the “strength” of a beam of light?
- Radiance: power *in a particular direction* per unit area when surface is orthogonal to direction


If the sensor is slanted it gets less energy
A larger sensor captures more energy

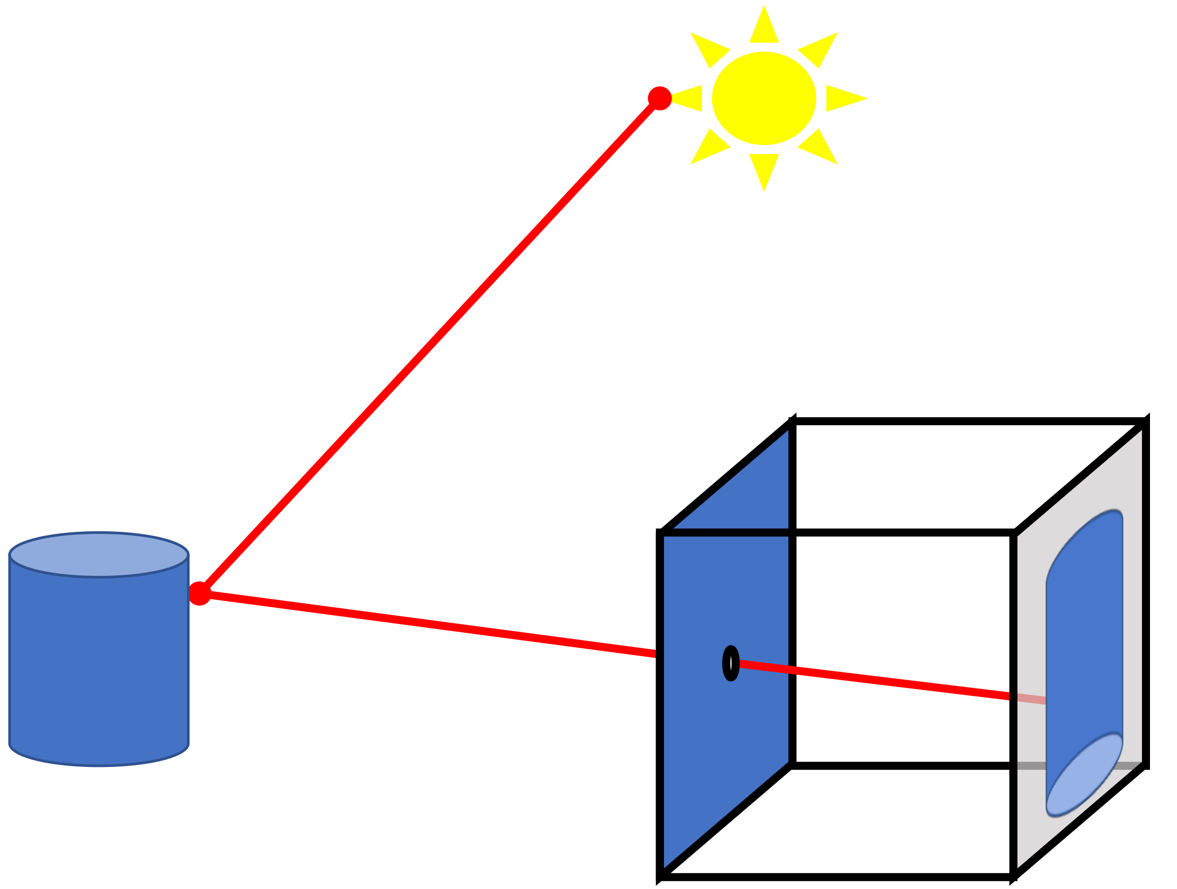
Radiance

- Pixels measure radiance



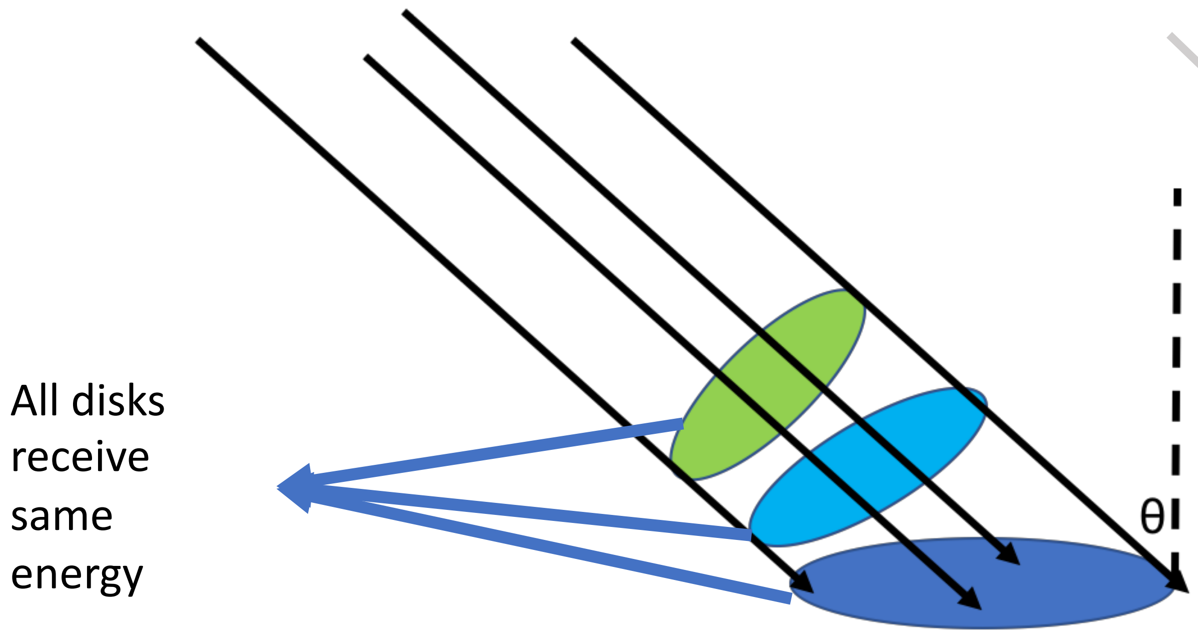
Where do the rays come from?

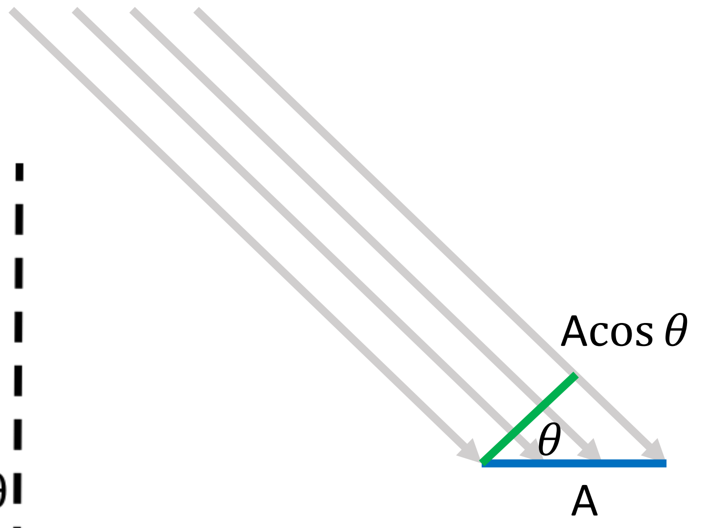
- Rays from the light source “reflect” off a surface and reach camera
- Surface gets some energy from the light source: *irradiance*
- Depending on paint, some of this energy is reflected back



Irradiance

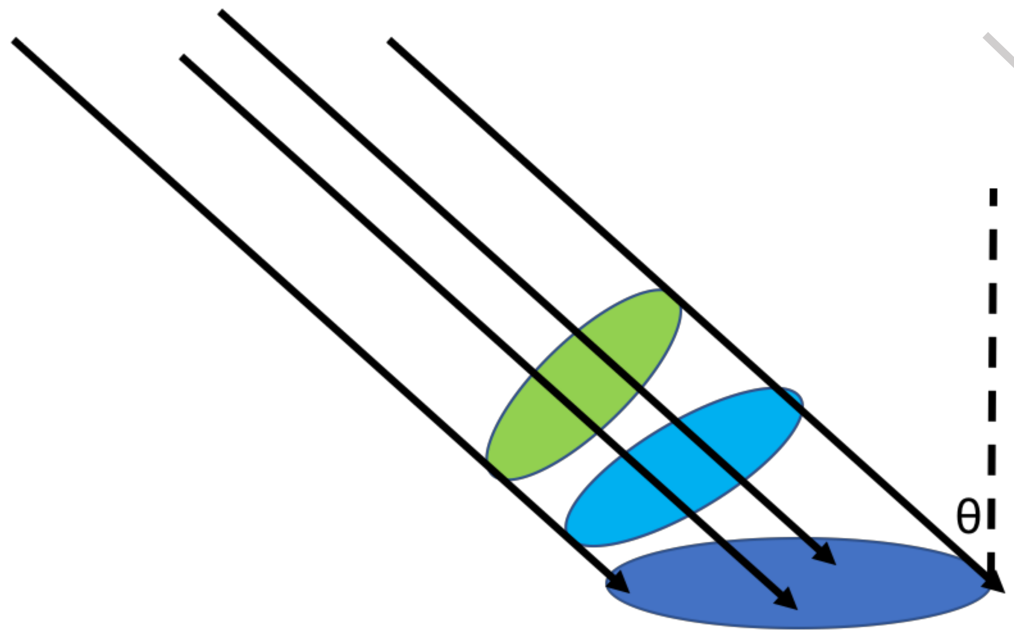
- What is the energy received by a surface from a light source?
- Depends on the area of the surface and its orientation relative to light

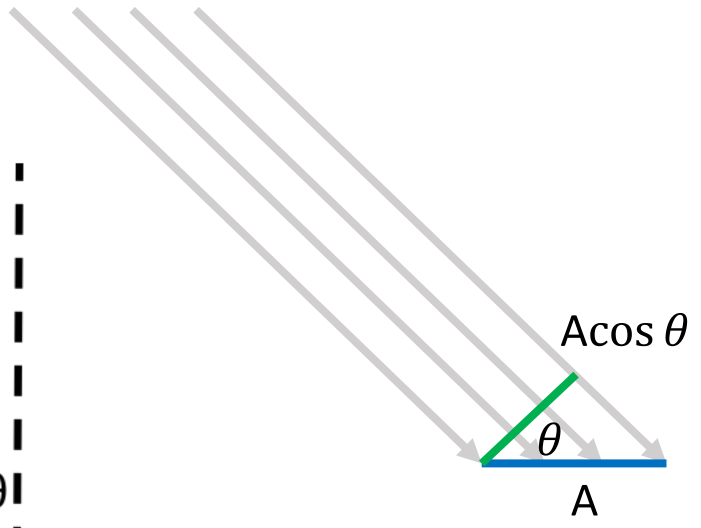




Irradiance

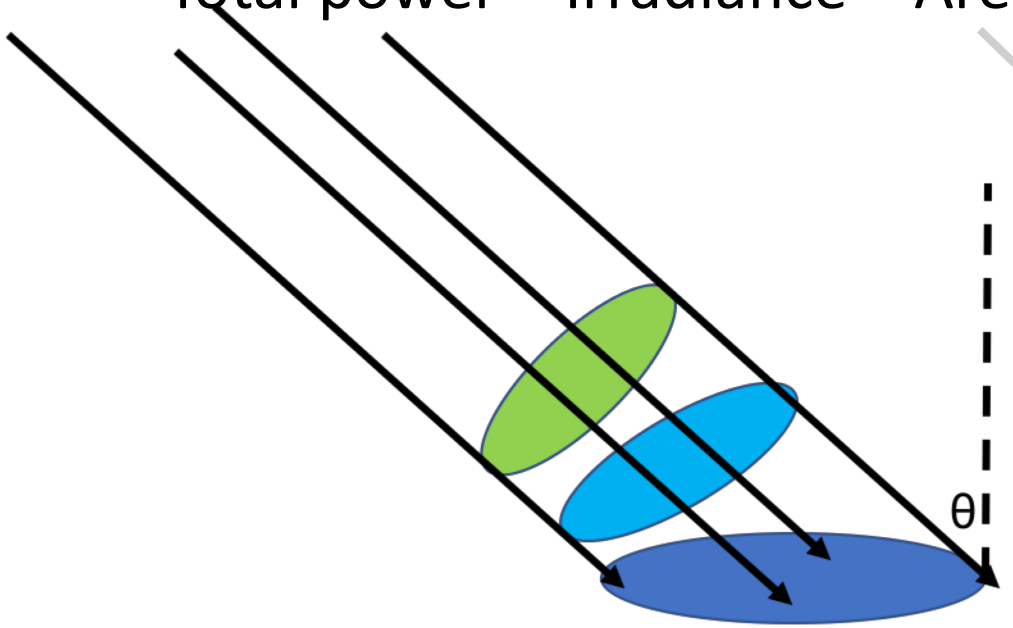
- Power received by a surface patch
    - of area A
    - from a beam of radiance L
    - coming at angle θ= LAcosθ

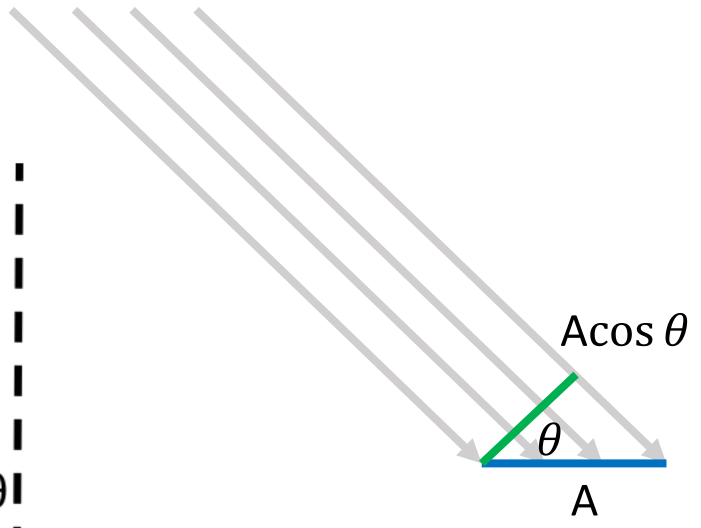




Irradiance

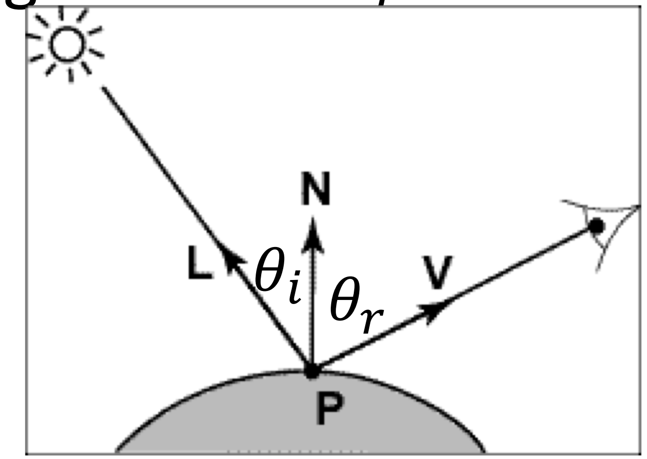
- Power received by a surface patch *of unit area*
  - from a beam of radiance L
  - coming at angle θ= Lcosθ
- Called Irradiance
- Irradiance = Radiance of ray* cosθ
- Total power = Irradiance * Area





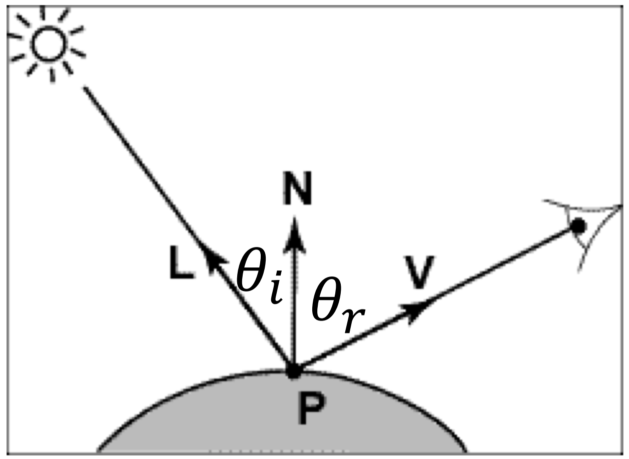
Light rays interacting with a surface

- Light of radiance L_i comes from light source at an incoming direction θ_i : incoming power = $L_i \cos \theta_i$
- Surface absorbs some of this energy and reflects a fraction in the outgoing direction θ_r
  - Fraction might depend on incoming light and outgoing light direction
  - Fraction = $\rho(\theta_i, \theta_r)$
- Outgoing radiance L_r = fraction * incoming power



- **N** is surface normal
- **L** is direction of light, making θ_i with normal
- **V** is viewing direction, making θ_r with normal

Light rays interacting with a surface



- **N** is surface normal
- **L** is direction of light, making θ_i with normal
- **V** is viewing direction, making θ_r with normal

$$L_r = \rho(\theta_i, \theta_r) L_i \cos\theta_i$$

Output radiance along **V**

Incoming irradiance along **L**

Bi-directional reflectance function (BRDF)

Light rays interacting with a surface

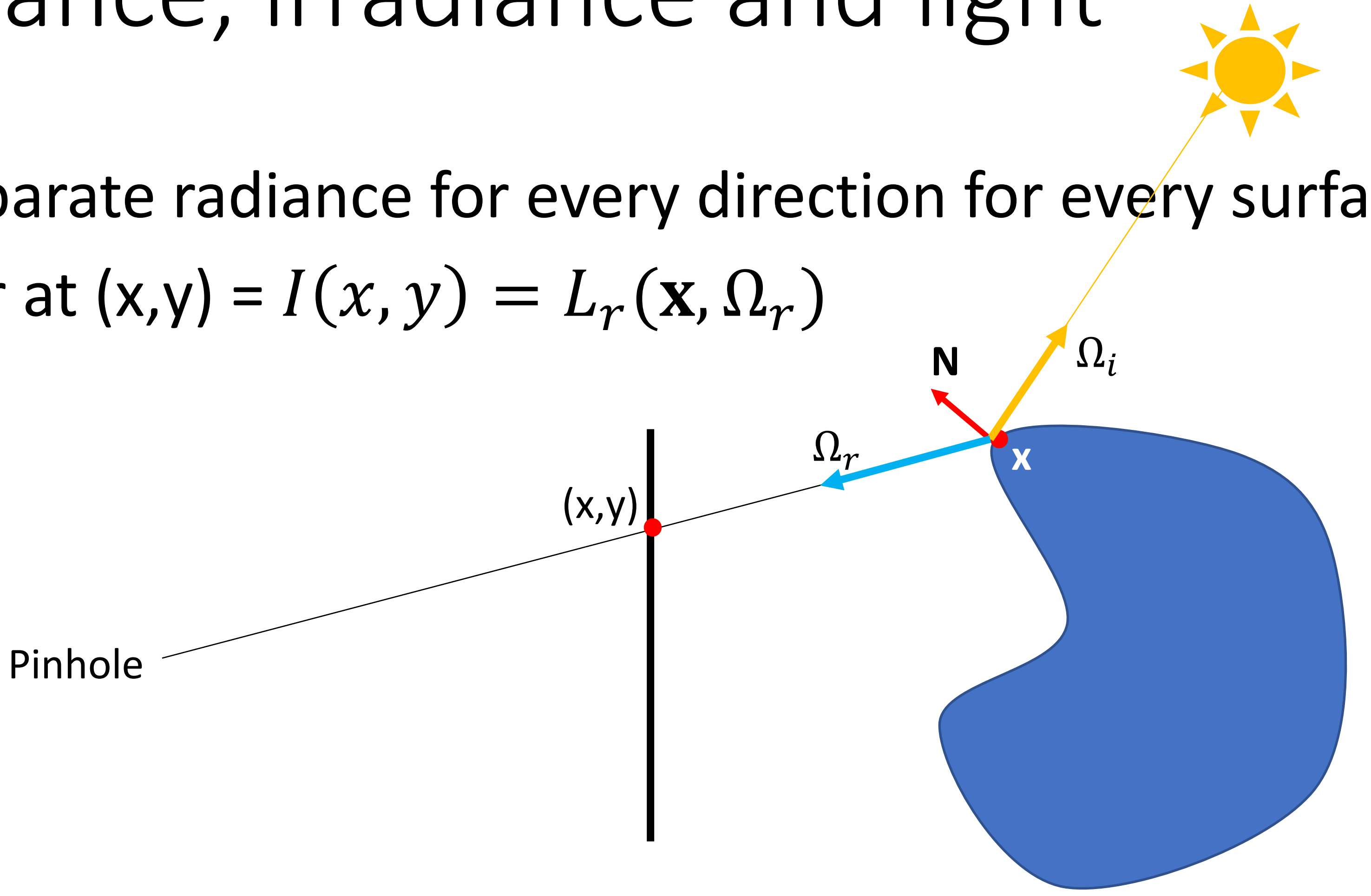
- In reality:
  - World is 3D, so incoming and outgoing directions are not angles θ_i, θ_r but general 3D directions Ω_i, Ω_r (represented by ”solid angles”)
  - Light might come from all directions with different radiance: need to integrate

- Final equation:

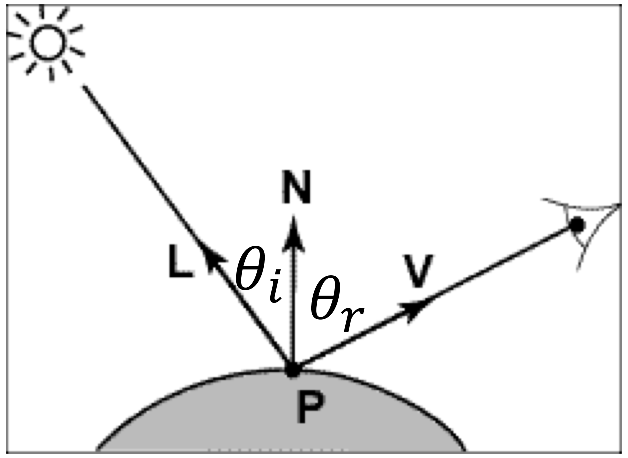
$$\underbrace{L_r(\Omega_r)}_{\text{Outgoing radiance at a point in direction } \Omega_r} = \underbrace{\int_{\Omega_i}}_{\text{Sum / Integral over incoming directions } \Omega_i} \overbrace{\rho(\Omega_i, \Omega_r)}^{\text{BRDF}} \underbrace{L_i \cos\theta_i}_{\text{Incoming irradiance in direction } \Omega_i} d\Omega_i$$

Radiance, irradiance and light

- A separate radiance for every direction for every surface point
- Color at (x,y) = $I(x,y) = L_r(\mathbf{x}, \Omega_r)$



What should BRDF be?

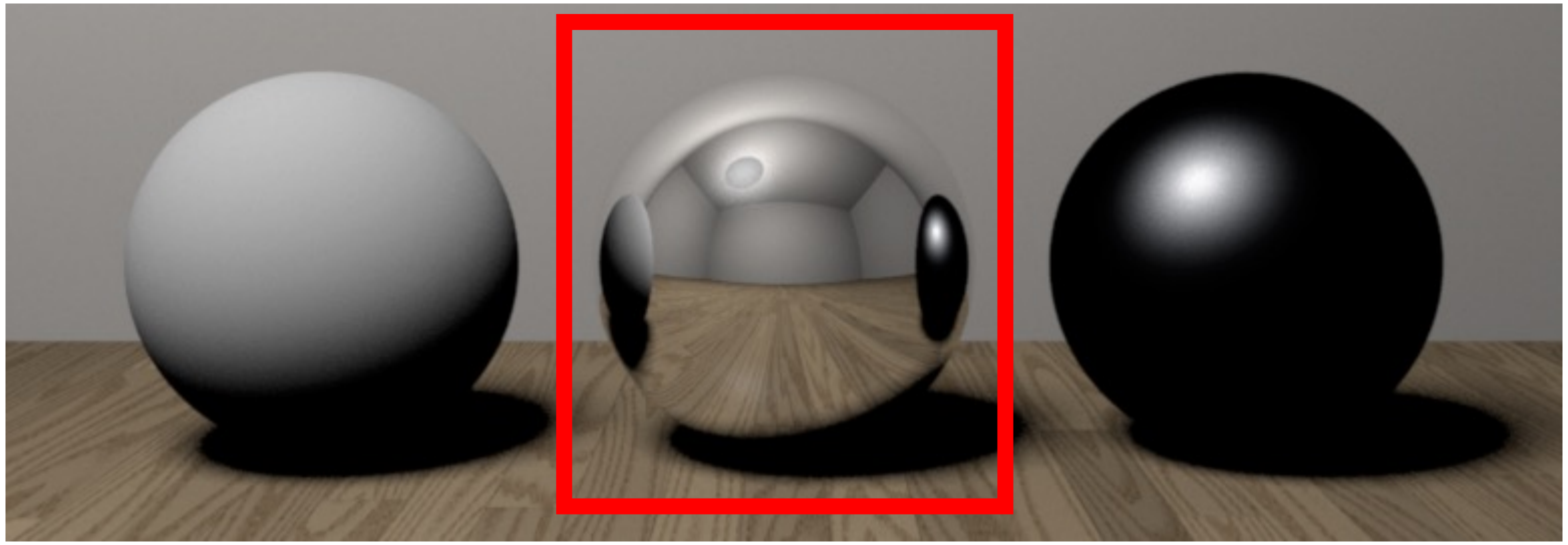


$$L_r = \rho(\theta_i, \theta_r) L_i \cos\theta_i$$

- Special case 1: Perfect mirror
  - $\rho(\theta_i, \theta_r)$ = 0 unless $\theta_i = \theta_r$
- Special case 2: Matte surface
  - $\rho(\theta_i, \theta_r)$ = ρ_0 (constant)

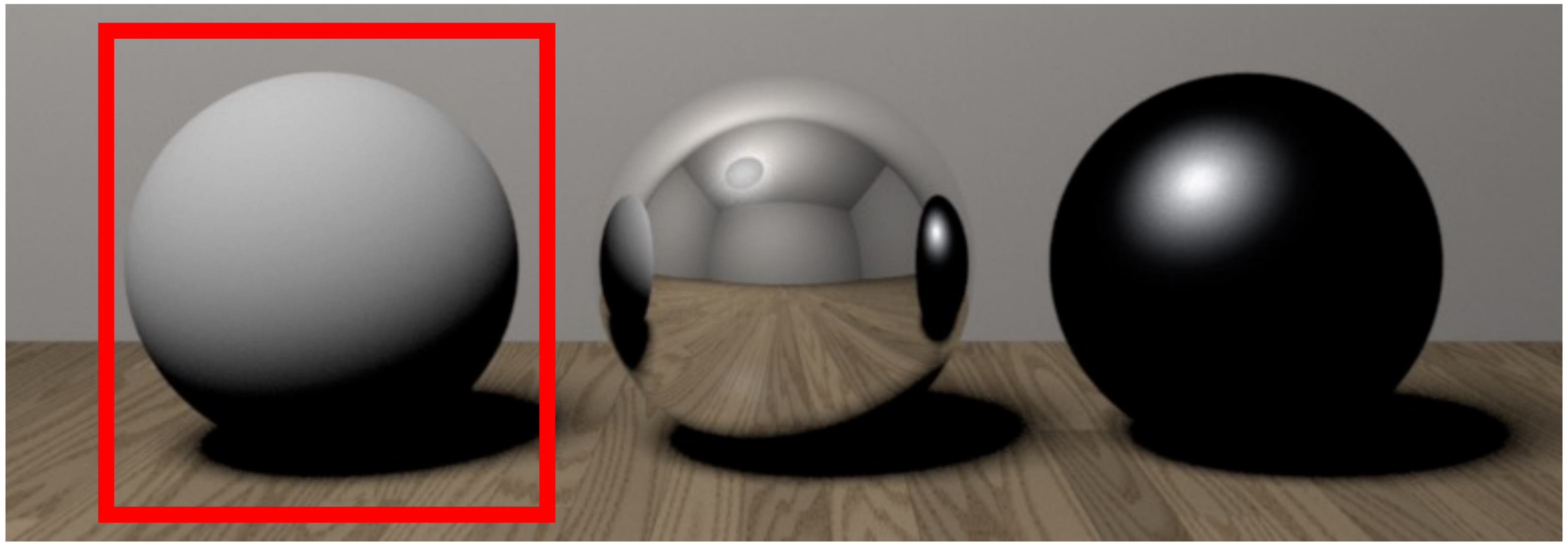
Special case 1: Perfect mirror

- $\rho(\theta_i, \theta_r)$ = 0 unless $\theta_i = \theta_r$
- Also called “Specular surfaces”
- Reflects light in a single, particular direction



Special case 2: Matte surface

- $\rho(\theta_i, \theta_r) = \rho_0$
- Also called “Lambertian surfaces”
- Reflected light is *independent of viewing direction*



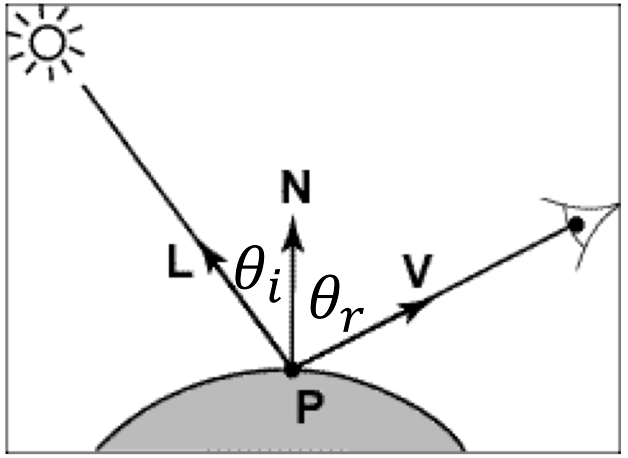
Lambertian surfaces

- For a lambertian surface:

$$L_r = \rho L_i \cos\theta_i$$

$$\Rightarrow L_r = \rho L_i \mathbf{L} \cdot \mathbf{N}$$

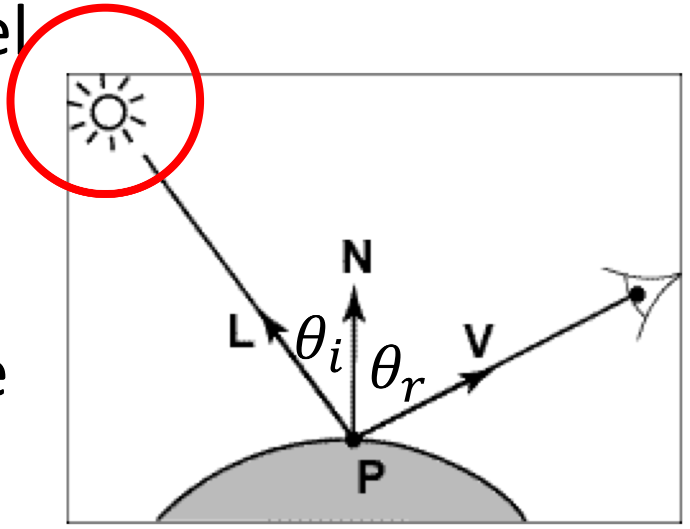
- **L** is direction to light source ($= \Omega_i$)
- L_i is intensity of light
- ρ is called *albedo*
  - Think of this as paint
  - High albedo: white colored surface
  - Low albedo: black surface
  - Varies from point to point



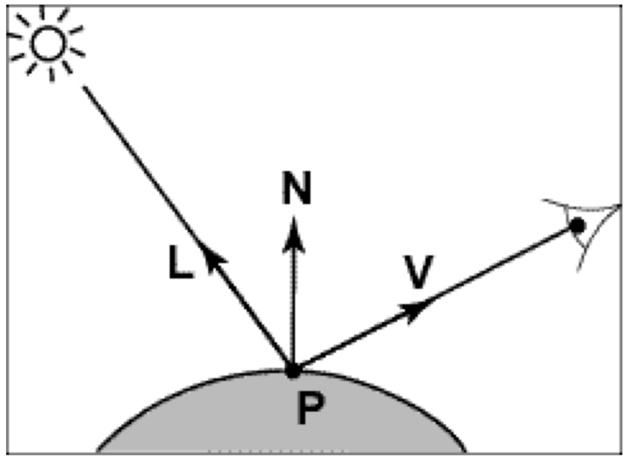
Lambertian surfaces

- Assume the light is directional: all rays from light source are parallel
  - Equivalent to a light source infinitely far away

- All pixels get light from the same direction **L** and of the same intensity L_i



Lambertian surfaces



Intrinsic Image Decomposition

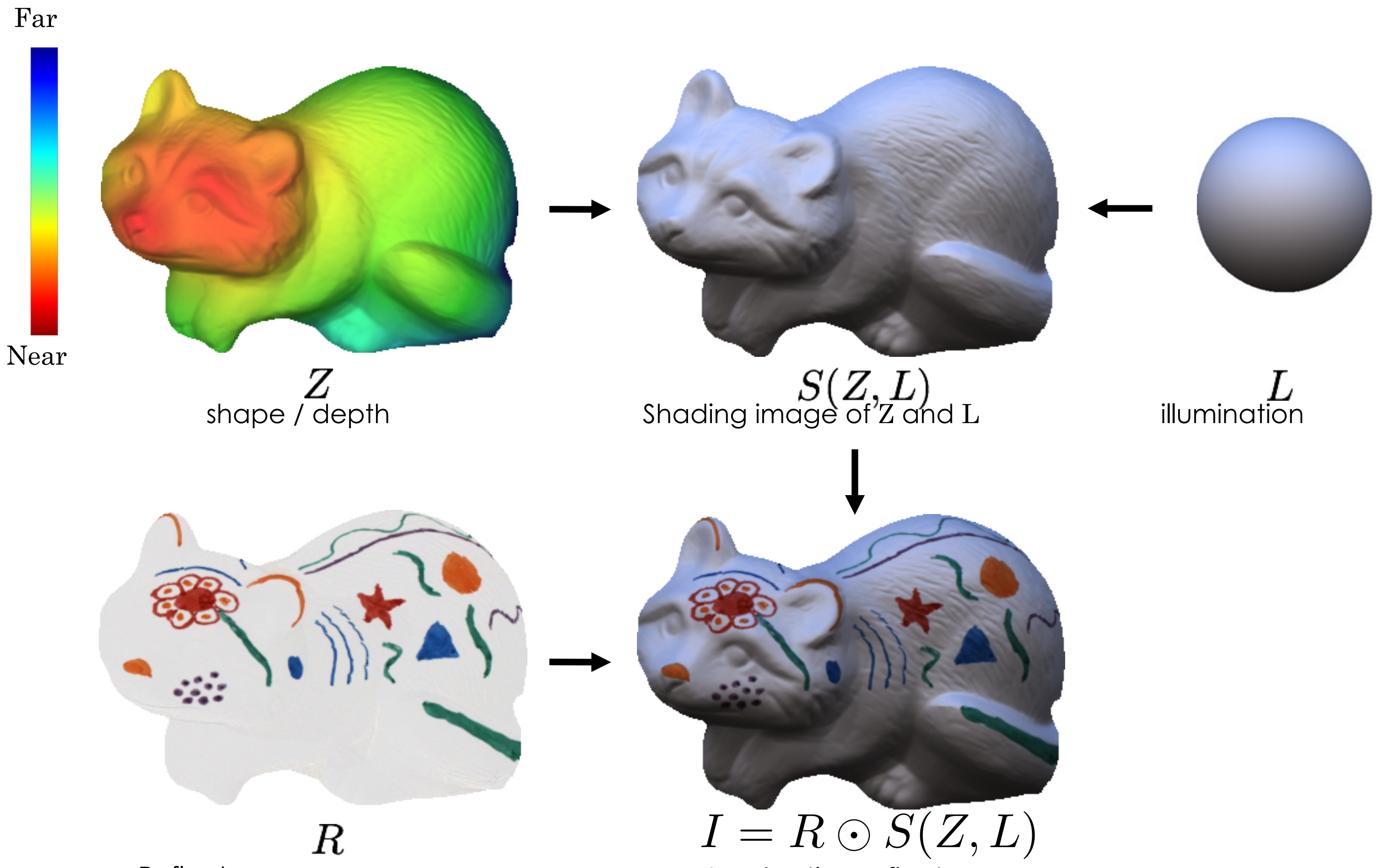
$$I(x,y) = \rho(x,y) L_i \mathbf{L} \cdot \mathbf{N}(x,y)$$

Reflectance image: albedo of surface corresponding to each pixel

Shading image: dot product between light and normal direction at each pixel

Lambertian surfaces

Lambertian surfaces



Reconstructing Lambertian surfaces

$$I(x, y) = \rho(x, y) L_i \mathbf{L} \cdot \mathbf{N}(x, y)$$

- Equation is a constraint on albedo and normals
- Can we solve for albedo and normals?

Recovery from multiple images

$$I(x,y) = \rho(x,y) L_i \mathbf{L} \cdot \mathbf{N}(x,y)$$

- Represents an equation in the albedo and normals
- Multiple images give constraints on albedo and normals
- Solve for albedo and normals
- Called *Photometric Stereo*

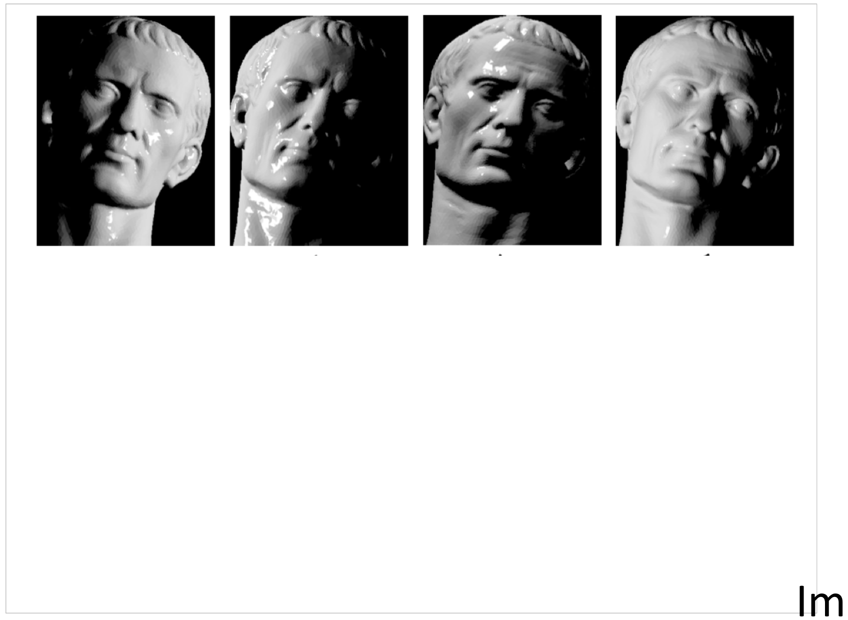
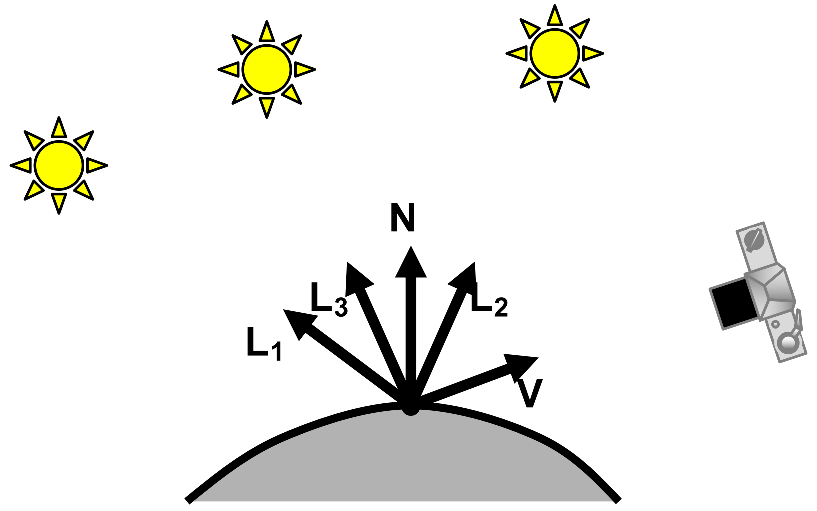


Image credit: Wikipedia

Multiple Images: Photometric Stereo



Photometric stereo - the math

$$I(x,y) = \rho(x,y) L_i \mathbf{L} \cdot \mathbf{N}(x,y)$$

- Consider single pixel
- Assume $L_i = 1$

$$I = \rho \mathbf{L} \cdot \mathbf{N}$$

$$I = \rho \mathbf{N}^T \mathbf{L}$$

- Write $\mathbf{G} = \rho \mathbf{N}$
- $\mathbf{G}$ is a 3-vector
    - Norm of **G** = ρ
    - Direction of **G** = **N**

Photometric stereo - the math

- Consider single pixel
- Assume $L_i = 1$

$$I = \rho \mathbf{N}^T \mathbf{L}$$

- Write $\mathbf{G} = \rho \mathbf{N}$
- $\mathbf{G}$ is a 3-vector
  - Norm of **G** = ρ
  - Direction of **G** = **N**

$$I = \mathbf{G}^T \mathbf{L} = \mathbf{L}^T \mathbf{G}$$

Photometric stereo - the math

$$I = \mathbf{L}^T \mathbf{G}$$

- Multiple images with different light sources but same viewing direction?

$$I_1 = \mathbf{L}_1^T \mathbf{G}$$

$$I_2 = \mathbf{L}_2^T \mathbf{G}$$

$$\vdots$$

$$I_k = \mathbf{L}_k^T \mathbf{G}$$

Photometric stereo - the math

$$I_1 = \mathbf{L}_1^T \mathbf{G}$$

$$I_2 = \mathbf{L}_2^T \mathbf{G}$$

$$\vdots$$

$$I_k = \mathbf{L}_k^T \mathbf{G}$$

- Assume lighting directions are known
- Each is a linear equation in G
- Stack everything up into a massive linear system of equations!

Photometric stereo - the math

$$
\begin{aligned}
I_1 &= \mathbf{L}_1^T \mathbf{G} \\
I_2 &= \mathbf{L}_2^T \mathbf{G} \\
&\vdots \\
I_k &= \mathbf{L}_k^T \mathbf{G}
\end{aligned}
\quad \Rightarrow \quad
\underbrace{\mathbf{I}}_{\text{k x 1 vector of intensities}} = \overbrace{\mathbf{L}^T}^{\text{k x 3 matrix of lighting directions}} \underbrace{\mathbf{G}}_{\text{3x1 vector of unknowns}}
$$

Photometric stereo - the math

$$\underset{k \times 1}{\mathbf{I}} = \underset{k \times 3}{\mathbf{L}^T} \underset{3 \times 1}{\mathbf{G}}$$

$$\mathbf{G} = \mathbf{L}^{-T}\mathbf{I}$$

- What is the minimum value of k to allow recovery of G?
- How do we recover G if the problem is overconstrained?

Photometric stereo - the math

- How do we recover G if the problem is overconstrained?
    - More than 3 lights: more than 3 images
- Least squares

$$\min_{\mathbf{G}} \|\mathbf{I} - \mathbf{L}^T\mathbf{G}\|^2$$

- Solved using normal equations

$$\mathbf{G} = (\mathbf{L}\mathbf{L}^T)^{-1}\mathbf{L}\mathbf{I}$$

Normal equations

$$\|\mathbf{I} - \mathbf{L}^T\mathbf{G}\|^2 = \mathbf{I}^T\mathbf{I} + \mathbf{G}^T\mathbf{L}\mathbf{L}^T\mathbf{G} - 2\mathbf{G}^T\mathbf{L}\mathbf{I}$$

- Take derivative with respect to **G** and set to 0

$$2\mathbf{L}\mathbf{L}^T\mathbf{G} - 2\mathbf{L}\mathbf{I} = 0$$

$$\Rightarrow \mathbf{G} = (\mathbf{L}\mathbf{L}^T)^{-1}\mathbf{L}\mathbf{I}$$

Estimating normals and albedo from **G**

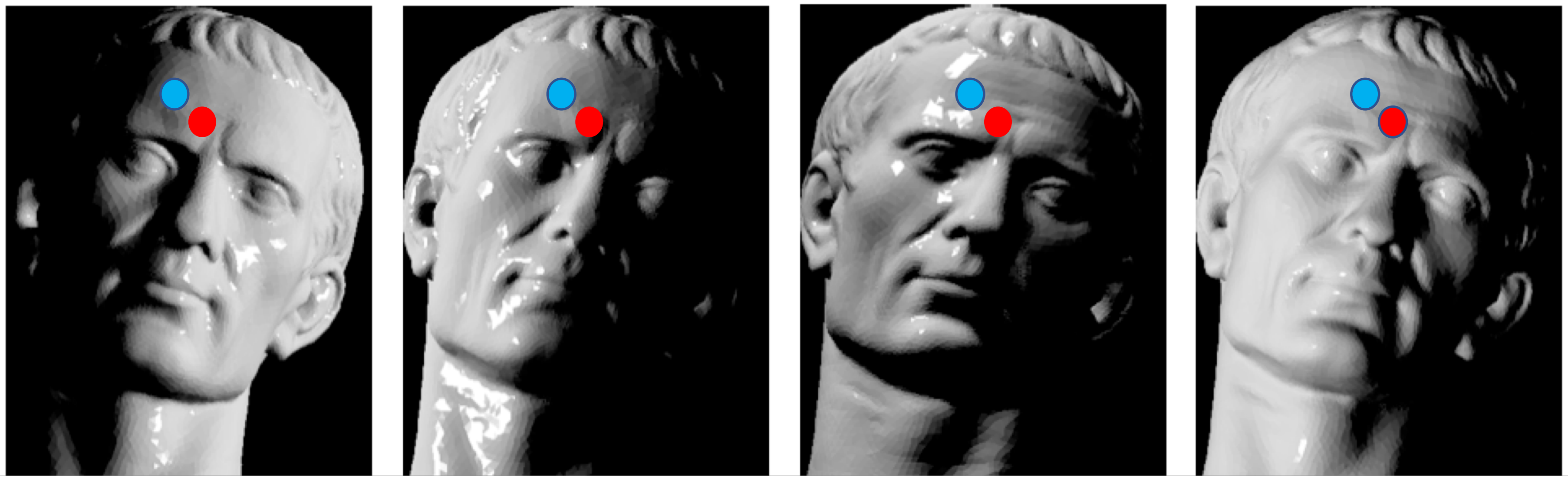
- Recall that $\mathbf{G} = \rho\mathbf{N}$

$$\|\mathbf{G}\| = \rho$$

$$\frac{\mathbf{G}}{\|\mathbf{G}\|} = \mathbf{N}$$

Multiple pixels

- We've looked at a single pixel till now
- How do we handle multiple pixels?
- Essentially independent equations!



Multiple pixels: matrix form

- Note that all pixels share the same set of lights

$$\mathbf{I}^{(1)} = \mathbf{L}^T \mathbf{G}^{(1)}$$

$$\mathbf{I}^{(2)} = \mathbf{L}^T \mathbf{G}^{(2)}$$

$$\vdots$$

$$\mathbf{I}^{(n)} = \mathbf{L}^T \mathbf{G}^{(n)}$$

Multiple pixels: matrix form

- Can stack these into *columns* of a matrix

$$\mathbf{I}^{(1)} = \mathbf{L}^T \mathbf{G}^{(1)}$$

$$\mathbf{I}^{(2)} = \mathbf{L}^T \mathbf{G}^{(2)}$$

$$\vdots$$

$$\mathbf{I}^{(n)} = \mathbf{L}^T \mathbf{G}^{(n)}$$

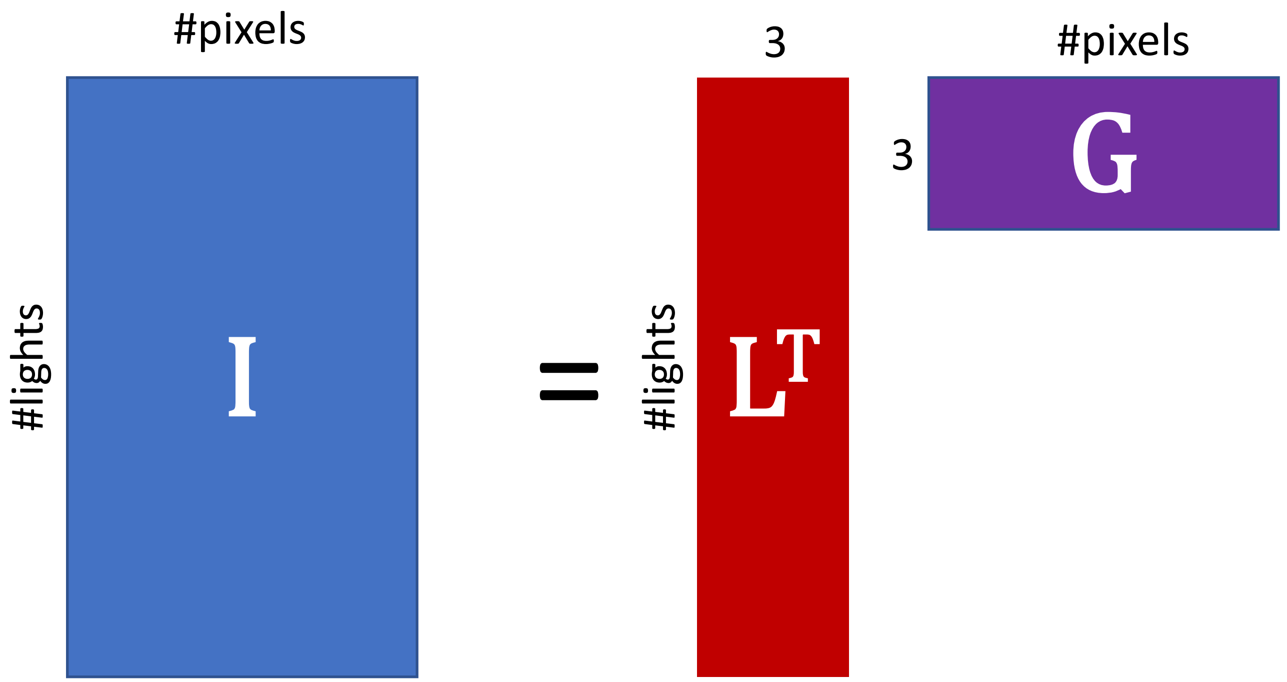
$$\Big\Downarrow$$

$$\begin{bmatrix}\mathbf{I}^{(1)} & \mathbf{I}^{(2)} & \cdots & \mathbf{I}^{(n)}\end{bmatrix} = \mathbf{L}^T \begin{bmatrix}\mathbf{G}^{(1)} & \mathbf{G}^{(2)} & \cdots & \mathbf{G}^{(n)}\end{bmatrix}$$

$$\boxed{\mathbf{I} = \mathbf{L}^T \mathbf{G}}$$

Multiple pixels: matrix form

$$\mathbf{I} = \mathbf{L}^T \mathbf{G}$$

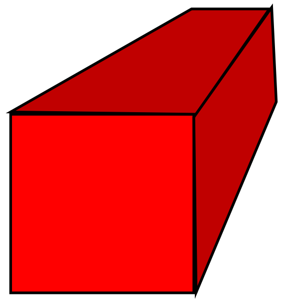


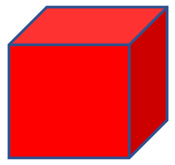
Estimating depth from normals

- So we got surface normals, can we get depth?
- Yes, given *boundary conditions*
- Normals provide information about the derivative

Brief detour: Orthographic projection

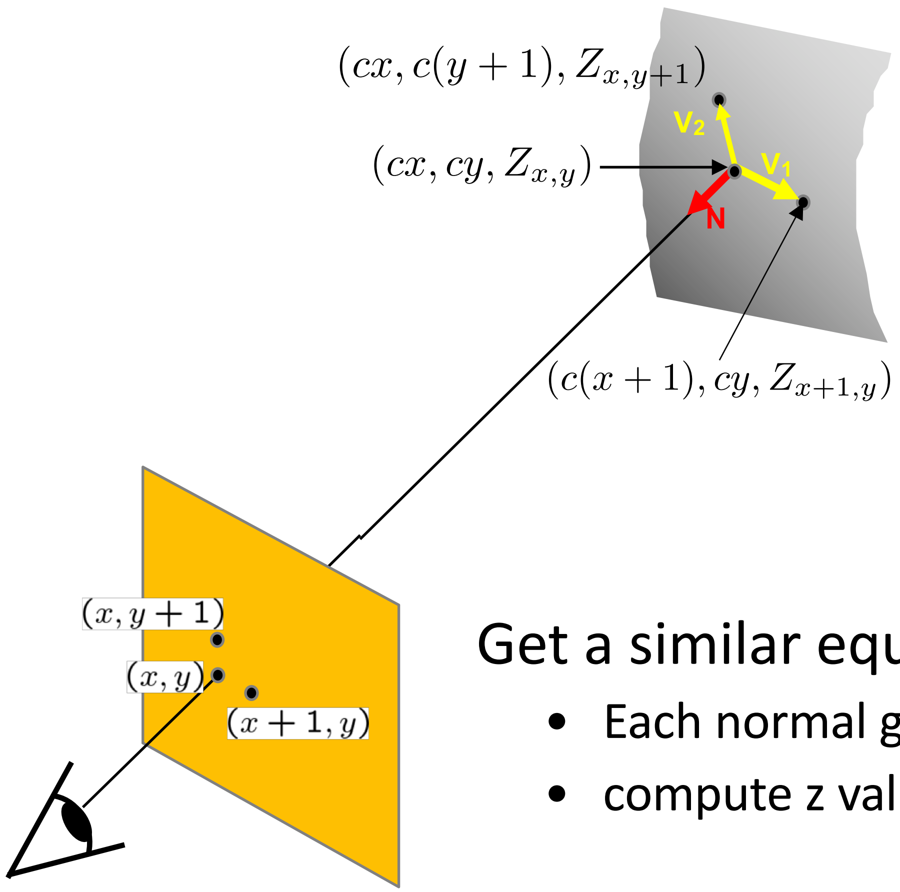
- Perspective projection
  - $x = \frac{X}{Z}, y = \frac{Y}{Z}$
- If all points have similar depth
  - $Z \approx Z_0$
  - $x \approx \frac{X}{Z_0}, y \approx \frac{Y}{Z_0}$
  - $x \approx cX, y \approx cY$
- A scaled version of orthographic projection
  - $x = X, y = Y$


Perspective


Scaled orthographic

Depth Map from Normal Map

- We now have a surface normal, but how do we get depth?



Assume a smooth surface

$$\begin{aligned} V_1 &= (c(x+1), cy, Z_{x+1,y}) - (cx, cy, Z_{x,y}) \\ &= (c, 0, Z_{x+1,y} - Z_{x,y}) \end{aligned}$$

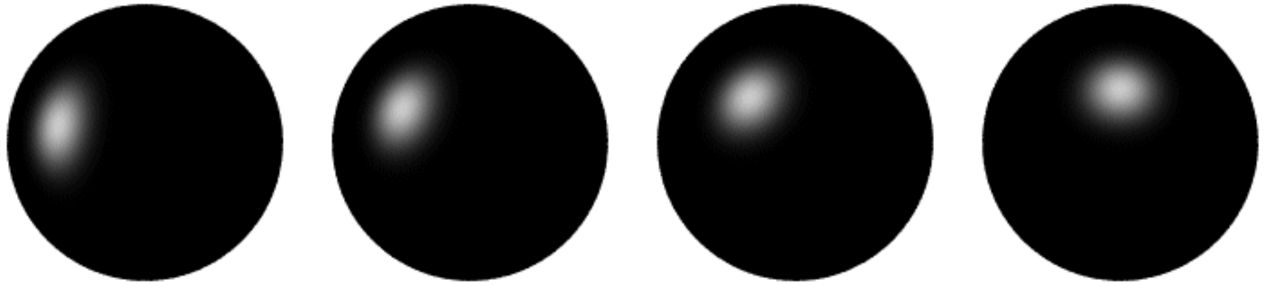
$$\begin{aligned} 0 &= N \cdot V_1 \\ &= (n_x, n_y, n_z) \cdot (c, 0, Z_{x+1,y} - Z_{x,y}) \\ &= cn_x + n_z(Z_{x+1,y} - Z_{x,y}) \end{aligned}$$

Get a similar equation for $\mathbf{V_2}$

- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation

Determining Light Directions

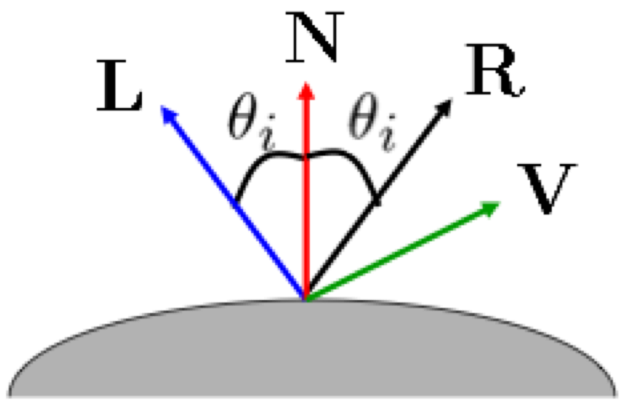
- Trick: Place a mirror ball in the scene.



- The location of the highlight is determined by the light source direction.
- Can relate the direction of highlight mathematically to direction of light source

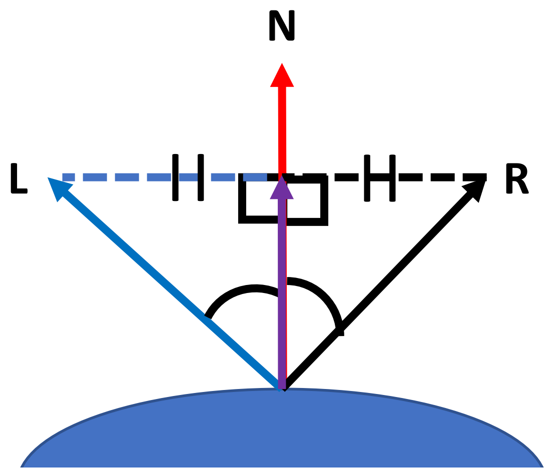
Optional: Determining Light Directions

- For a perfect mirror, the light is reflected across N:



$$I_e = \begin{cases} I_i & \text{if } \mathbf{V} = \mathbf{R} \\ 0 & \text{otherwise} \end{cases}$$

Optional: Determining Light Directions



$$\longrightarrow = (N \cdot R)N$$

$$\text{-------} = R - (N \cdot R)N$$

$$\text{-------} = R - (N \cdot R)N$$

$$\longrightarrow = \longrightarrow - 2 \ \text{-------}$$

$$= R - 2(R - N \cdot R)N$$

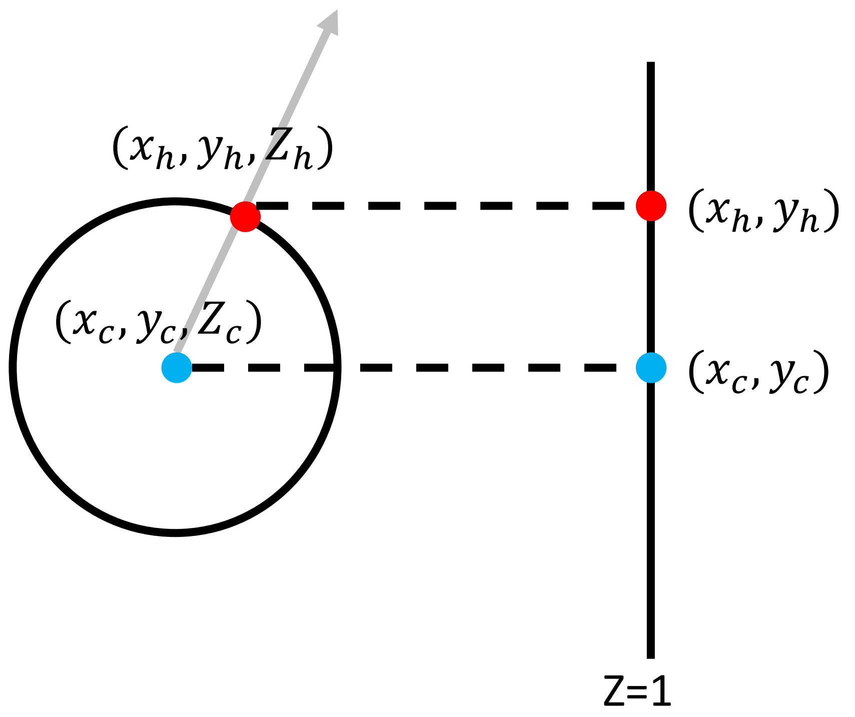
$$= 2(N \cdot R)N - R$$

So the light source direction is given by:

$$L = 2(N \cdot R)N - R$$

Optional: Determining Light Directions

- Assume orthographic projection
- Viewing direction R = [0,0,-1]
- Normal?



Z_h and Z_c are unknown, but:

$$(x_h - x_c)^2 + (y_h - y_c)^2 + (Z_h - Z_c)^2 = r^2$$

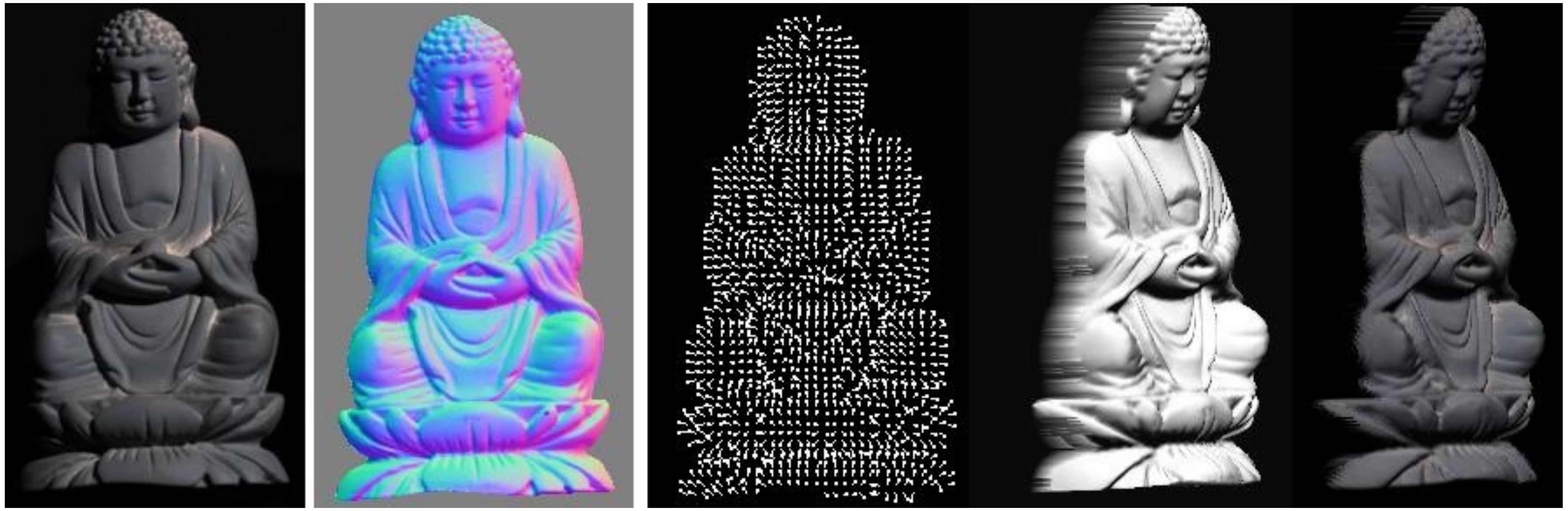
$(Z_h - Z_c)$ can be computed

$(x_h - x_c, y_h - y_c, Z_h - Z_c)$ is the normal

$$L = 2(N \cdot R)N - R$$

Photometric Stereo

What results can you get?



Input
(1 of 12)

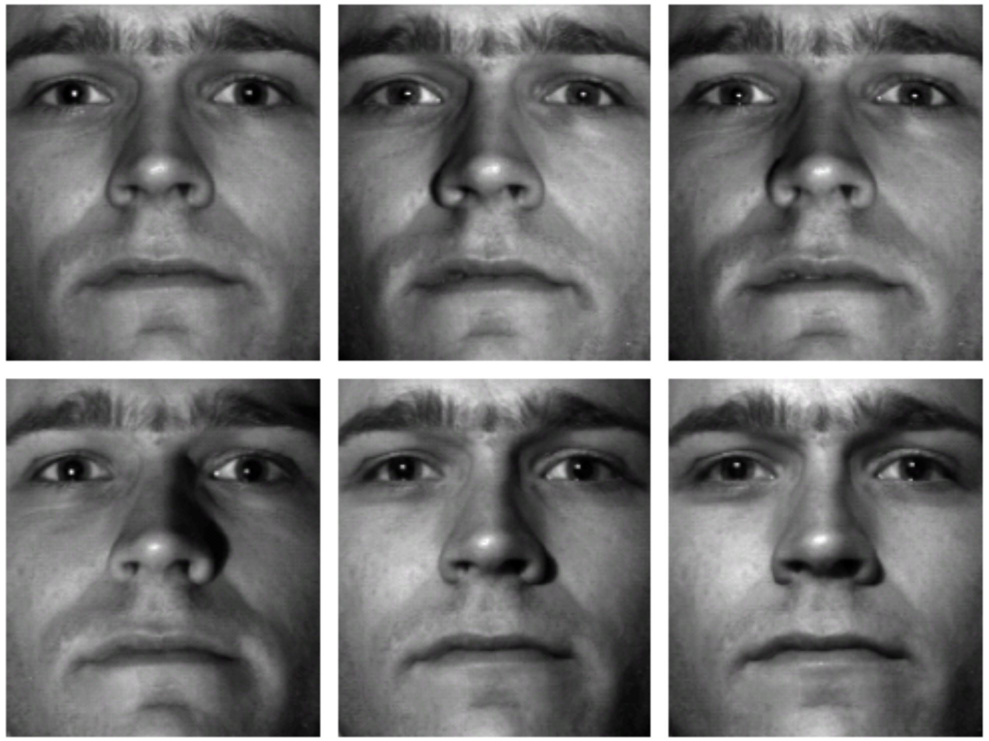
Normals (RGB
colormap)

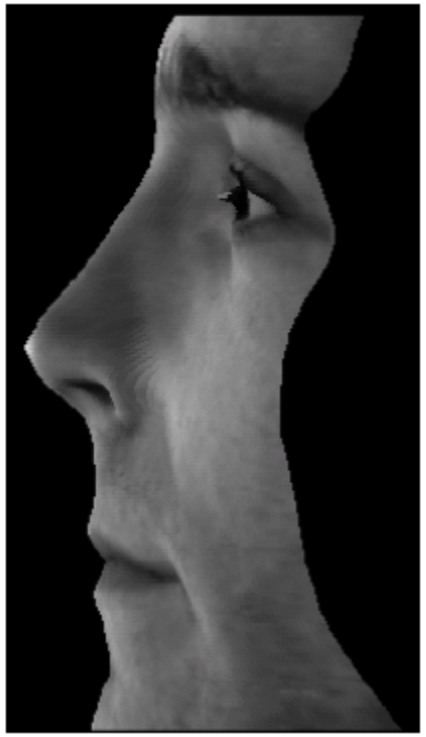
Normals (vectors)

Shaded 3D
rendering

Textured 3D
rendering

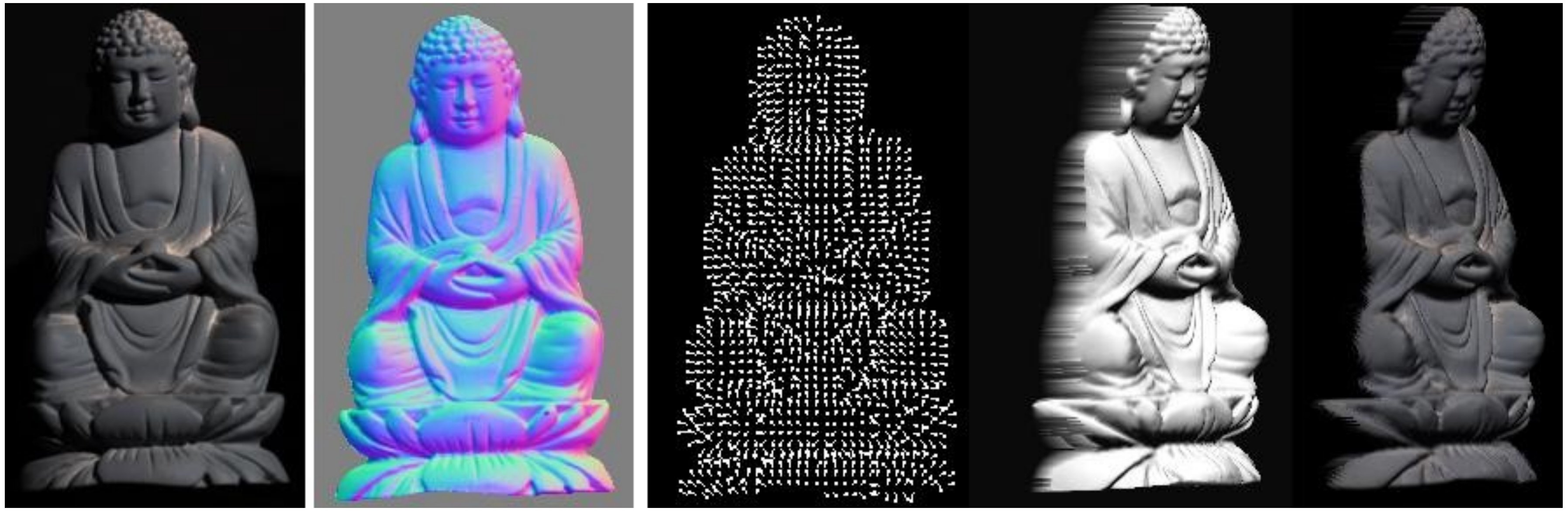
Results





from Athos Georghiades

Results



Input
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Normals (RGB
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