Correspondence: Feature detection

A general pipeline for correspondence

1. If sparse correspondences are enough, *choose points for which we will search for correspondences (feature points)*
2. For each point (or every pixel if dense correspondence), describe point using a *feature descriptor*
3. Find best matching descriptors across two images (*feature matching*)
4. Use feature matches to perform downstream task, e.g., pose estimation

Characteristics of good feature points

- Repeatability / invariance
  - The same feature point can be found in several images despite geometric and photometric transformations
- Saliency / distinctiveness
  - Each feature point is distinctive
  - Fewer ”false” matches

Goal: repeatability

- We want to detect (at least some of) the same points in both images.


No chance to find true matches!

- Yet we have to be able to run the detection procedure *independently* per image.

Kristen Grauman

Goal: distinctiveness

- The feature point should be distinctive enough that it is easy to match
    - Should *at least* be distinctive from other patches nearby

The aperture problem

The aperture problem

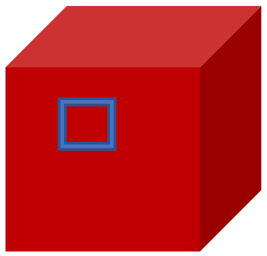
- Individual pixels are ambiguous
- Idea: Look at whole patches!

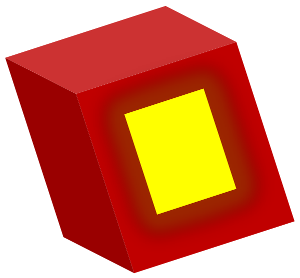
The aperture problem

- Individual pixels are ambiguous
- Idea: Look at whole patches!

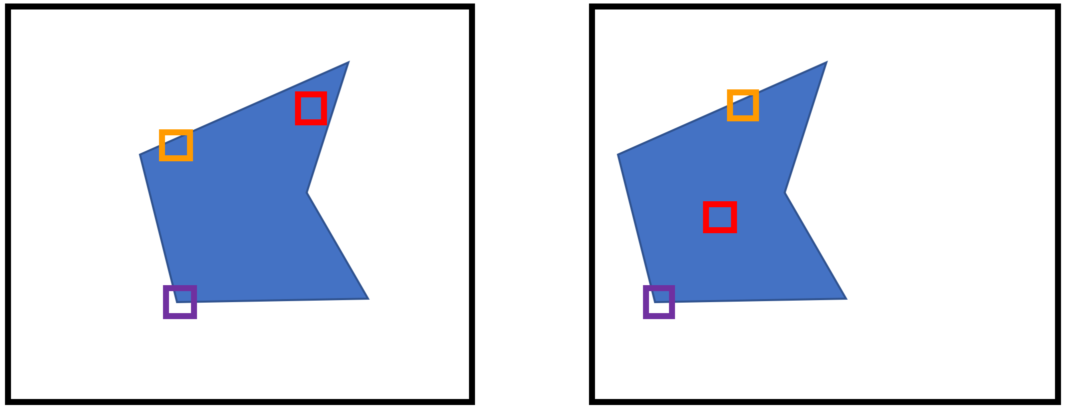
The aperture problem

- *Some local neighborhoods* are ambiguous



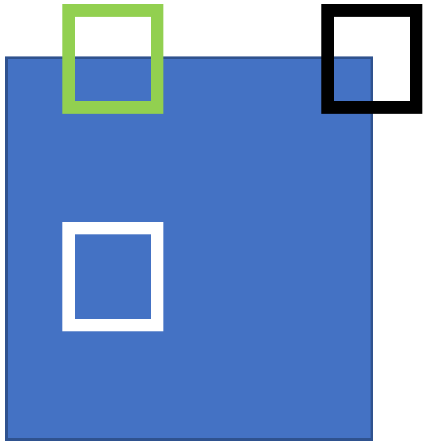


The aperture problem



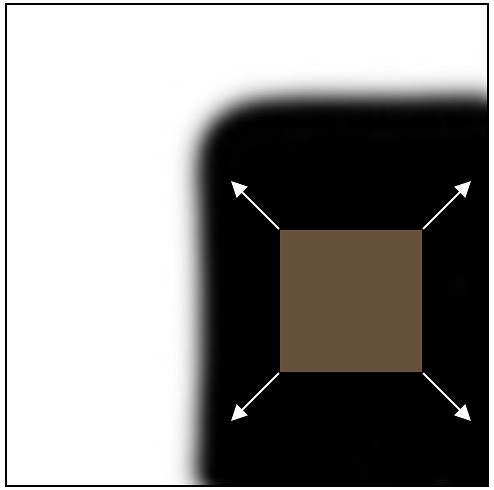
Corner detection

- Main idea: Translating window should cause large differences in patch appearance

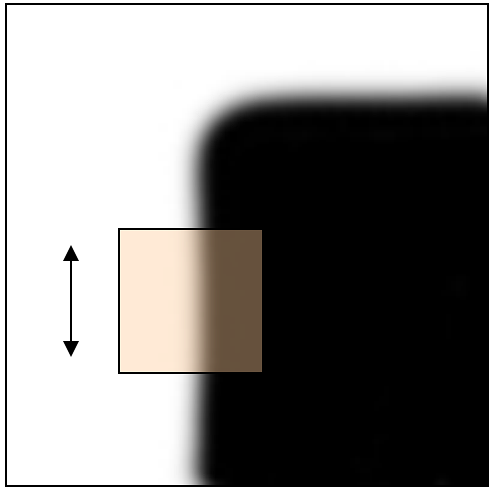


Corner Detection: Basic Idea

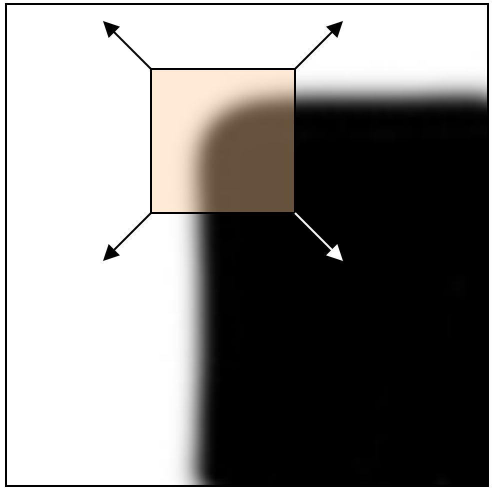
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions



"edge":
no change along the edge direction

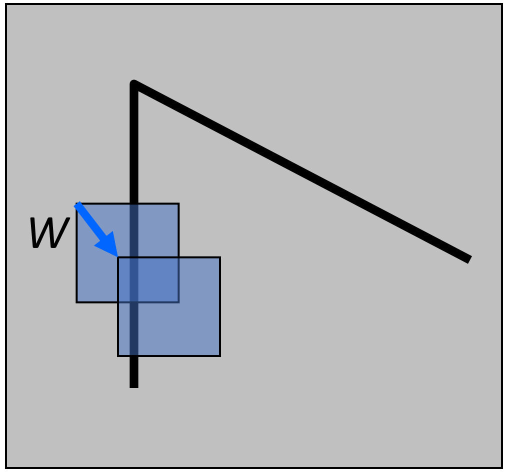


"corner":
significant change in all directions

Source: A. Efros

Corner detection the math

- Consider shifting the window *W* by (*u*,*v*)
  - how do the pixels in W change?
- Write pixels in window as a vector:



$$\phi_0 = [I(0,0), I(0,1), \ldots, I(n,n)]$$

$$\phi_1 = [I(0+u, 0+v), I(0+u, 1+v), \ldots, I(n+u, n+v)]$$

$$E(u,v) = \|\phi_0 - \phi_1\|_2^2$$

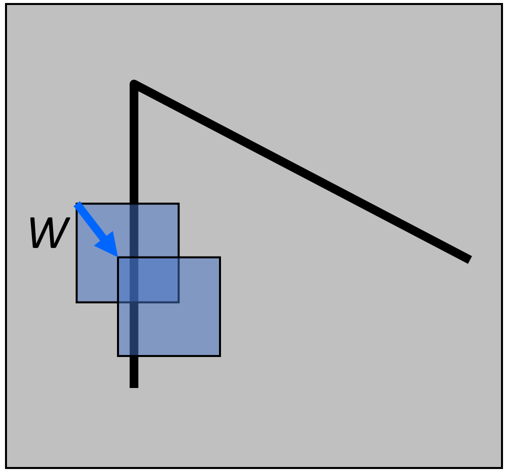
Corner detection: the math

Consider shifting the window *W* by (*u,v*)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD "error" *E*(*u,v*):

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^2$$

- We want E(u,v) to be *as high as possible for all u, v!*



Small motion assumption

Taylor Series expansion of *I*:

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u,v) is small, then first order approximation is good

$$I(x+u, y+v) \approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

$$\approx I(x, y) + [I_x \;\; I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

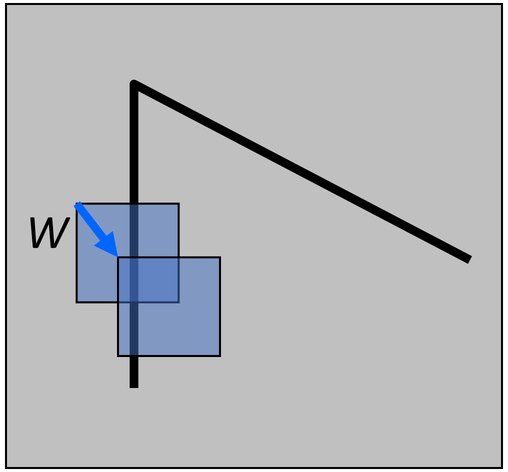
shorthand: $I_x = \frac{\partial I}{\partial x}$

Plugging this into the formula on the previous slide…

Corner detection: the math

Consider shifting the window *W* by (*u,v*)

- define an SSD "error" *E(u,v)*:

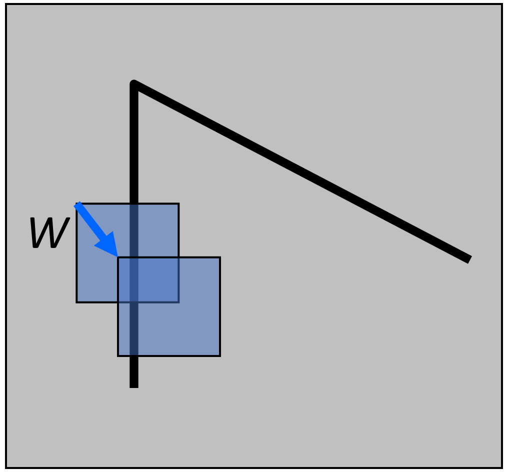


$$
\begin{aligned}
E(u,v) &= \sum_{(x,y)\in W} \left[I(x+u,y+v) - I(x,y)\right]^2 \\
&\approx \sum_{(x,y)\in W} \left[I(x,y) + I_x u + I_y v - I(x,y)\right]^2 \\
&\approx \sum_{(x,y)\in W} \left[I_x u + I_y v\right]^2
\end{aligned}
$$

Corner detection: the math

Consider shifting the window *W* by (*u,v*)

- define an "error" *E(u,v)*:



$$E(u,v) \approx \sum_{(x,y)\in W} [I_x u + I_y v]^2$$

$$\approx Au^2 + 2Buv + Cv^2$$

$$A = \sum_{(x,y)\in W} I_x^2 \qquad B = \sum_{(x,y)\in W} I_x I_y \qquad C = \sum_{(x,y)\in W} I_y^2$$

- Thus, *E*(*u*,*v*) is locally approximated as a quadratic error function

A more general formulation

- Maybe all pixels in the patch are not equally important
- Consider a “window function” $w(x, y)$ that acts as weights
- $E(u, v) = \sum_{(x,y)\in W} w(x, y)[I(x + u, y + v) - I(x, y)]^2$
- Case till now:
    - w(x,y) = 1 inside the window, 0 otherwise

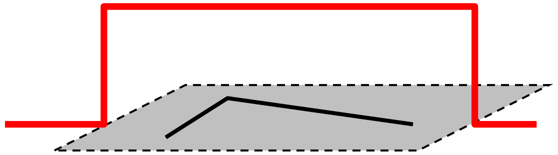
Using a window function

- Change in appearance of window w(x,y) for the shift [u,v]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

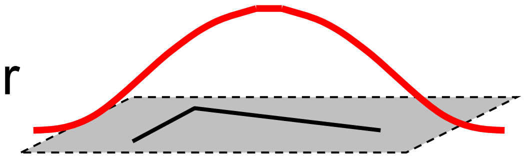
Window function — Shifted intensity — Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Source: R. Szeliski

Redoing the derivation using a window function

$$E(u, v) = \sum_{x,y \in W} w(x, y)\,[I(x+u, y+v) - I(x, y)]^2$$

$$\approx \sum_{x,y \in W} w(x, y)\big[I(x, y) + uI_x(x, y) + vI_y(x, y) - I(x, y)\big]^2$$

$$= \sum_{x,y \in W} w(x, y)\big[uI_x(x, y) + vI_y(x, y)\big]^2$$

$$= \sum_{x,y \in W} w(x, y)[u^2 I_x(x, y)^2 + v^2 I_y(x, y)^2 + 2uvI_x(x, y)I_y(x, y)]$$

Redoing the derivation using a window function

-

$$E(u,v) \approx \sum_{x,y \in W} w(x,y)[u^2 I_x(x,y)^2 + v^2 I_y(x,y)^2 + 2uvI_x(x,y)I_y(x,y)]$$

$$= Au^2 + 2Buv + Cv^2$$

$$A = \sum_{x,y \in W} w(x,y)I_x(x,y)^2$$

$$B = \sum_{x,y \in W} w(x,y)I_x(x,y)I_y(x,y)$$

$$C = \sum_{x,y \in W} w(x,y)I_y(x,y)^2$$

The second moment matrix

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \underbrace{\sum_{x,y \in W} w(x,y) \begin{bmatrix} I_x(x,y)^2 & I_x(x,y)I_y(x,y) \\ I_x(x,y)I_y(x,y) & I_y(x,y)^2 \end{bmatrix}}_{\text{Second moment matrix}}$$

Second moment matrix

The second moment matrix

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\underbrace{M = \sum_{x,y \in W} w(x,y) \begin{bmatrix} I_x(x,y)^2 & I_x(x,y)I_y(x,y) \\ I_x(x,y)I_y(x,y) & I_y(x,y)^2 \end{bmatrix}}_{\text{Second moment matrix}}$$

Second moment matrix

Recall that we want E(u,v) to be as large as possible for all u,v

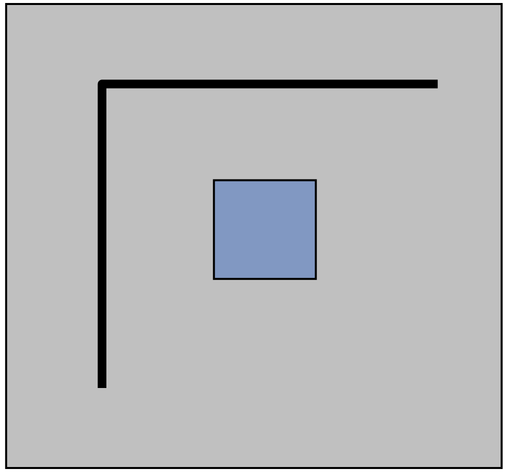
What does this mean in terms of M?

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Flat patch: $I_x = 0$ $I_y = 0$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

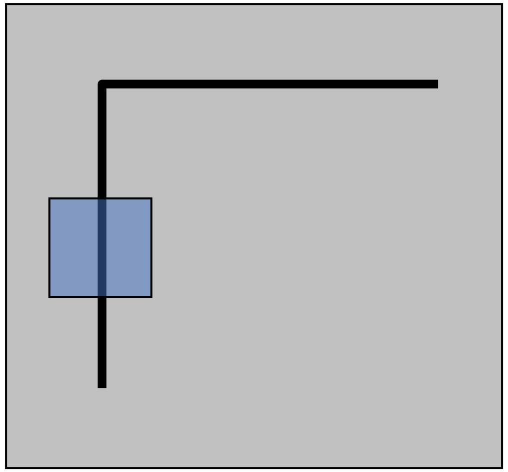
$$E(u,v) = 0 \quad \forall u, v$$

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Vertical edge: $I_y = 0$

$$M = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$$

$$M \begin{bmatrix} 0 \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

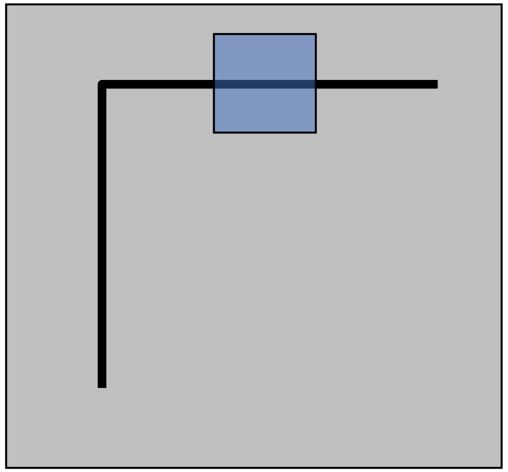
$$E(0,v) = 0 \ \ \forall v$$

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y) \in W} I_x^2$$

$$B = \sum_{(x,y) \in W} I_x I_y$$

$$C = \sum_{(x,y) \in W} I_y^2$$



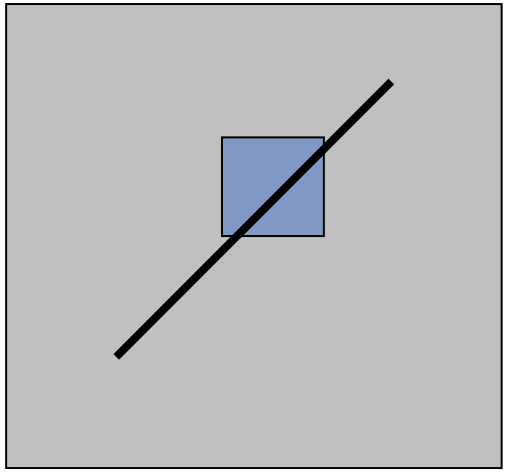
Horizontal edge: $I_x = 0$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$

$$M \begin{bmatrix} u \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$E(u,0) = 0 \; \forall u$$

What about edges in arbitrary orientation?



$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

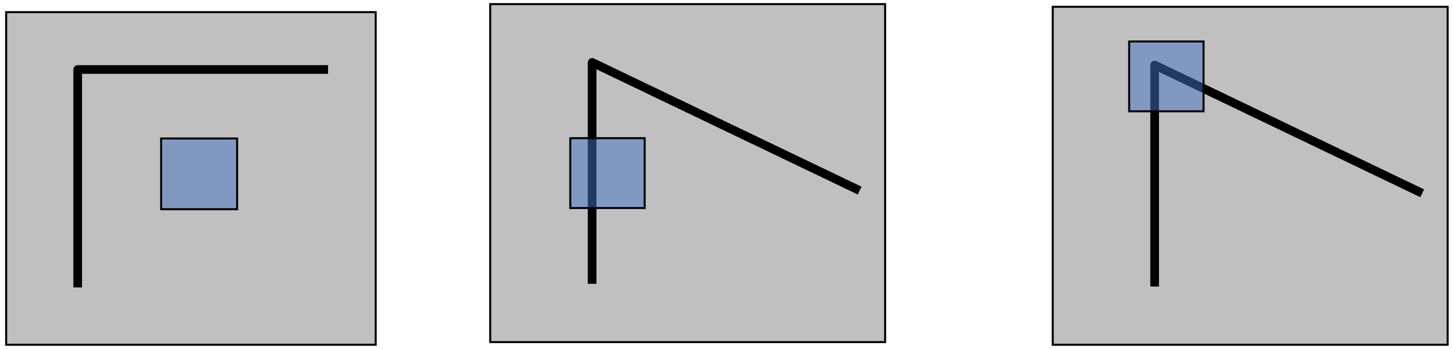
$$M \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Leftrightarrow E(u,v) = 0$$

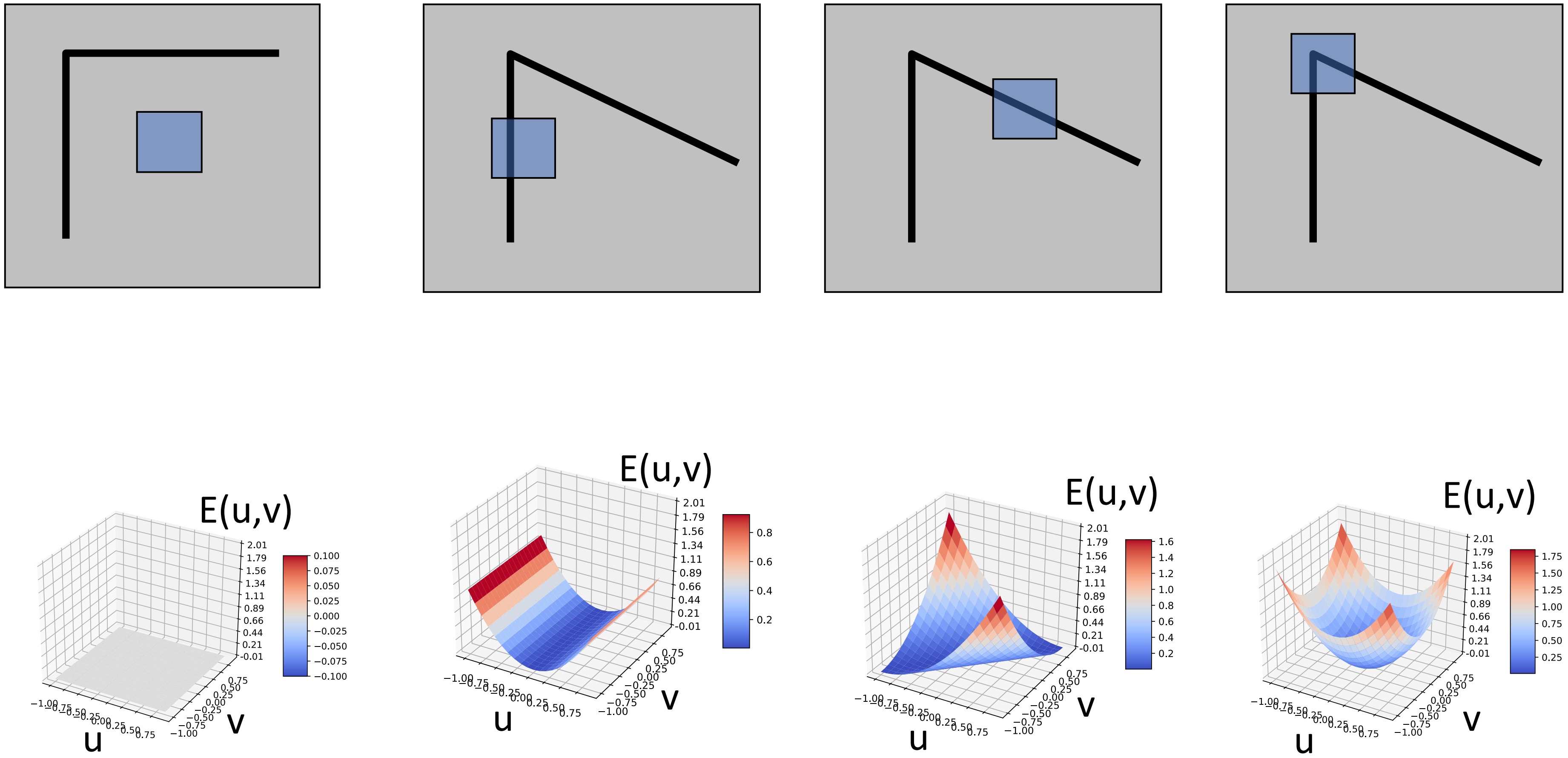
Solutions to Mx = 0 are directions for which E is 0: window can slide in this direction without changing appearance

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

Solutions to Mx = 0 are directions for which E is 0: window can slide in this direction without changing appearance

For corners, we want no such directions to exist



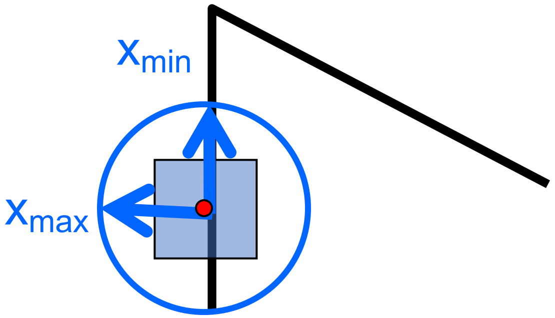

E(u,v)
u
v
E(u,v)
u
v
E(u,v)
u
v
E(u,v)
u
v

Eigenvalues and eigenvectors of M

- $Mx = 0 \Rightarrow Mx = \lambda x$: x is an eigenvector of M with eigenvalue 0
- M is 2 x 2, so it has 2 eigenvalues $(\lambda_{max}, \lambda_{min})$ with eigenvectors (x_{max}, x_{min})
- $E(x_{max}) = \ x_{max}^T M x_{max} = \lambda_{max}||x_{max}||^2 = \lambda_{max}$ (eigenvectors have unit norm)
- $E(x_{min}) = x_{min}^T M x_{min} = \lambda_{min}||x_{min}||^2 = \lambda_{min}$

Eigenvalues and eigenvectors of M

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$



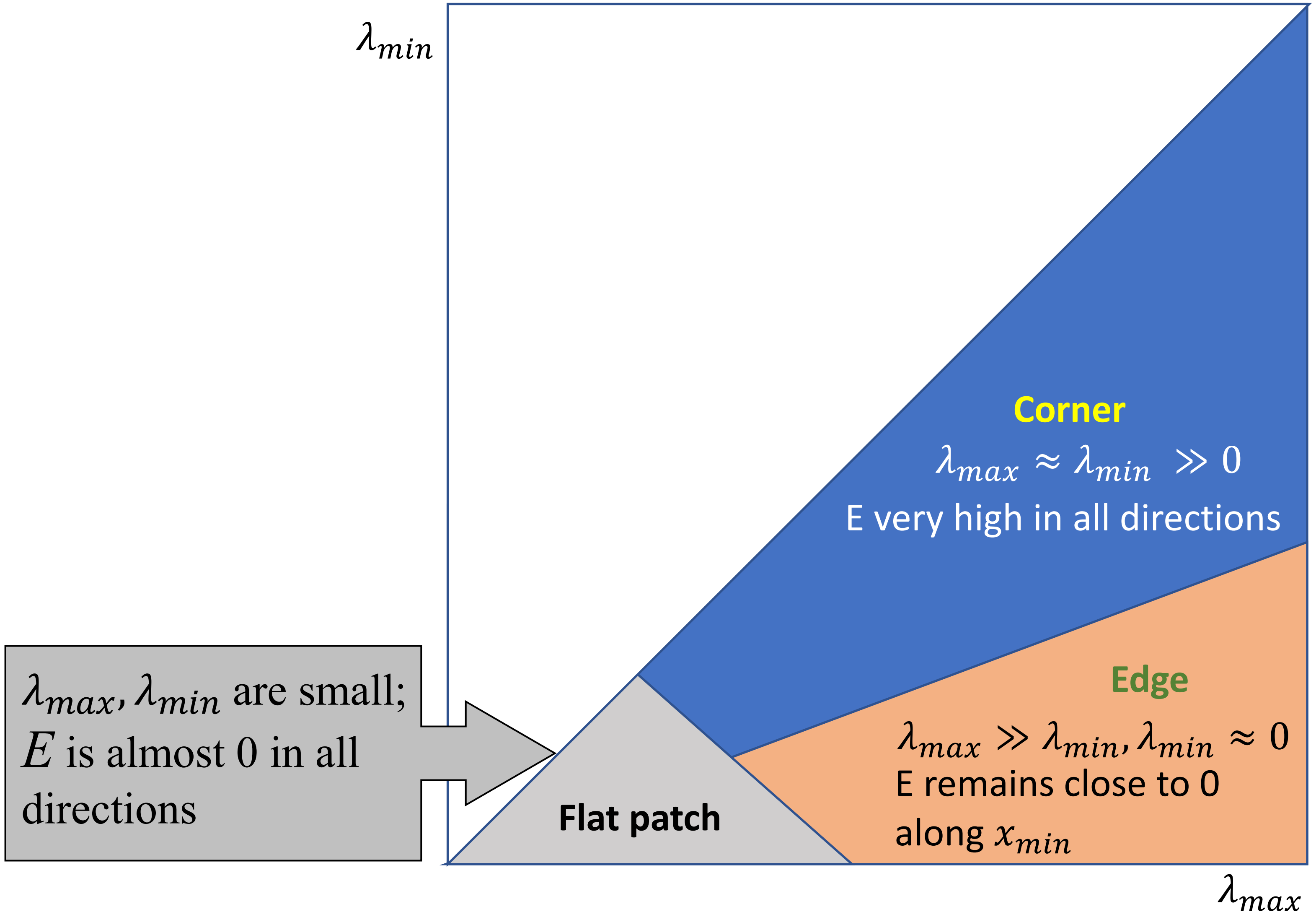
$$M\, x_{\max} = \lambda_{\max} x_{\max}$$

$$M\, x_{\min} = \lambda_{\min} x_{\min}$$

Eigenvalues and eigenvectors of M

- Define shift directions with the smallest and largest change in error
- x_{max} = direction of largest increase in *E*
- λ_{max} = amount of increase in direction x_{max}
- x_{min} = direction of smallest increase in *E*
- λ_{min} = amount of increase in direction x_{min}

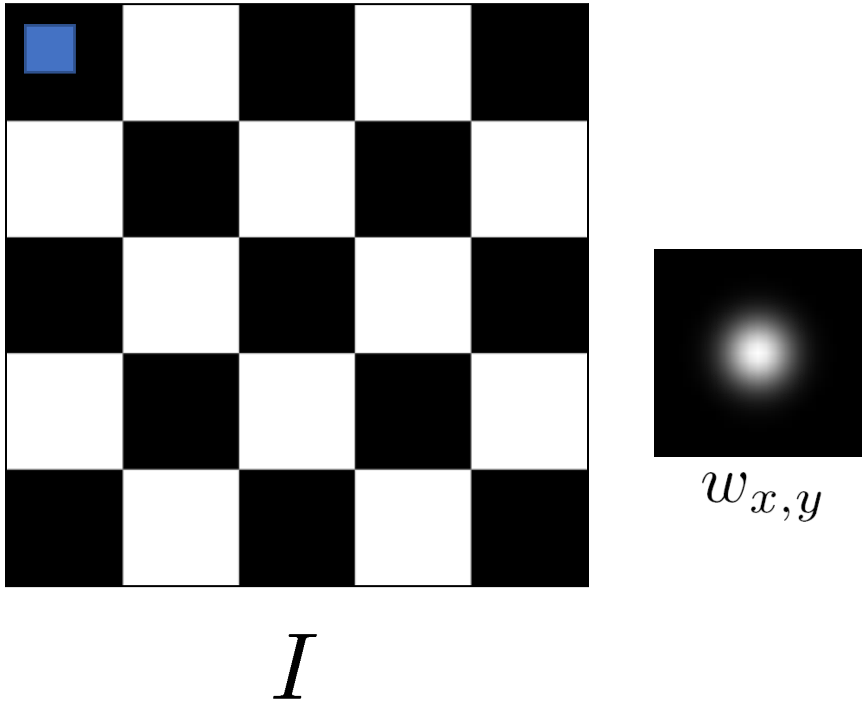
Interpreting the eigenvalues



Computing the second moment matrix efficiently

$$M = \sum_{x,y \in W} w(x,y) \begin{bmatrix} I_x(x,y)^2 & I_x(x,y)I_y(x,y) \\ I_x(x,y)I_y(x,y) & I_y(x,y)^2 \end{bmatrix}$$

- Window function w(x,y) typically a Gaussian centered on the window
  - $w(x,y) = e^{-\frac{(x-x_0)^2}{\sigma^2} - \frac{(y-y_0)^2}{\sigma^2}}$
- Need to compute this matrix efficiently for *every* window location



Computing the second moment matrix efficiently

$$M = \sum_{x,y \in W} w(x,y) \begin{bmatrix} I_x(x,y)^2 & I_x(x,y)I_y(x,y) \\ I_x(x,y)I_y(x,y) & I_y(x,y)^2 \end{bmatrix}$$

- Step 1: Place k x k window
- Step 2: Compute $\sum_{x,y \in W} w(x,y) I_x(x,y)^2 = \sum_{x,y} e^{-\frac{(x-x_0)^2}{\sigma^2} - \frac{(y-y_0)^2}{\sigma^2}} I_x(x,y)^2$ (similarly other terms)
- This can be expressed as a convolution!

Computing the second moment matrix

- Compute image gradients I_x, I_y (both of these are images)
  - Might want to blur with a Gaussian before doing this. Why?
- Compute $I_x^2, I_y^2, I_x I_y$ (these are images too)
- Convolve with windowing function (typically Gaussian)
- Assemble second moment matrix at every pixel

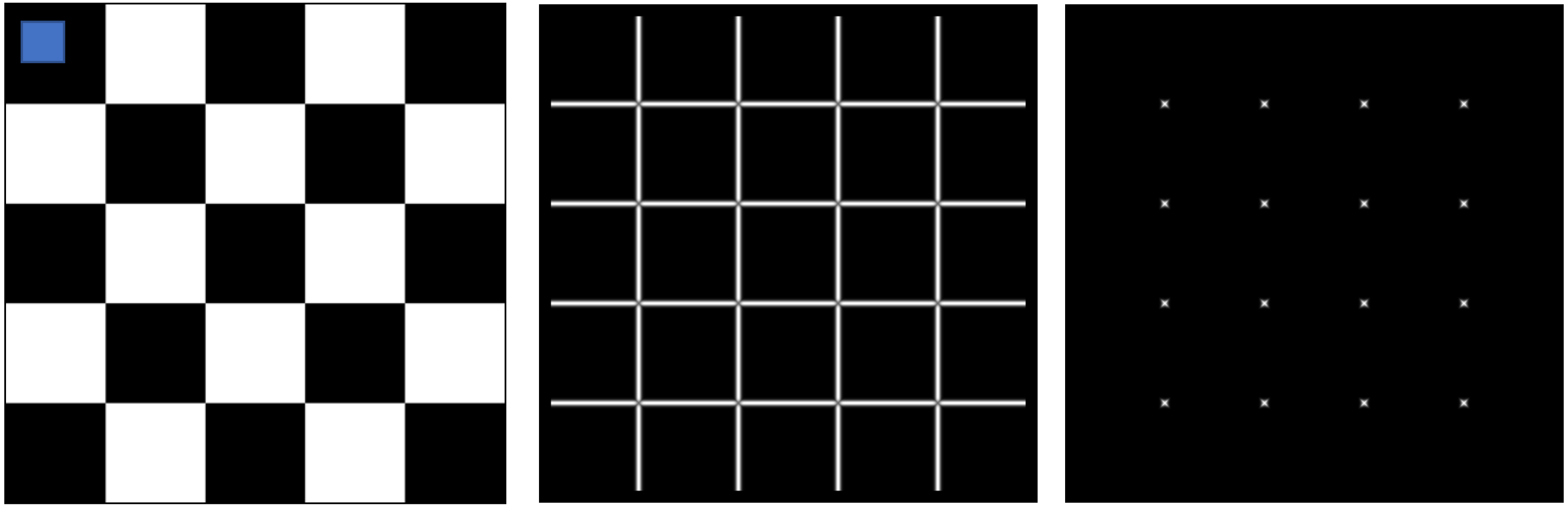
Corner detection: the math

How are λ_{max}, x_{max}, λ_{min}, and x_{min} relevant for feature detection?

- Need a feature scoring function

Want $E(u,v)$ to be large for small shifts in all directions

- the minimum of $E(u,v)$ should be large, over all unit vectors $[u\ v]$
- this minimum is given by the smaller eigenvalue (λ_{min}) of M

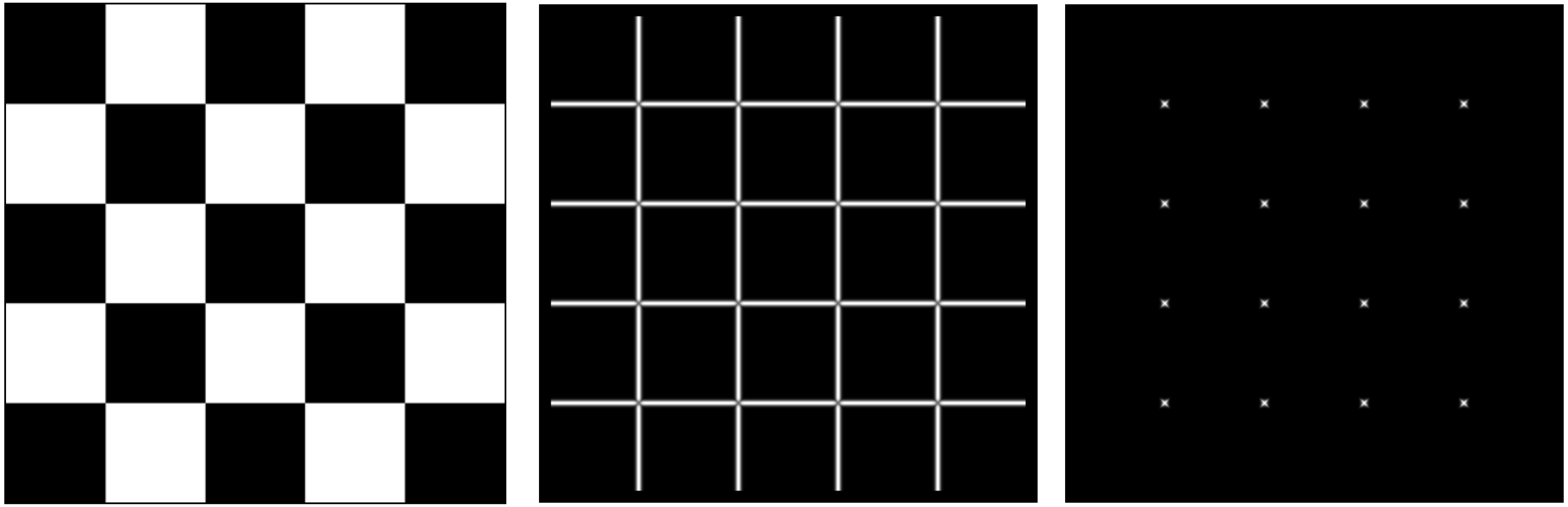


I $\quad\quad$ $\lambda_{\max}$ $\quad\quad$ $\lambda_{\min}$

Corner detection summary

Here's what you do

- Compute the gradient at each point in the image
- Create the *M* matrix from the entries in the gradient
- Compute the eigenvalues
- Find points with large response (λ_{min} > threshold)
- Choose those points where λ_{min} is a local maximum as features

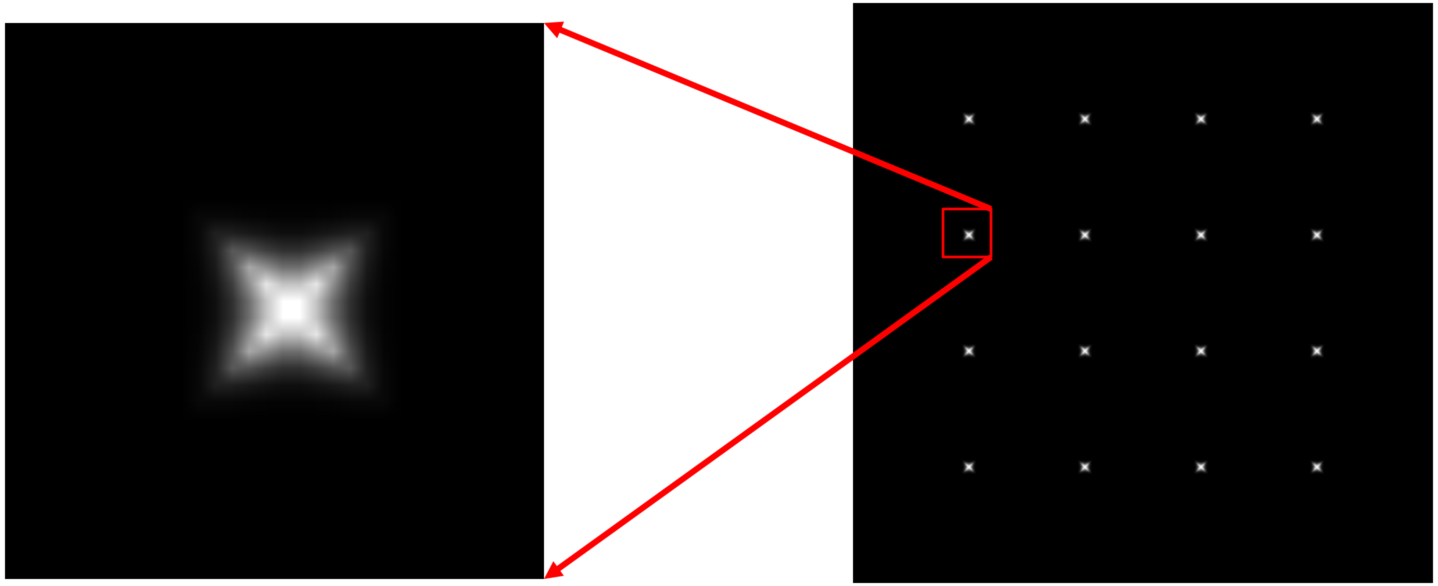


I $\lambda_{\max}$ $\lambda_{\min}$

Corner detection summary

Here's what you do

- Compute the gradient at each point in the image
- Create the *H* matrix from the entries in the gradient
- Compute the eigenvalues.
- Find points with large response (λ_{min} > threshold)
- Choose those points where λ_{min} is a local maximum as features



λ_{min}

The Harris operator

λ_{min} is a variant of the "Harris operator" for feature detection

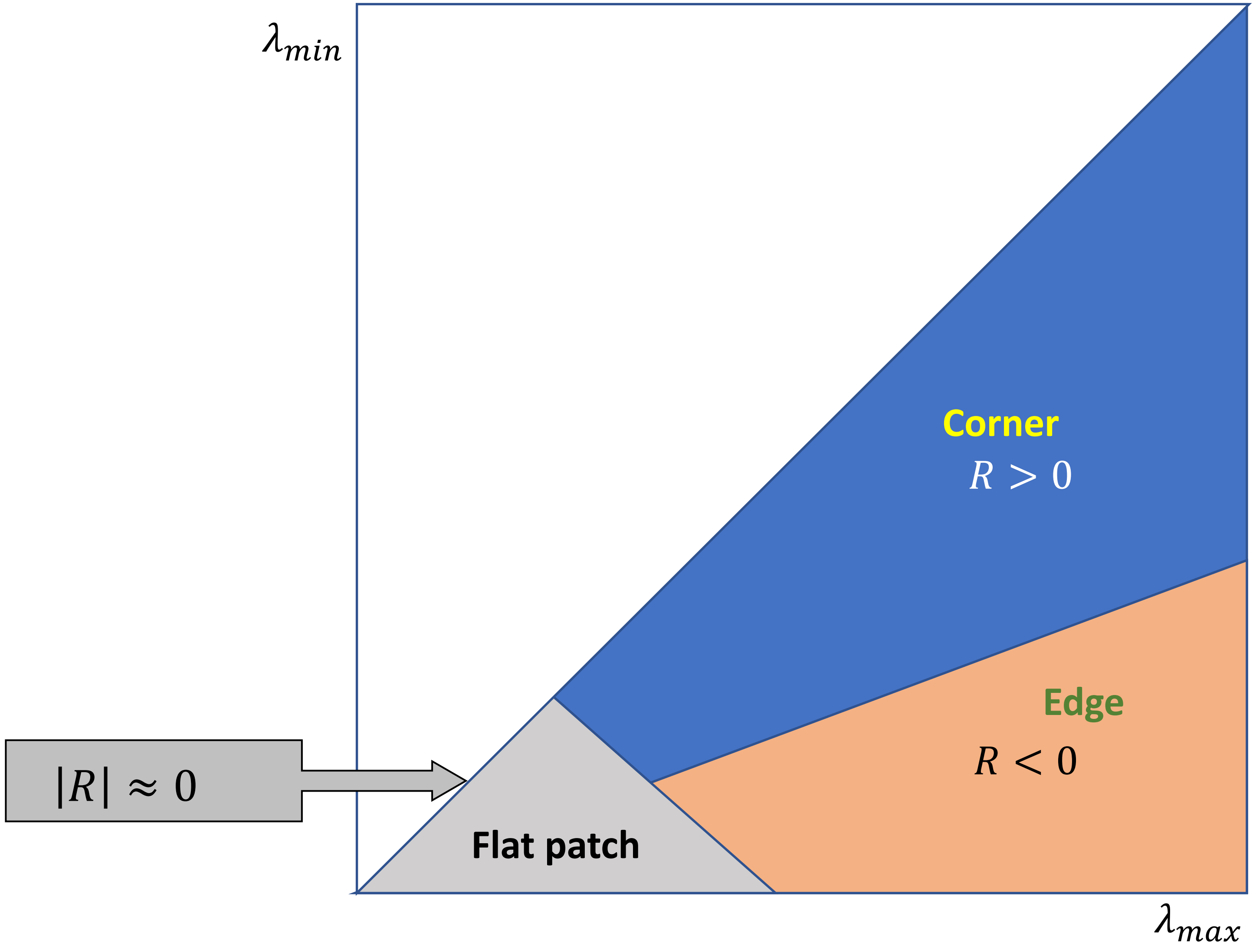
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

$$= \frac{determinant(H)}{trace(H)}$$

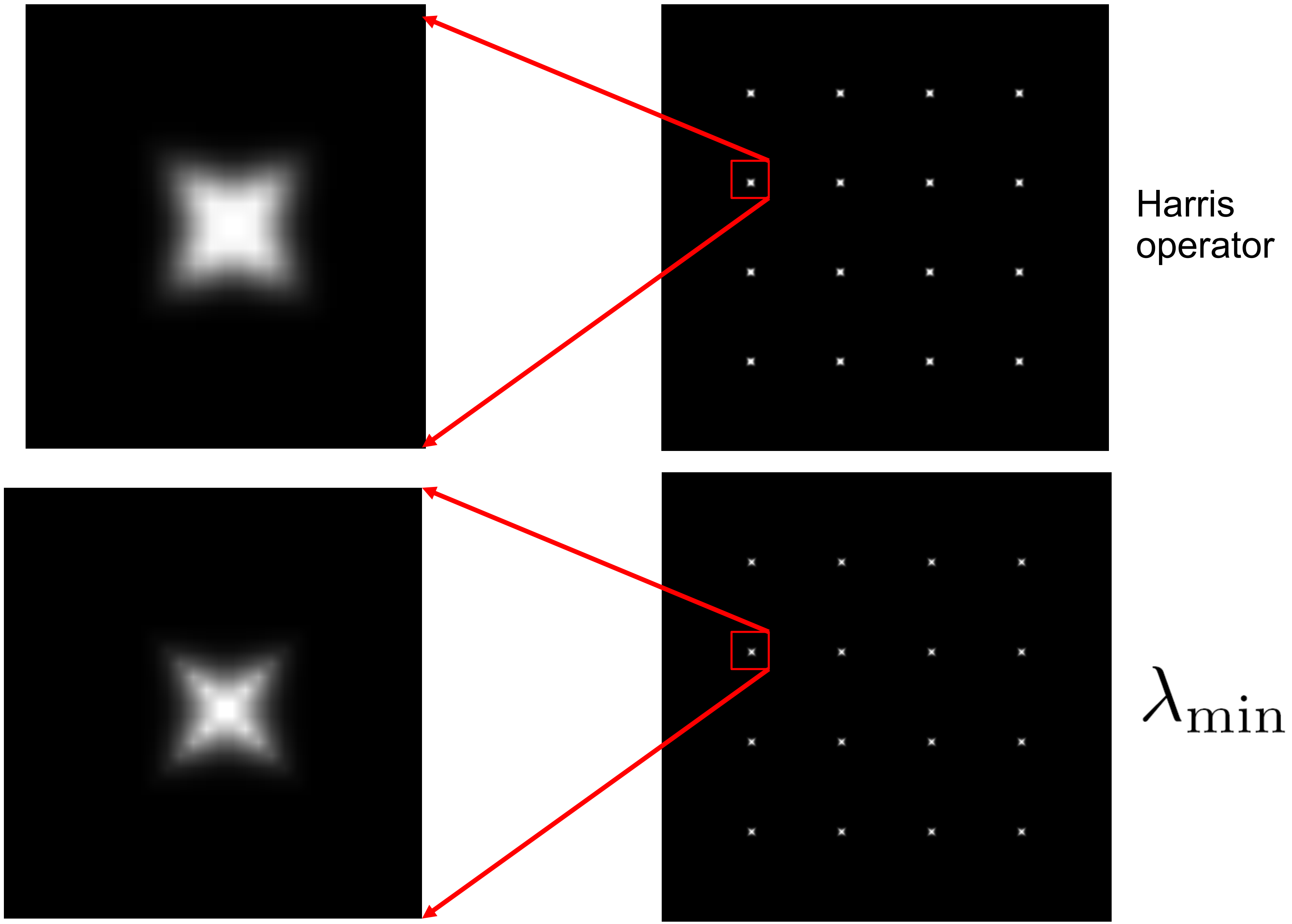
- The *trace* is the sum of the diagonals, i.e., $trace(H) = h_{11} + h_{22}$
- Very similar to λ_{min} but less expensive (no square root)
- Called the "Harris Corner Detector" or "Harris Operator"
  - Actually the Noble variant of the Harris Corner Detector
- Lots of other detectors, this is one of the most popular

Corner response function

$$R = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

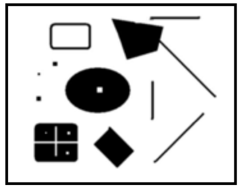


The Harris operator



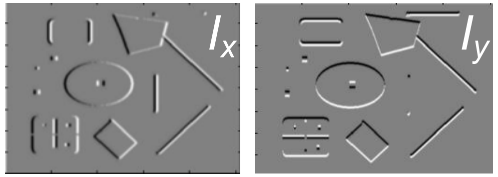
Harris Detector [Harris88]

- Second moment matrix



$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

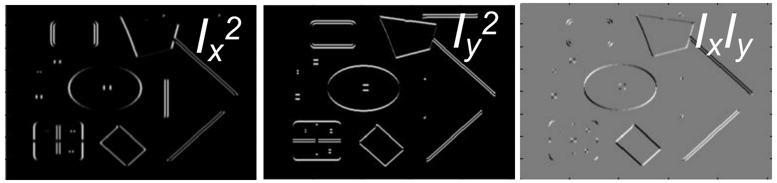
1. Image derivatives (optionally, blur first)



$$\det M = \lambda_1 \lambda_2$$

$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

2. Square of derivatives

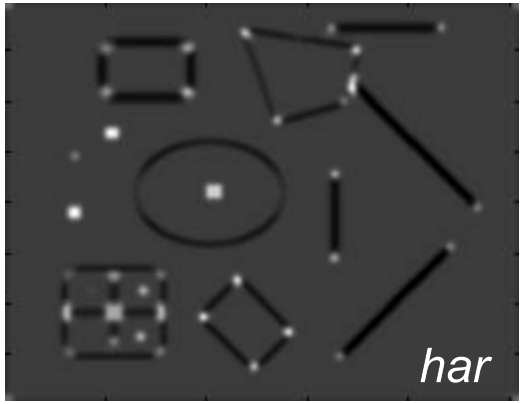


3. Gaussian filter $g(\sigma_I)$

4. Cornerness function – both eigenvalues are strong

$$har = \det[\mu(\sigma_I, \sigma_D)] - \alpha[\operatorname{trace}(\mu(\sigma_I, \sigma_D))^2] =$$

$$g(I_x^2)g(I_y^2) - [g(I_xI_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$



5. Non-maxima suppression

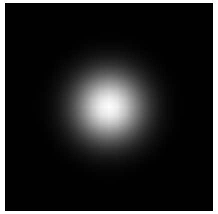
Weighting the derivatives

- In practice, using a simple window *W* doesn't work too well

$$H = \sum_{(x,y)\in W} \left[\begin{array}{cc} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{array} \right]$$

- Instead, we'll *weight* each derivative value based on its distance from the center pixel

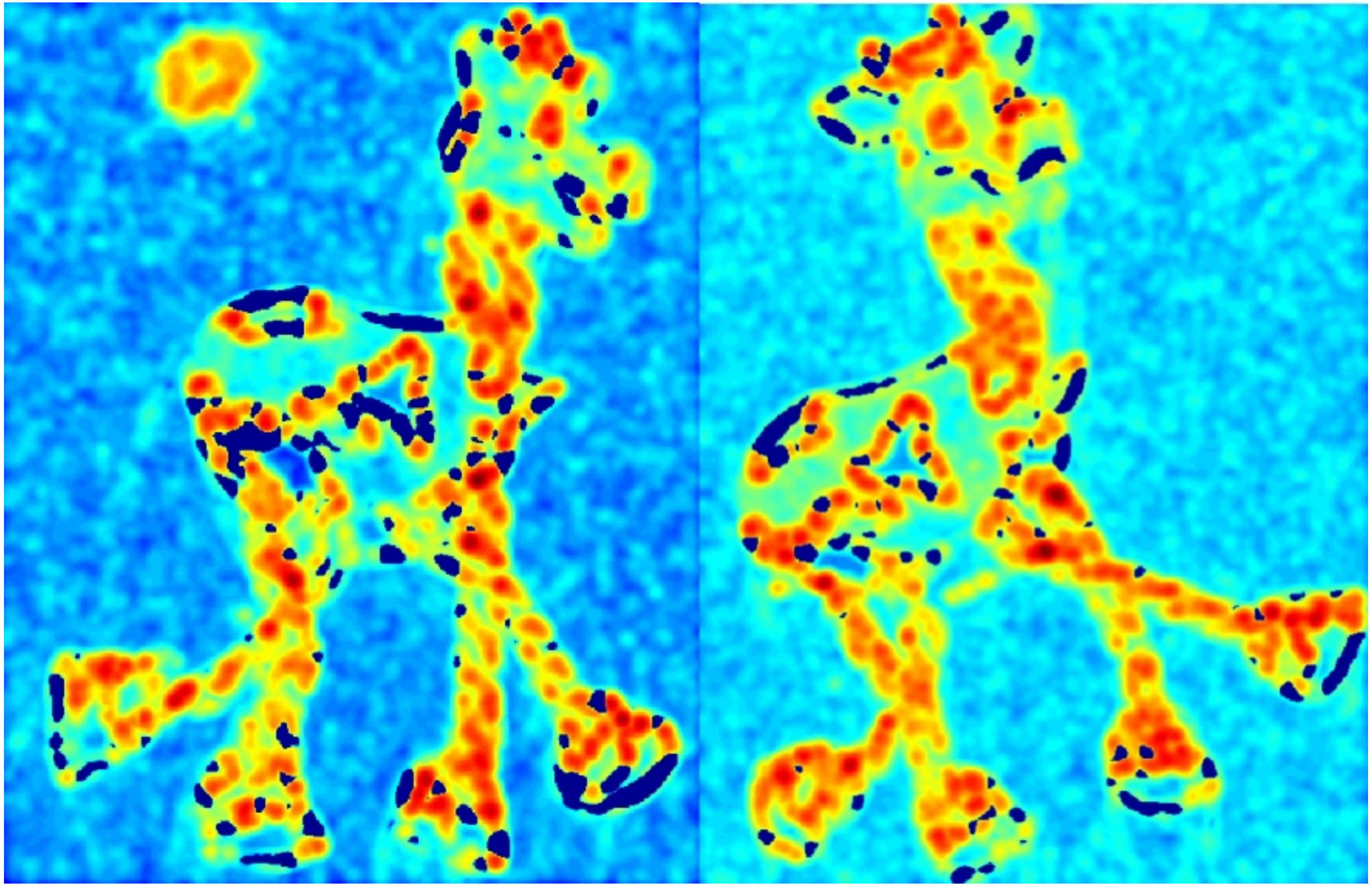
$$H = \sum_{(x,y)\in W} w_{x,y} \left[\begin{array}{cc} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{array} \right]$$



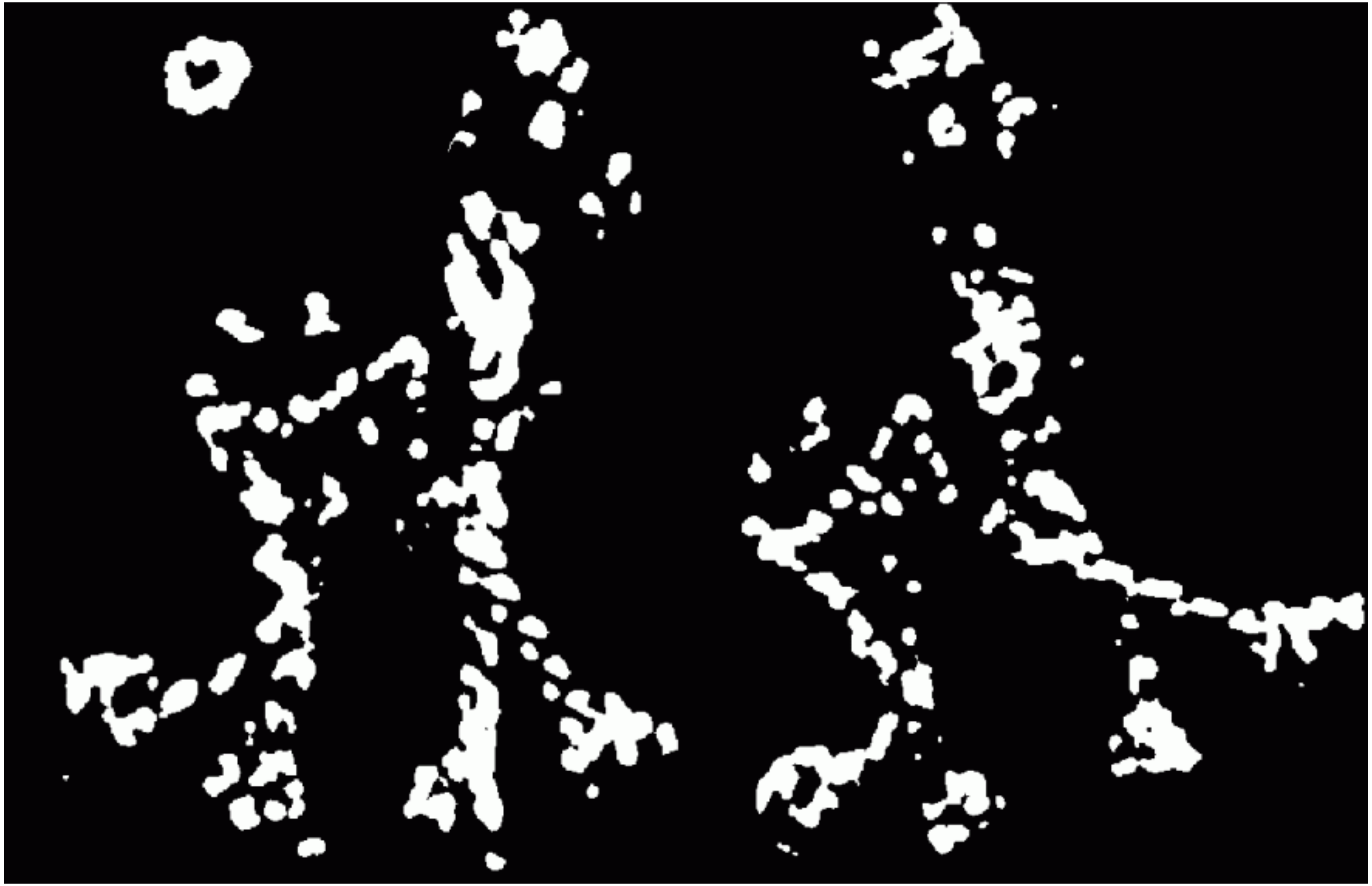
$w_{x,y}$

Harris detector example

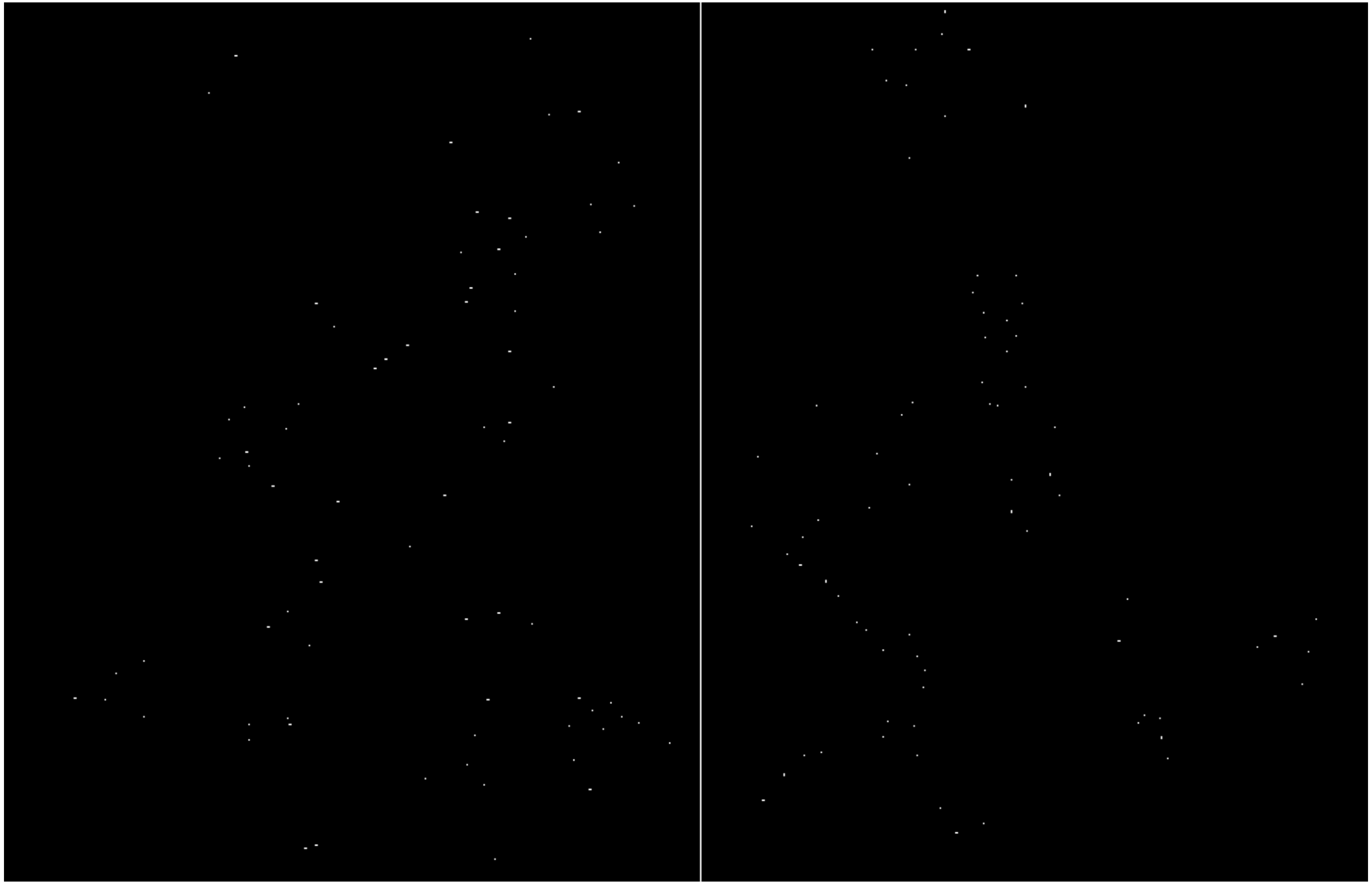
f value (red high, blue low)



Threshold (f > value)



Find local maxima of f



Harris features (in red)