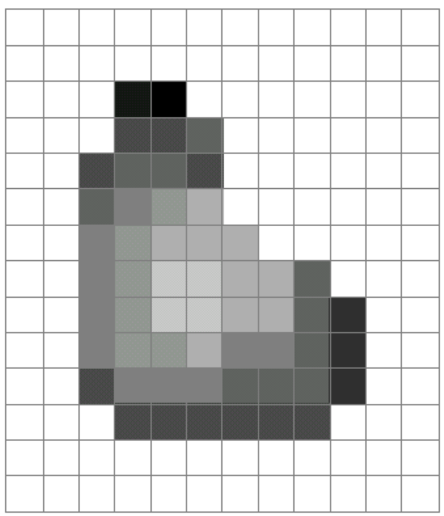
Lecture 2: Image filtering

What is an image?

- A grid (matrix) of intensity values



=

255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255
255	255	127	145	145	175	127	127	95	47	255	255
255	255	74	127	127	127	95	95	95	47	255	255
255	255	255	74	74	74	74	74	74	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

(common to use one byte per value: 0 = black, 255 = white)

Images as functions

- An image contains discrete numbers of pixels

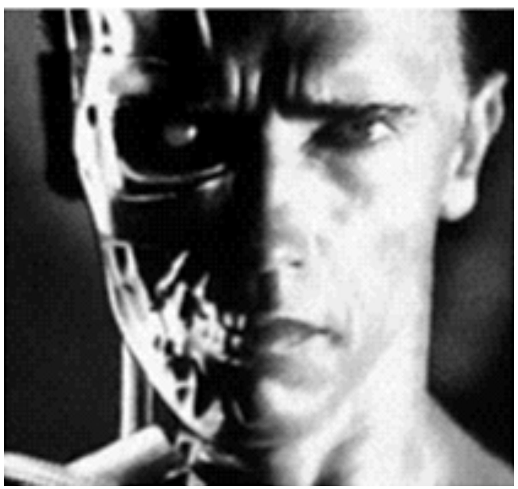
- Pixel value
  - grayscale/intensity
    - [0,255]
  - Color
    - RGB [R, G, B], where [0,255] per channel
    - Lab [L, a, b]: Lightness, a and b are color-opponent dimensions
    - HSV [H, S, V]: Hue, saturation, value

Images as functions

- Can think of image as a **function**, f, from R^2 to R or R^M:
  - Grayscale: $f(x,y)$ gives **intensity** at position (x,y)
    - f: [a,b] x [c,d] ➔[0,255]
  - Color: $f(x,y) = [r(x,y), g(x,y), b(x,y)]$

What is an image?

A **digital** image is a discrete (**sampled**, **quantized**) version of this function



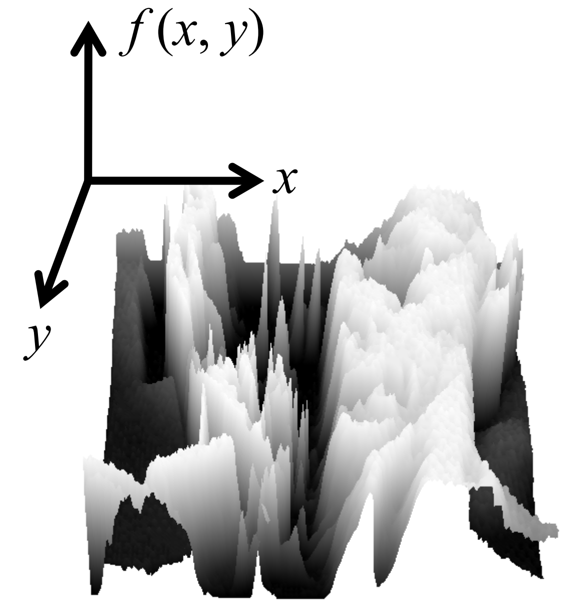
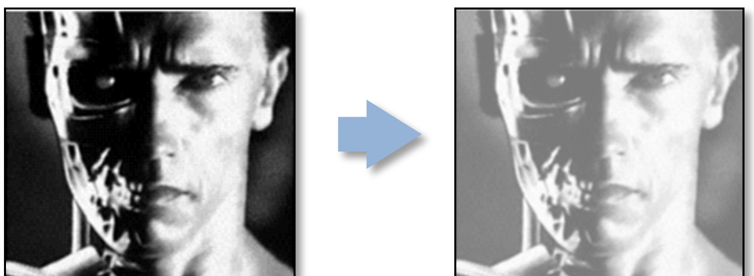
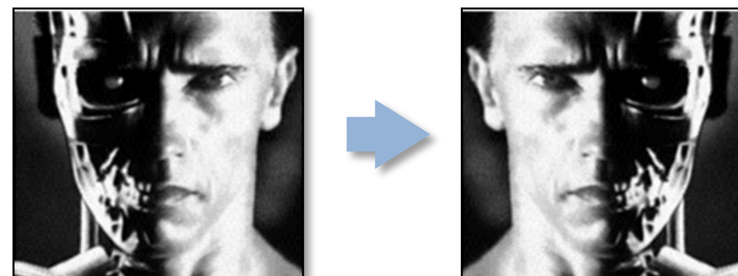


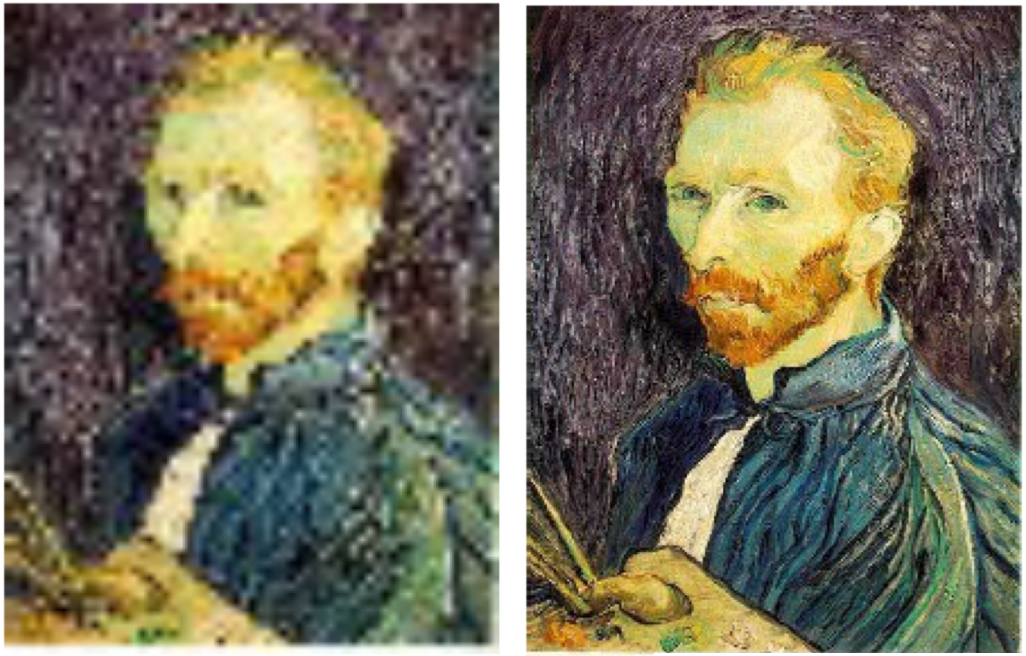
Image transformations



$g(x,y) = f(x,y) + 20$



$g(x,y) = f(-x,y)$



Super-resolution

Noise reduction

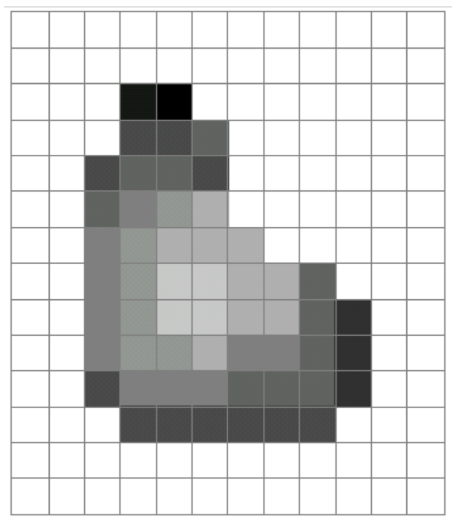
Image denoising

Why would images have noise?

- Sensor noise
  - Sensors count photons: noise in count
- Dead pixels
- Old photographs
- …

What is an image?

- A grid (matrix) of intensity values: 1 color or 3 colors



=

255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255
255	255	127	145	145	175	127	127	95	47	255	255
255	255	74	127	127	127	95	95	95	47	255	255
255	255	255	74	74	74	74	74	74	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

(common to use one byte per value: 0 = black, 255 = white)

An assumption about noise

- Let us assume noise at a pixel is
  - independent of other pixels
  - distributed according to a Gaussian distribution
    - i.e., low noise values are more likely than high noise values
    - “grainy images”

Noise reduction

- Nearby pixels are likely to belong to same object
  - thus likely to have similar color
- Replace each pixel by *average of neighbors*

Mean filtering

0	0	0	0	0	0	0	0	0	0
0	0	0	10	10	10	0	0	0	0
0	0	10	20	20	20	10	40	0	0
0	10	20	30	0	20	10	0	0	0
0	10	0	30	40	30	20	10	0	0
0	10	20	30	40	30	20	10	0	0
0	10	20	10	40	30	20	10	0	0
0	10	20	30	30	20	10	0	0	0
0	0	10	20	20	0	10	0	20	0
0	0	0	10	10	10	0	0	0	0

$$(0 + 0 + 0 + 10 + 40 + 0 + 10 + 0 + 0)/9 = 6.66$$

Mean filtering

0	0	0	0	0	0	0	0	0	0
0	0	0	10	10	10	0	0	0	0
0	0	10	20	20	20	10	40	0	0
0	10	20	30	0	20	10	0	0	0
0	10	0	30	40	30	20	10	0	0
0	10	20	30	40	30	20	10	0	0
0	10	20	10	40	30	20	10	0	0
0	10	20	30	30	20	10	0	0	0
0	0	10	20	20	0	10	0	20	0
0	0	0	10	10	10	0	0	0	0

(0 + 0 + 0 + 0 + 0 + 10 + 0 + 0 + 0 + 0 + 20 + 10 + 40 + 0 + 0 + 20 + 10 + 0 + 0 + 0 + 30 + 20 + 10 + 0 + 0)/25 = 6.8

Mean filtering

0	0	0	0	0	0	0	0	0	0
0	0	0	10	10	10	0	0	0	0
0	0	10	20	20	20	10	40	0	0
0	10	20	30	0	20	10	0	0	0
0	10	0	30	40	30	20	10	0	0
0	10	20	30	40	30	20	10	0	0
0	10	20	10	40	30	20	10	0	0
0	10	20	30	30	20	10	0	0	0
0	0	10	20	20	0	10	0	20	0
0	0	0	10	10	10	0	0	0	0

0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$(0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 10)/9 = 1.11$$

Mean filtering

0	0	0	0	0	0	0	0	0	0
0	0	0	10	10	10	0	0	0	0
0	0	10	20	20	20	10	40	0	0
0	10	20	30	0	20	10	0	0	0
0	10	0	30	40	30	20	10	0	0
0	10	20	30	40	30	20	10	0	0
0	10	20	10	40	30	20	10	0	0
0	10	20	30	30	20	10	0	0	0
0	0	10	20	20	0	10	0	20	0
0	0	0	10	10	10	0	0	0	0

0	0	0	0	0	0	0	0	0	0
0	1	4	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$(0 + 0 + 0 + 0 + 0 + 10 + 0 + 10 + 20)/9 = 4.44$$

Mean filtering

0	0	0	0	0	0	0	0	0	0
0	0	0	10	10	10	0	0	0	0
0	0	10	20	20	20	10	40	0	0
0	10	20	30	0	20	10	0	0	0
0	10	0	30	40	30	20	10	0	0
0	10	20	30	40	30	20	10	0	0
0	10	20	10	40	30	20	10	0	0
0	10	20	30	30	20	10	0	0	0
0	0	10	20	20	0	10	0	20	0
0	0	0	10	10	10	0	0	0	0

0	0	0	0	0	0	0	0	0	0
0	1	4	8	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

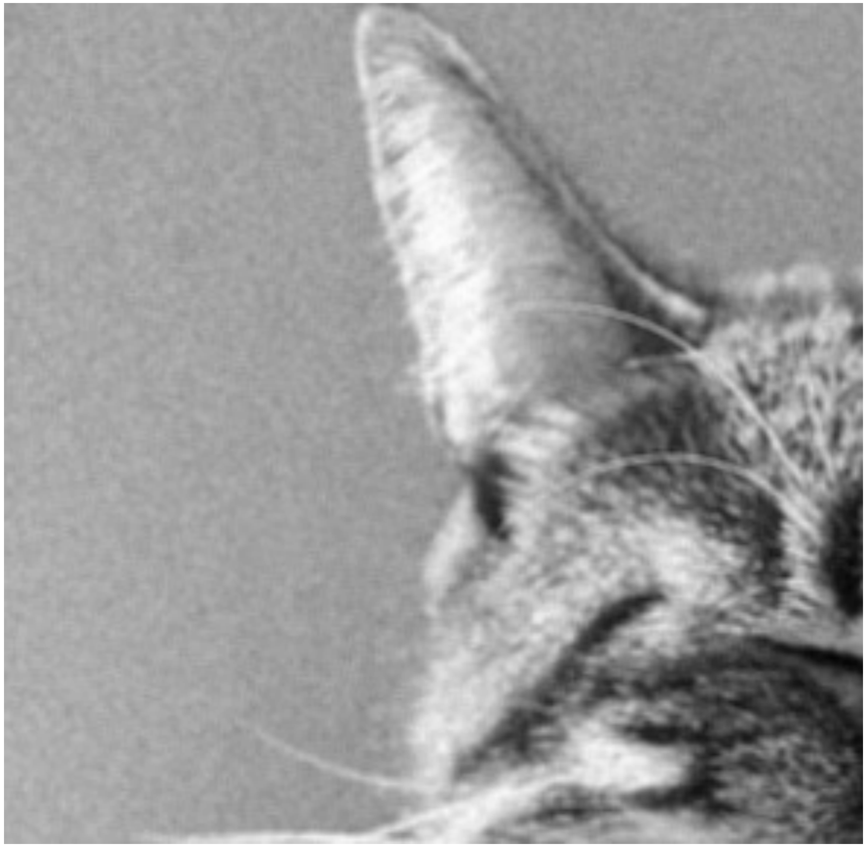
$$(0 + 0 + 0 + 0 + 10 + 10 + 10 + 20 + 20)/9 = 7.77$$

Mean filtering

0	0	0	0	0	0	0	0	0	0
0	0	0	10	10	10	0	0	0	0
0	0	10	20	20	20	10	40	0	0
0	10	20	30	0	20	10	0	0	0
0	10	0	30	40	30	20	10	0	0
0	10	20	30	40	30	20	10	0	0
0	10	20	10	40	30	20	10	0	0
0	10	20	30	30	20	10	0	0	0
0	0	10	20	20	0	10	0	20	0
0	0	0	10	10	10	0	0	0	0

0	0	0	0	0	0	0	0	0	0
0	1	4	8	10	8	9	6	4	0
0	4	11	13	16	11	12	7	4	0
0	6	14	19	23	19	18	10	6	0
0	8	18	23	28	23	17	8	2	0
0	8	16	26	31	30	20	10	3	0
0	10	18	27	29	27	17	8	2	0
0	8	14	22	22	20	11	8	3	0
0	4	11	17	17	12	6	4	2	0
0	0	0	0	0	0	0	0	0	0

Noise reduction using mean filtering



Filters

- Filtering
  - Form a new image whose pixels are a combination of the original pixels
- Why?
  - To get useful information from images
    - E.g., extract edges or contours (to understand shape)
  - To enhance the image
    - E.g., to blur to remove noise
    - E.g., to sharpen to “enhance image” a la CSI

Mean filtering

- Replace pixel by mean of neighborhood

10	5	3
4	5	1
1	1	7

Local image data

f

	4.1	

Modified image data

$S[f]$

$$S[f](m,n) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} f(m+i, n+j)/9$$

A more general version

10	5	3
4	5	1
1	1	7

Local image data

	7	

Kernel / filter

$$S[f](m,n) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} \textcolor{red}{w(i,j)} f(m+i, n+j)$$

A more general version

0	10	5	7	0
5	11	6	8	3
9	22	4	5	1
2	9	14	6	7
3	10	15	12	9

Local image data

→

		7		

Kernel size = 2k+1

$$S[f](m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

A more general version

$$S[f](m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

- w(i,j) = $1/(2k+1)^2$ for mean filter
- If w(i,j) >=0 and sum to 1, *weighted mean*
- But w(i,j) can be *arbitrary real numbers!*

Properties: Linearity

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$f'(m,n) = af(m,n)$$

$$(w \otimes f')(m,n) = a(w \otimes f)(m,n)$$

Properties: Linearity

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$f' = af$$

$$(w \otimes f') = a(w \otimes f)$$

Properties: Linearity

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$f' = af + bg$$

$$w \otimes f' = a(w \otimes f) + b(w \otimes g)$$

Properties: Linearity

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$w' = aw + bv$$

$$w' \otimes f = a(w \otimes f) + b(v \otimes f)$$

Properties: Shift invariance

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$f'(m,n) = f(m - m_0, n - n_0)$$

f

f'

Shift invariance

$$(w \otimes f)(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

$$f'(m,n) = f(m-m_0, n-n_0)$$

$$(w \otimes f')(m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f'(m+i, n+j)$$

$$= \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i-m_0, n+j-n_0)$$

$$= (w \otimes f)(m-m_0, n-n_0)$$

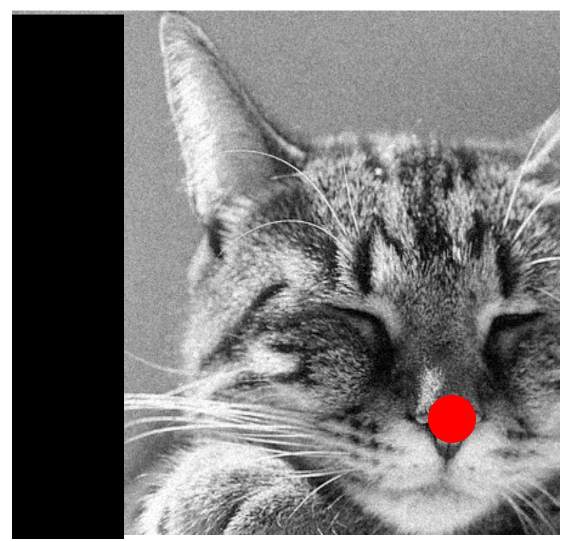
Shift invariance

$$f'(m,n) = f(m-m_0, n-n_0)$$

$$(w \otimes f')(m,n) = (w \otimes f)(m-m_0, n-n_0)$$

- Shift, then convolve = convolve, then shift
- Output of convolution does not depend on where the pixel is

f



f'

Convolution and cross-correlation

- Cross correlation

$$S[f] = w \otimes f$$

$$S[f](m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m+i, n+j)$$

- Convolution

$$S[f] = w * f$$

$$S[f](m,n) = \sum_{i=-k}^{k} \sum_{j=-k}^{k} w(i,j) f(m-i, n-j)$$

Cross-correlation

1	2	3
4	5	6
7	8	9

w

1	2	3
4	5	6
7	8	9

f

1*1 + 2*2 + 3*3 + 4*4 + 5*5 + 6*6 + 7*7 + 8*8 + 9*9

Convolution

1	2	3
4	5	6
7	8	9

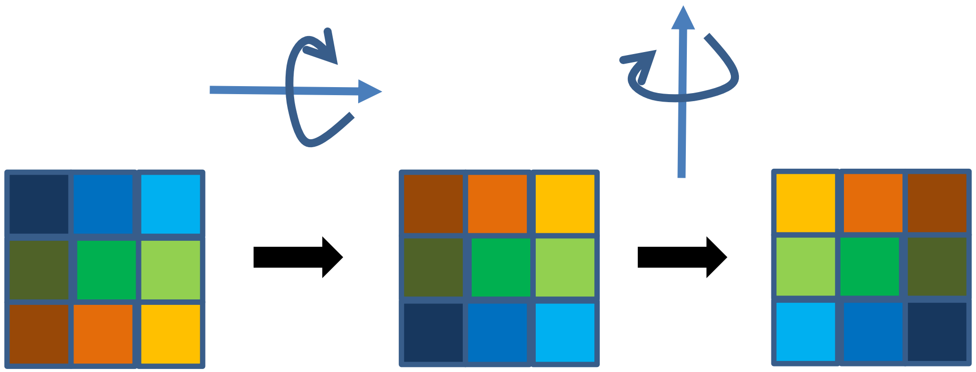
w

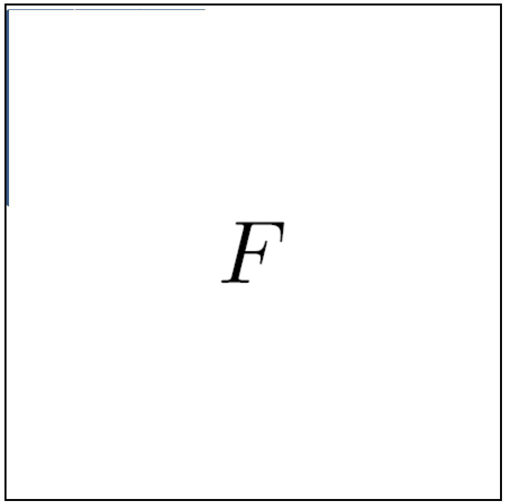
1	2	3
4	5	6
7	8	9

f

$$1*9 + 2*8 + 3*7 + 4*6 + 5*5 + 6*4 + 7*3 + 8*2 + 9*1$$

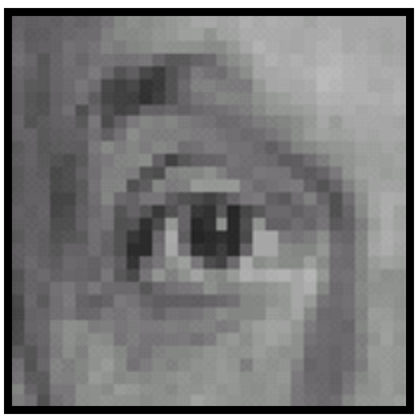
Convolution





Adapted from F. Durand

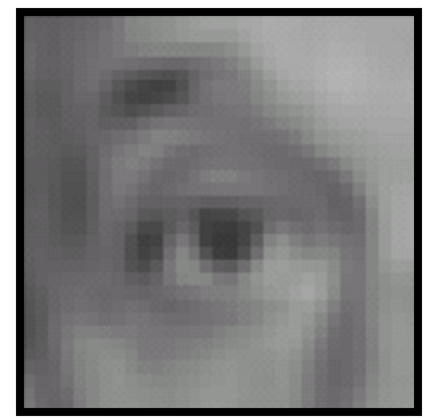
Filters: examples



Original (f)

$$* \ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} \ =$$

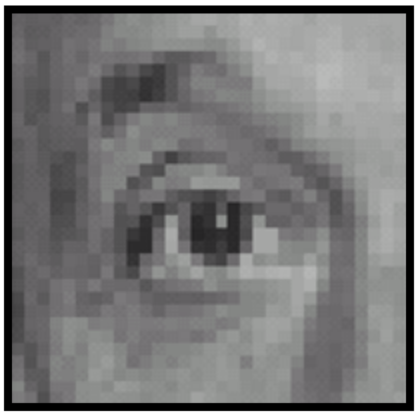
Kernel (k)



Blur (with a mean filter) (g)

Source: D. Lowe

Filters: examples



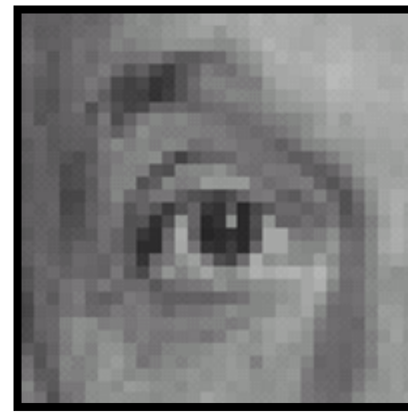
Original (f)

*

0	0	0
0	1	0
0	0	0

Kernel (k)

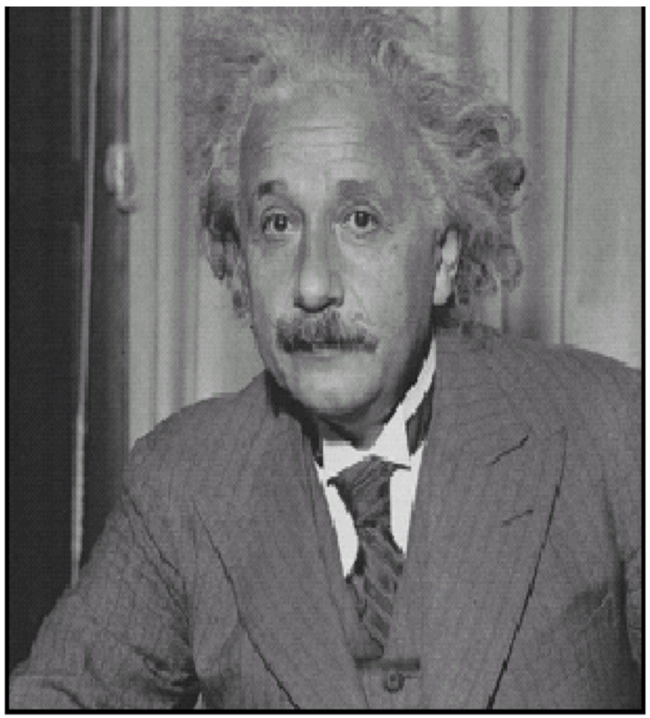
=



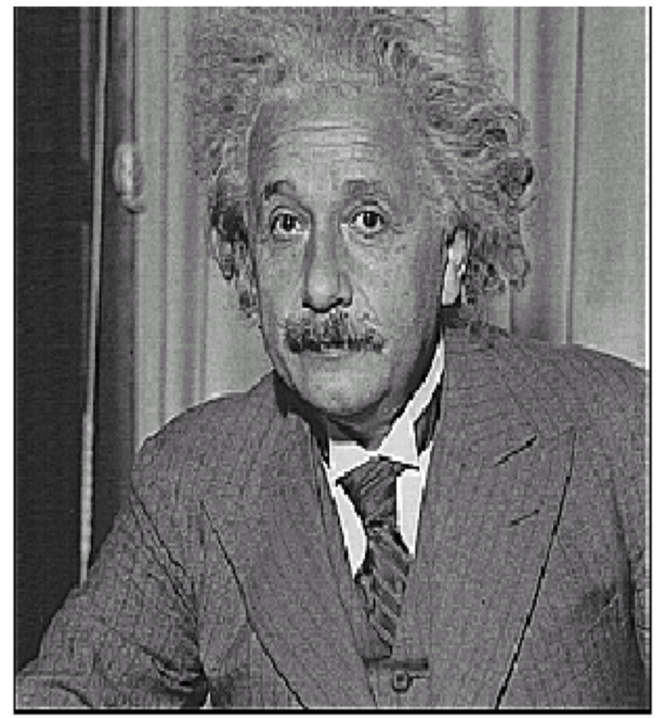
Identical image (g)

Source: D. Lowe

Sharpening



before

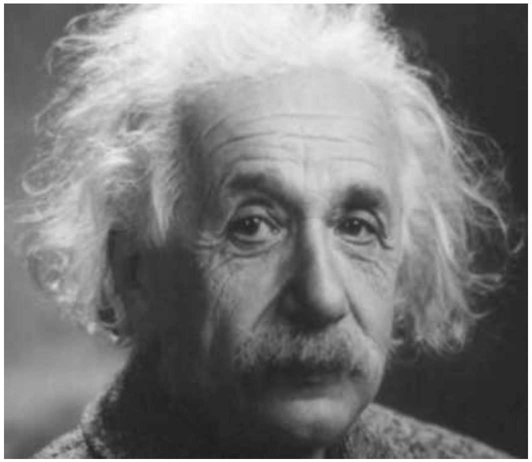


after

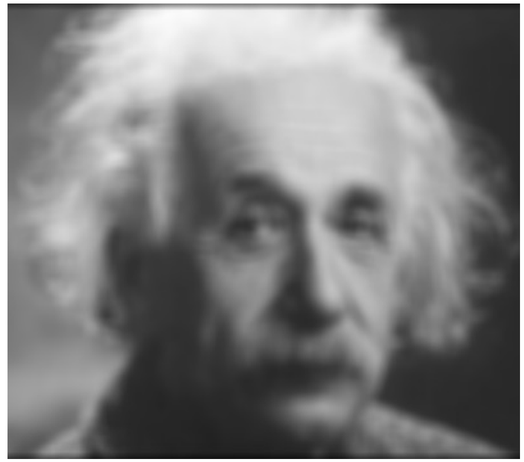
Source: D. Lowe

Sharpening

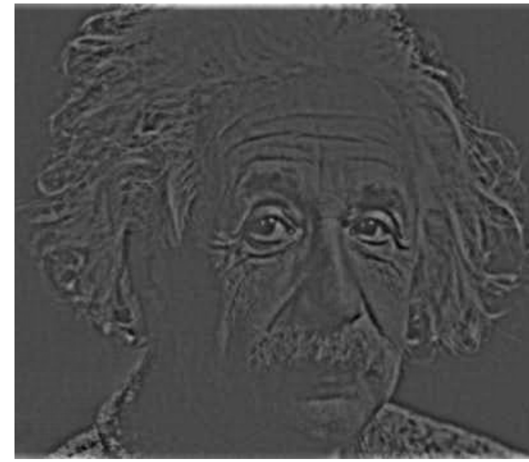
- What does blurring take away?



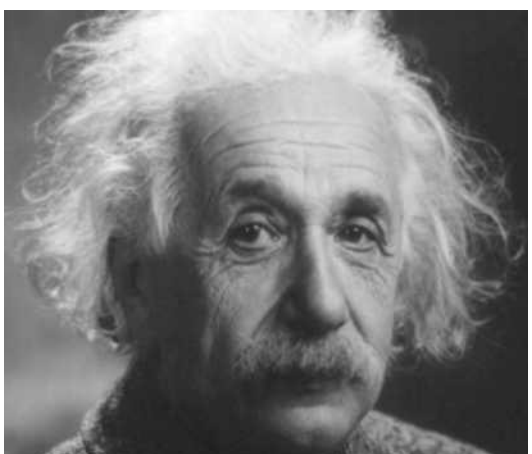
–



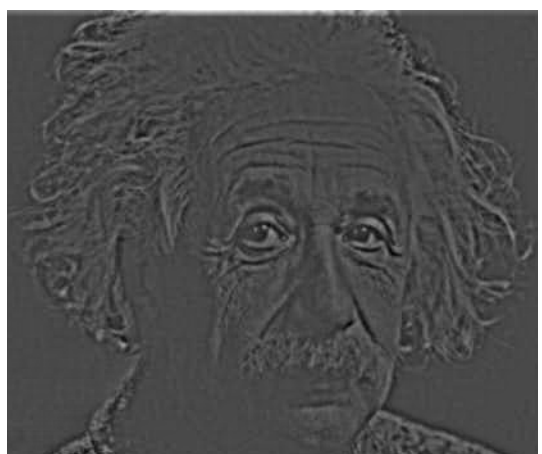
=



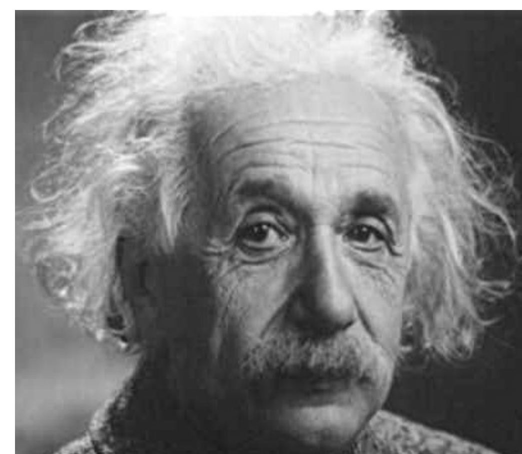
Let's add it back:



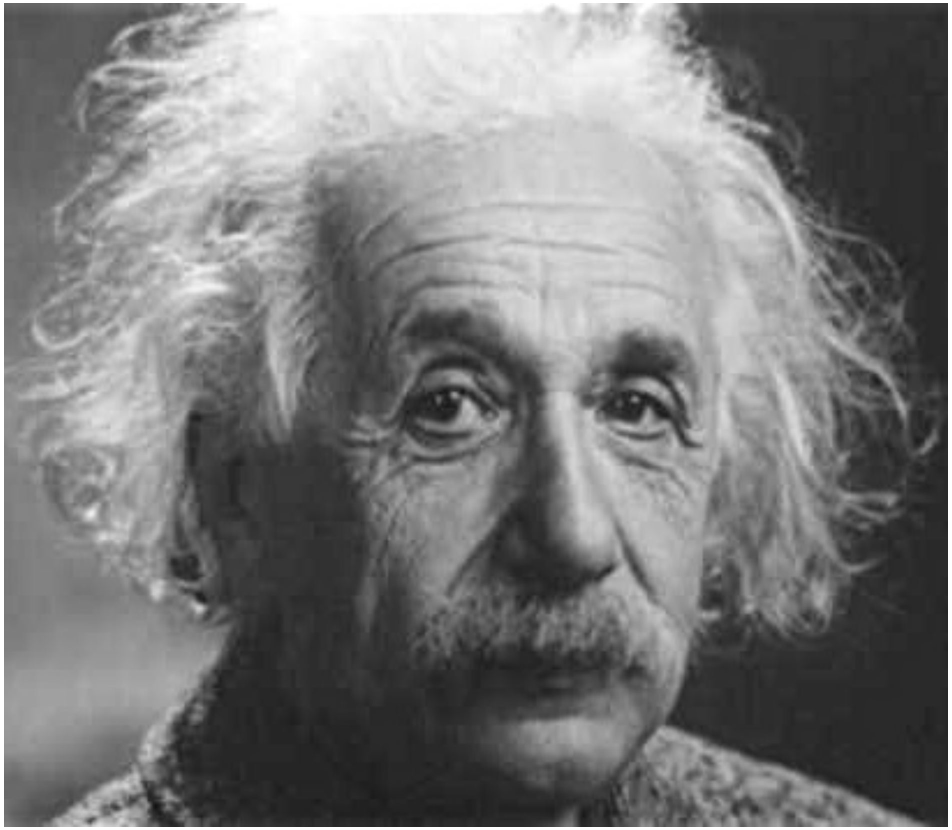
+ α



=

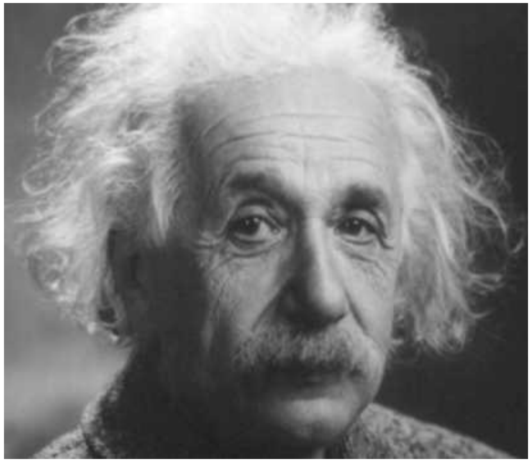


Sharpening

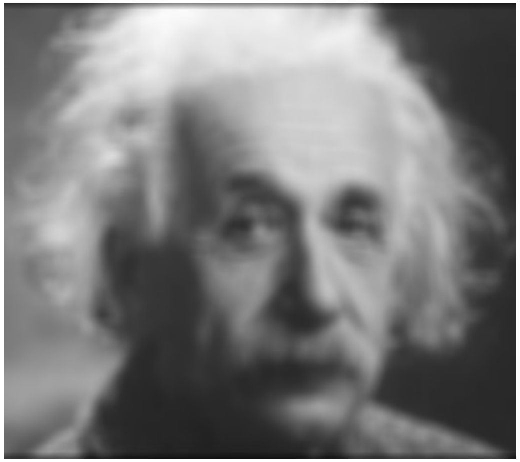


Sharpening

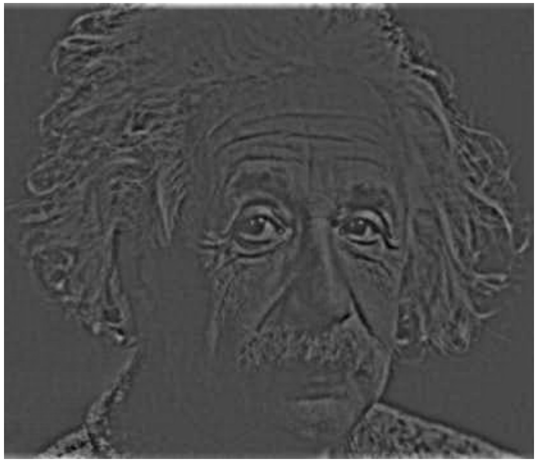
- What does blurring take away?



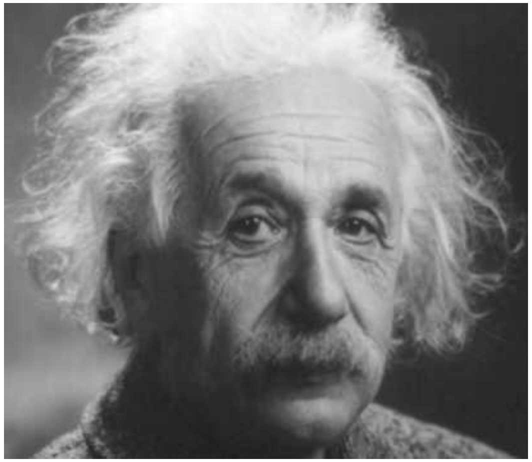
–



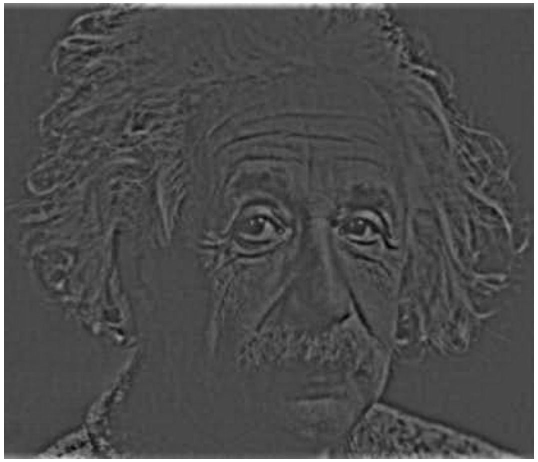
=



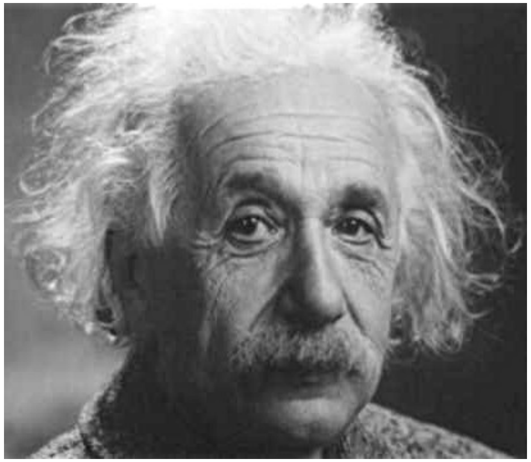
Let’s add it back:



+ α



=



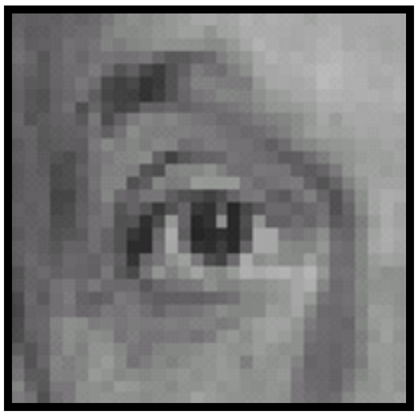
Sharpening

$$
\begin{aligned}
f_{sharp} &= f + \alpha(f - f_{blur}) \\
&= (1+\alpha)f - \alpha f_{blur} \\
&= (1+\alpha)(w * f) - \alpha(v * f)
\end{aligned}
$$

$$
w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad v = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}
$$

$$
= ((1+\alpha)w - \alpha v) * f
$$

Sharpening filter



Original

$$* \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$

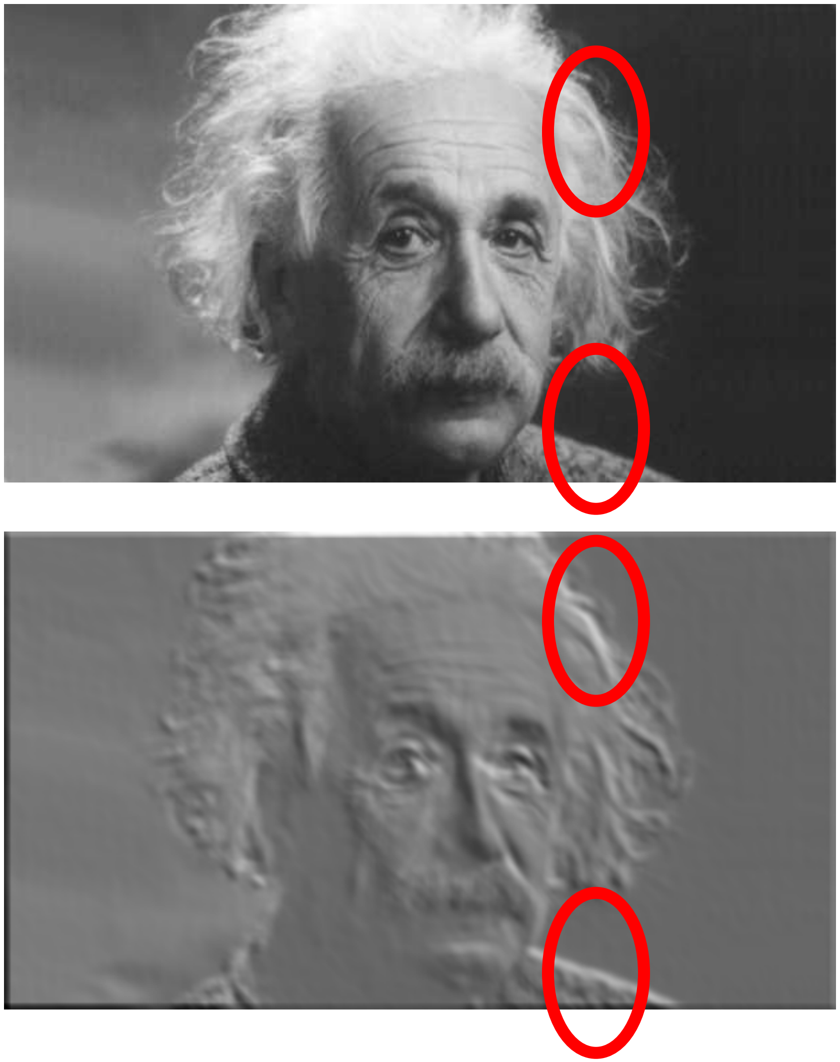
Sharpening filter
(accentuates edges)

Source: D. Lowe

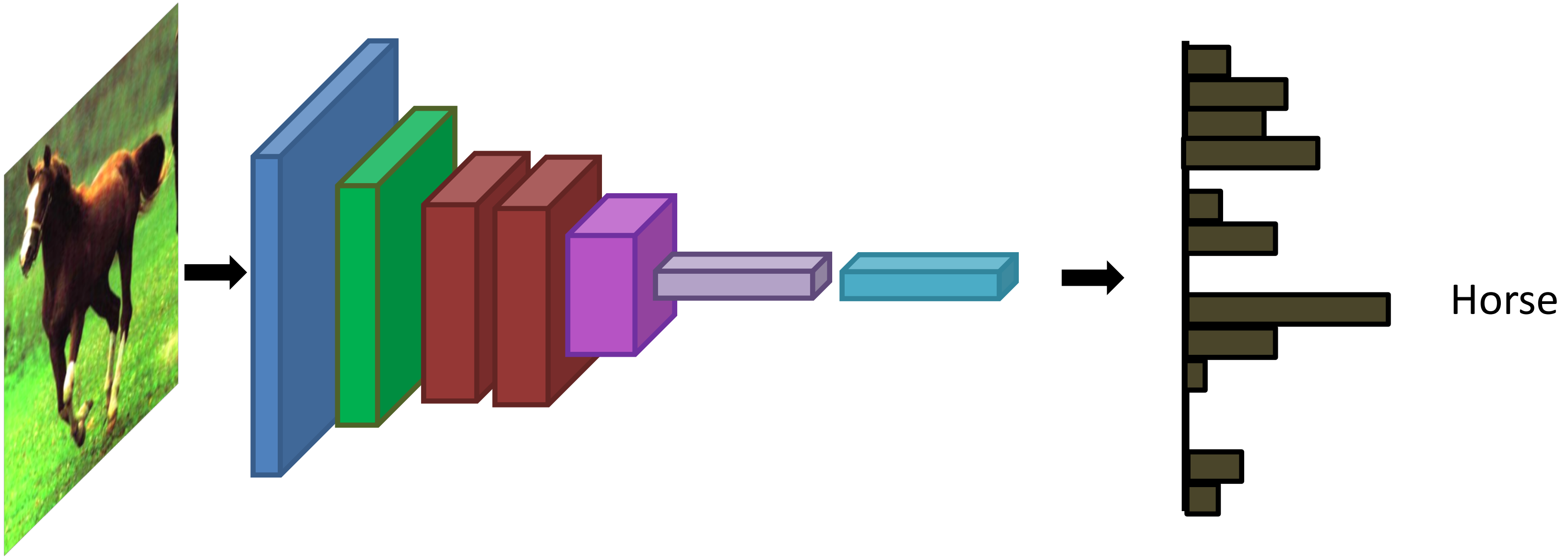
Another example

-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	1	-1	-1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1	-1
1	1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	1	1	-1	-1	-1	-1
1	1	1	1	1	1	1	1	-1	-1	-1
1	1	1	1	1	1	1	1	1	-1	-1
1	1	1	1	1	1	1	1	1	1	-1

Another example

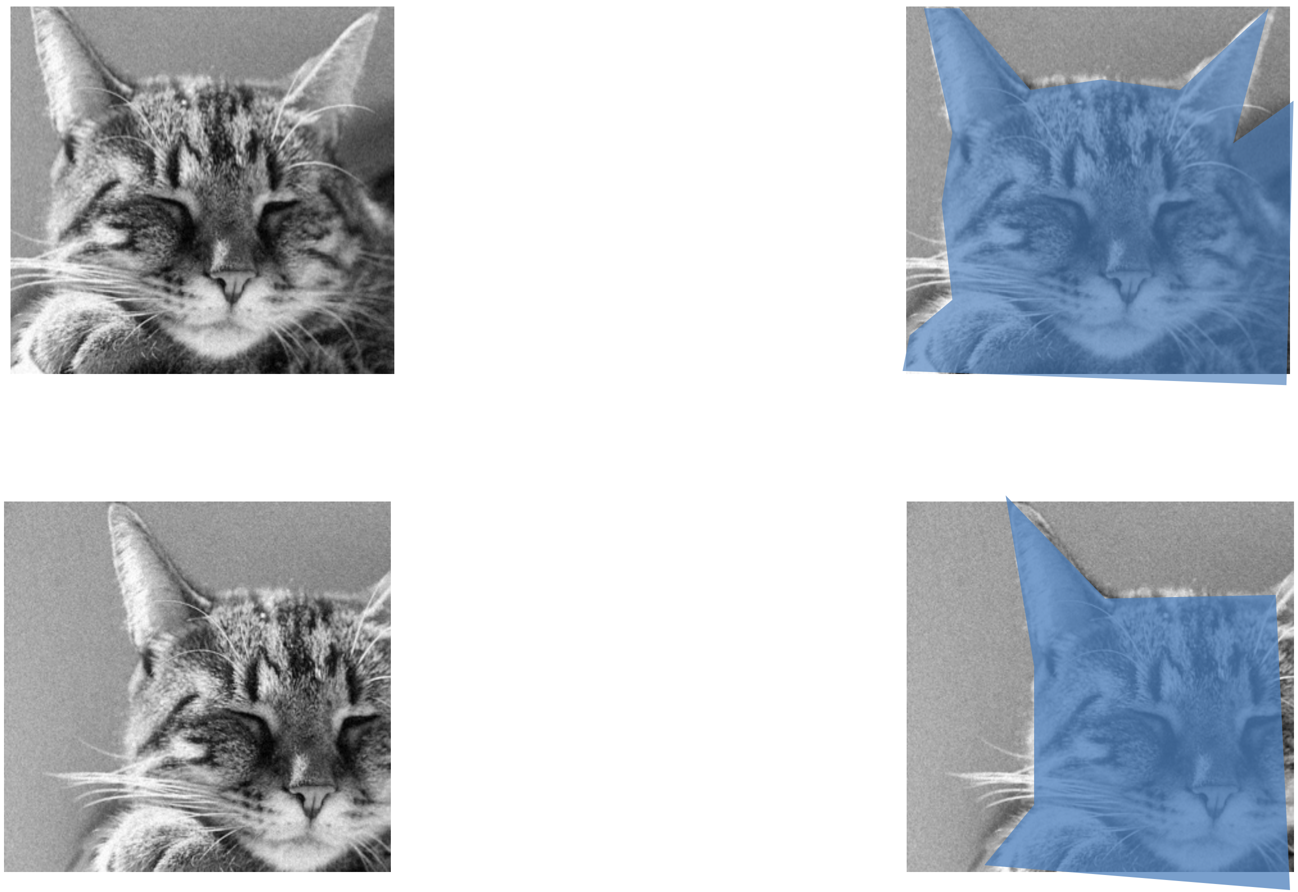


Convolution is everywhere



Why is convolution important?

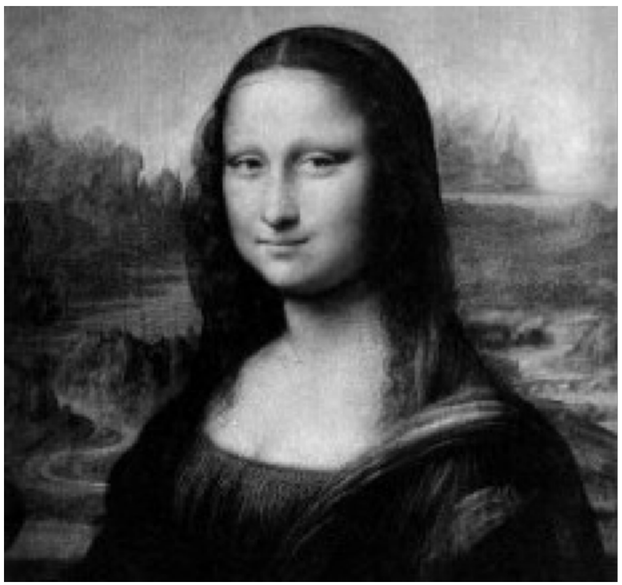
- Shift invariance is a crucial property



Why is convolution important?

- We *like* linearity
  - Linear functions behave predictably when input changes
  - Lots of theory just easier with linear functions
- *All linear shift-invariant systems can be expressed as a convolution*

Non-linear filters: Thresholding



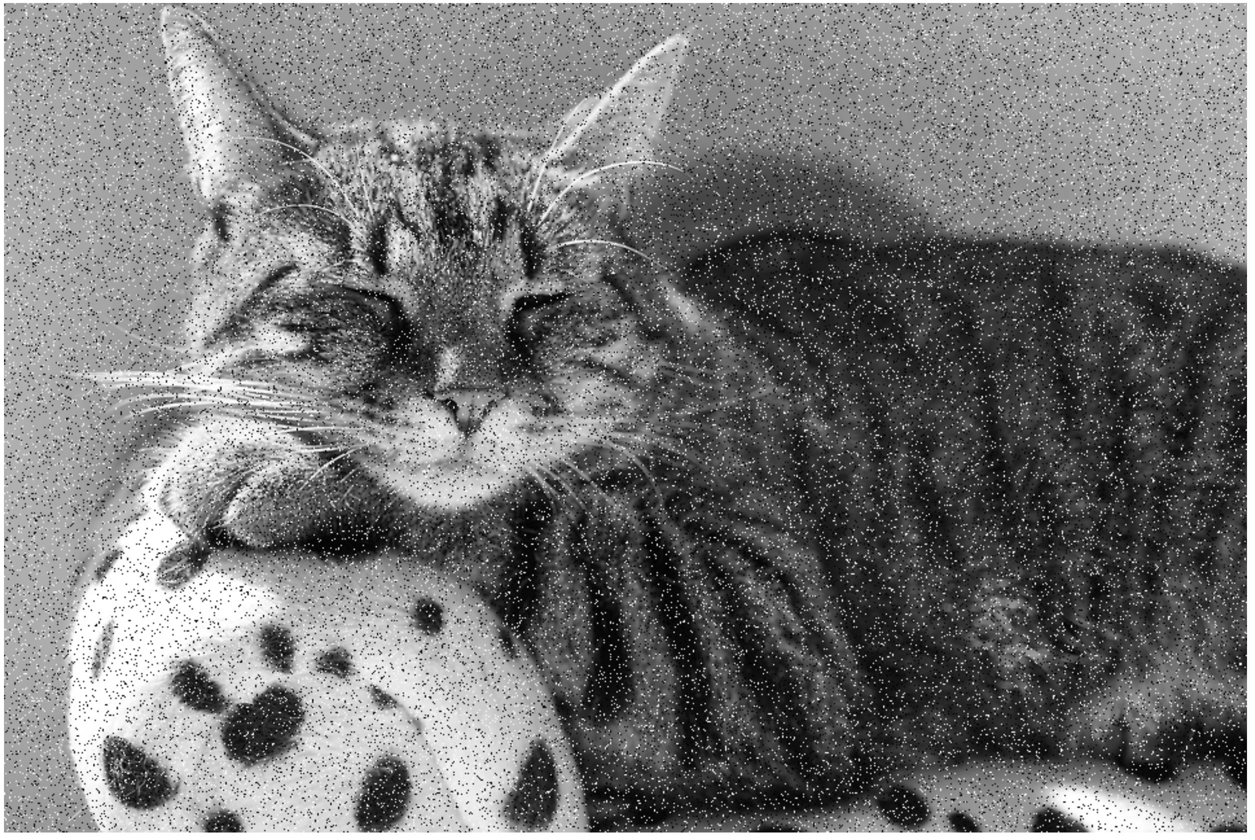
$$g(m,n) = \begin{cases} 255, & f(m,n) > A \\ 0 & otherwise \end{cases}$$

Non-linear filters: Rectification

- $g(m,n) = \max(f(m,n), 0)$
- Crucial component of modern convolutional networks

Non-linear filters

- Sometimes mean filtering does not work



Non-linear filters

- Sometimes mean filtering does not work

Non-linear filters

- Mean is sensitive to outliers
- Median filter: Replace pixel by *median* of neighbors

Non-linear filters

Takeaway

- Two general recipes:
  - convolution
  - cross-correlation
- Properties
  - Shift-invariant: a sensible thing to require
  - Linearity: convenient
- Can be used for smoothing, sharpening
- Also main component of CNNs

Next up

- Back to linear filters
- Signal processing view of filtering
- Filtering for detecting edges etc