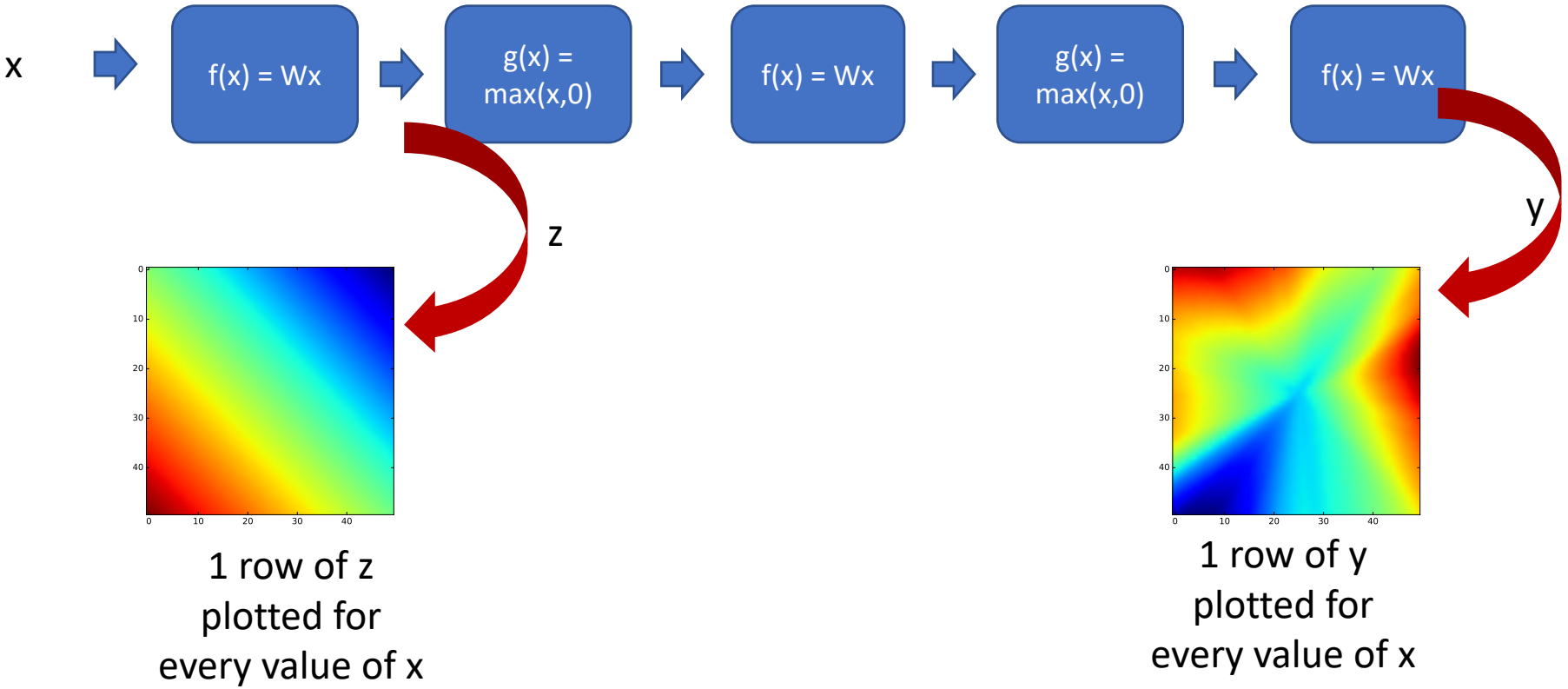
Convolutional network training

Multilayer perceptrons

- Key idea: build complex functions by composing simple functions



Linear function + translation invariance = *convolution*

- Local connectivity determines kernel size

5.4	0.1	3.6
1.8	2.3	4.5
1.1	3.4	7.2

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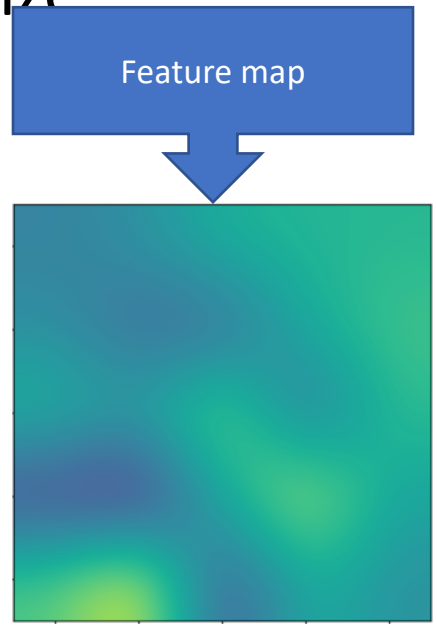
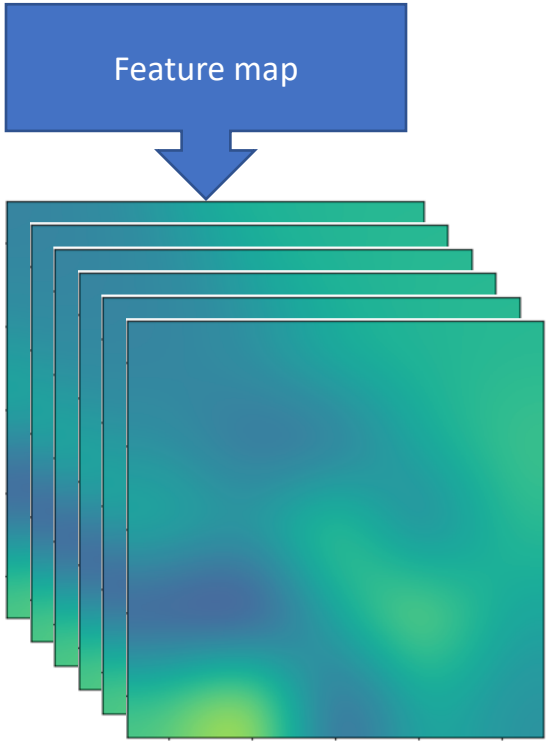
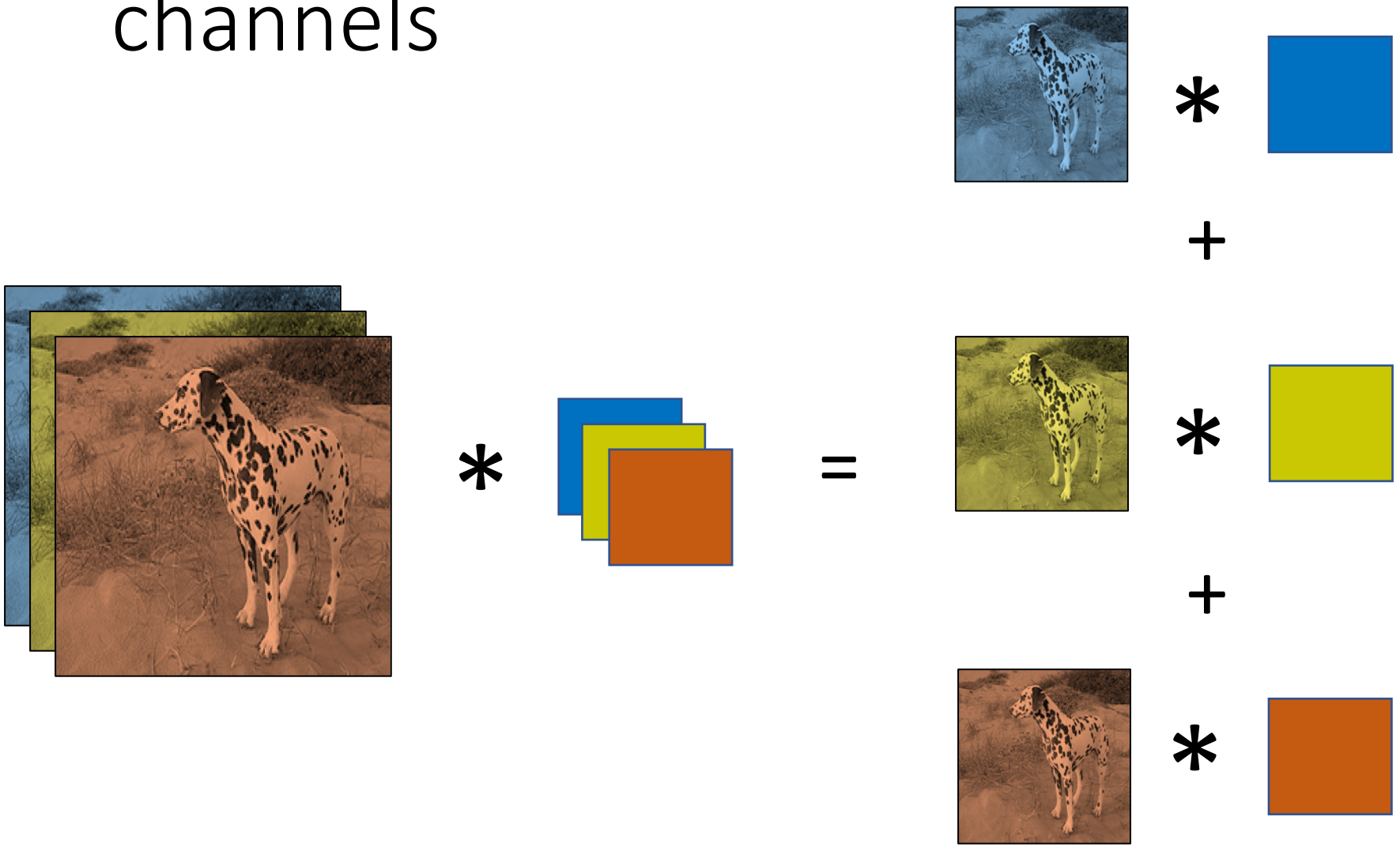


Convolution with multiple filters

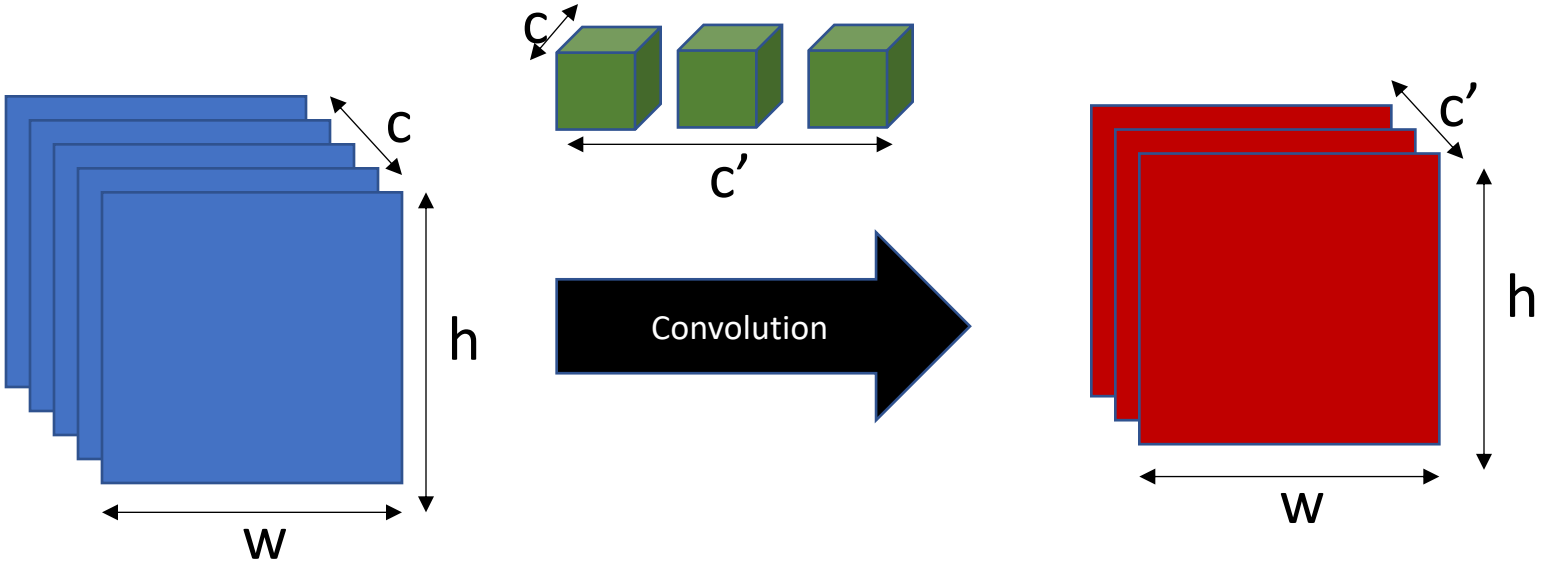
5.4	0.1	3.6
1.8	2.3	4.5
1.1	3.4	7.2



Convolution over multiple channels



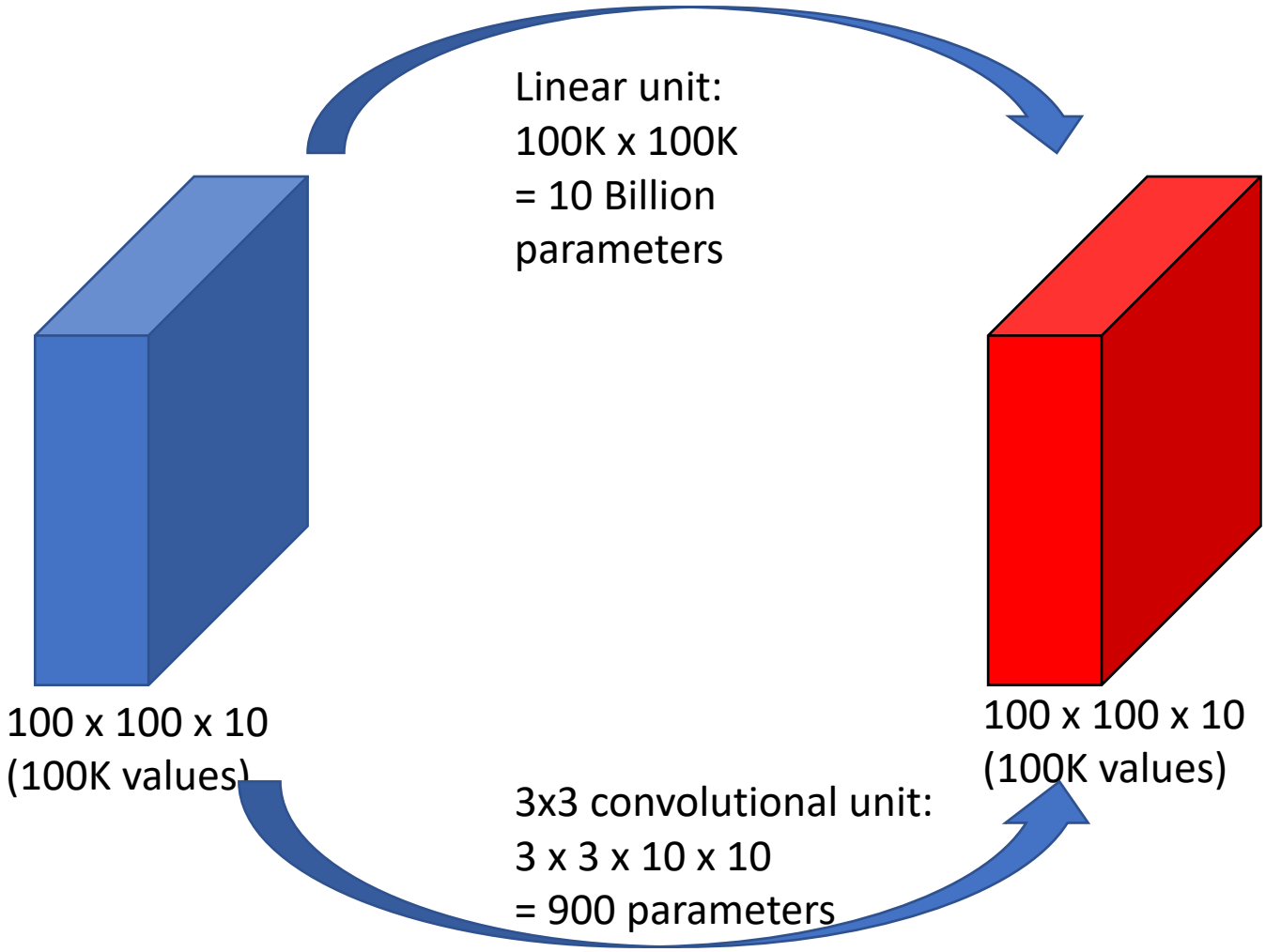
Convolution as a primitive



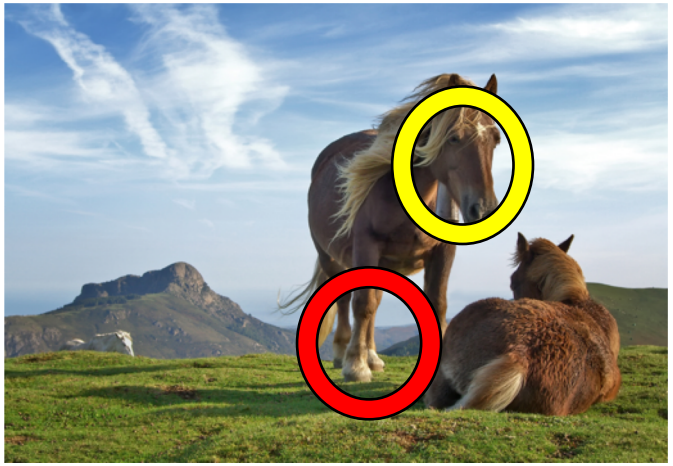
The convolution unit

- Each convolutional unit takes *a collection of feature maps* as input, and produces *a collection of feature maps* as output
- Parameters: Filters (+bias)
- If c_{in} input feature maps and c_{out} output feature maps
  - Each filter is k x k x c_{in}
  - There are c_{out} such filters
- Other hyperparameters: padding

Convolution vs Linear unit

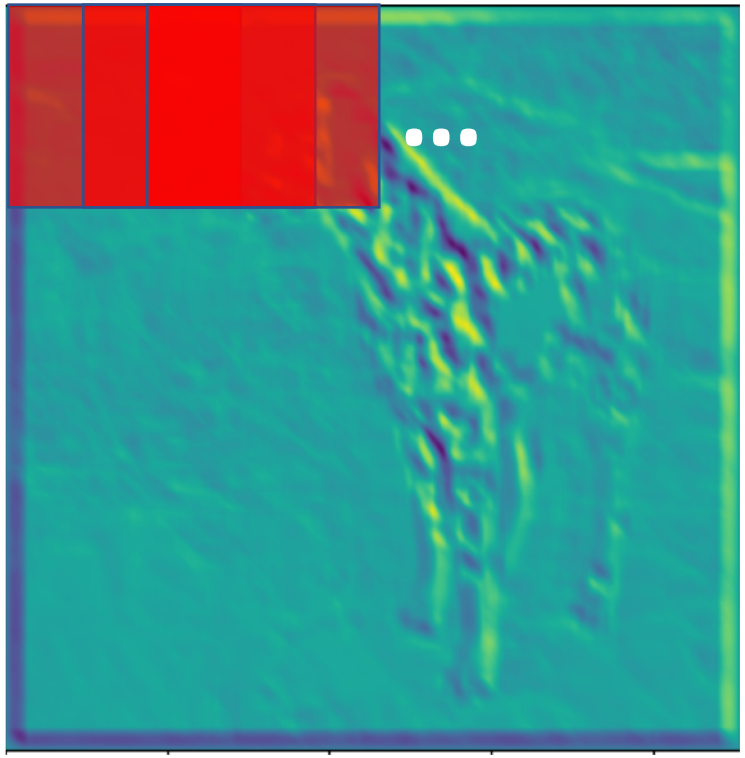


Invariance to distortions

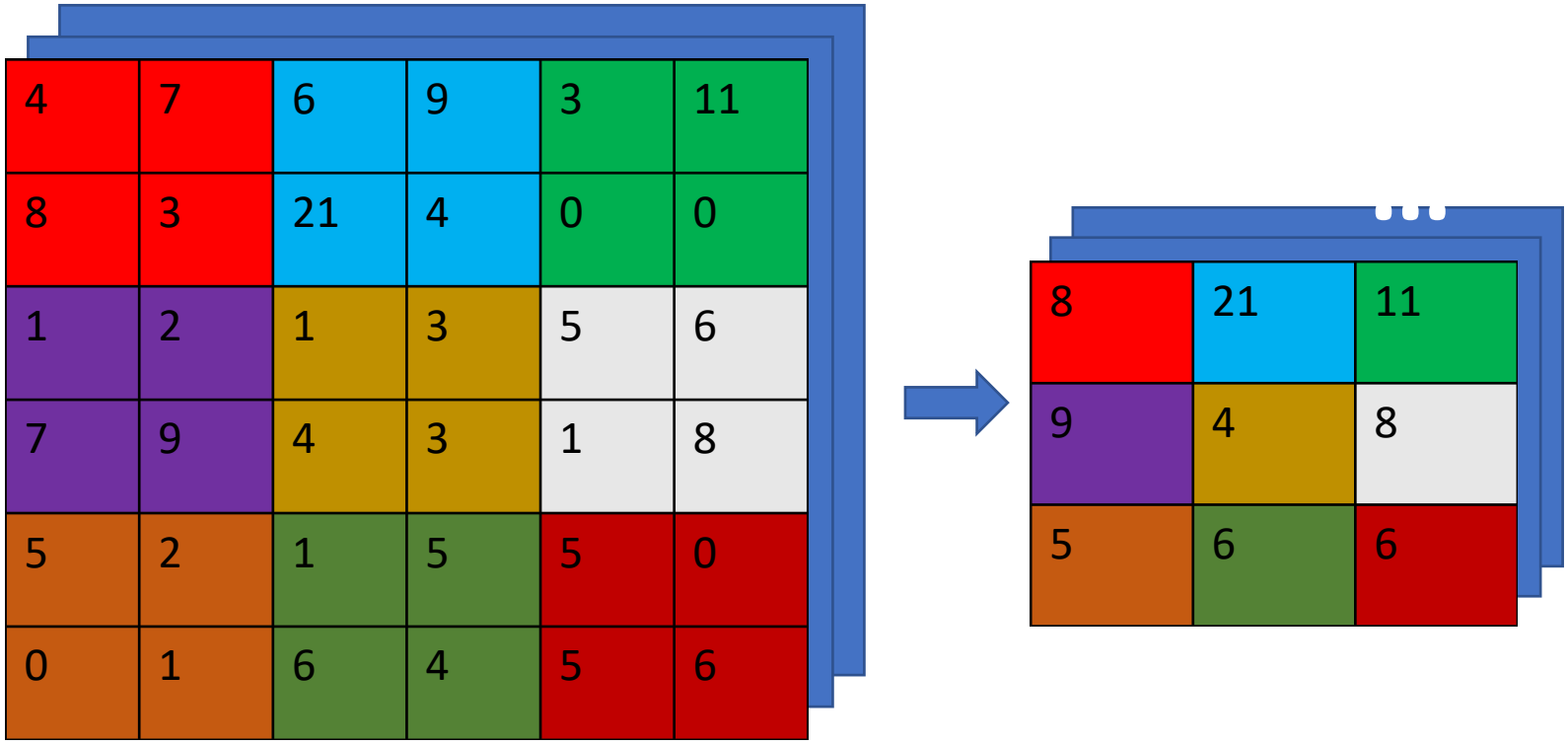


Invariance to distortions: Pooling

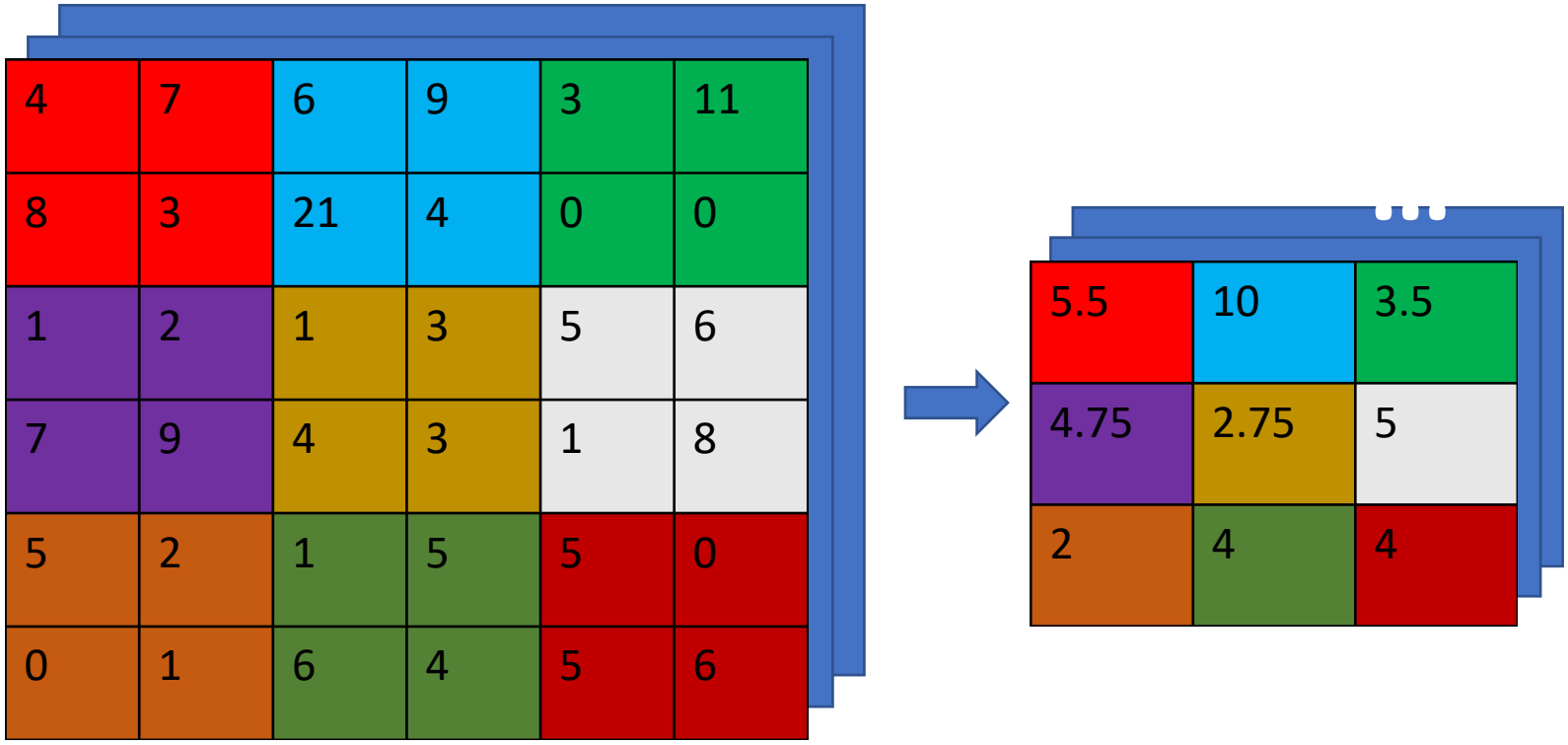
- Each window is reduced to a single value, example max or average



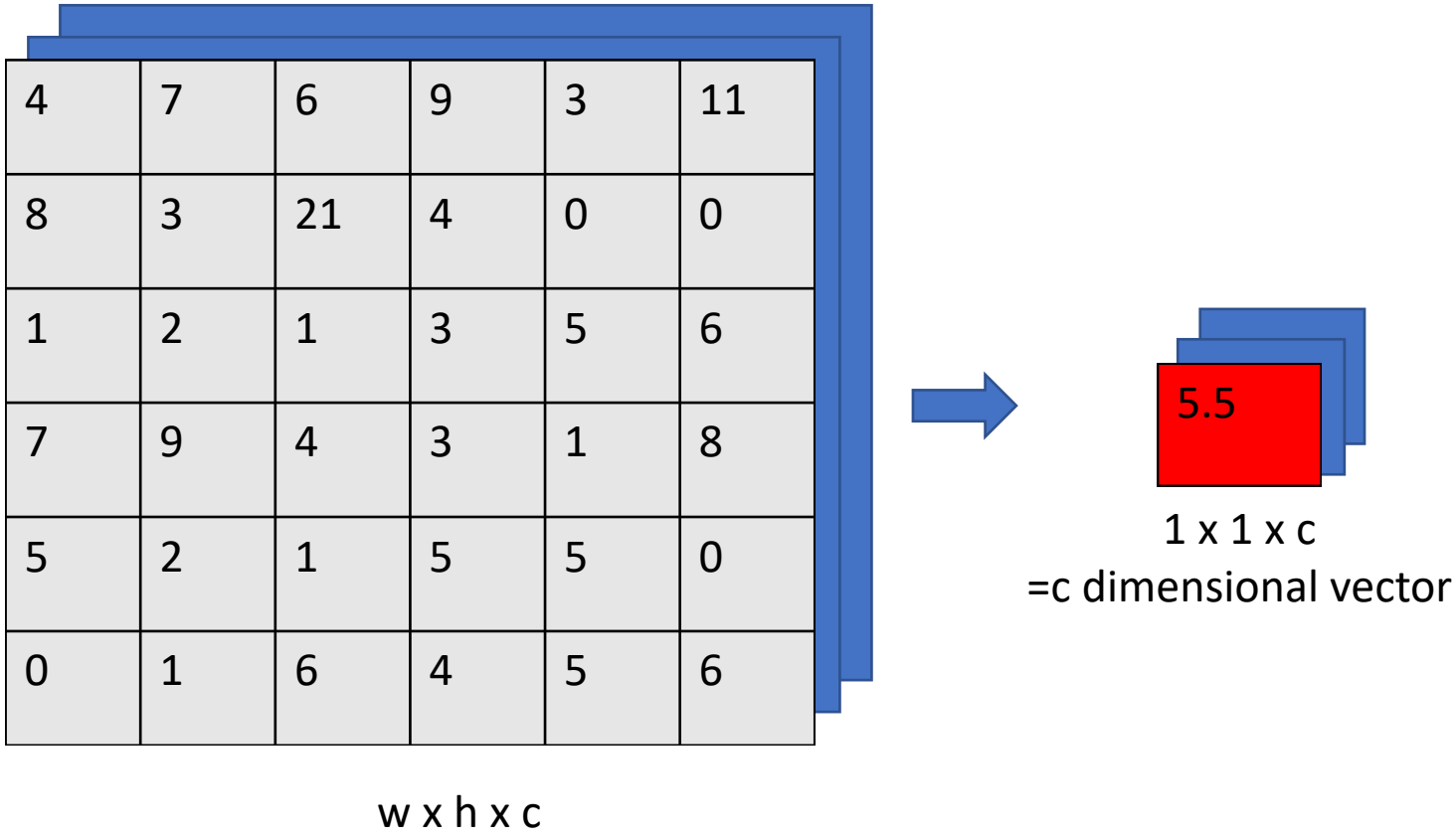
Invariance to distortions: Max Pooling



Invariance to distortions: Average Pooling



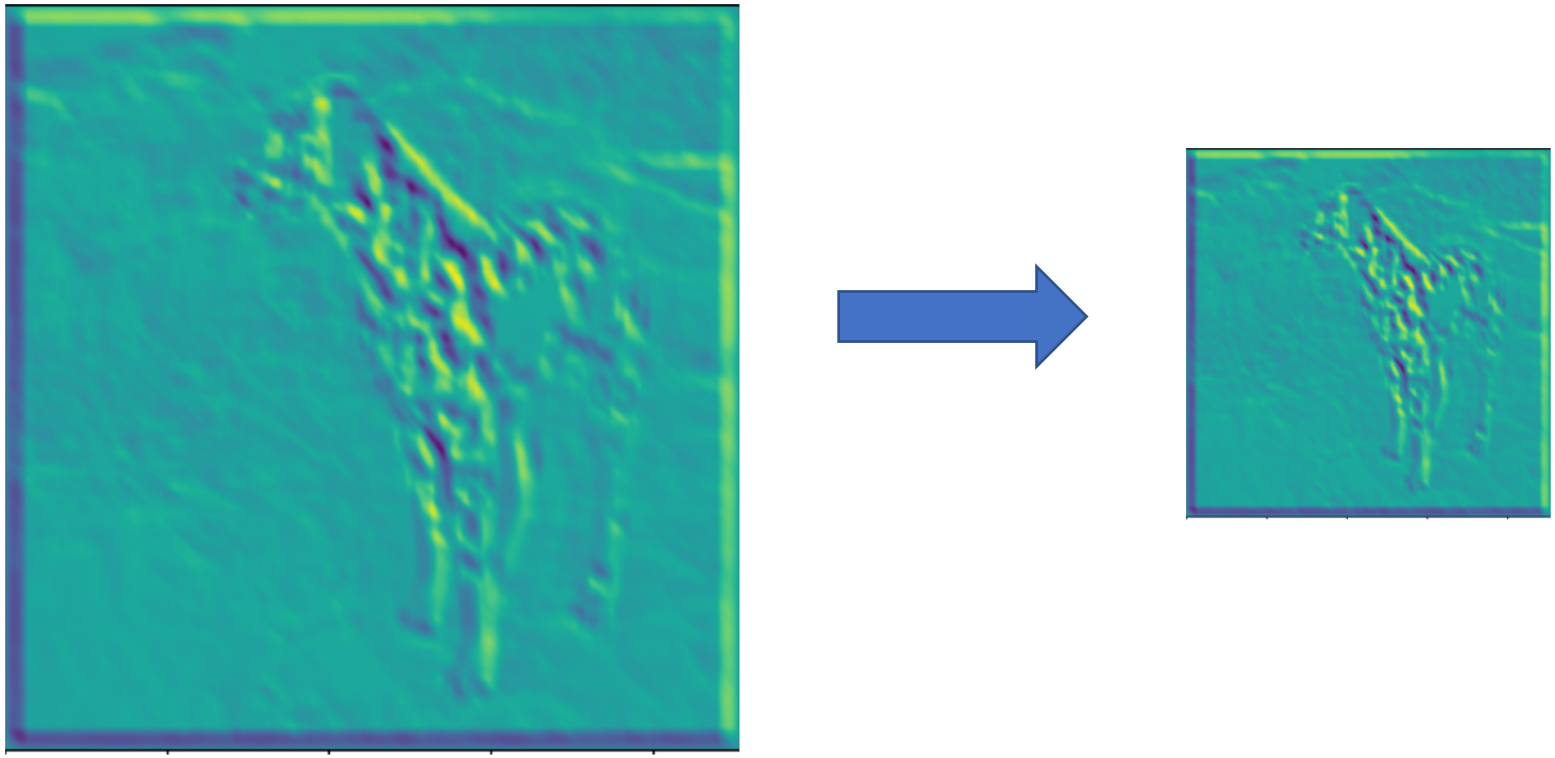
Global average pooling



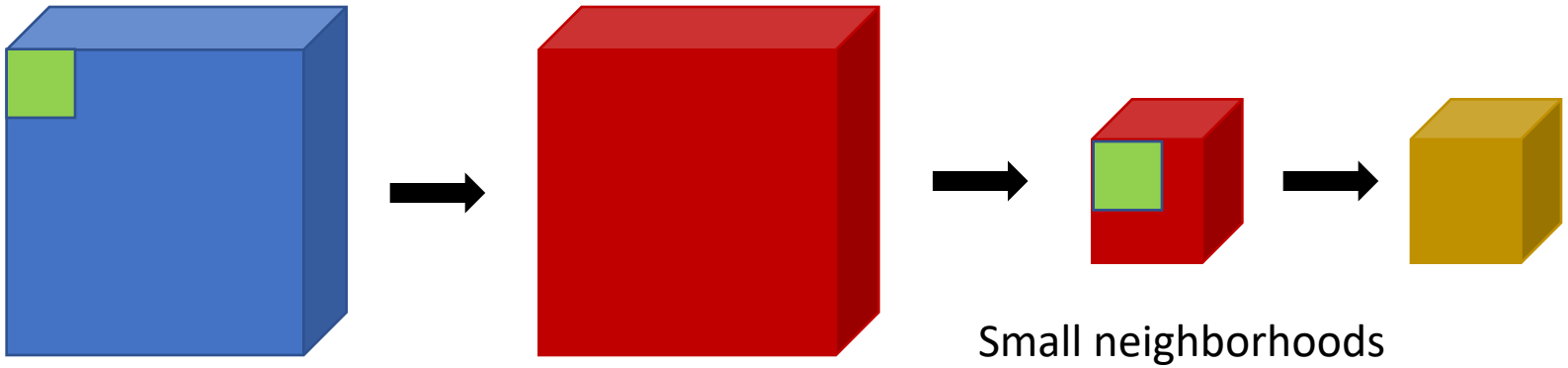
The pooling unit

- Each pooling unit takes *a collection of feature maps* as input and produces *a collection of feature maps* as output
- Output feature maps are usually smaller in height / width
- Parameters: None

Invariance to distortions: Subsampling



Convolution subsampling convolution



Small neighborhoods on subsampled feature map = large neighborhood on original image

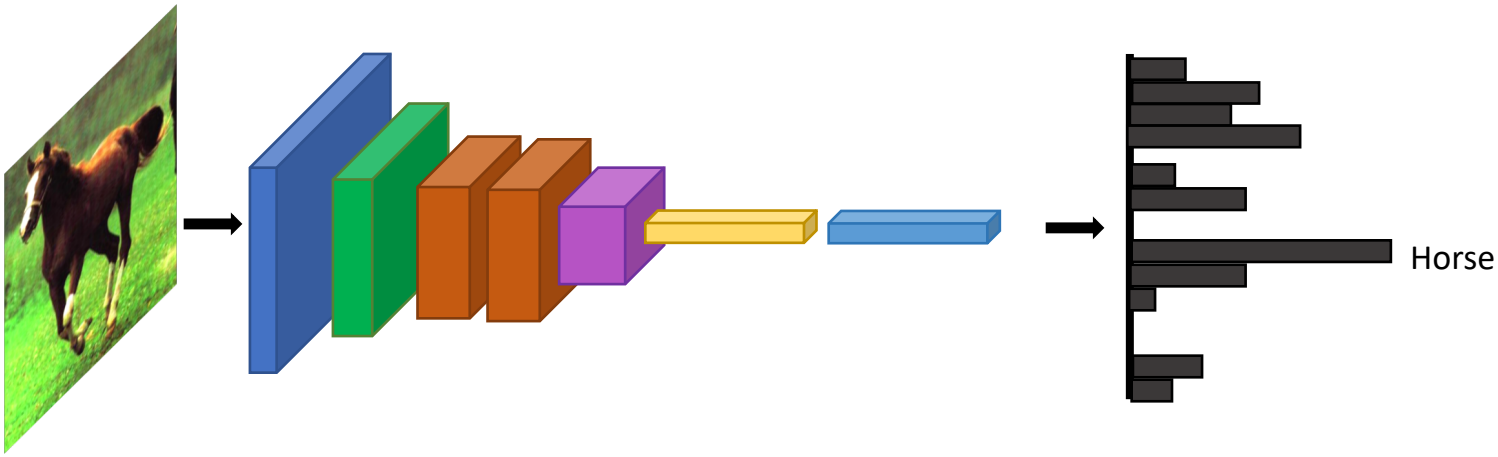
Convolution subsampling convolution

- Convolution in earlier steps detects *more local* patterns *less resilient* to distortion
- Convolution in later steps detects *more global* patterns *more resilient* to distortion
- Subsampling allows capture of *larger, more invariant* patterns

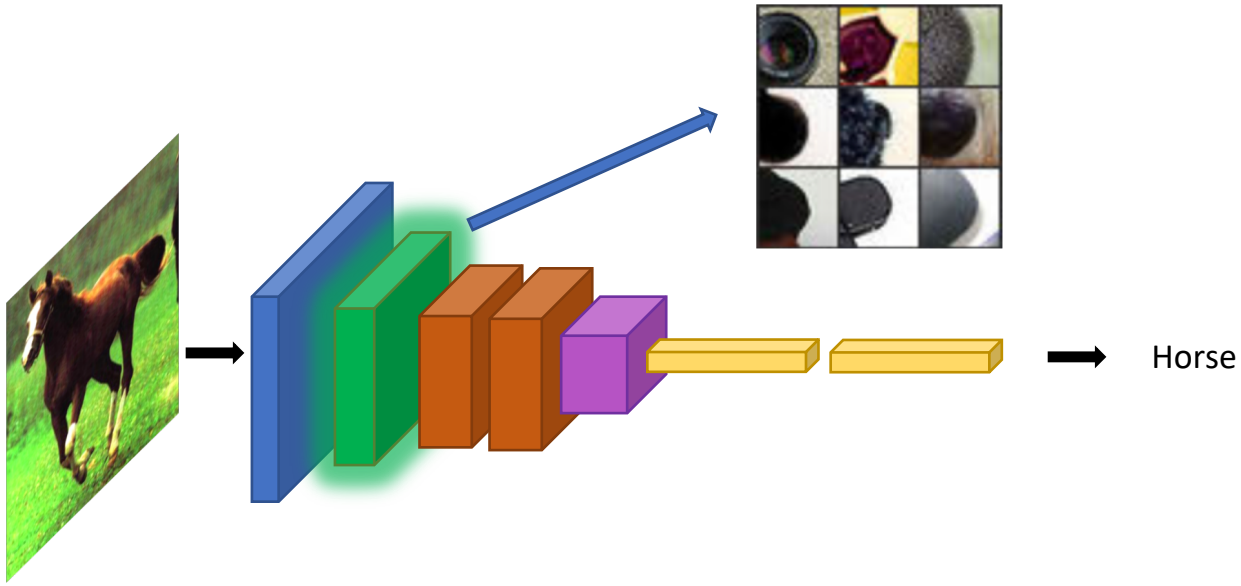
Strided convolution

- Convolution with stride s = standard convolution + subsampling by picking 1 value every s values
- Example: convolution with stride 2 = standard convolution + subsampling by a factor of 2

Convolutional networks

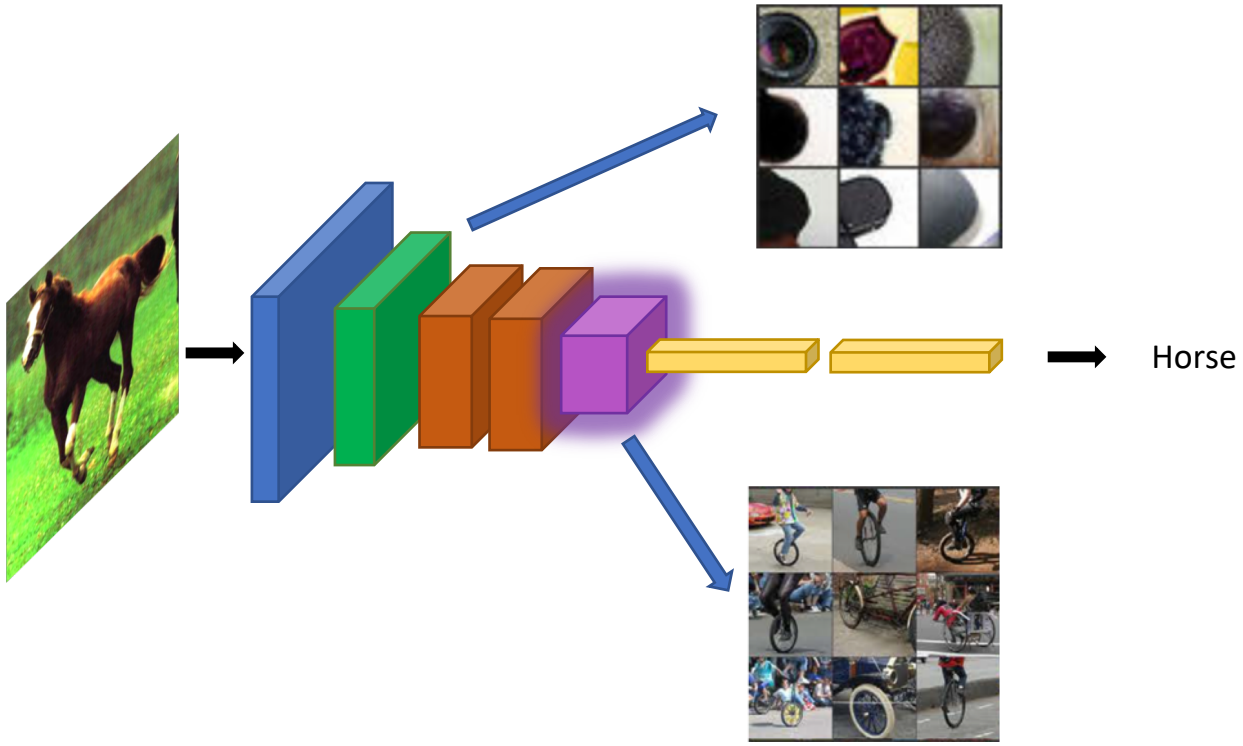


Convolutional networks



Visualizations from : M. Zeiler and R. Fergus. Visualizing and Understanding Convolutional Networks. In ECCV 2014.

Convolutional networks



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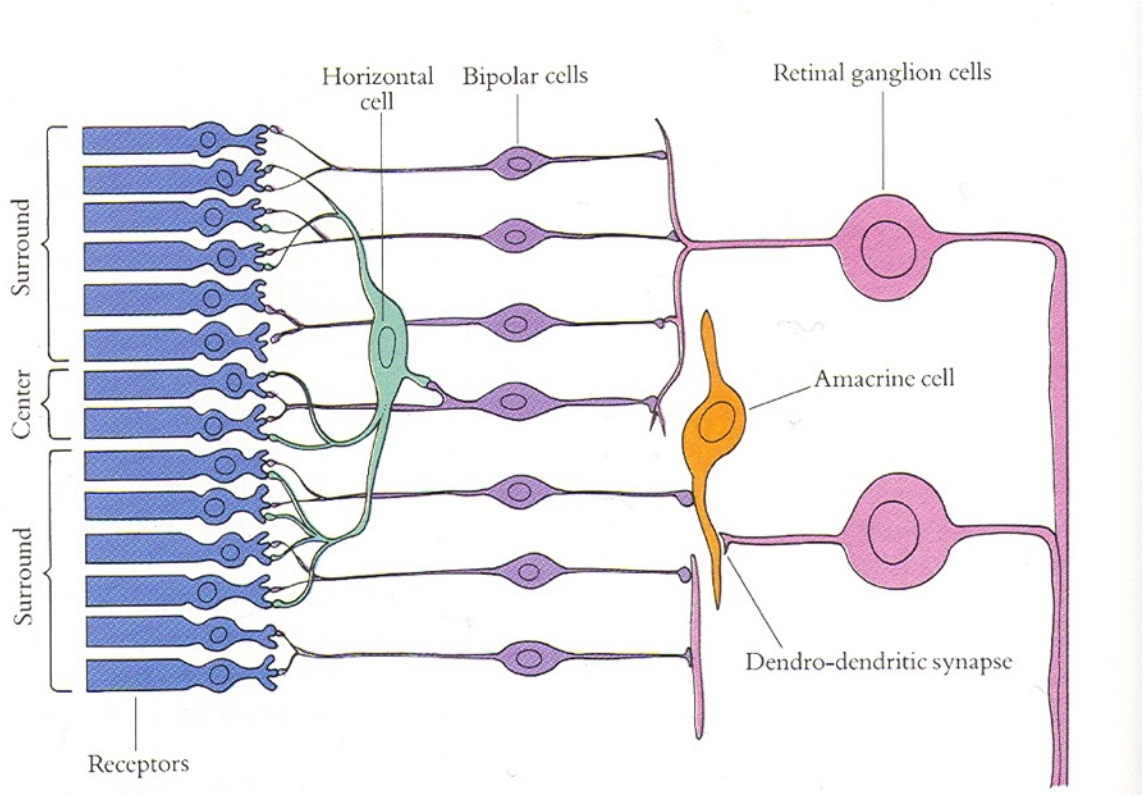
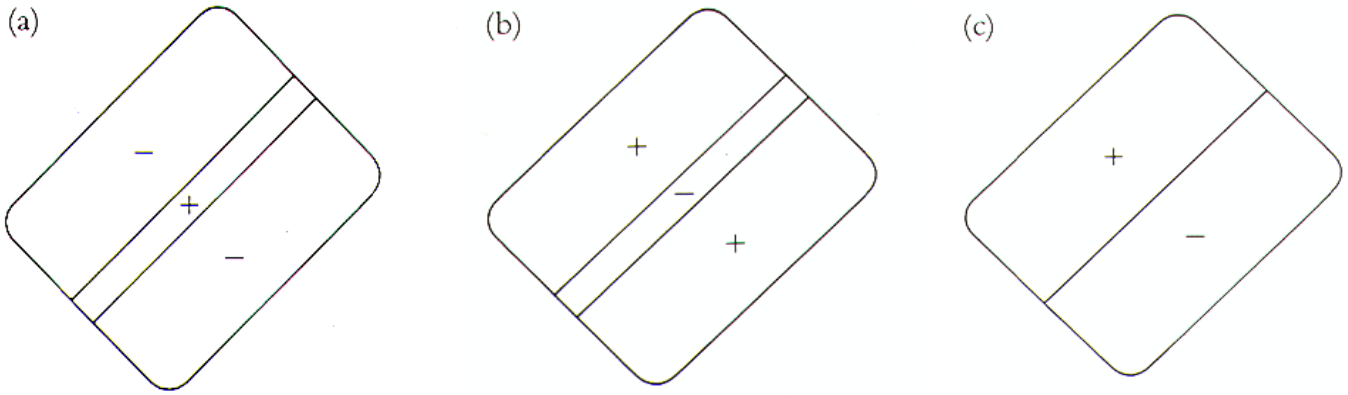
Convolutional Networks and the Brain



Slide credit: Jitendra Malik

Receptive fields of simple cells (discovered by Hubel & Wiesel)



Slide credit: Jitendra Malik

Convolutional networks

Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE* 86.11 (1998): 2278-2324.

Convolutional networks

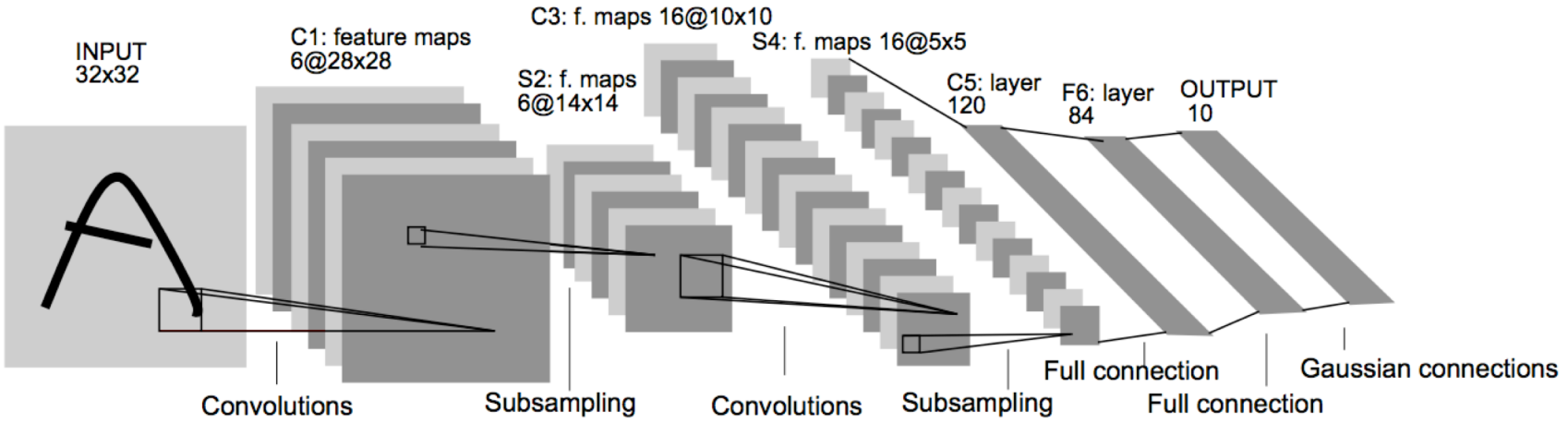
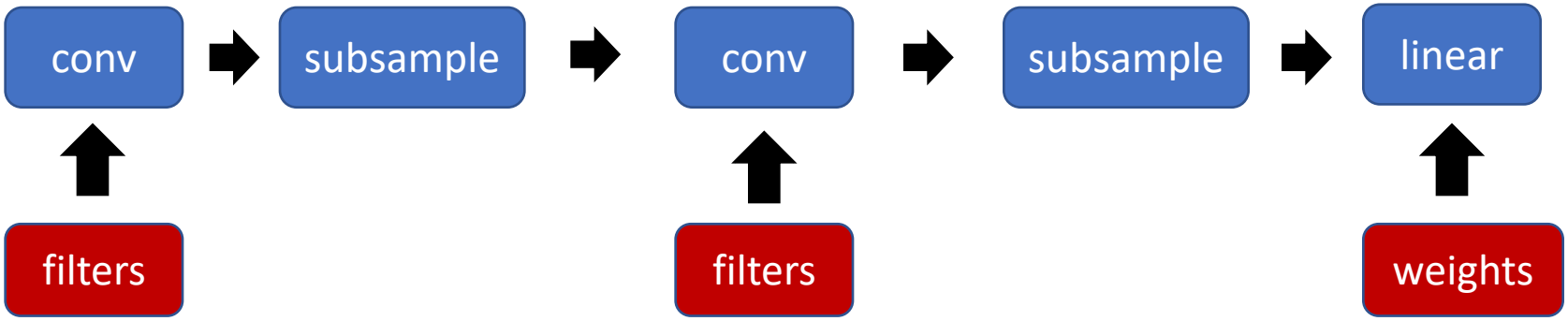


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

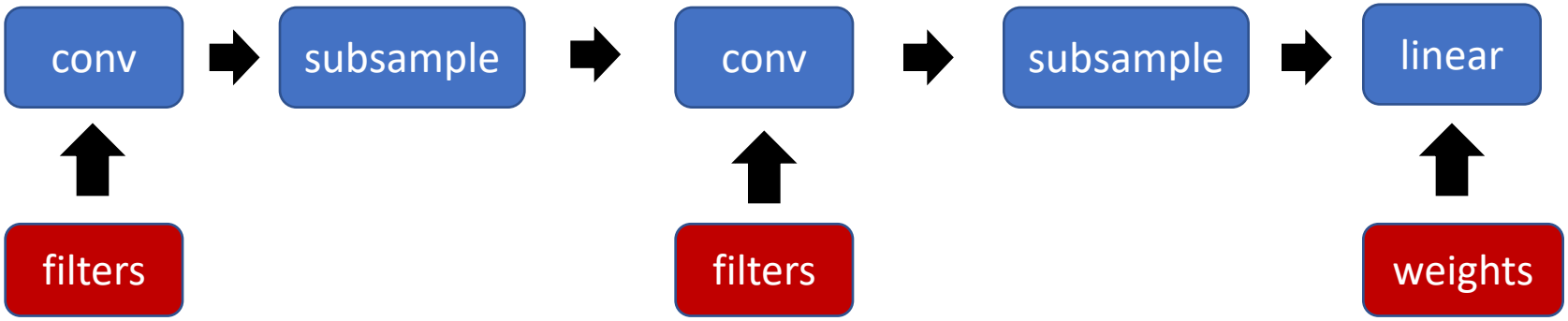
Convolutional networks



Last time

- Linear classifiers on pixels bad, need non-linear classifiers
- Multi-layer perceptrons overparametrized
- Reduce parameters by local connections and shift invariance => Convolution
- Intersperse subsampling to capture ever larger deformations
- Stick a final classifier

Convolutional networks



Empirical Risk Minimization

$$\min_{\boldsymbol{\theta}} \frac{1}{N} \sum_{i=1}^{N} L(h(x_i; \boldsymbol{\theta}), y_i)$$

Convolutional network

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \lambda \frac{1}{N} \sum_{i=1}^{N} \nabla L(h(x_i; \boldsymbol{\theta}), y_i)$$

Gradient descent update

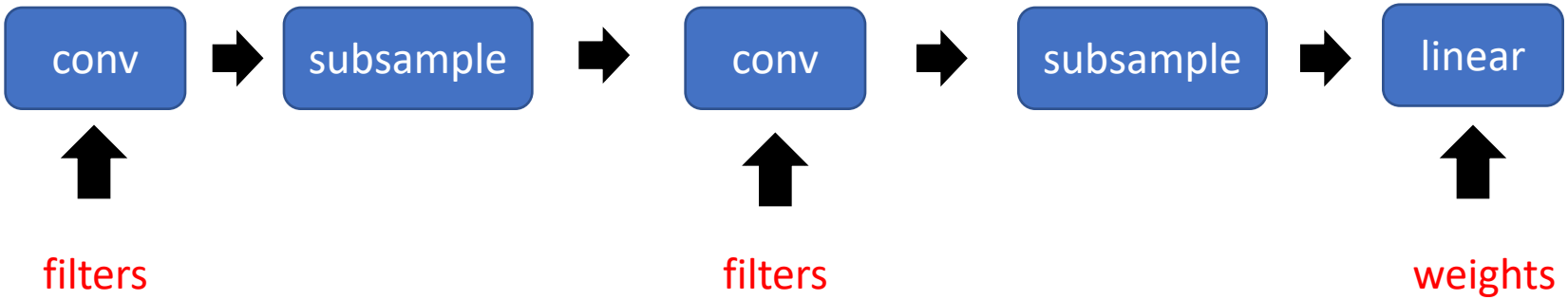
Computing the gradient of the loss

$$\nabla L(h(x;\boldsymbol{\theta}), y)$$

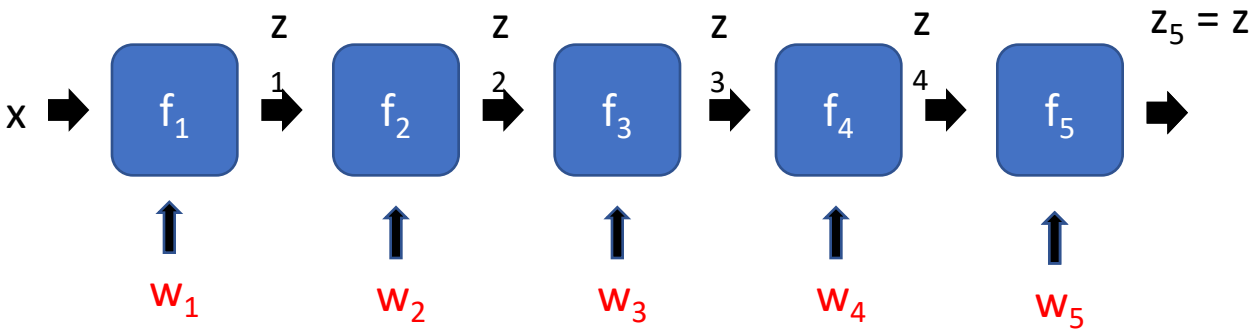
$$z = h(x;\boldsymbol{\theta})$$

$$\nabla_{\boldsymbol{\theta}} L(z, y) = \frac{\partial L(z, y)}{\partial z} \frac{\partial z}{\partial \boldsymbol{\theta}}$$

Convolutional networks



The gradient of convnets



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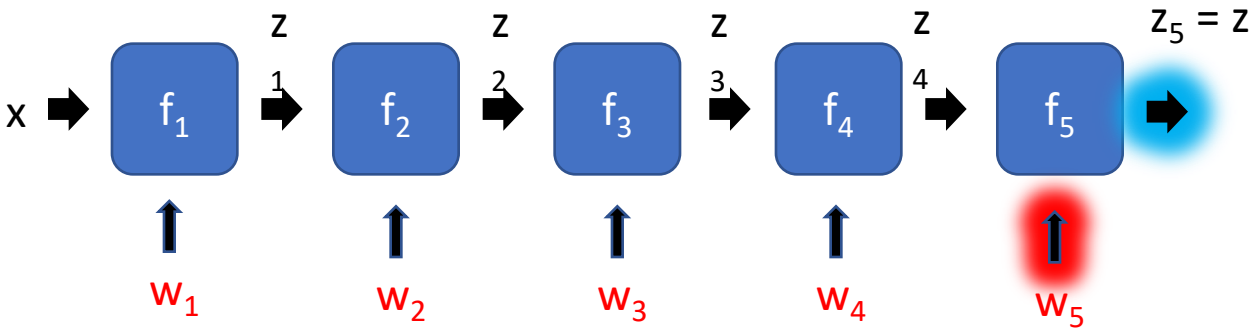
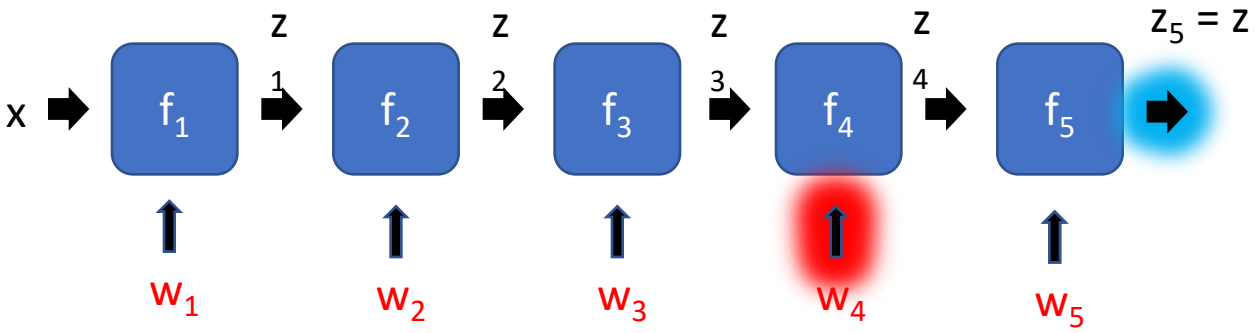


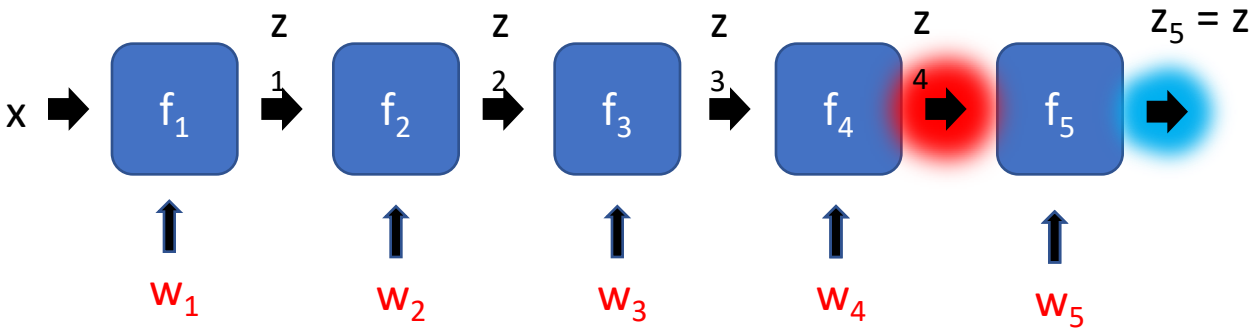
$$\frac{\partial z}{\partial w_5}$$

The gradient of convnets



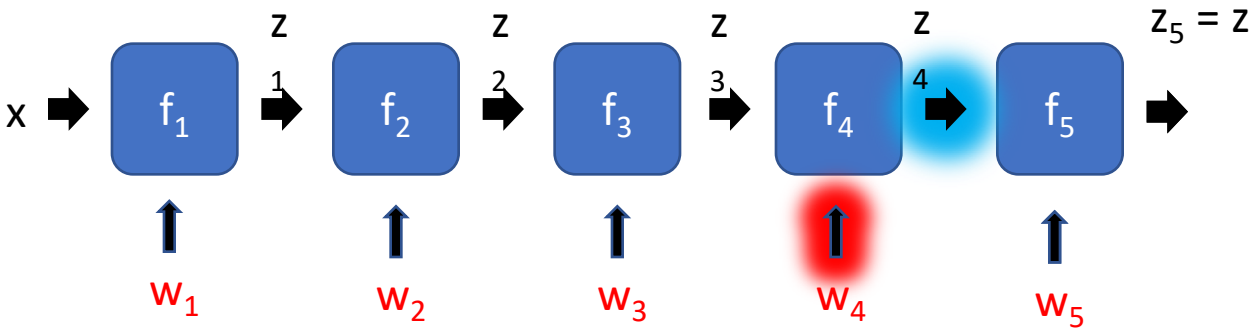
$$\frac{\partial z}{\partial w_4}$$

The gradient of convnets



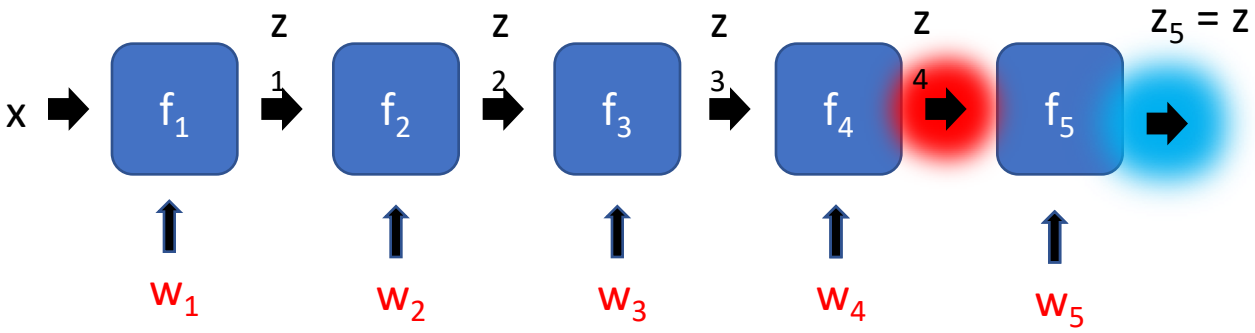
$$\frac{\partial z}{\partial w_4} = \frac{\partial z}{\partial z_4}\frac{\partial z_4}{\partial w_4}$$

The gradient of convnets



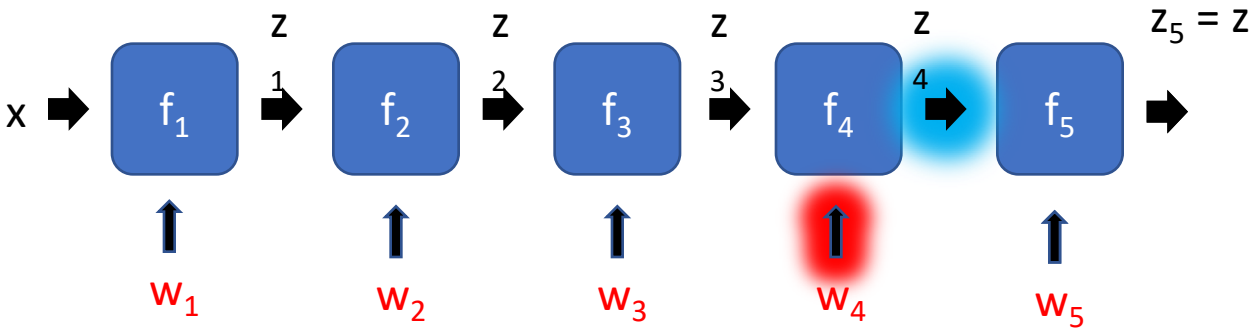
$$\frac{\partial z}{\partial w_4} = \frac{\partial z}{\partial z_4}\frac{\partial z_4}{\partial w_4}$$

The gradient of convnets



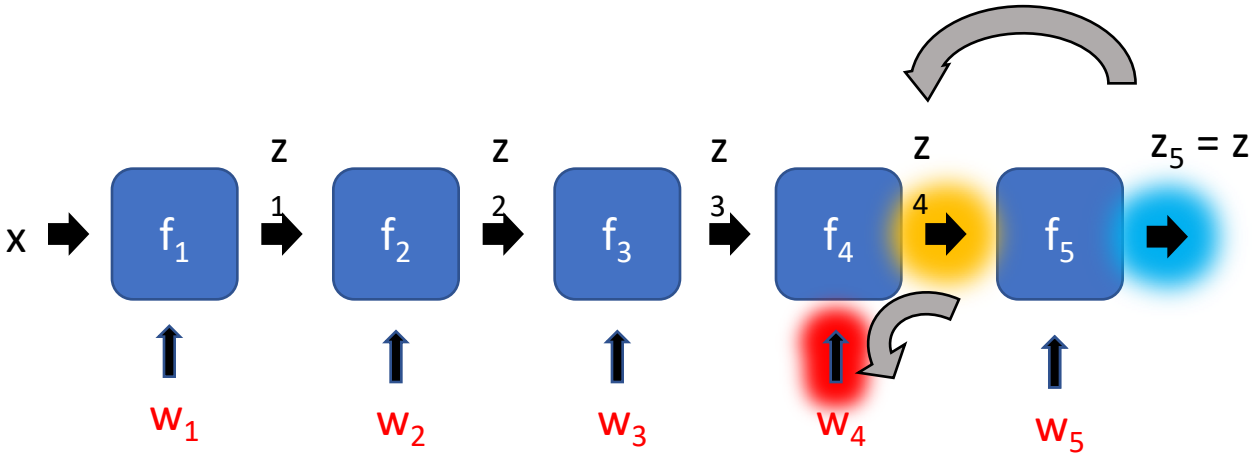
$$\frac{\partial z}{\partial w_4} = \frac{\partial z}{\partial z_4}\frac{\partial z_4}{\partial w_4} = \frac{\partial f_5(z_4, w_5)}{\partial z_4}\frac{\partial f_4(z_3, w_4)}{\partial w_4}$$

The gradient of convnets



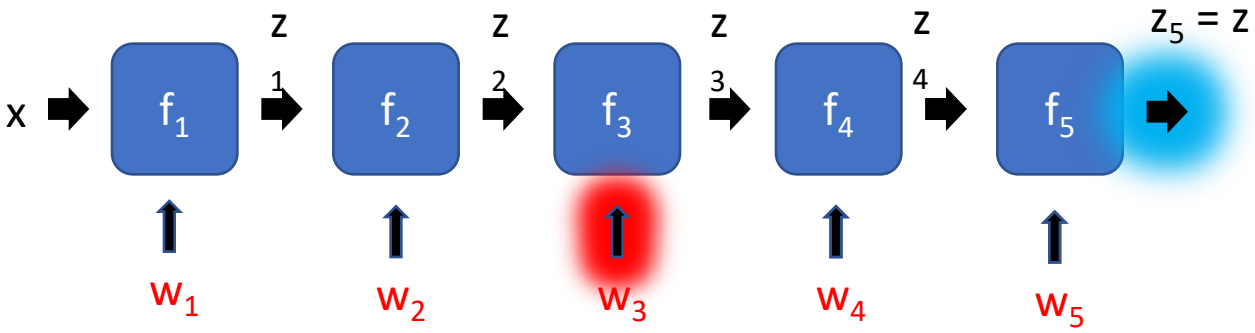
$$\frac{\partial z}{\partial w_4} = \frac{\partial z}{\partial z_4}\frac{\partial z_4}{\partial w_4} = \frac{\partial f_5(z_4, w_5)}{\partial z_4}\frac{\partial f_4(z_3, w_4)}{\partial w_4}$$

The gradient of convnets



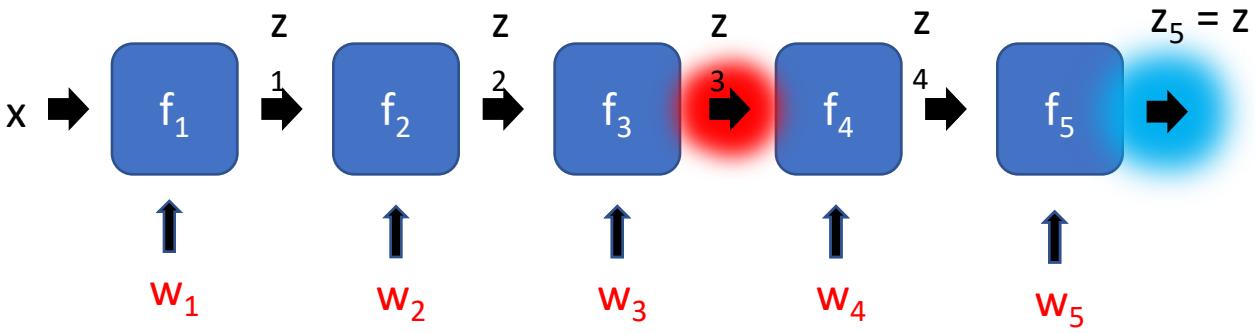
$$\frac{\partial z}{\partial w_4} = \frac{\partial z}{\partial z_4}\frac{\partial z_4}{\partial w_4} = \frac{\partial f_5(z_4, w_5)}{\partial z_4}\frac{\partial f_4(z_3, w_4)}{\partial w_4}$$

The gradient of convnets



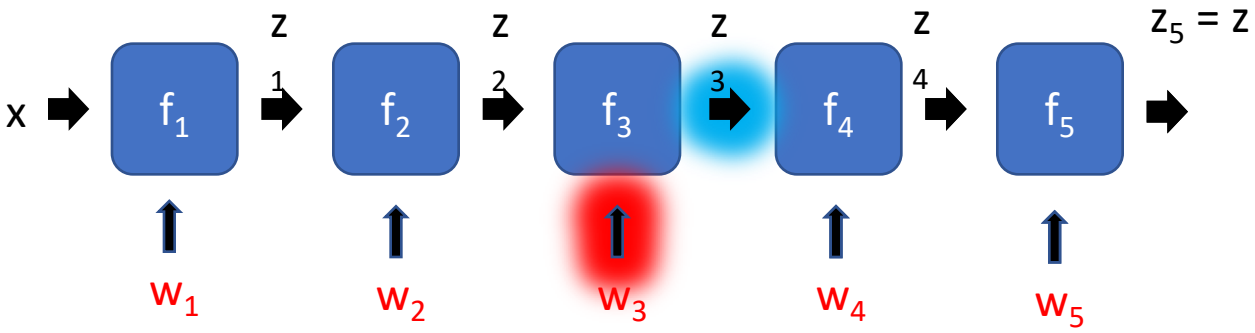
$$\frac{\partial z}{\partial w_3}$$

The gradient of convnets



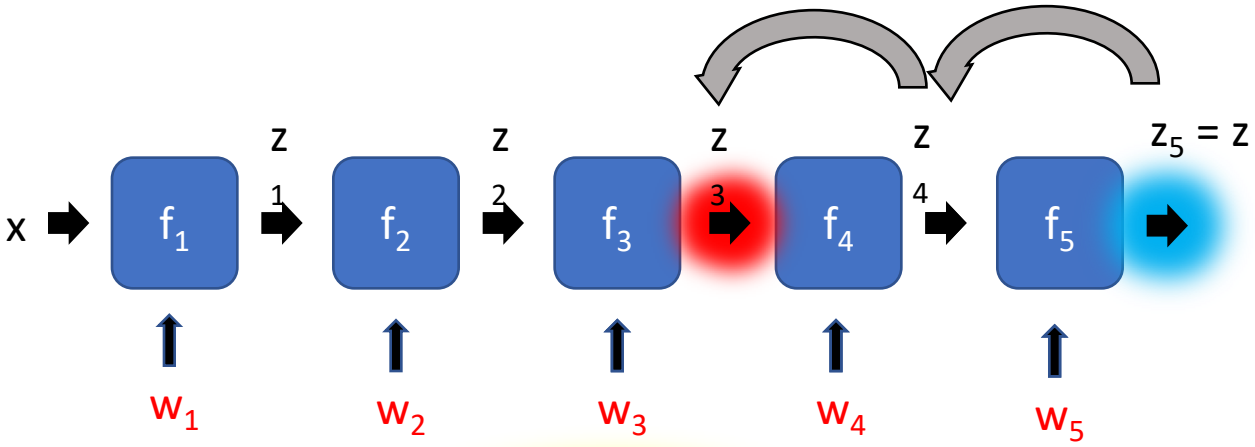
$$\frac{\partial z}{\partial w_3} = \frac{\partial z}{\partial z_3}\frac{\partial z_3}{\partial w_3}$$

The gradient of convnets



$$\frac{\partial z}{\partial w_3} = \frac{\partial z}{\partial z_3}\frac{\partial z_3}{\partial w_3}$$

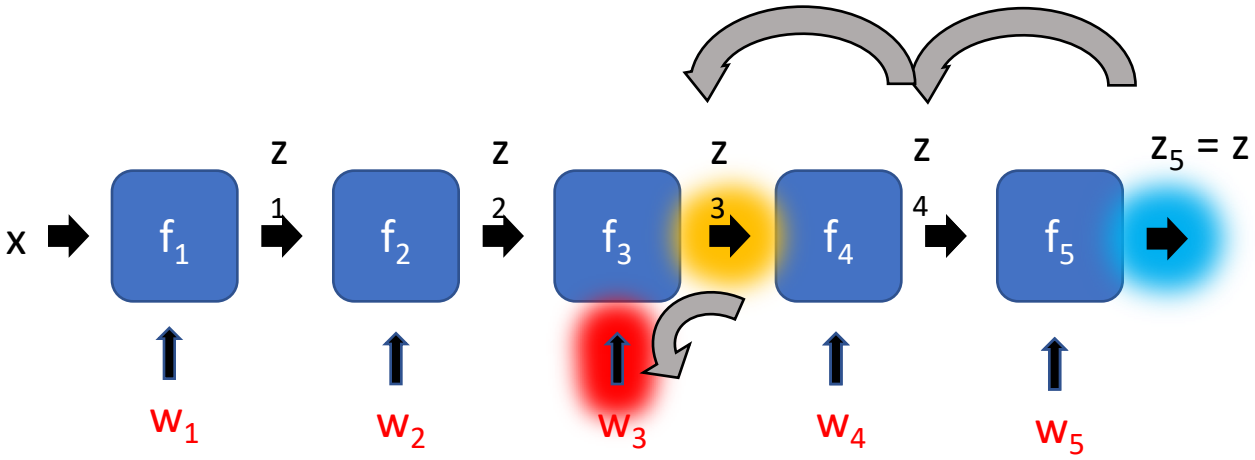
The gradient of convnets



$$\frac{\partial z}{\partial z_3} = \frac{\partial z}{\partial z_4}\frac{\partial z_4}{\partial z_3}$$

$$\frac{\partial z}{\partial w_3} = \frac{\partial z}{\partial z_3}\frac{\partial z_3}{\partial w_3}$$

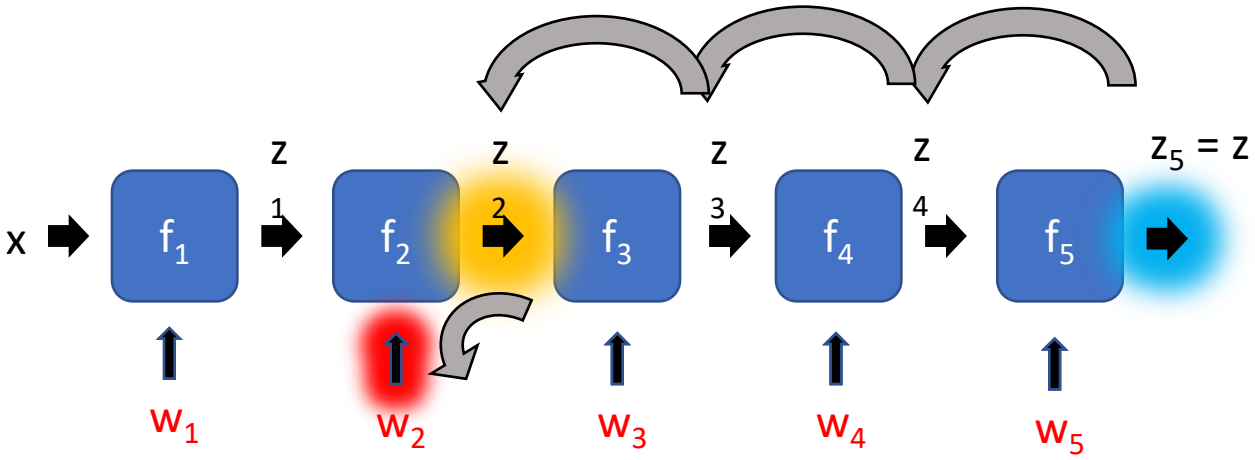
The gradient of convnets



$$\frac{\partial z}{\partial z_3} = \frac{\partial z}{\partial z_4}\frac{\partial z_4}{\partial z_3}$$

$$\frac{\partial z}{\partial w_3} = \frac{\partial z}{\partial z_3}\frac{\partial z_3}{\partial w_3}$$

The gradient of convnets

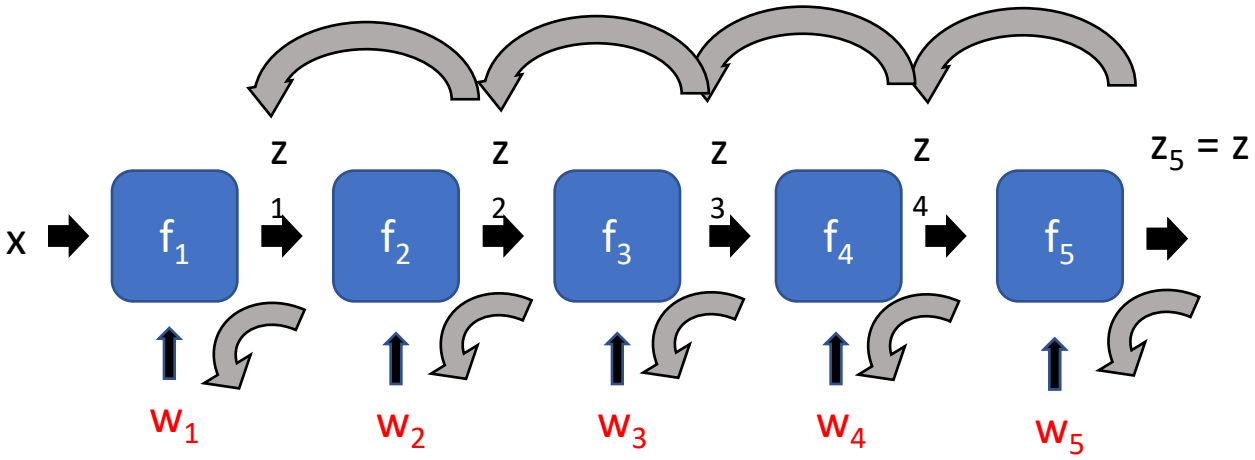


$$\frac{\partial z}{\partial z_2} = \frac{\partial z}{\partial z_3}\frac{\partial z_3}{\partial z_2}$$

$$\frac{\partial z}{\partial w_2} = \frac{\partial z}{\partial z_2}\frac{\partial z_2}{\partial w_2}$$

Recurrence going backward!!

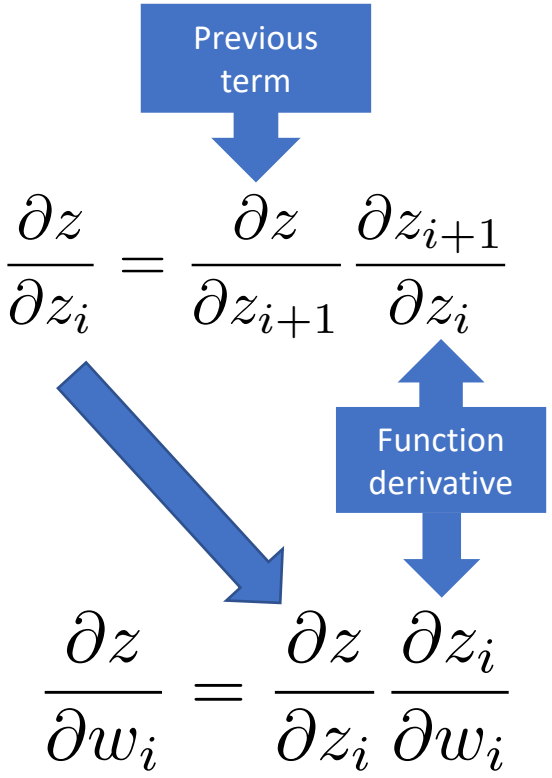
The gradient of convnets



Backpropagation

Backpropagation for a sequence of functions

$$z_i = f_i(z_{i-1}, w_i)$$
$$z_0 = x$$
$$z = z_n$$



Backpropagation for a sequence of functions

$$z_i = f_i(z_{i-1}, w_i) \qquad z_0 = x \qquad z = z_n$$

- Assume we can compute partial derivatives of each function

$$\frac{\partial z_i}{\partial z_{i-1}} = \frac{\partial f_i(z_{i-1}, w_i)}{\partial z_{i-1}} \qquad \frac{\partial z_i}{\partial w_i} = \frac{\partial f_i(z_{i-1}, w_i)}{\partial w_i}$$

- Use $g(z_i)$ to store gradient of z w.r.t z_i, $g(w_i)$ for w_i
- Calculate g_i by iterating backwards

$$g(z_n) = \frac{\partial z}{\partial z_n} = 1 \qquad g(z_{i-1}) = \frac{\partial z}{\partial z_i}\frac{\partial z_i}{\partial z_{i-1}} = g(z_i)\frac{\partial z_i}{\partial z_{i-1}}$$

- Use gi to compute gradient of parameters

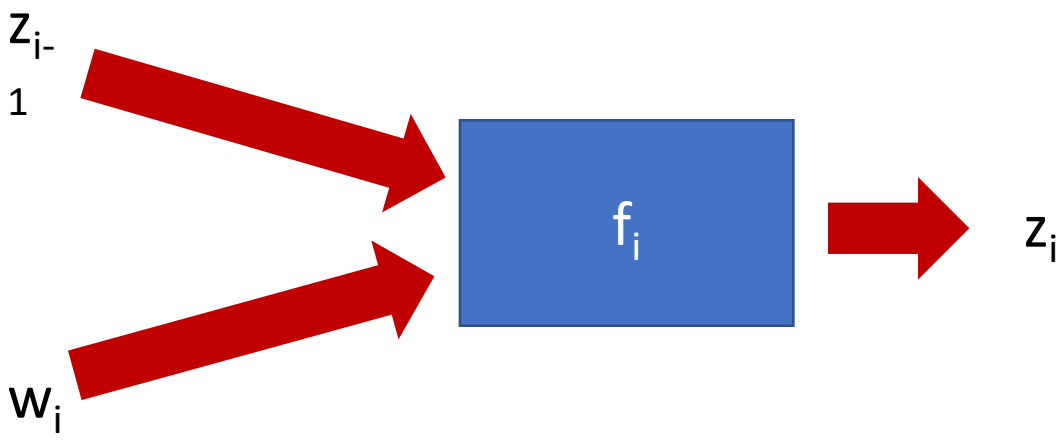
$$g(w_i) = \frac{\partial z}{\partial z_i}\frac{\partial z_i}{\partial w_i} = g(z_i)\frac{\partial z_i}{\partial w_i}$$

Backpropagation for a sequence of functions

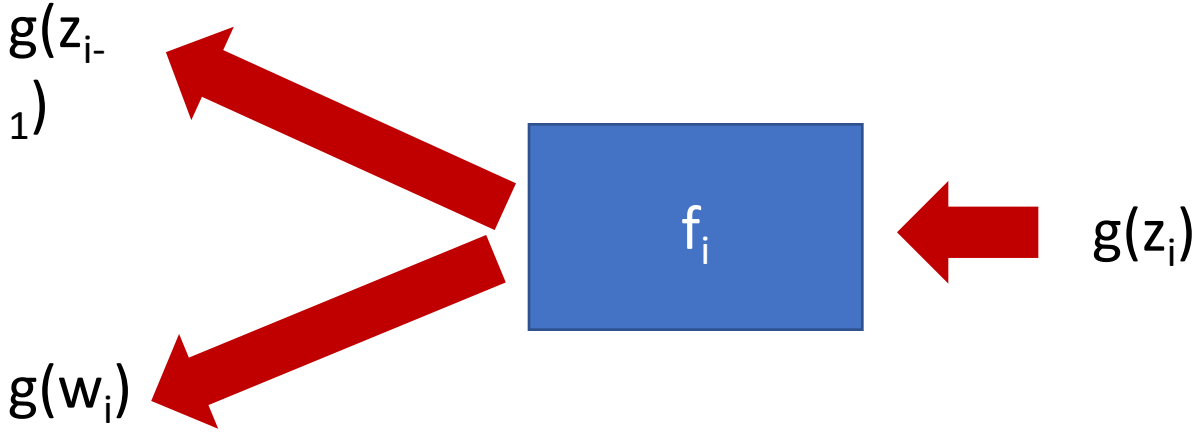
- Each “function” has a “forward” and “backward” module
- Forward module for f_i
  - takes z_{i-1} and weight w_i as input
  - produces z_i as output
- Backward module for f_i
  - takes $g(z_i)$ as input
  - produces $g(z_{i-1})$ and $g(w_i)$ as output

$$g(z_{i-1}) = g(z_i)\frac{\partial z_i}{\partial z_{i-1}} \qquad g(w_i) = g(z_i)\frac{\partial z_i}{\partial w_i}$$

Backpropagation for a sequence of functions



Backpropagation for a sequence of functions



Chain rule for vectors

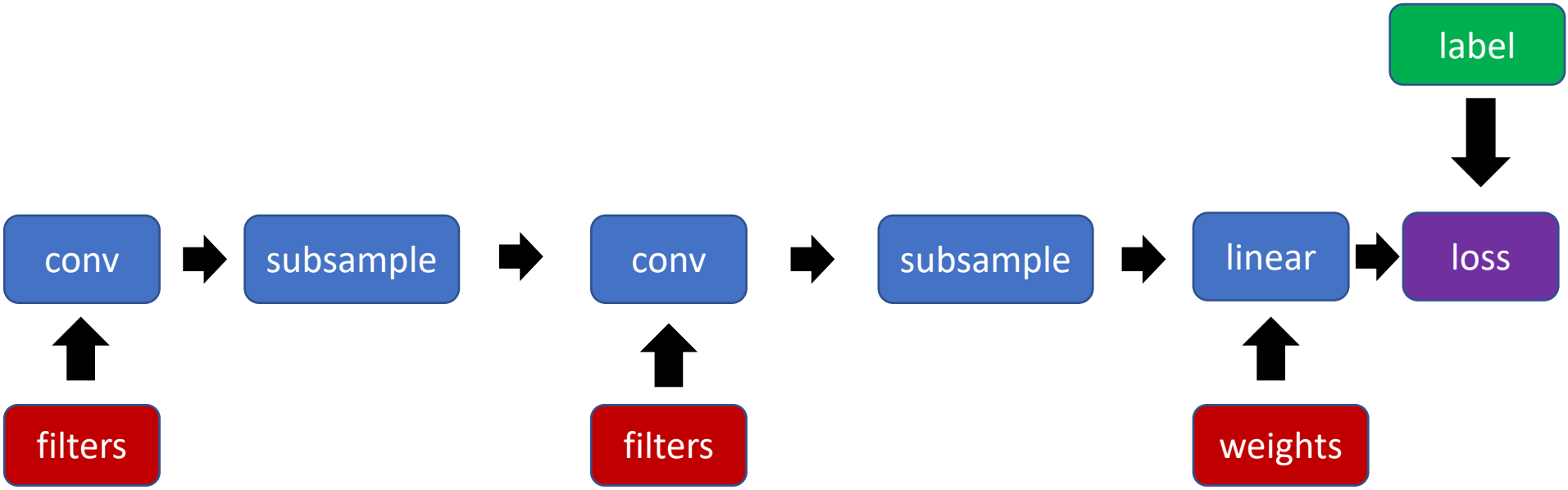
$$\frac{\partial a}{\partial b} = \frac{\partial a}{\partial c}\frac{\partial c}{\partial b} \qquad \frac{\partial a_i}{\partial b_j} = \sum_k \frac{\partial a_i}{\partial c_k}\frac{\partial c_k}{\partial b_j}$$

$$\frac{\partial \mathbf{a}}{\partial \mathbf{b}}(i,j) = \frac{\partial a_i}{\partial b_j}$$

Jacobian

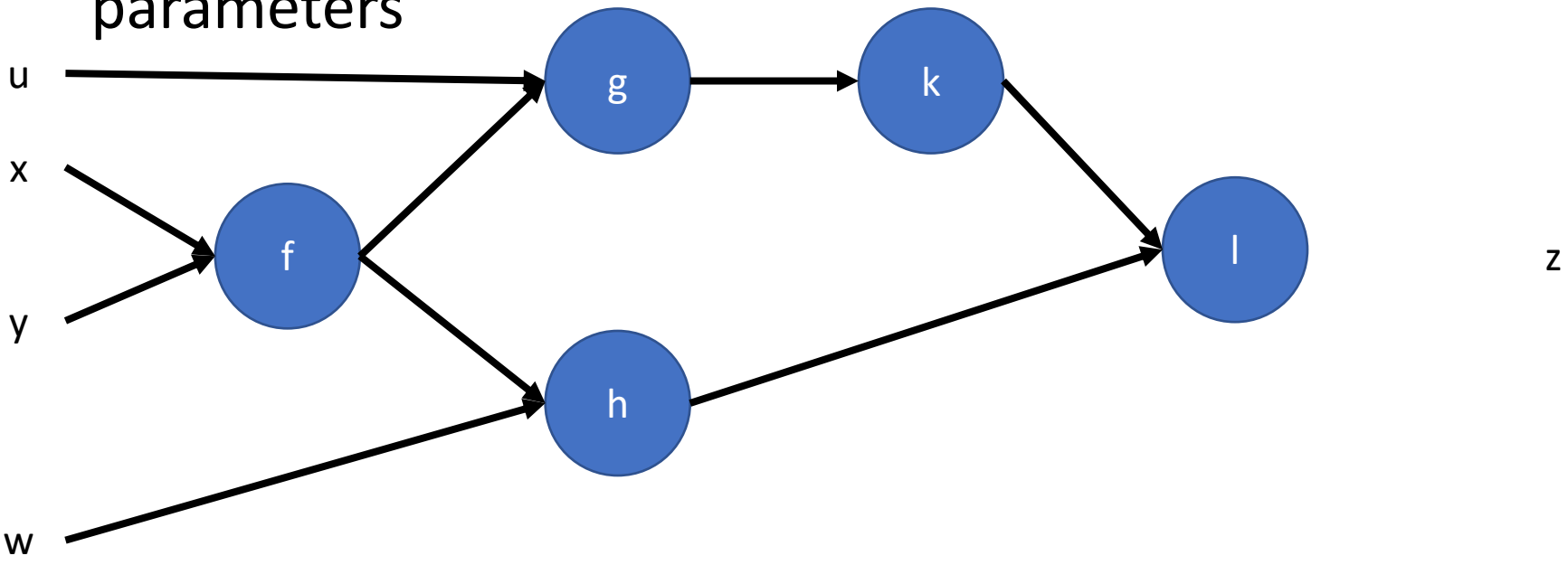
$$\frac{\partial \mathbf{a}}{\partial \mathbf{b}} = \frac{\partial \mathbf{a}}{\partial \mathbf{c}}\frac{\partial \mathbf{c}}{\partial \mathbf{b}}$$

Loss as a function



Beyond sequences: computation graphs

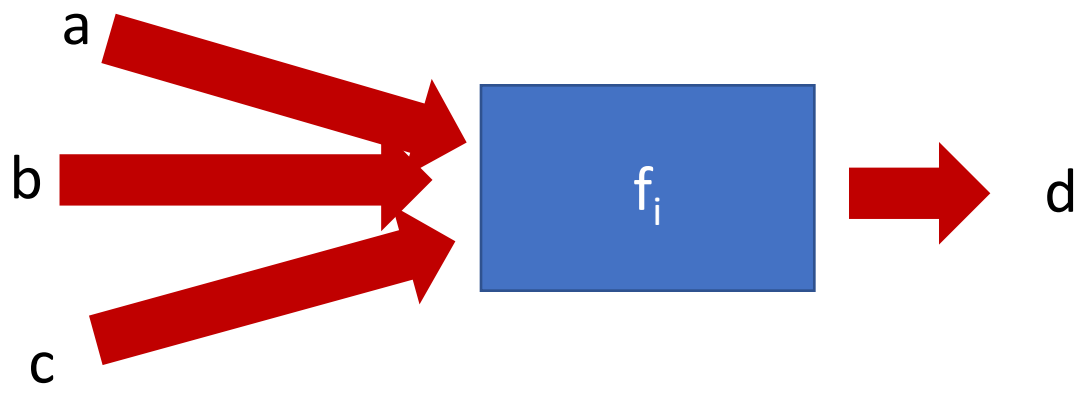
- Arbitrary *graphs* of functions
- No distinction between intermediate outputs and parameters



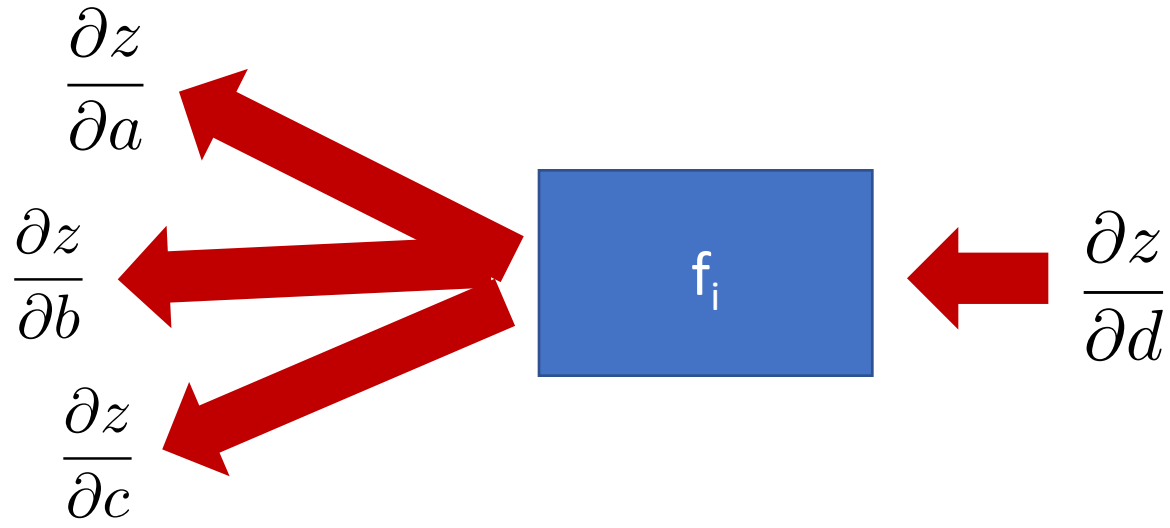
Computation graph - Functions

- Each node implements two functions
  - A “forward”
    - Computes output given input
  - A “backward”
    - Computes derivative of z w.r.t input, given derivative of z w.r.t output

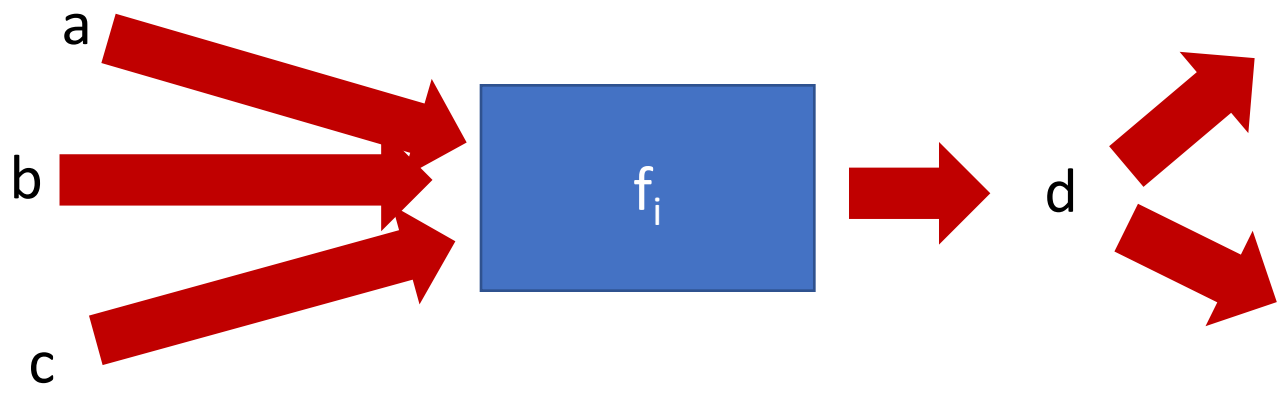
Computation graphs



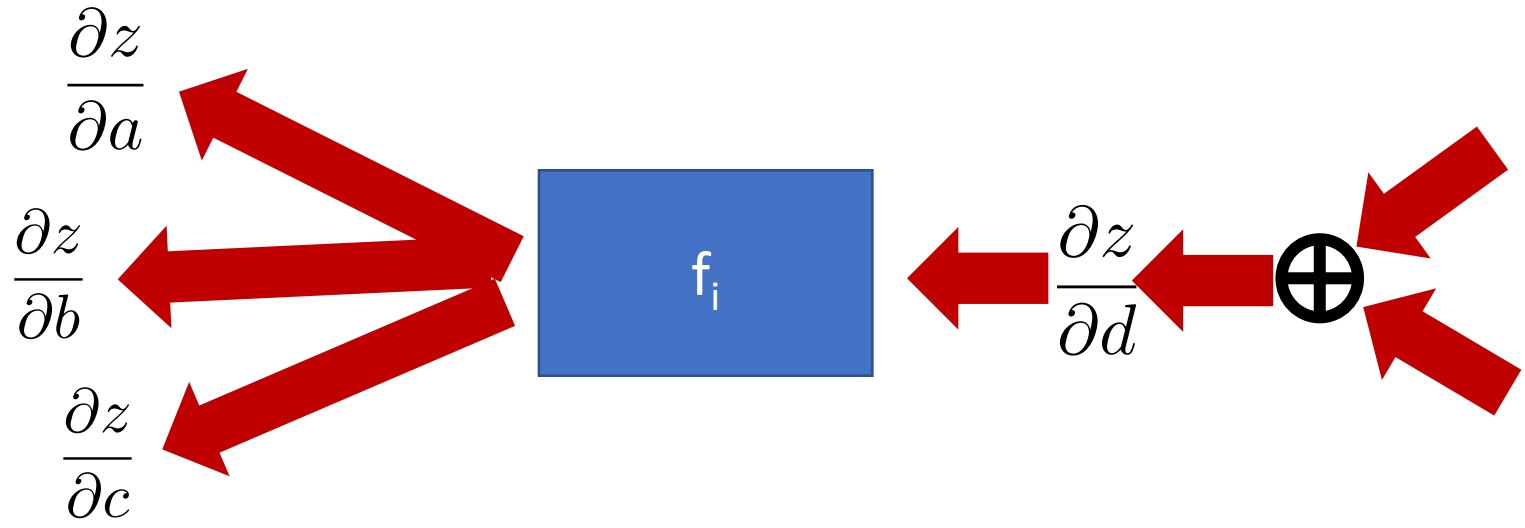
Computation graphs



Computation graphs



Computation graphs



Neural network frameworks