Recognition - III

General recipe

Logistic Regression!

- Fix hypothesis class

$$h(x; \mathbf{w}, b) = \sigma(\mathbf{w}^T \phi(x) + b)$$

- Define loss function

$$L(h(x; \mathbf{w}, b), y) = -(y \log h(x; \mathbf{w}, b) + (1 - y) \log(1 - h(x; \mathbf{w}, b))$$

- Minimize total loss on the training set

$$\min_{\mathbf{w}, b} \sum_{i=1}^{N} L(h(x_i; \mathbf{w}, b), y_i)$$

- Equivalent to minimizing the average loss

$$\min_{\mathbf{w}, b} \frac{1}{N} \sum_{i=1}^{N} L(h(x_i, \mathbf{w}, b), y_i)$$

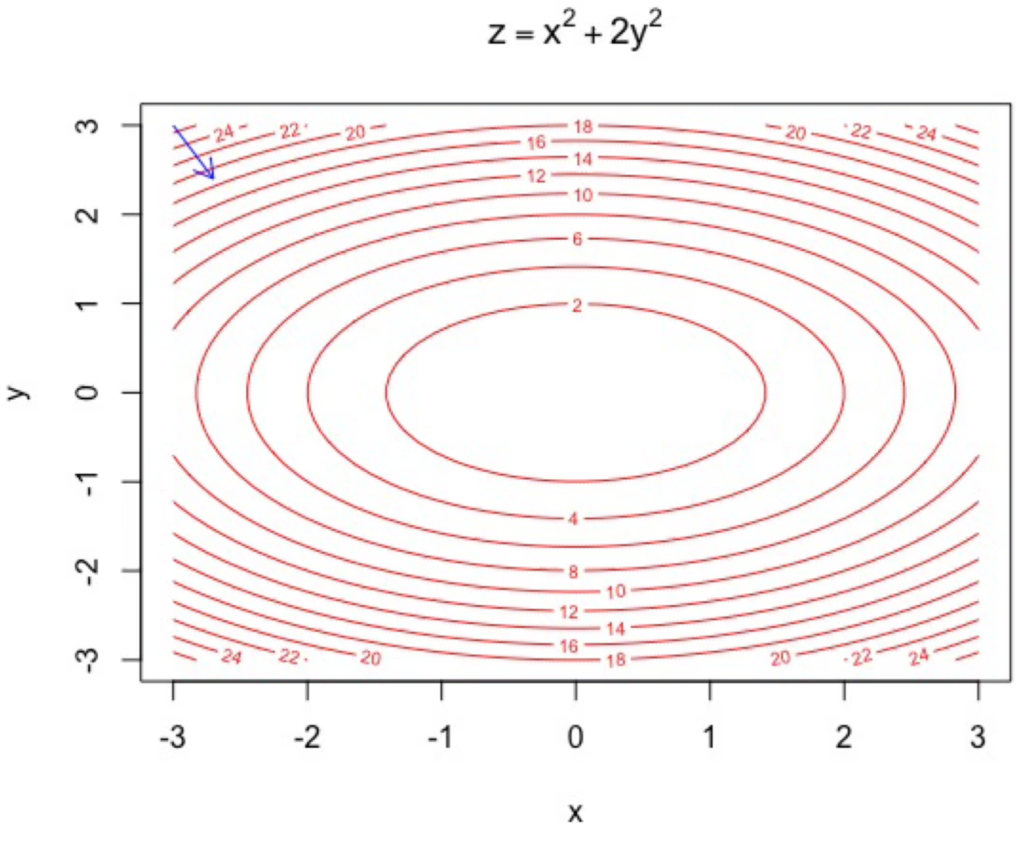
Machine learning is optimization

$$\min_{\mathbf{w},b} \frac{1}{N} \sum_{i=1}^{N} L(h(x_i, \mathbf{w}, b), y_i) \equiv \min_{\boldsymbol{\theta}} F(\boldsymbol{\theta})$$

Optimization using gradient descent

- Randomly initialize $\boldsymbol{\theta}^{(0)}$
- For i = 1 to max_iterations:
  - Compute gradient of F at $\boldsymbol{\theta}^{(t)}$
  - $\boldsymbol{\theta}^{(t+1)} \leftarrow \boldsymbol{\theta}^{(t)} - \lambda \nabla F(\boldsymbol{\theta}^{(t)})$
    - Function value will decrease by $\lambda ||\nabla F(\boldsymbol{\theta}^{(t)})||^2$
  - Repeat until $||\nabla F(\boldsymbol{\theta}^{(t)})||^2$ drops below a threshold

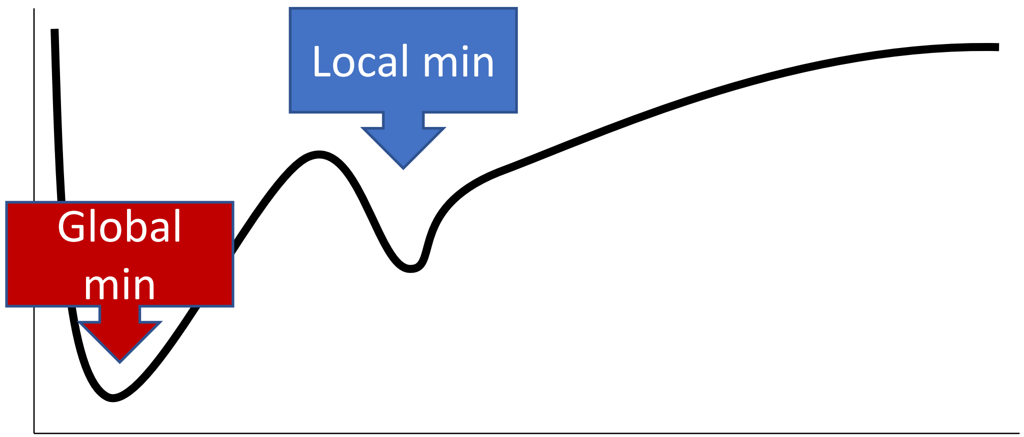
Gradient descent



https://yihui.name/animation/example/grad-desc/

Gradient descent - convergence

- Every step leads to a reduction in the function value
- If function is bounded below, we will eventually stop
- But will we stop at the right “global minimum”?
  - Not necessarily: local optimum!



Gradient descent in machine learning

$$\min_{\mathbf{w},b} \frac{1}{N} \sum_{i=1}^{N} L(h(x_i, \mathbf{w}, b), y_i) \equiv \min_{\boldsymbol{\theta}} F(\boldsymbol{\theta})$$

$$\nabla F(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^{N} \nabla L(h(x_i, \mathbf{w}, b), y_i)$$

- Computing the gradient requires a *loop over all training examples*
- Very expensive for large datasets

Stochastic gradient descent

$$\nabla F(\boldsymbol{\theta}) = \frac{1}{N}\sum_{i=1}^{N}\nabla L(h(x_i, \mathbf{w}, b), y_i)$$

$$\nabla F(\boldsymbol{\theta}) \approx \frac{1}{K}\sum_{k=1}^{K}\nabla L(h(x_{i_k}, \mathbf{w}, b), y_{i_k})$$

- Randomly sample small subset of examples
- Compute gradient on small subset
    - *Unbiased estimate of true gradient*
- Take step along estimated gradient

General recipe

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- Minimize average loss on the training set using SGD

$$\min_{\mathbf{w}, b} \frac{1}{N} \sum_{i=1}^{N} L(h(x_i, \mathbf{w}, b), y_i)$$

General recipe

- Fix hypothesis class

$$h(x; \mathbf{w}, b) = \sigma(\mathbf{w}^T \phi(x) + b)$$

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$$\min_{\mathbf{w}, b} \frac{1}{N} \sum_{i=1}^{N} L(h(x_i, \mathbf{w}, b), y_i)$$

- *Why should this work?*

Why should this work?

- Let us look at the objective more carefully

$$\min_{\mathbf{w},b} \frac{1}{N} \sum_{i=1}^{N} L(h(x_i, \mathbf{w}, b), y_i)$$

- We are minimizing average loss on the training set
- Is this what we actually care about?

Risk

- Given:
  - Distribution $\mathcal{D}$ over (x,y) pairs
  - A hypothesis $h \in H$ from hypothesis class H
  - Loss function L
- We are interested in Expected Risk:

$$R(h) = \mathbb{E}_{(x,y)\sim\mathcal{D}} L(h(x), y)$$

- Given training set S, and a particular hypothesis h, Empirical Risk:

$$\hat{R}(S, h) = \frac{1}{|S|} \sum_{(x,y)\in S} L(h(x), y)$$

Risk

$$R(h) = \mathbb{E}_{(x,y)\sim\mathcal{D}} L(h(x), y) \qquad \hat{R}(S, h) = \frac{1}{|S|} \sum_{(x,y)\in S} L(h(x), y)$$

- Left: true quantity of interest, right: estimate
- How good is this estimate?
- If h is *randomly chosen,* actually a pretty good estimate!
  - In statistics-speak, it is an *unbiased estimator* : correct in expectation

$$\mathbb{E}_{S\sim\mathcal{D}^n} \hat{R}(S, h) = R(h)$$

Risk

- Empirical risk unbiased estimate of expected risk
- Want to minimize expected risk
- Idea: Minimize *empirical risk* instead
- This is the **Empirical Risk Minimization Principle**

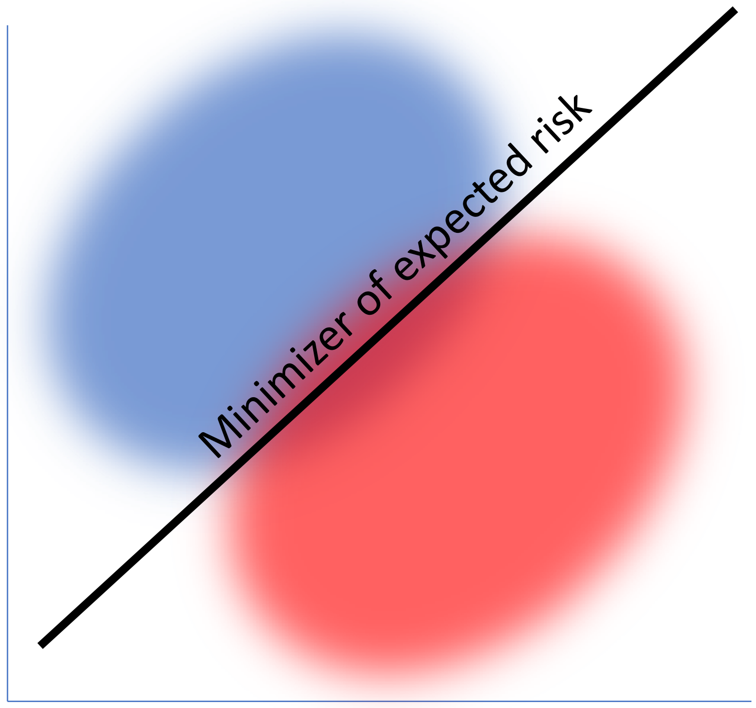
$$R(h) = \mathbb{E}_{(x,y)\sim\mathcal{D}} L(h(x), y) \qquad \hat{R}(S, h) = \frac{1}{|S|} \sum_{(x,y)\in S} L(h(x), y)$$

$$\boxed{h^* = \arg\min_{h\in H} \hat{R}(S, h)}$$

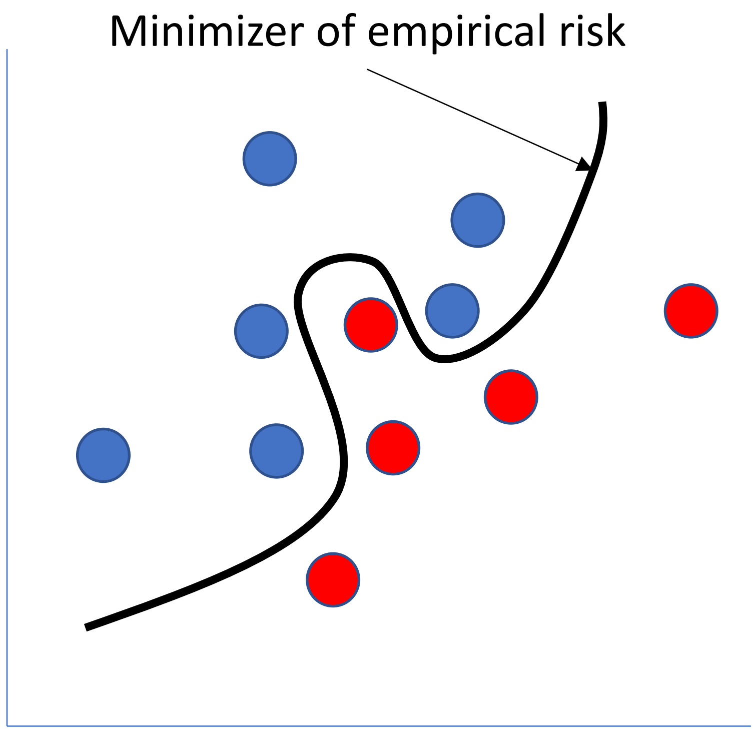
Overfitting

- For *randomly chosen* h, empirical risk (training error) good estimate of expected risk
- But we are *choosing* h by minimizing training error
- Empirical risk of chosen hypothesis *no longer* unbiased estimate:
  - We chose hypothesis based on S
  - Might have chosen h for which S is a special case
- Overfitting:
  - Minimize training error, but generalization error *increases*

Overfitting = fitting the noise



True distribution



Sampled training set

Generalization

$$R(h) = \mathbb{E}_{(x,y)\sim\mathcal{D}} L(h(x), y) \qquad \hat{R}(S,h) = \frac{1}{|S|} \sum_{(x,y)\in S} L(h(x), y)$$

$$R(h) = \hat{R}(S,h) + (R(h) - \hat{R}(S,h))$$

Training error

Generalization error

Controlling generalization error

- How do we know we are overfitting?
  - Use a *held-out* “validation set”
  - To be an unbiased sample, must be completely *unseen*

Controlling generalization error

- Variance of empirical risk inversely proportional to size of S
  - Choose very large S!
- *Larger* the hypothesis class H, *Higher* the chance of hitting bad hypotheses with low training error and high generalization error
  - Choose small H!
- For many models, can *bound* generalization error using some property of parameters
  - Regularize during optimization!
  - Eg. L2 regularization

Controlling the size of the hypothesis class

$$h(x; \mathbf{w}, b) = \sigma(\mathbf{w}^T \phi(x) + b)$$

- How many parameters (**w,** b) are there to find?
- Depends on dimensionality of ϕ
- Large dimensionality = large number of parameters = more chance of overfitting
- Rule of thumb: size of training set should be at least 10x number of parameters
- Often training sets are much smaller

Regularization

- Old objective

$$\min_{\mathbf{w},b} \sum_{i=1}^{N} L(h(x_i;\mathbf{w},b),y_i)$$

- New objective

$$\min_{\mathbf{w},b} \sum_{i=1}^{N} L(h(x_i;\mathbf{w},b),y_i) + \lambda\|\mathbf{w}\|^2$$

- Why does this help?

Regularization

$$\min_{\mathbf{w},b} \sum_{i=1}^{N} L(h(x_i; \mathbf{w}, b), y_i) + \lambda \|\mathbf{w}\|^2$$

- Ensures classifier does not weigh any one feature too highly
- Makes sure classifier scores *vary slowly* when image changes

$$|\mathbf{w}^T \phi(x_1) - \mathbf{w}^T \phi(x_2)| \leq \|\mathbf{w}\| \|\phi(x_1) - \phi(x_2)\|$$

- Prevents “crazy hypotheses” that are unlikely

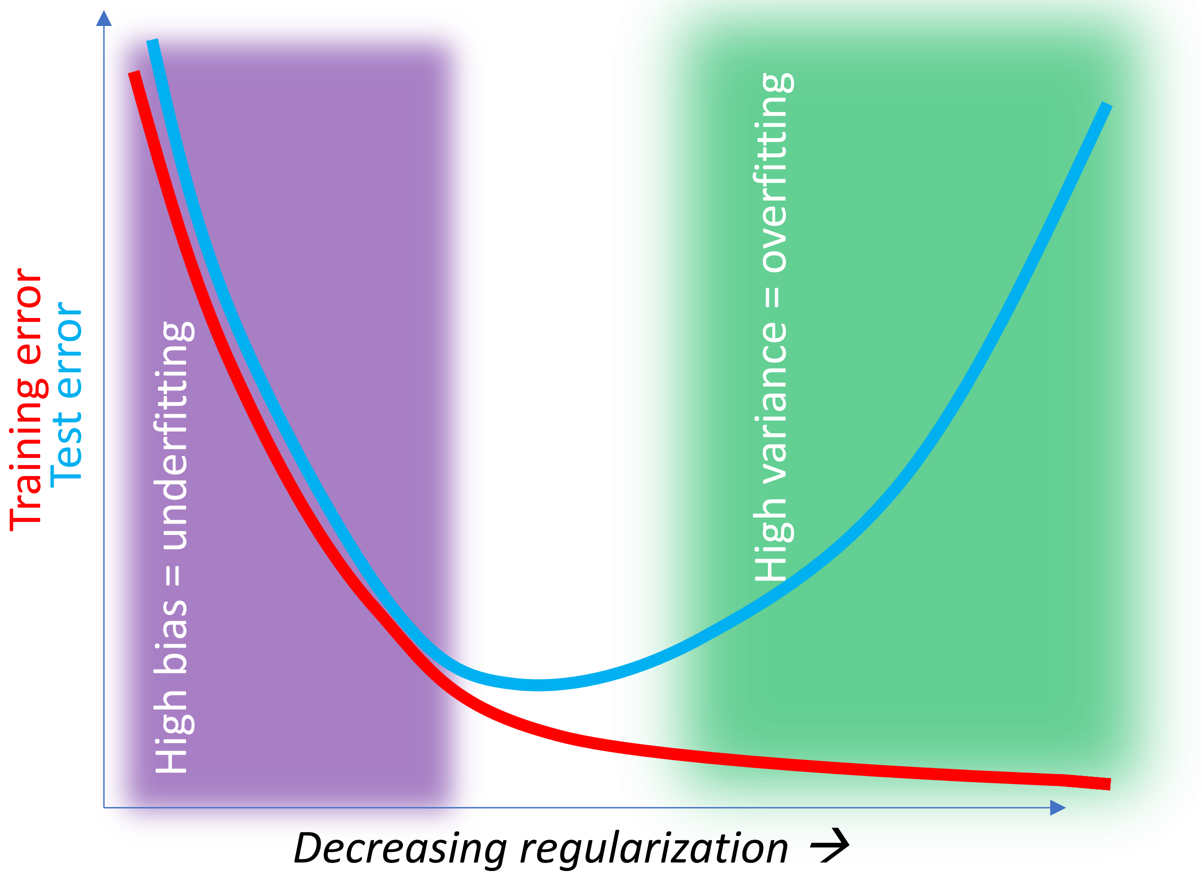
Generalization error and priors

- Regularization can be thought of as introducing prior knowledge into the model
  - L2-regularization: model output varies slowly as image changes
  - *Biases* the training to consider some hypotheses more than others
- What if bias is wrong?

Bias and variance

- Two things characterize a learning algorithm
- *Variance*
  - How sensitive is the algorithm to the training set?
  - High variance = learnt model varies a lot depending on training set
  - High variance = *overfitting*, i.e., high *generalization error*
- *Bias*
  - How much prior knowledge has been put in?
  - If prior knowledge is wrong, model learnt will not be able to achieve low loss (favors bad hypotheses in general)
  - High bias = *underfitting,* i.e., high *training error*

Bias and variance



Putting it all together

- Want model with least expected risk = expected loss
- But expected risk hard to evaluate
- Empirical Risk Minimization: minimize empirical risk in training set
- Might end up picking special case: overfitting
- Avoid overfitting by:
  - Constructing large training sets
  - Reducing size of model class
  - Regularization

Putting it all together

- Collect training set and validation set
- Pick hypothesis class
- Pick loss function
- Minimize empirical risk (+ regularization)
- Measure performance on held-out validation set
- Profit!

Loss functions and hypothesis classes

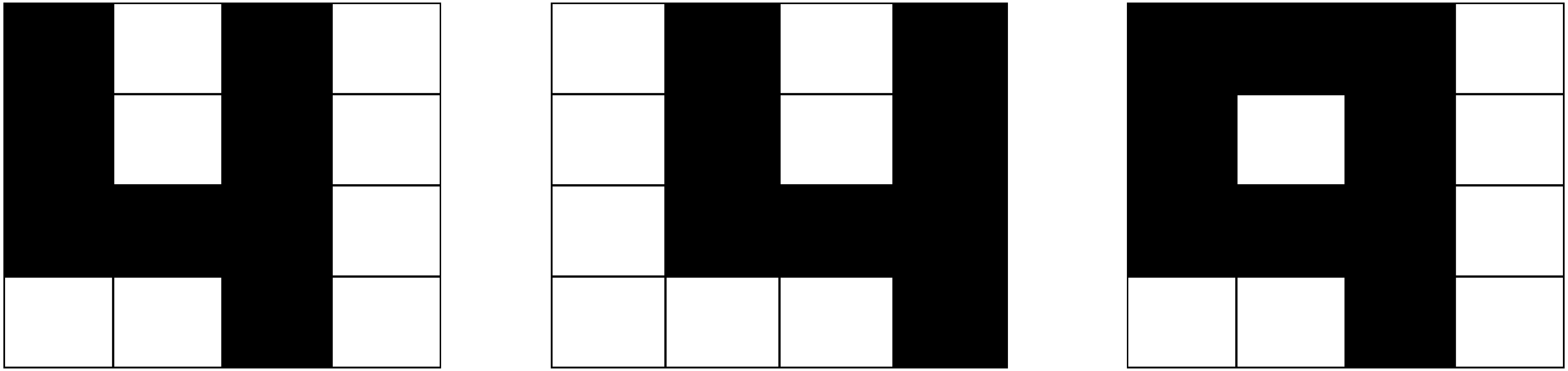
Loss function	Problem	Range of h	$\mathcal{Y}$	Formula
Log loss	Binary Classification	$\mathbb{R}$	$\{0,1\}$	$\log(1+e^{-yh(x)})$
Negative log likelihood	Multiclass classification	$[0,1]^k$	$\{1,\ldots,k\}$	$-\log h_y(x)$
Hinge loss	Binary Classification	$\mathbb{R}$	$\{0,1\}$	$\max(0,1-yh(x))$
MSE	Regression	$\mathbb{R}$	$\mathbb{R}$	$(y-h(x))^2$

Back to images

$$h(x; \mathbf{w}, b) = \sigma(\mathbf{w}^T \phi(x) + b)$$

- What should ϕ be?
- Simplest solution: string 2D image intensity values into vector

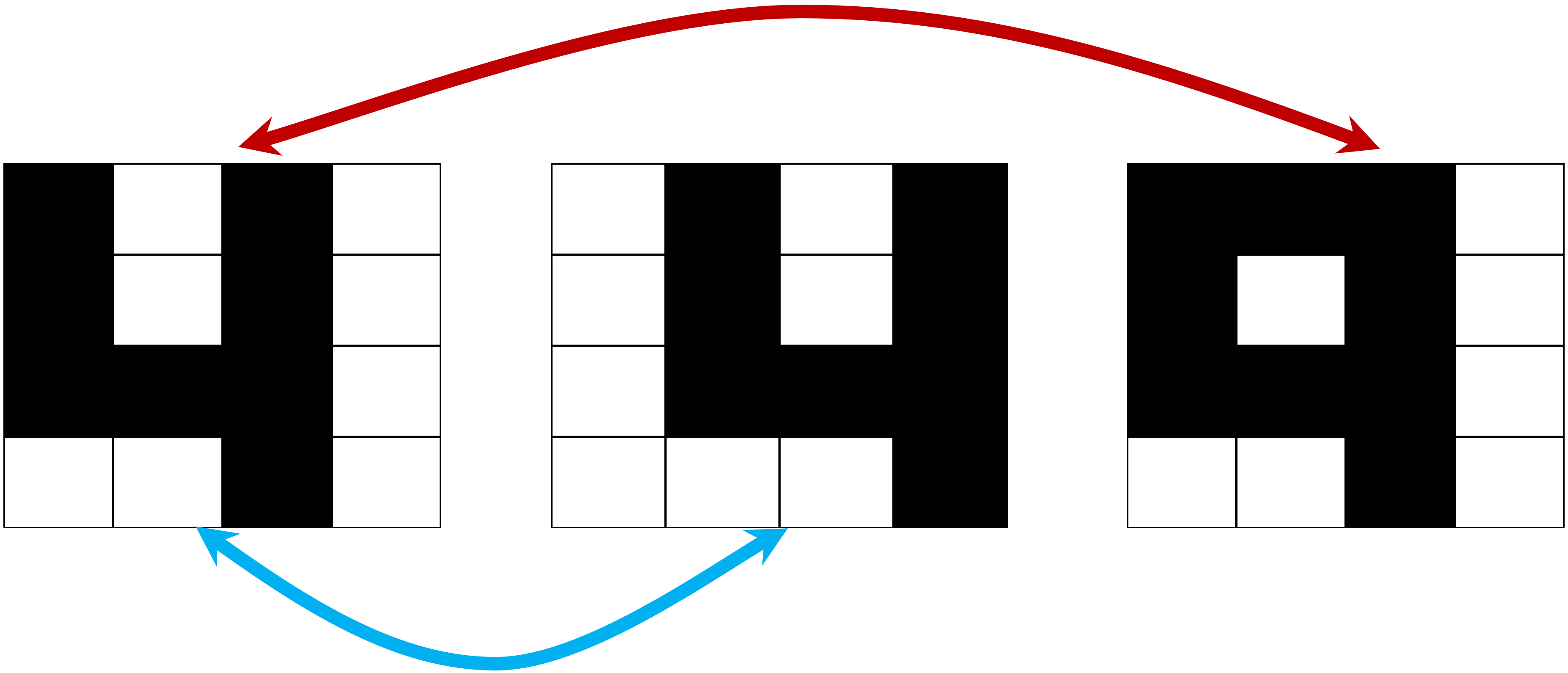
Linear classifiers on pixels are bad



- Solution 1: Better feature vectors
- Solution 2: Non-linear classifiers

Better feature vectors

These must have different feature vectors: *discriminability*

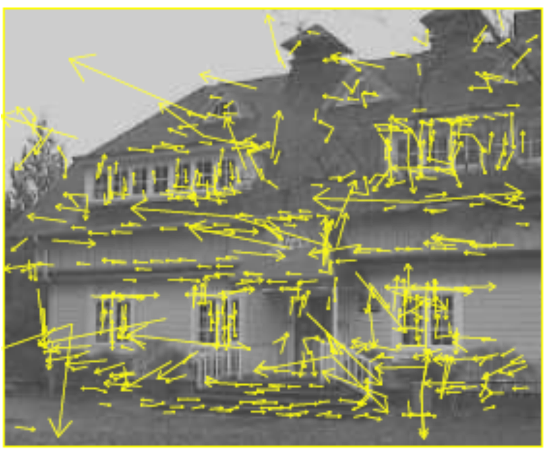


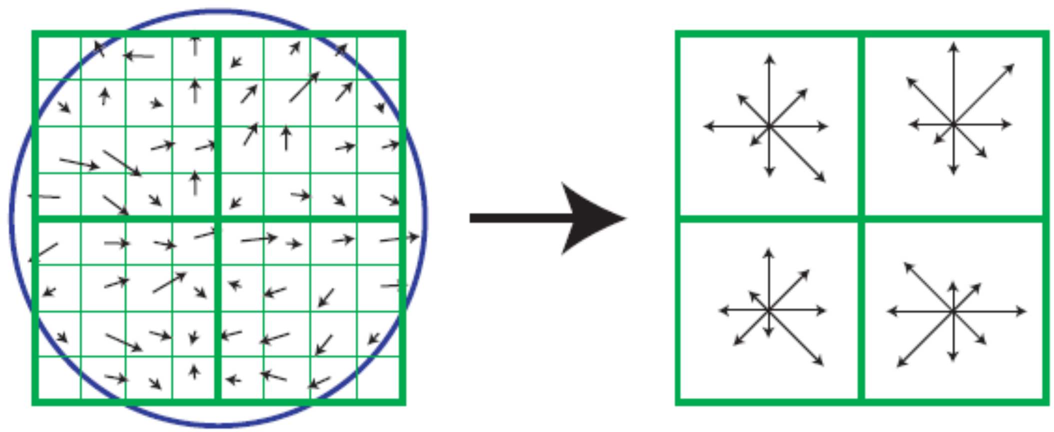
These must have similar feature vectors: *invariance*

SIFT

- Match *pattern of edges*
  - Edge orientation – clue to shape
- Be resilient to *small deformations*
  - Deformations might move pixels around, but slightly
  - Deformations might change edge orientations, but slightly

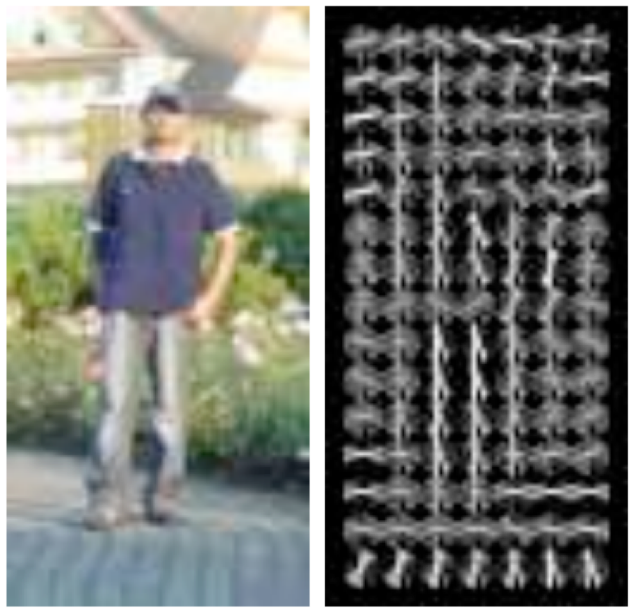
The SIFT descriptor





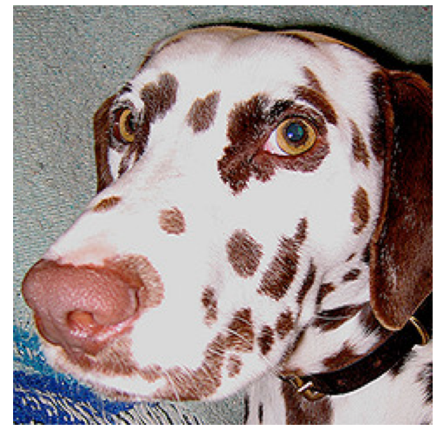
SIFT – Lowe IJCV 2004

Same but different: HOG



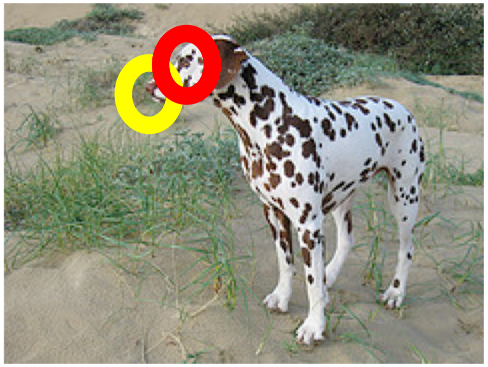
Histogram of oriented gradients
Same as SIFT but without orientation normalization. Why?

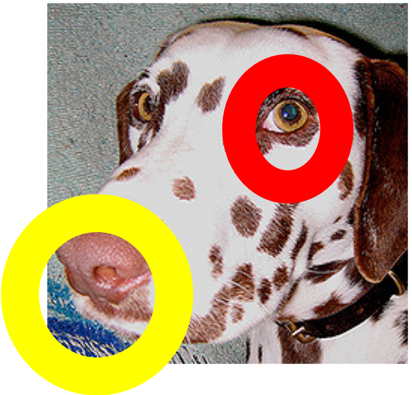
Invariance to large deformations



Invariance to large deformations

- Large deformations can cause objects / object parts to move a lot (much more than single grid cell)
- Yet, object parts themselves have precise appearance

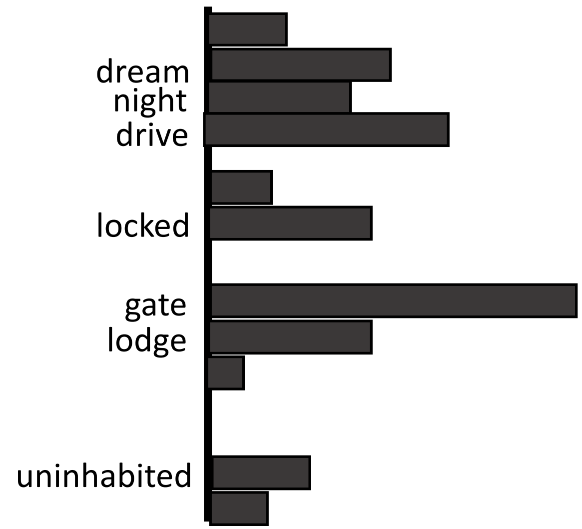




- Idea: want to represent the image as a “bag of object parts”

Bags of words

Last night I dreamt I went to Manderley again. It seemed to me I stood by the iron gate leading to the drive, and for a while I could not enter, for the way was barred to me. There was a padlock and a chain upon the gate. I called in my dream to the lodge-keeper, and had no answer, and peering closer through the rusted spokes of the gate I saw that the lodge was uninhabited….



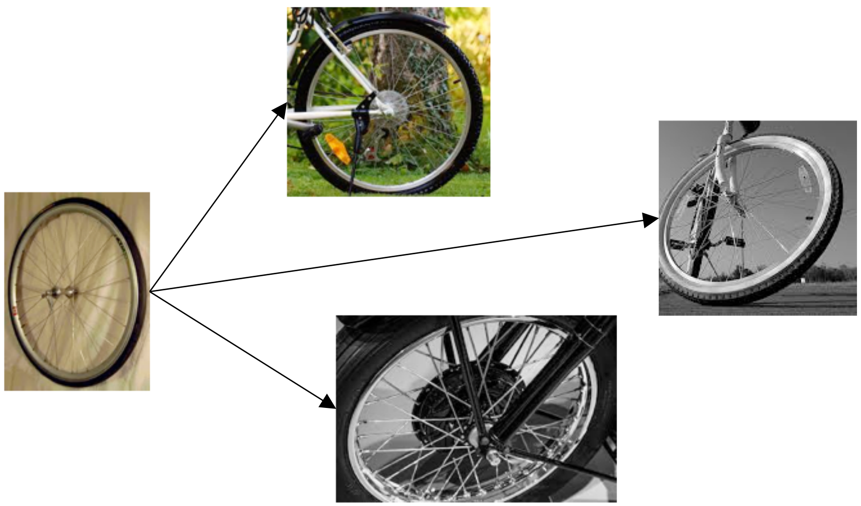
Bags of visual words

What should be visual words?

- A word is a sequence of letters that commonly occurs
  - cthn is not a word, cotton is
- Typically such a sequence of letters means something
- Visual words are image patches that frequently occur
- How do we get these visual words?

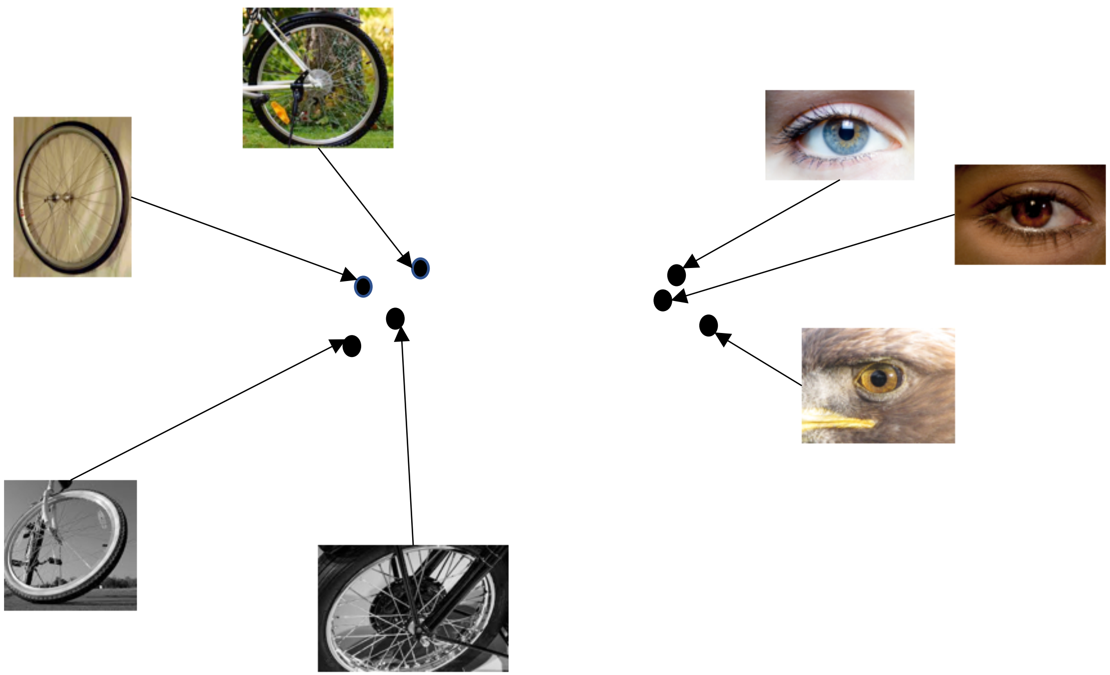
What should be visual words?

- “Image patches that occur frequently”
- ..but obviously under small variations of color and deformations
- Each occurrence of image patch is slightly different



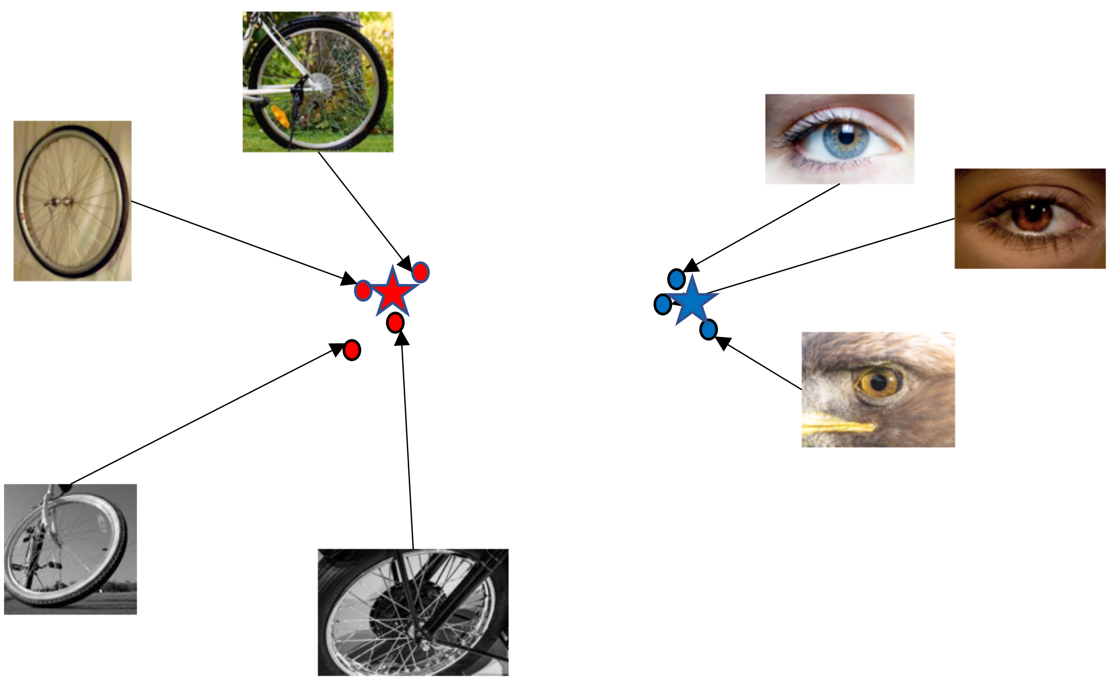
What should be visual words?

- Consider representing each image patch with SIFT descriptors
- Consider plotting them out in feature space



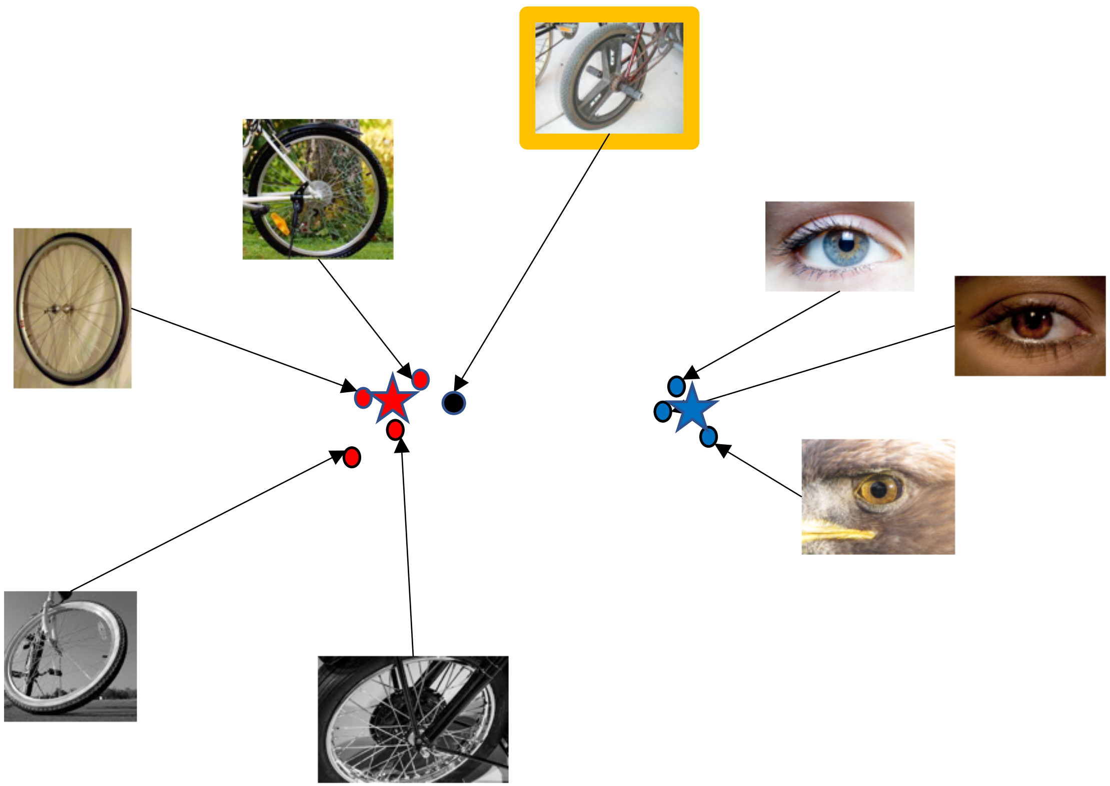
What should be visual words?

- Consider plotting SIFT feature vectors *and clustering them using k-means*



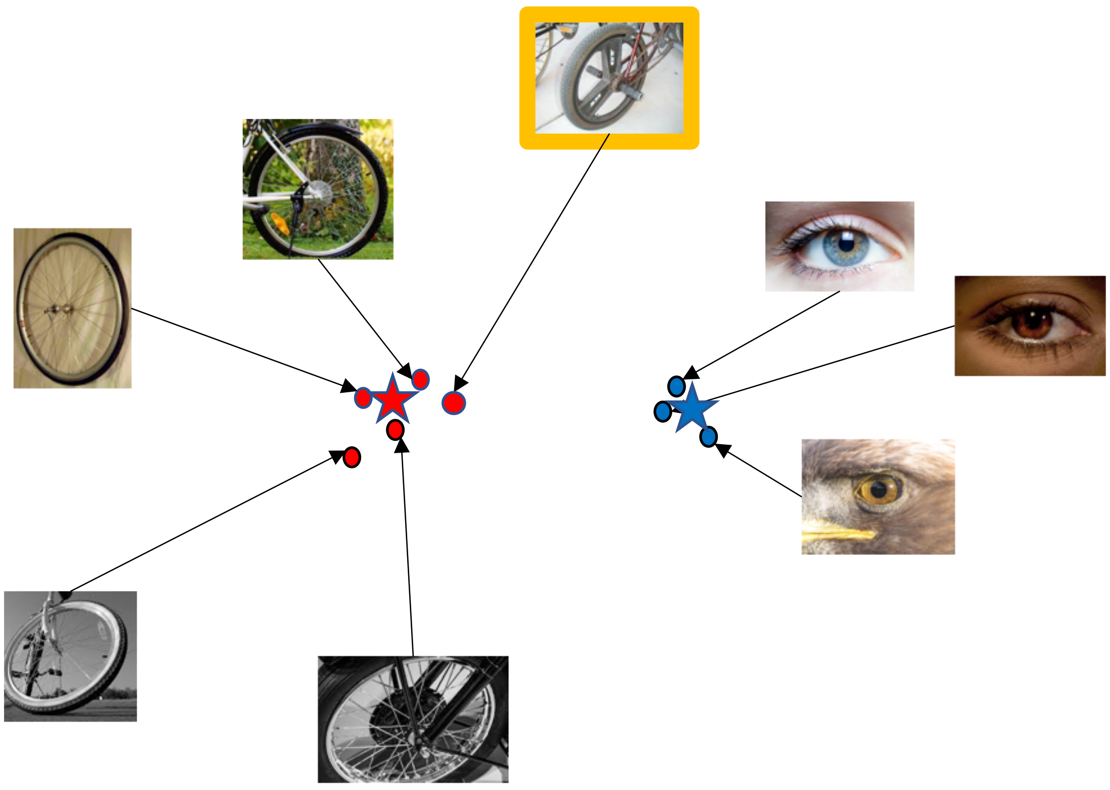
What should be visual words?

- Given a new patch, we can assign each patch to the closest center



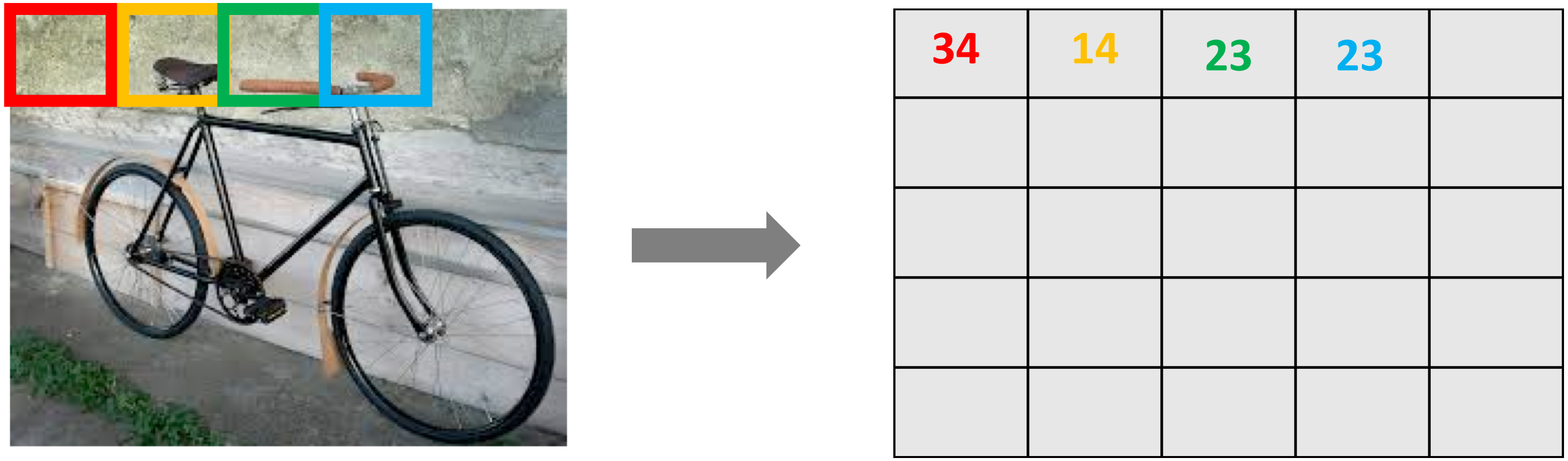
Identifying the words in an image

- Given a new patch, we can assign each patch to the closest center



Identifying the words in an image

- Given an image, take every patch and assign it to the closest k-means center
  - Each k-means center is a “word”



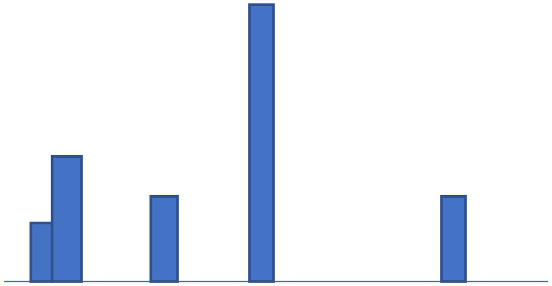
Identifying the words in an image

- Given an image, take every patch and assign it to the closest k-means center
  - Each k-means center is a "word"

34	14	23	23	34
34	19	7	8	34
34	56	7	24	56
45	13	98	45	38
7	7	34	77	29

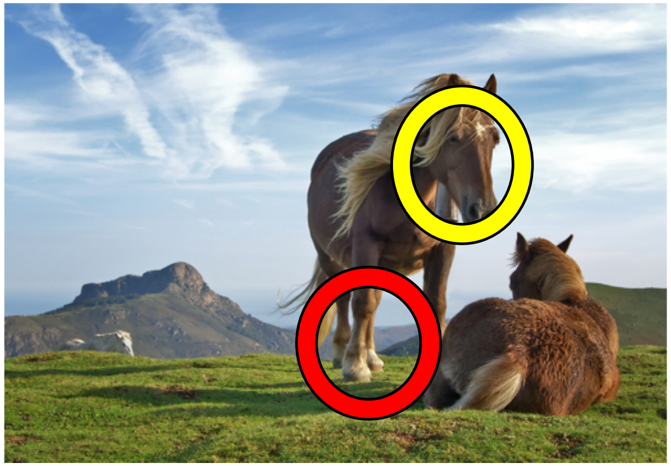
Encoding images as bag of words

- Densely extract image patches from image
- Compute SIFT vector for each patch
- Assign each patch to a visual word
- Compute histogram of occurrence



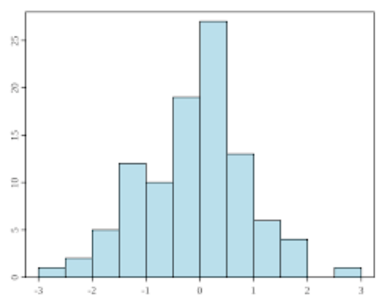
Too much invariance?

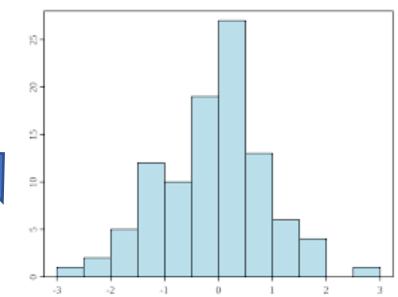
- Object parts appear in somewhat fixed relationships

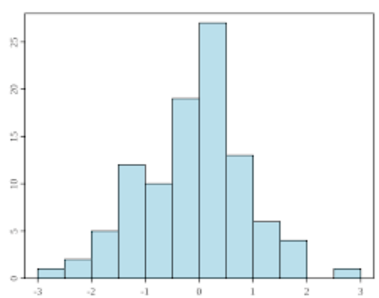


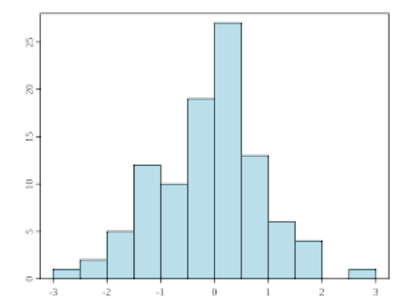
Idea: Spatial pyramids

- Divide the image into four parts
- Compute separate histogram in each part
- Concatenate into a single feature vector

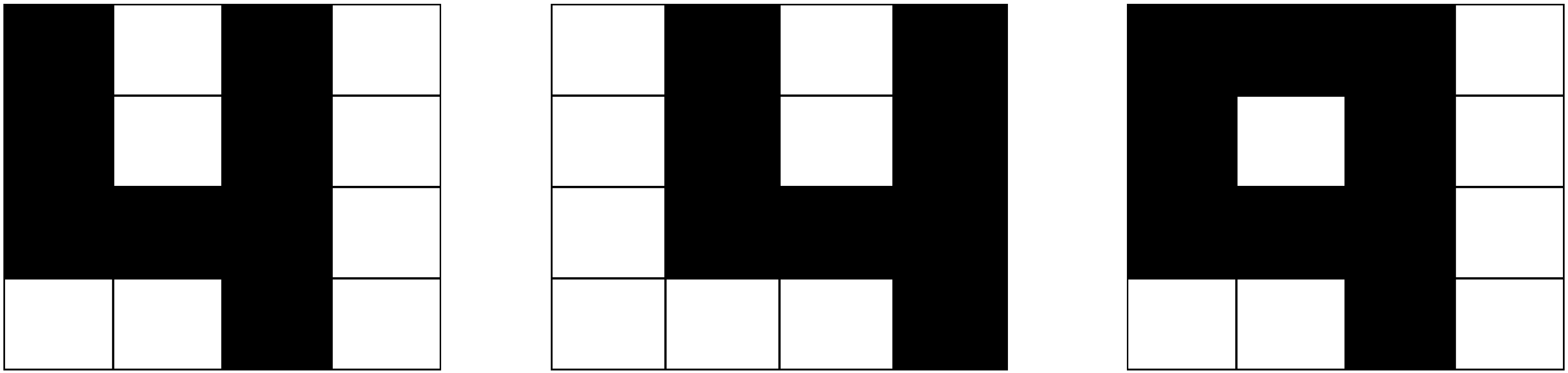








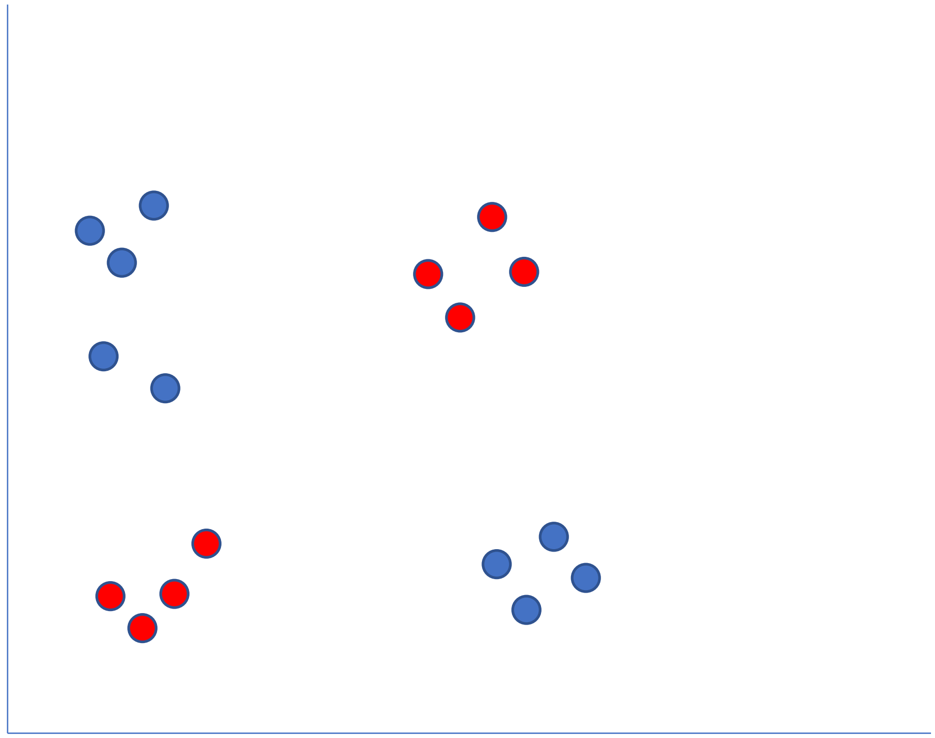
Linear classifiers on pixels are bad



- Solution 1: Better feature vectors
- Solution 2: Non-linear classifiers

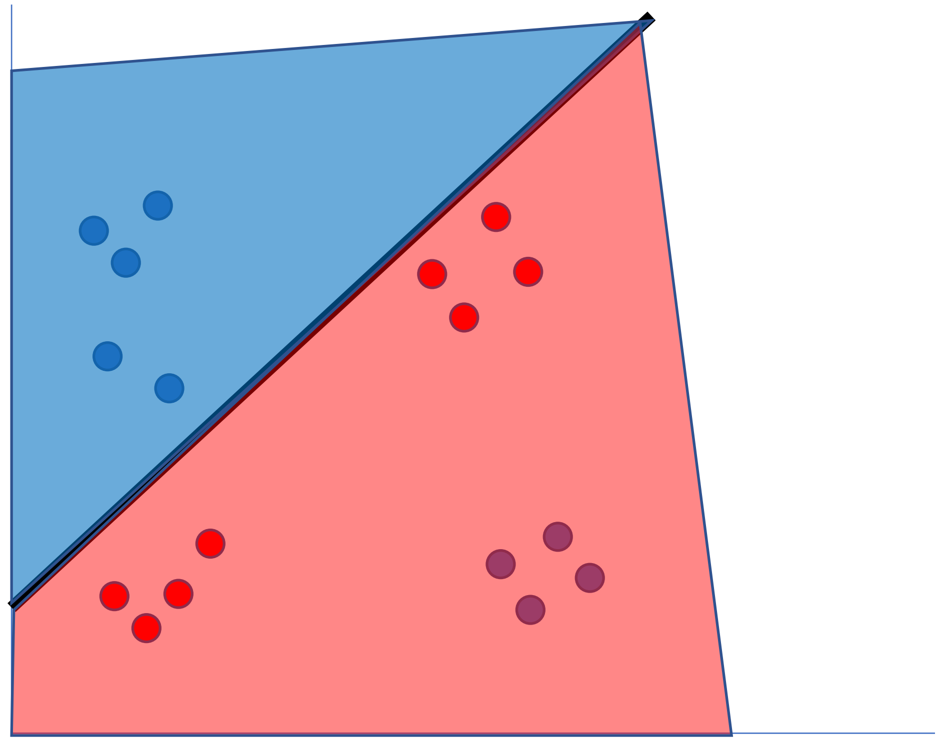
Non-linear classifiers

- Suppose we have a feature vector for every image



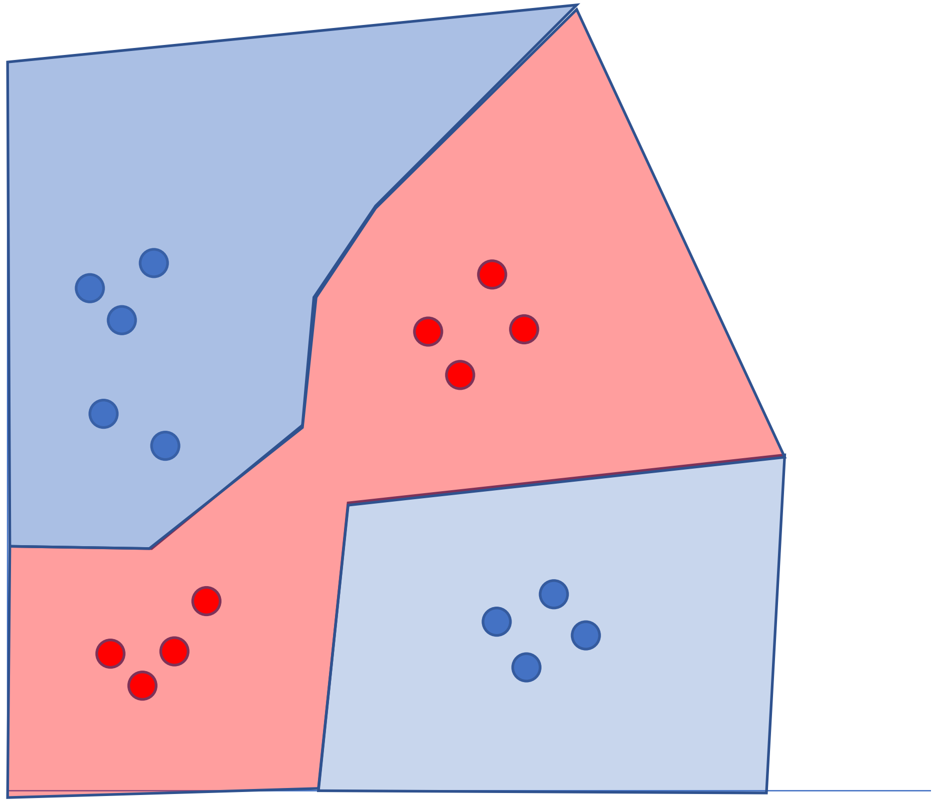
Non-linear classifiers

- Suppose we have a feature vector for every image
  - Linear classifier



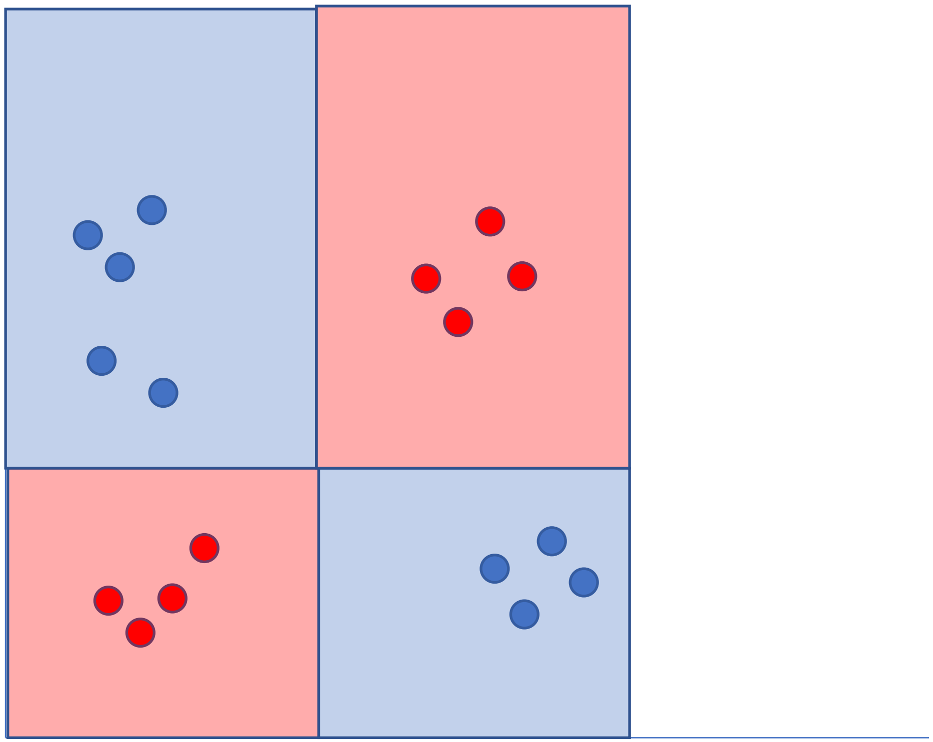
Non-linear classifiers

- Suppose we have a feature vector for every image
  - Linear classifier
  - Nearest neighbor: assign each point the label of the nearest neighbor



Non-linear classifiers

- Suppose we have a feature vector for every image
  - Linear classifier
  - Nearest neighbor: assign each point the label of the nearest neighbor
  - Decision tree: series of if-then-else statements on different features



Non-linear classifiers

- Suppose we have a feature vector for every image
  - Linear classifier
  - Nearest neighbor: assign each point the label of the nearest neighbor
  - Decision tree: series of if-then-else statements on different features
  - Neural networks