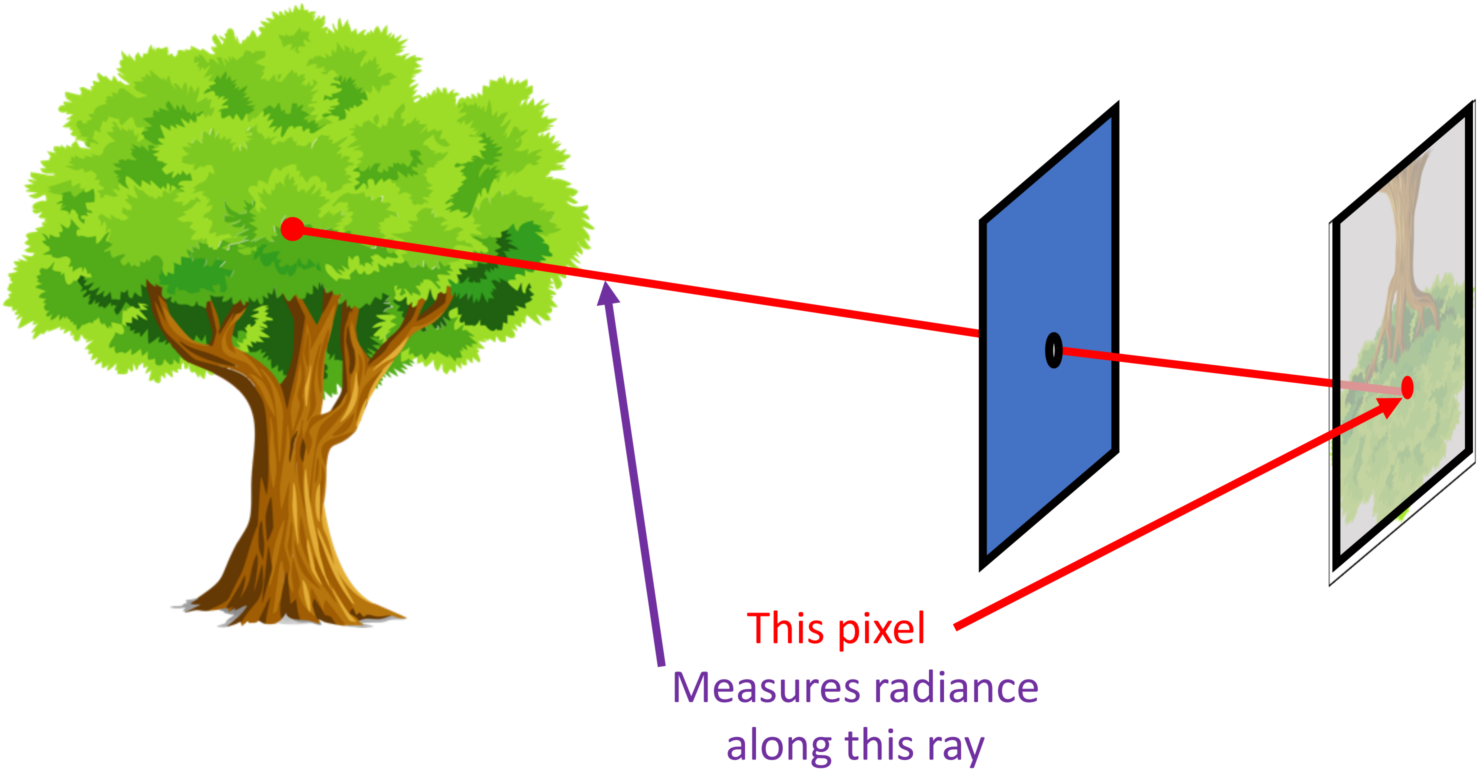
Photometric stereo

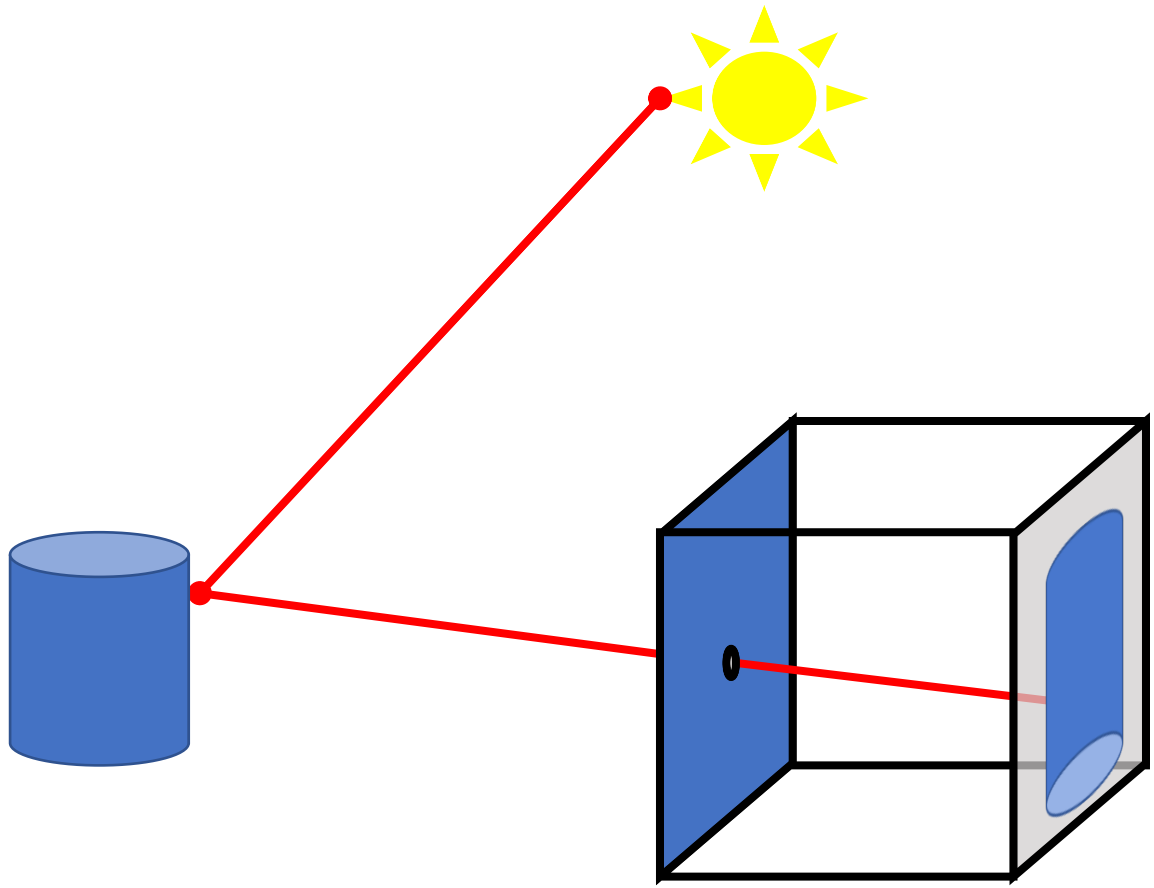
Radiance

- Pixels measure radiance



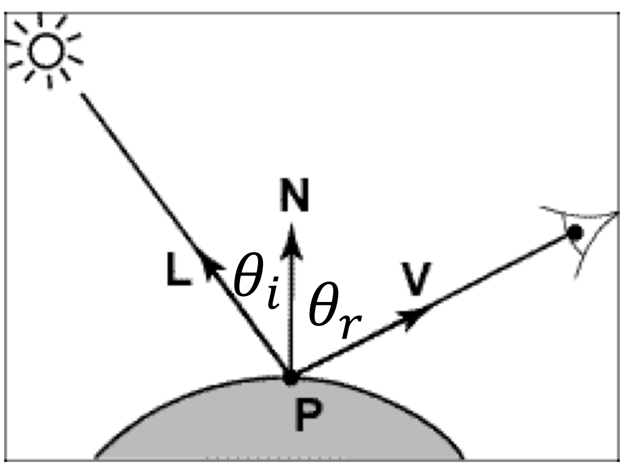
Where do the rays come from?

- Rays from the light source “reflect” off a surface and reach camera
- Reflection: Surface absorbs light energy and radiates it back



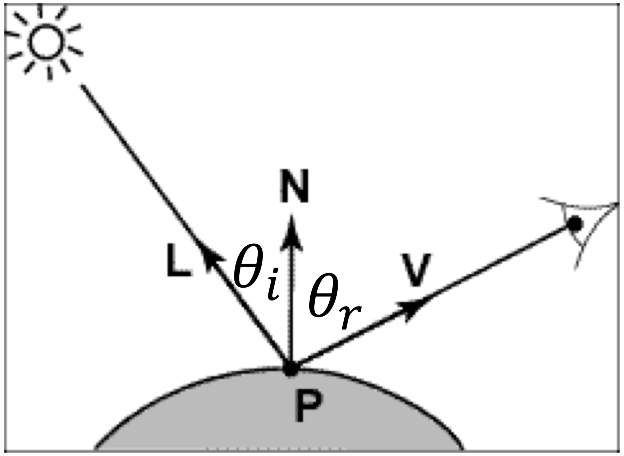
Light rays interacting with a surface

- Light of radiance L_i comes from light source at an incoming direction θ_i
- It sends out a ray of radiance L_r in the outgoing direction θ_r
- How does L_r relate to L_i ?



- **N** is surface normal
- **L** is direction of light, making θ_i with normal
- **V** is viewing direction, making θ_r with normal

Light rays interacting with a surface



- **N** is surface normal
- **L** is direction of light, making θ_i with normal
- **V** is viewing direction, making θ_r with normal

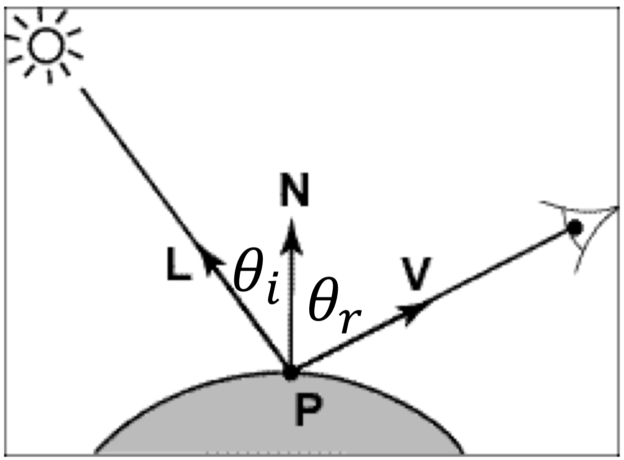
Output radiance along **V**

$$L_r = \rho(\theta_i, \theta_r) L_i \cos\theta_i$$

Incoming irradiance along **L**

Bi-directional reflectance function (BRDF)

Light rays interacting with a surface

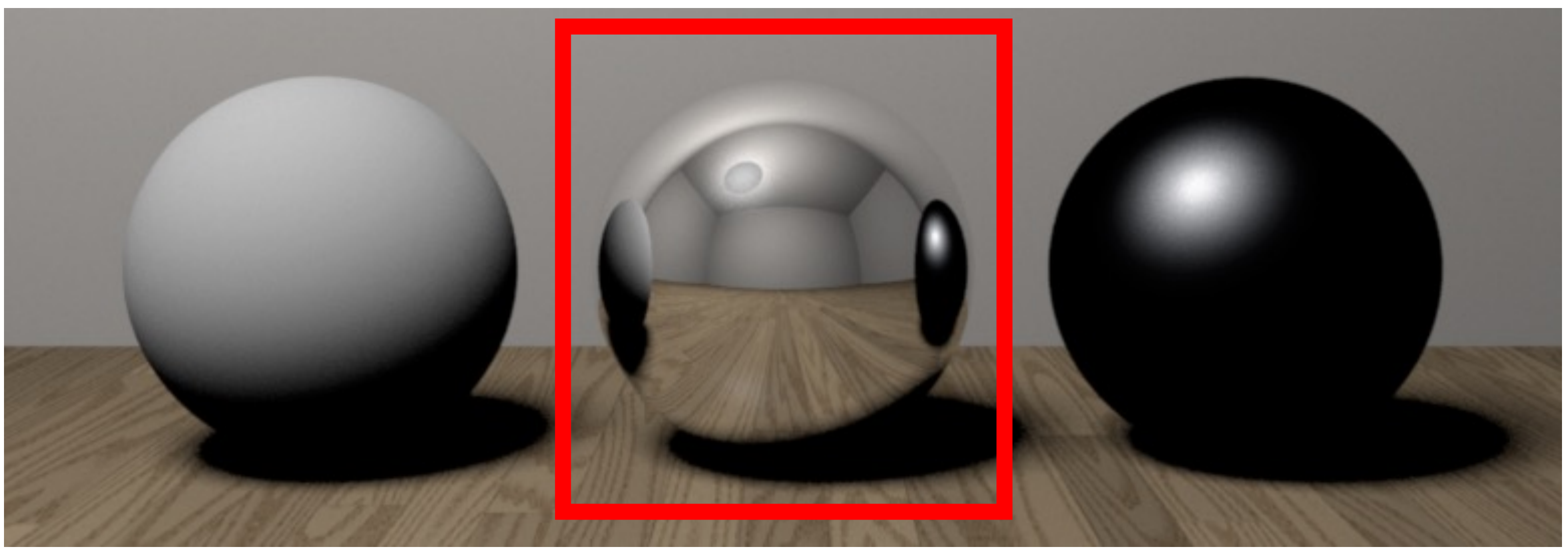


$$L_r = \rho(\theta_i, \theta_r) L_i \cos\theta_i$$

- Special case 1: Perfect mirror
  - $\rho(\theta_i, \theta_r)$ = 0 unless $\theta_i = \theta_r$
- Special case 2: Matte surface
  - $\rho(\theta_i, \theta_r)$ = ρ_0 (constant)

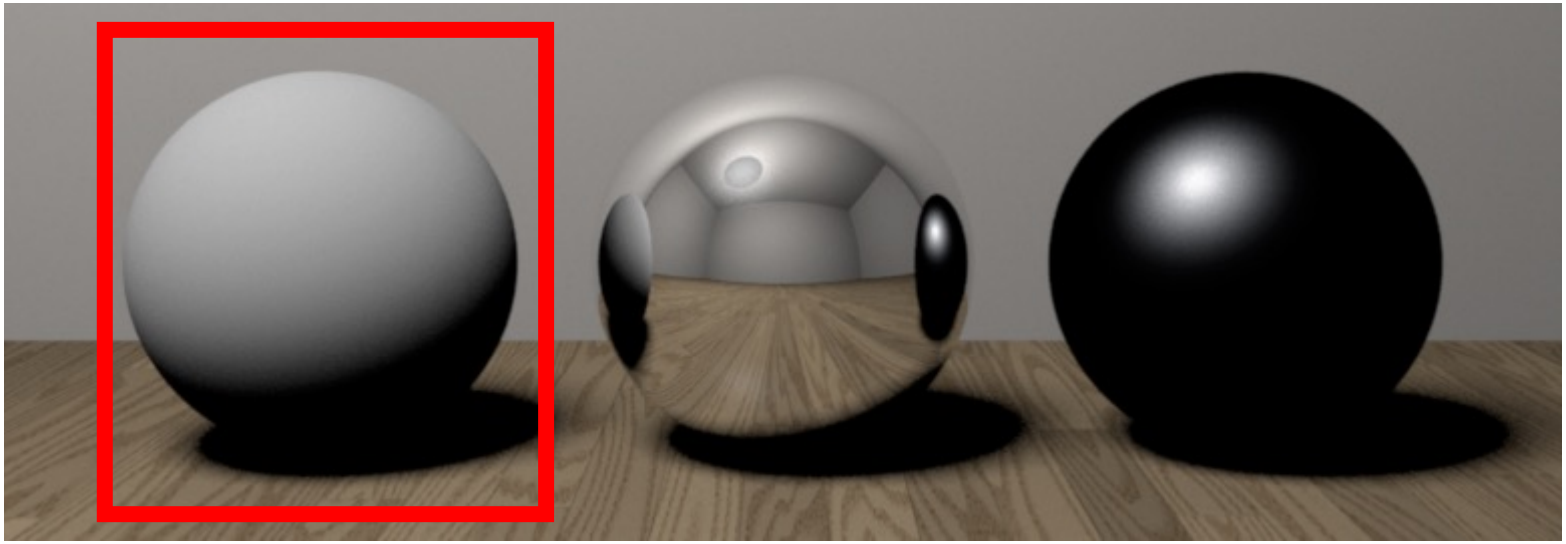
Special case 1: Perfect mirror

- $\rho(\theta_i, \theta_r)$ = 0 unless $\theta_i = \theta_r$
- Also called “Specular surfaces”
- Reflects light in a single, particular direction



Special case 2: Matte surface

- $\rho(\theta_i, \theta_r) = \rho_0$
- Also called “Lambertian surfaces”
- Reflected light is *independent of viewing direction*

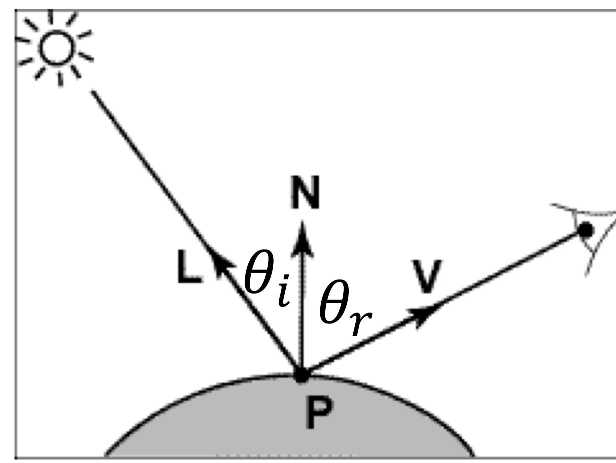


Lambertian surfaces

- For a lambertian surface:

$$L_r = \rho L_i \cos \theta_i$$

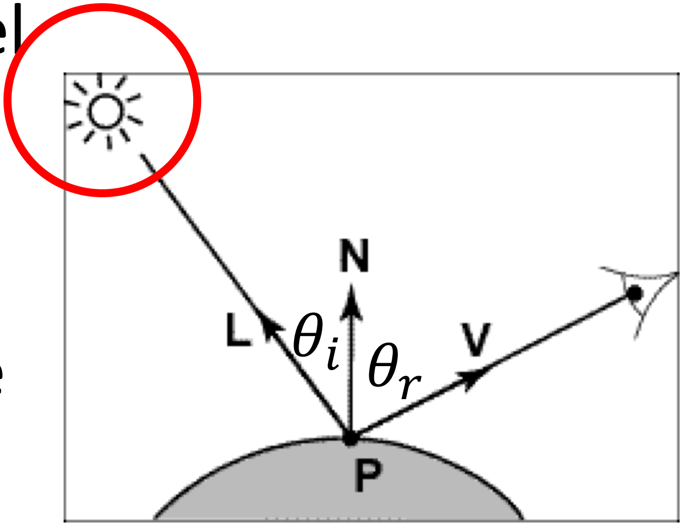
$$\Rightarrow L_r = \rho L_i \mathbf{L} \cdot \mathbf{N}$$



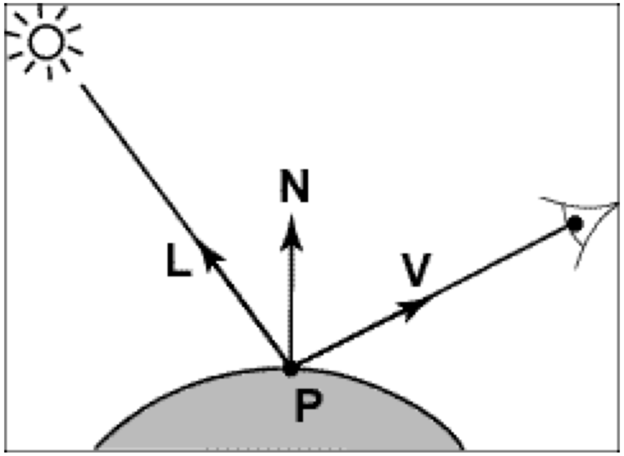
- ρ is called *albedo*
  - Think of this as paint
  - High albedo: white colored surface
  - Low albedo: black surface
  - Varies from point to point

Lambertian surfaces

- Assume the light is directional: all rays from light source are parallel
  - Equivalent to a light source infinitely far away
- All pixels get light from the same direction **L** and of the same intensity L_i



Lambertian surfaces



Intrinsic Image Decomposition

$$I(x,y) = \rho(x,y) L_i \mathbf{L} \cdot \mathbf{N}(x,y)$$

Reflectance image

Shading image

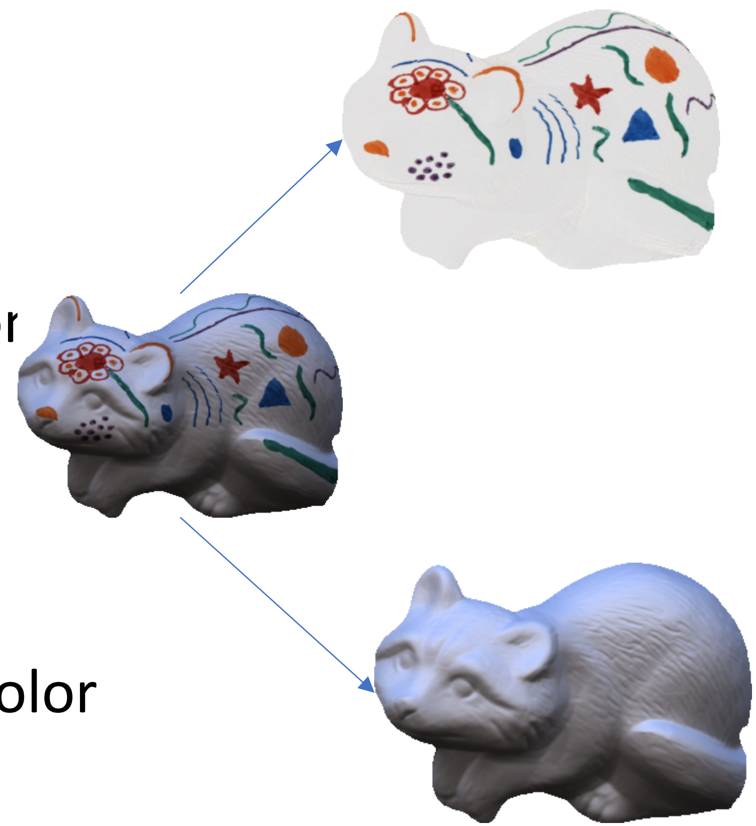
Reconstructing Lambertian surfaces

$$I(x, y) = \rho(x, y) L_i \mathbf{L} \cdot \mathbf{N}(x, y)$$

- Equation is a constraint on albedo and normals
- Can we solve for albedo and normals?

Solution 1: Recovery from a single image

- Step 1: Intrinsic image decomposition
  - Reflectance image $\rho(x, y)$
  - Shading image $L_i \mathbf{L} \cdot \mathbf{N}(x, y)$
  - Decomposition relies on priors on reflectance image
- What kind of priors?
  - Reflectance image captures the "paint" on an object surface
  - Surfaces tend to be of uniform color with sharp edges when color changes



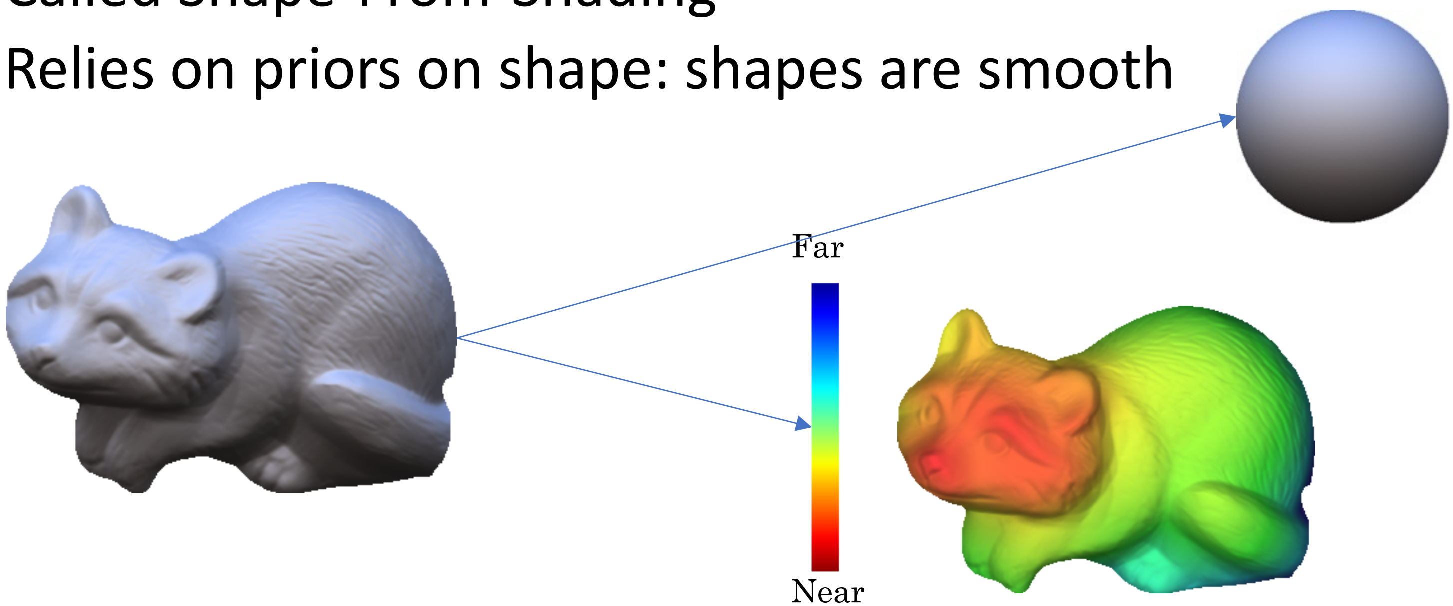
Images from Barron et al, TPAMI 13

Solution 1: Recovery from a single image

- Step 2: Decompose shading image into illumination and normals

  $$L_i \mathbf{L} \cdot \mathbf{N}(x, y)$$

  - Called Shape-From-Shading
  - Relies on priors on shape: shapes are smooth



Solution 2: Recovery from multiple images

$$I(x, y) = \rho(x, y) L_i \mathbf{L} \cdot \mathbf{N}(x, y)$$

- Represents an equation in the albedo and normals
- Multiple images give constraints on albedo and normals
- Called *Photometric Stereo*

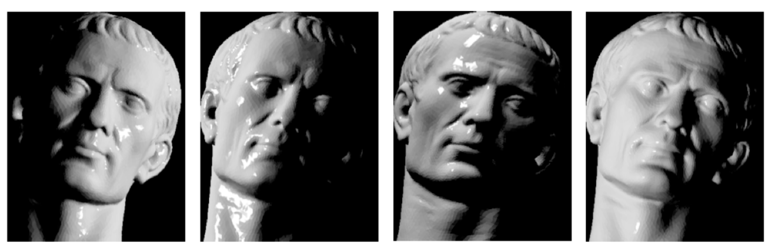
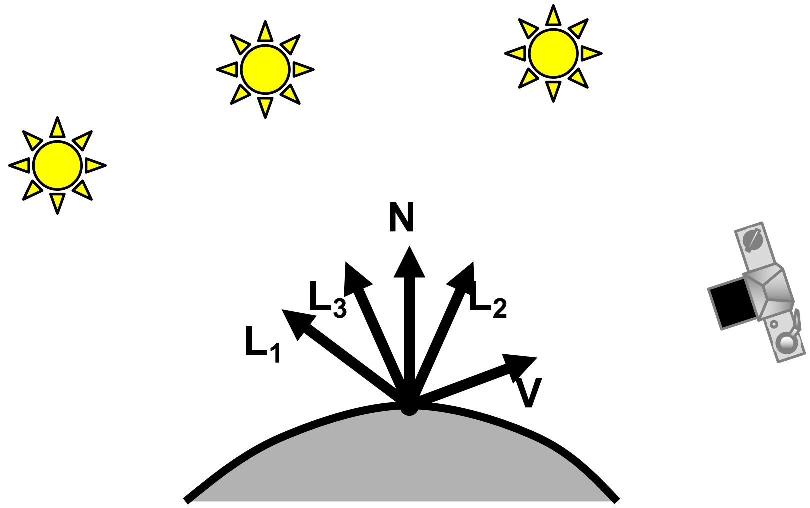


Image credit: Wikipedia

Multiple Images: Photometric Stereo



Photometric stereo - the math

$$I(x,y) = \rho(x,y) L_i \mathbf{L} \cdot \mathbf{N}(x,y)$$

- Consider single pixel
- Assume $L_i = 1$

$$I = \rho \mathbf{L} \cdot \mathbf{N}$$

$$I = \rho \mathbf{N}^T \mathbf{L}$$

- Write $\mathbf{G} = \rho \mathbf{N}$
- **G** is a 3-vector
  - Norm of **G** = ρ
  - Direction of **G** = **N**

Photometric stereo - the math

- Consider single pixel
- Assume $L_i = 1$

$$I = \rho \mathbf{N}^T \mathbf{L}$$

- Write $\mathbf{G} = \rho \mathbf{N}$
- **G** is a 3-vector
    - Norm of **G** = ρ
    - Direction of **G** = **N**

$$I = \mathbf{G}^T \mathbf{L} = \mathbf{L}^T \mathbf{G}$$

Photometric stereo - the math

$$I = \mathbf{L}^T \mathbf{G}$$

- Multiple images with different light sources but same viewing direction?

$$I_1 = \mathbf{L}_1^T \mathbf{G}$$

$$I_2 = \mathbf{L}_2^T \mathbf{G}$$

$$\vdots$$

$$I_k = \mathbf{L}_k^T \mathbf{G}$$

Photometric stereo - the math

$$I_1 = \mathbf{L}_1^T \mathbf{G}$$

$$I_2 = \mathbf{L}_2^T \mathbf{G}$$

$$\vdots$$

$$I_k = \mathbf{L}_k^T \mathbf{G}$$

- Assume lighting directions are known
- Each is a linear equation in G
- Stack everything up into a massive linear system of equations!

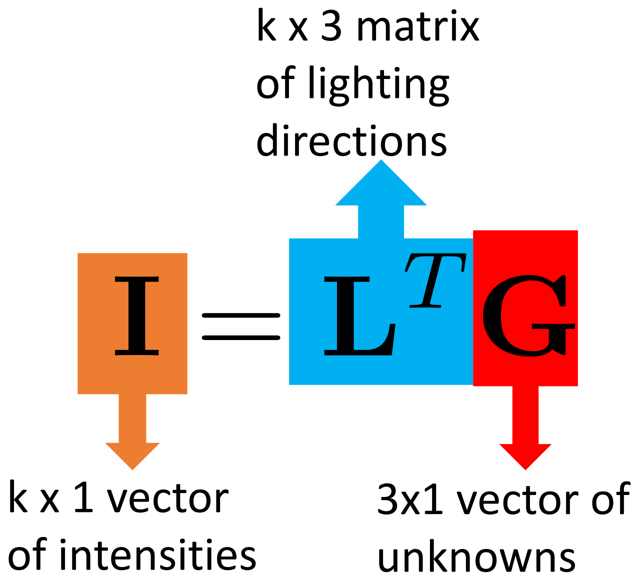
Photometric stereo - the math

$$I_1 = \mathbf{L}_1^T \mathbf{G}$$

$$I_2 = \mathbf{L}_2^T \mathbf{G}$$

$$\vdots$$

$$I_k = \mathbf{L}_k^T \mathbf{G}$$



Photometric stereo - the math

$$\mathbf{I} = \mathbf{L}^T \mathbf{G}$$

k x 1   k x 3   3 x 1

$$\mathbf{G} = \mathbf{L}^{-T} \mathbf{I}$$

- What is the minimum value of k to allow recovery of G?
- How do we recover G if the problem is overconstrained?

Photometric stereo - the math

- How do we recover G if the problem is overconstrained?
  - More than 3 lights: more than 3 images

- Least squares

$$\min_{\mathbf{G}} \|\mathbf{I} - \mathbf{L}^T \mathbf{G}\|^2$$

- Solved using normal equations

$$\mathbf{G} = (\mathbf{L}\mathbf{L}^T)^{-1}\mathbf{L}\mathbf{I}$$

Normal equations

$$\|\mathbf{I} - \mathbf{L}^T\mathbf{G}\|^2 = \mathbf{I}^T\mathbf{I} + \mathbf{G}^T\mathbf{L}\mathbf{L}^T\mathbf{G} - 2\mathbf{G}^T\mathbf{L}\mathbf{I}$$

- Take derivative with respect to **G** and set to 0

$$2\mathbf{L}\mathbf{L}^T\mathbf{G} - 2\mathbf{L}\mathbf{I} = 0$$

$$\Rightarrow \mathbf{G} = (\mathbf{L}\mathbf{L}^T)^{-1}\mathbf{L}\mathbf{I}$$

Estimating normals and albedo from G

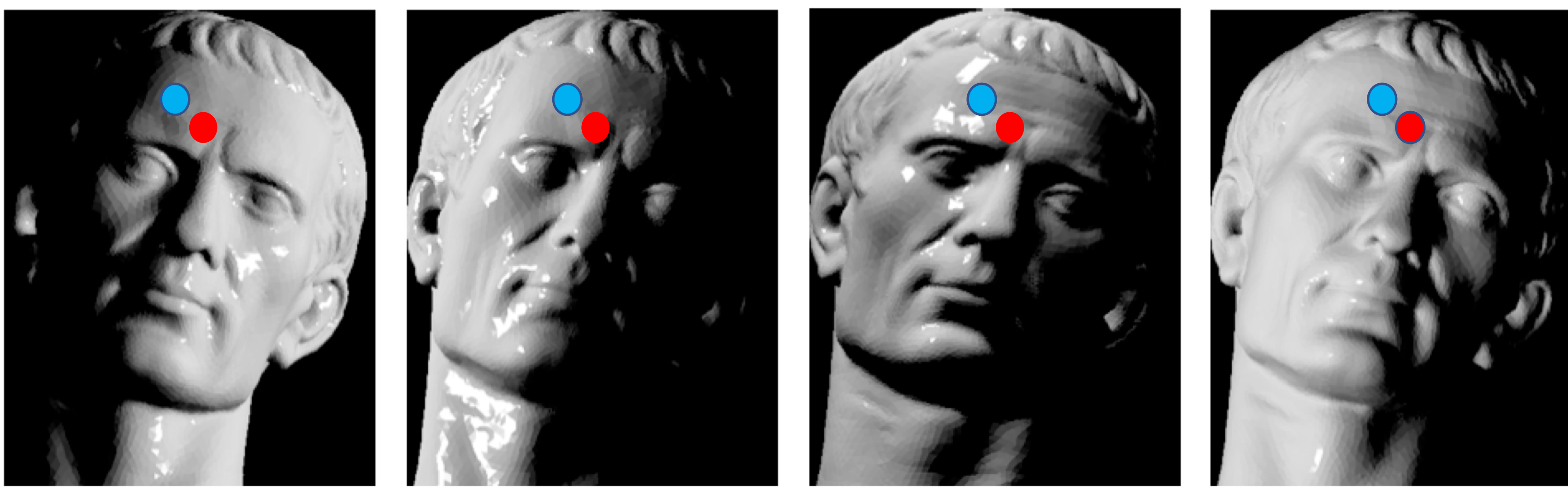
- Recall that $\mathbf{G} = \rho \mathbf{N}$

$$\|\mathbf{G}\| = \rho$$

$$\frac{\mathbf{G}}{\|\mathbf{G}\|} = \mathbf{N}$$

Multiple pixels

- We've looked at a single pixel till now
- How do we handle multiple pixels?
- Essentially independent equations!



Multiple pixels: matrix form

- Note that all pixels share the same set of lights

$$\mathbf{I}^{(1)} = \mathbf{L}^T \mathbf{G}^{(1)}$$

$$\mathbf{I}^{(2)} = \mathbf{L}^T \mathbf{G}^{(2)}$$

$$\vdots$$

$$\mathbf{I}^{(n)} = \mathbf{L}^T \mathbf{G}^{(n)}$$

Multiple pixels: matrix form

- Can stack these into *columns* of a matrix

$$\mathbf{I}^{(1)} = \mathbf{L}^T \mathbf{G}^{(1)}$$

$$\mathbf{I}^{(2)} = \mathbf{L}^T \mathbf{G}^{(2)}$$

$$\vdots$$

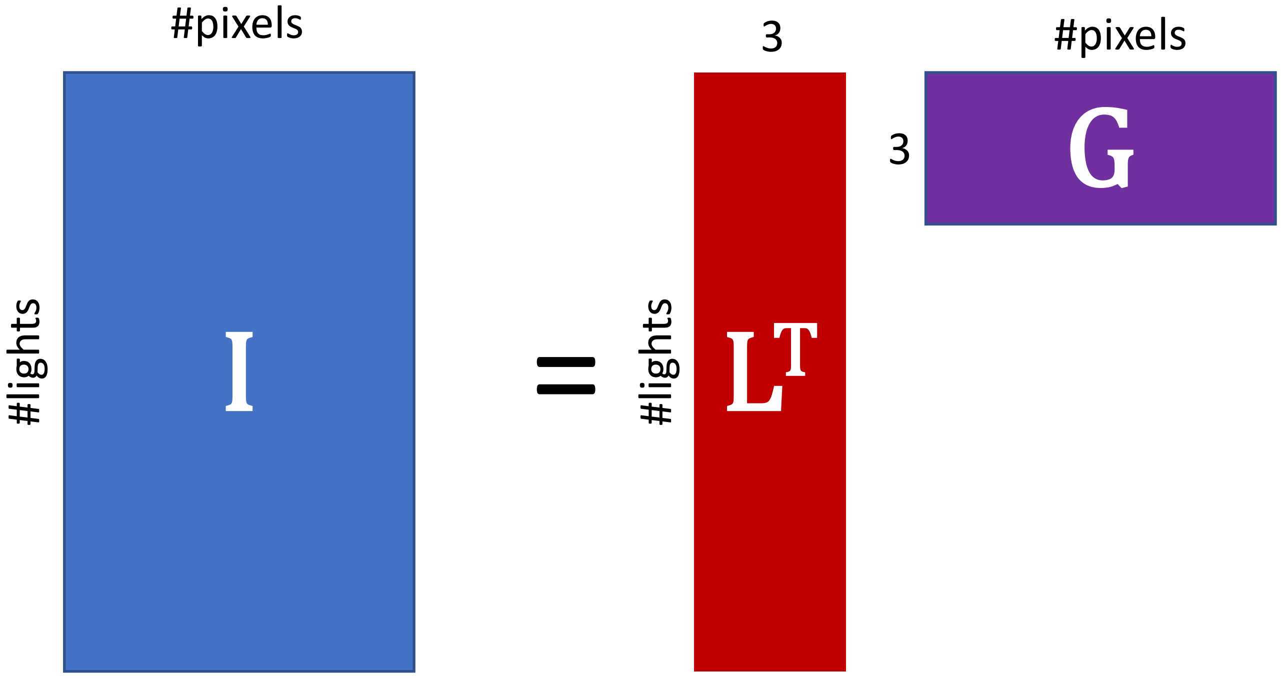
$$\mathbf{I}^{(n)} = \mathbf{L}^T \mathbf{G}^{(n)}$$

$$\begin{bmatrix} \mathbf{I}^{(1)} & \mathbf{I}^{(2)} & \cdots & \mathbf{I}^{(n)} \end{bmatrix} = \mathbf{L}^T \begin{bmatrix} \mathbf{G}^{(1)} & \mathbf{G}^{(2)} & \cdots & \mathbf{G}^{(n)} \end{bmatrix}$$

$$\boxed{\mathbf{I} = \mathbf{L}^T \mathbf{G}}$$

Multiple pixels: matrix form

$$\mathbf{I} = \mathbf{L}^T \mathbf{G}$$

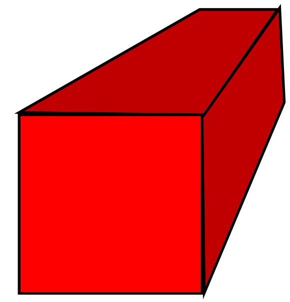


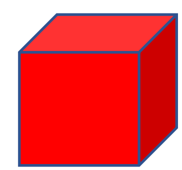
Estimating depth from normals

- So we got surface normals, can we get depth?
- Yes, given *boundary conditions*
- Normals provide information about the derivative

Brief detour: Orthographic projection

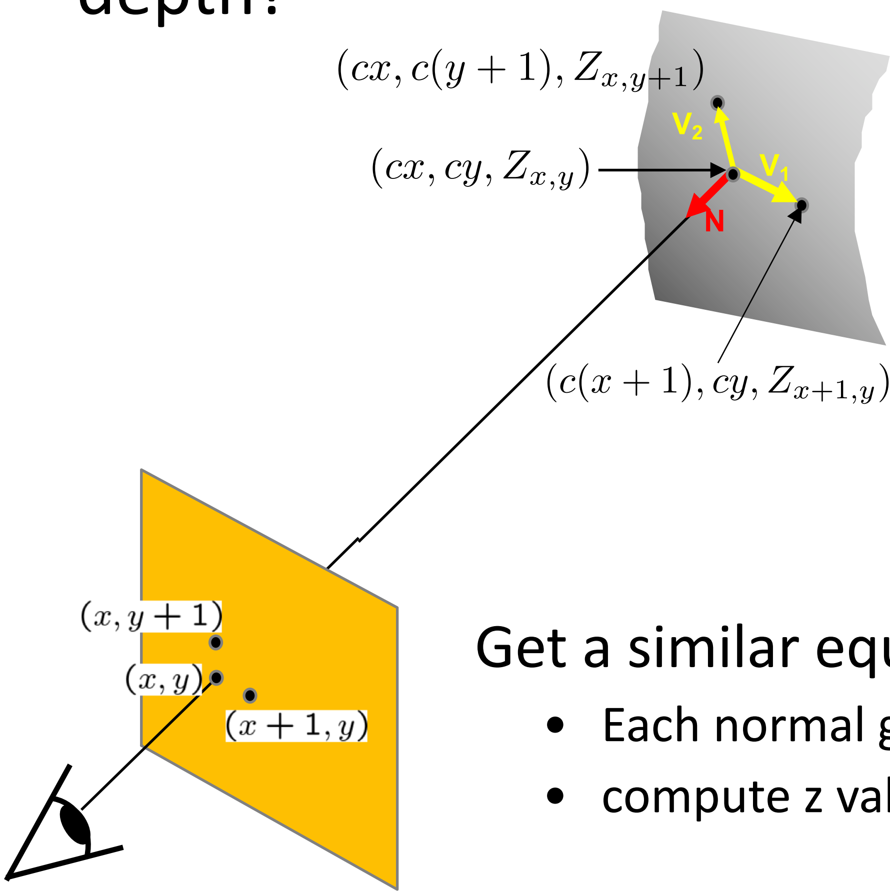
- Perspective projection
  - $x = \frac{X}{Z}, y = \frac{Y}{Z}$
- If all points have similar depth
  - $Z \approx Z_0$
  - $x \approx \frac{X}{Z_0}, y \approx \frac{Y}{Z_0}$
  - $x \approx cX, y \approx cY$
- A scaled version of orthographic projection
  - $x = X, y = Y$


Perspective


Scaled orthographic

Depth Map from Normal Map

- We now have a surface normal, but how do we get depth?



Assume a smooth surface

$$
\begin{aligned}
V_1 &= (c(x+1), cy, Z_{x+1,y}) - (cx, cy, Z_{x,y}) \\
&= (c, 0, Z_{x+1,y} - Z_{x,y})
\end{aligned}
$$

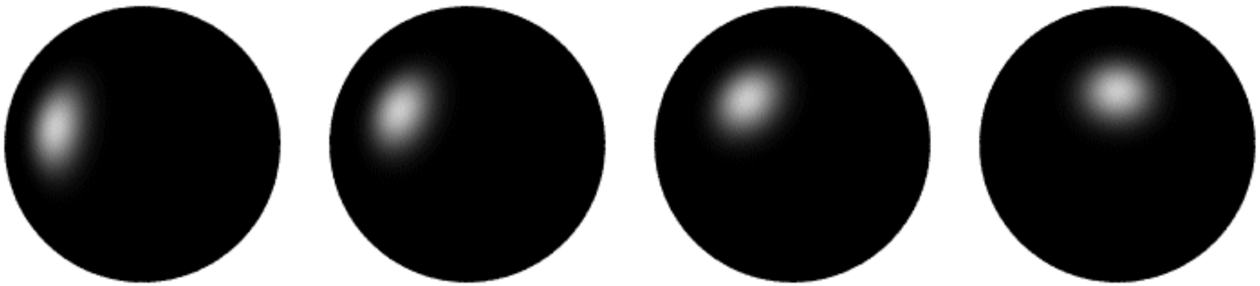
$$
\begin{aligned}
0 &= N \cdot V_1 \\
&= (n_x, n_y, n_z) \cdot (c, 0, Z_{x+1,y} - Z_{x,y}) \\
&= cn_x + n_z(Z_{x+1,y} - Z_{x,y})
\end{aligned}
$$

Get a similar equation for $\mathbf{V_2}$

- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation

Determining Light Directions

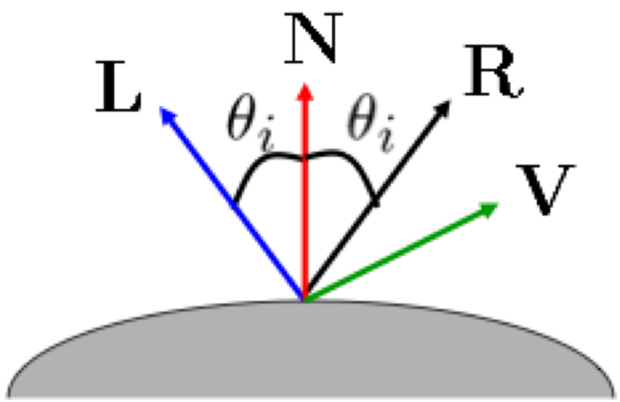
- Trick: Place a mirror ball in the scene.



- The location of the highlight is determined by the light source direction.

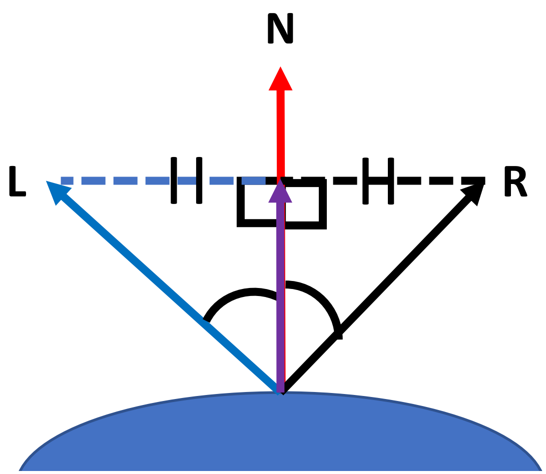
Determining Light Directions

- For a perfect mirror, the light is reflected across N:



$$I_e = \begin{cases} I_i & \text{if } \mathbf{V} = \mathbf{R} \\ 0 & \text{otherwise} \end{cases}$$

Determining Light Directions



$$= (N \cdot R)N$$

$$= R - (N \cdot R)N$$

$$= R - (N \cdot R)N$$

= - 2

$$= R - 2(R - N \cdot R)N$$

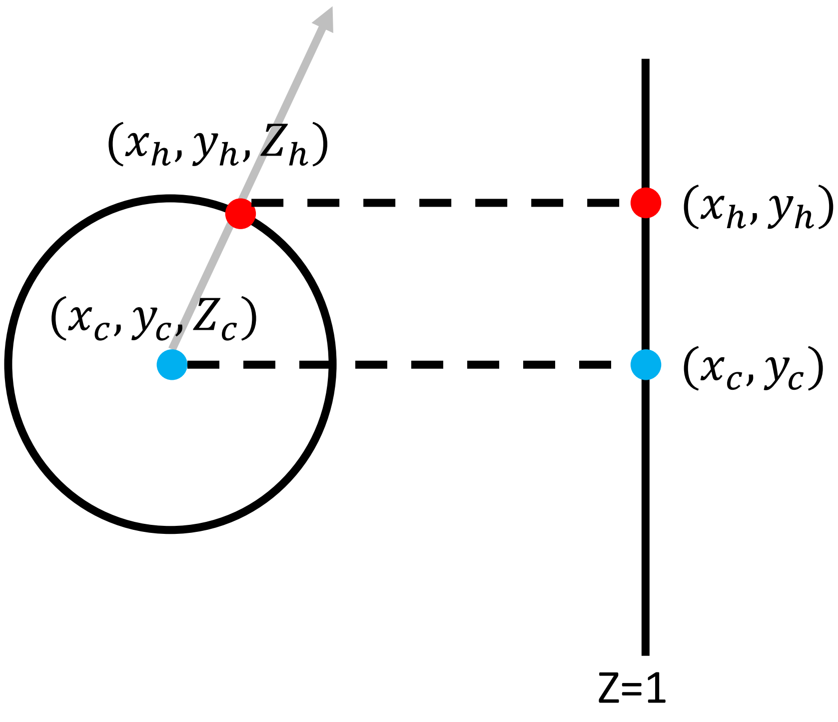
$$= 2(N \cdot R)N - R$$

So the light source direction is given by:

$$L = 2(N \cdot R)N - R$$

Determining Light Directions

- Assume orthographic projection
- Viewing direction R = [0,0,-1]
- Normal?



Z_h and Z_c are unknown, but:

$$(x_h - x_c)^2 + (y_h - y_c)^2 + (Z_h - Z_c)^2 = r^2$$

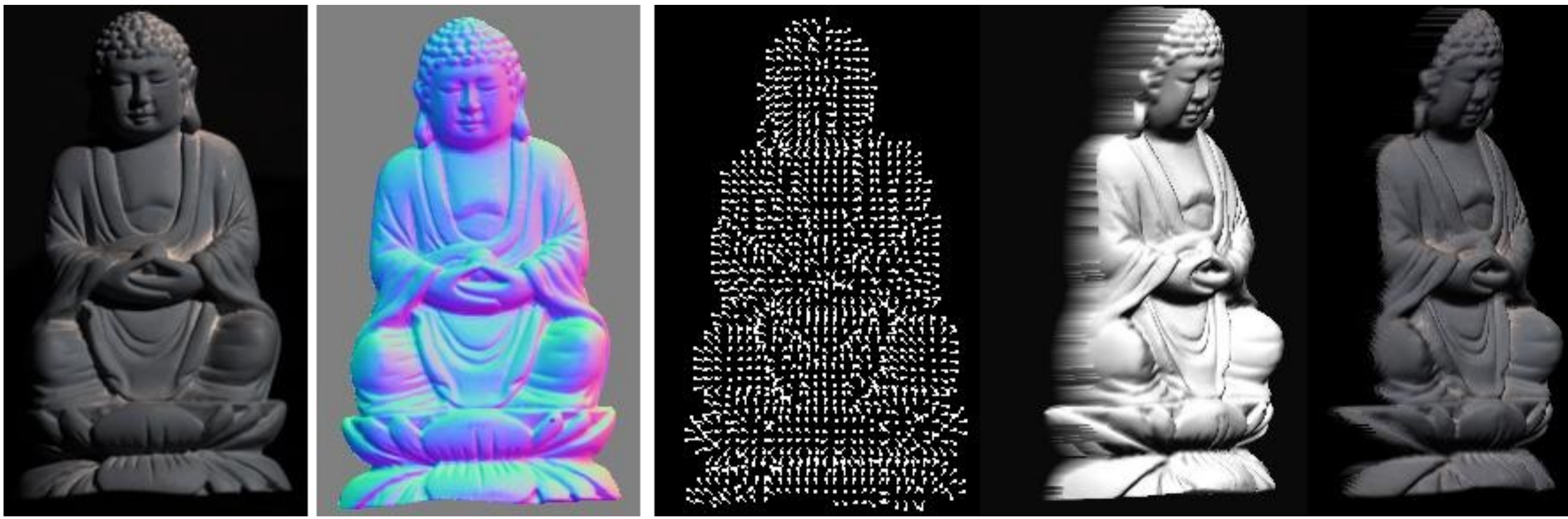
$(Z_h - Z_c)$ can be computed

$(x_h - x_c, y_h - y_c, Z_h - Z_c)$ is the normal

$$L = 2(N \cdot R)N - R$$

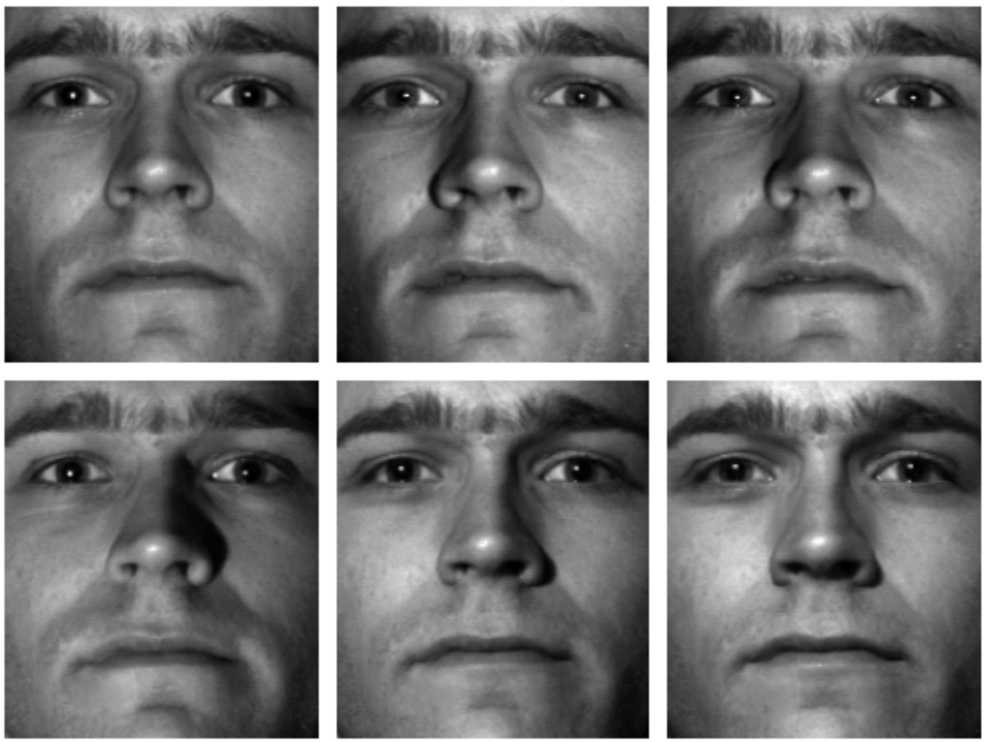
Photometric Stereo

What results can you get?



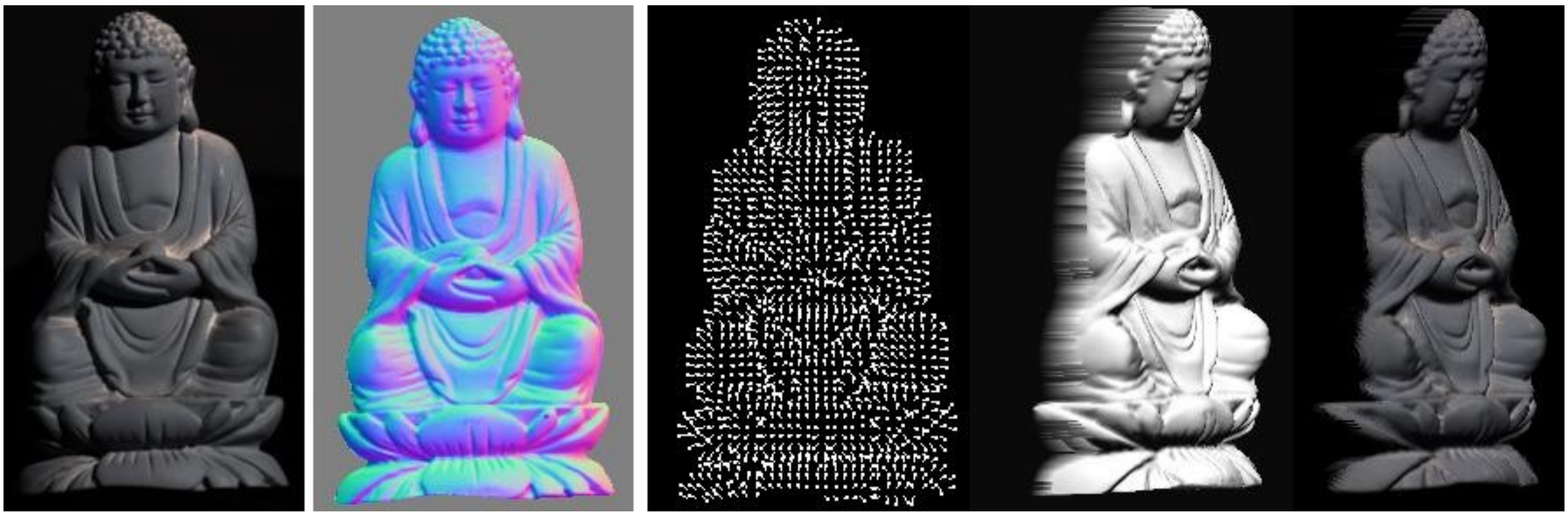
Input (1 of 12) | Normals (RGB colormap) | Normals (vectors) | Shaded 3D rendering | Textured 3D rendering

Results



from Athos Georghiades

Results



Input (1 of 12) | Normals (RGB colormap) | Normals (vectors) | Shaded 3D rendering | Textured 3D rendering

Questions?