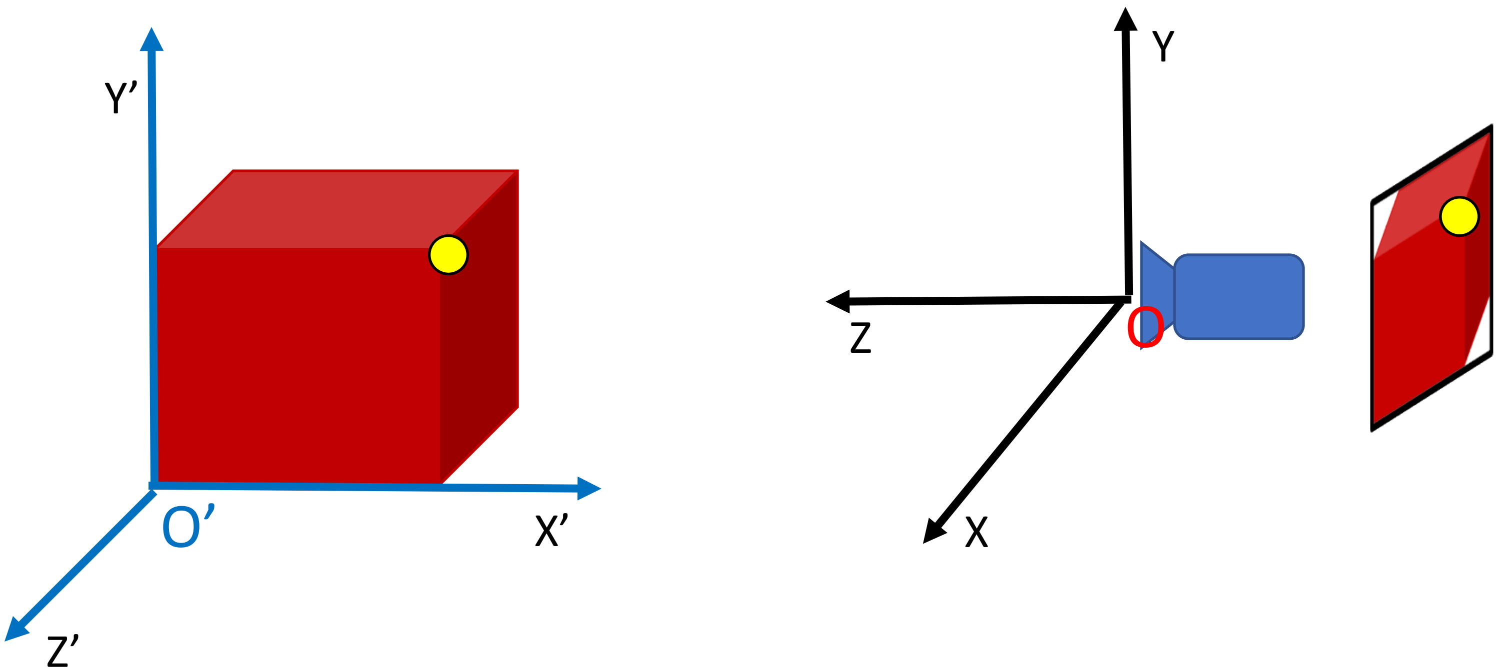
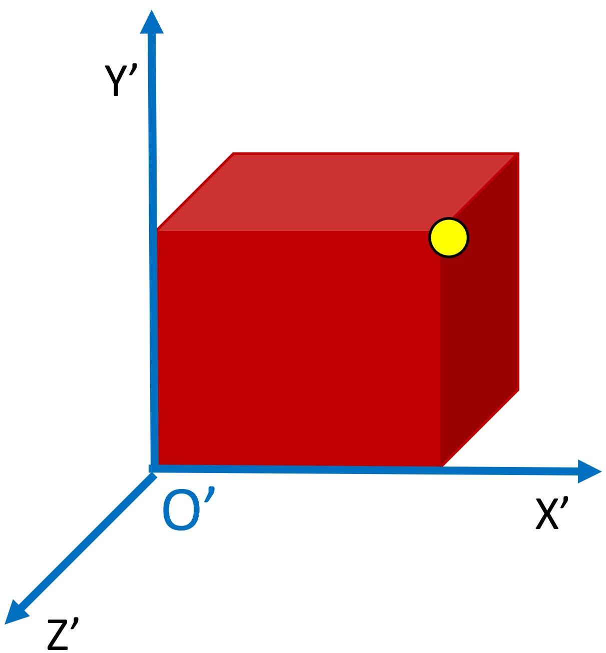
A special case of calibration

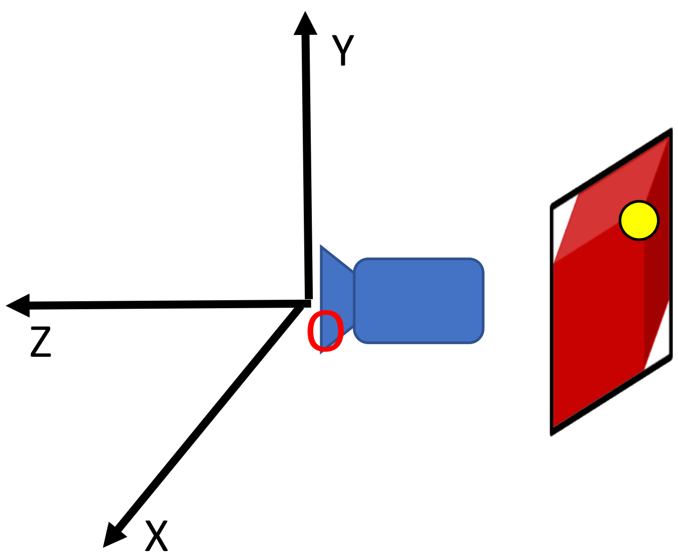
Camera calibration



Camera calibration = pose estimation

- Estimating where camera is relative to object in world
- = Estimating where object is relative to camera

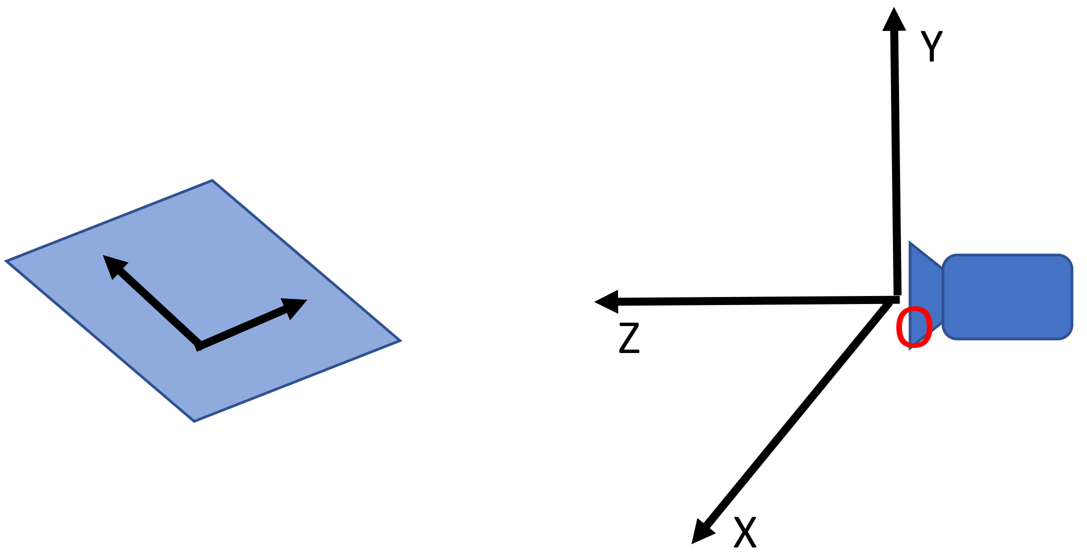




What if object of interest is plane?

- Not that uncommon....

What if object of interest is plane?



What if object of interest is a plane?

- Imagine that plane is equipped with two axes.
- Points on the plane are represented by *two* euclidean coordinates
- …Or 3 homogenous coordinates

3D object

$$\vec{\mathbf{x}}_{img} \equiv P\vec{\mathbf{x}}_w$$

3 x 4

2D object (plane)

$$\vec{\mathbf{x}}_{img} \equiv H\vec{\mathbf{x}}_w$$

3 x 3

What if object of interest is a plane?

Homography

$$\vec{\mathbf{x}}_{img} \equiv H\vec{\mathbf{x}}_w$$

3 x 3

- Homography maps points on the plane to pixels in the image

Fitting homographies

- How many parameters does a homography have?
- Given a single point on the plane and corresponding image location, what does that tell us?

$$\vec{\mathbf{x}}_{img} \equiv H\vec{\mathbf{x}}_w$$

$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ 1 \end{bmatrix}$$

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- Convince yourself that this gives 2 linear equations!

Fitting homographies

- Homography has 9 parameters
- But can't determine scale factor, so only 8: 4 points!

$$A\mathbf{h} = 0 \text{ s.t } \|\mathbf{h}\| = 1$$

- Or because we will have noise:

$$\min_{\mathbf{h}} \|A\mathbf{h}\|^2 \text{ s.t } \|\mathbf{h}\| = 1$$

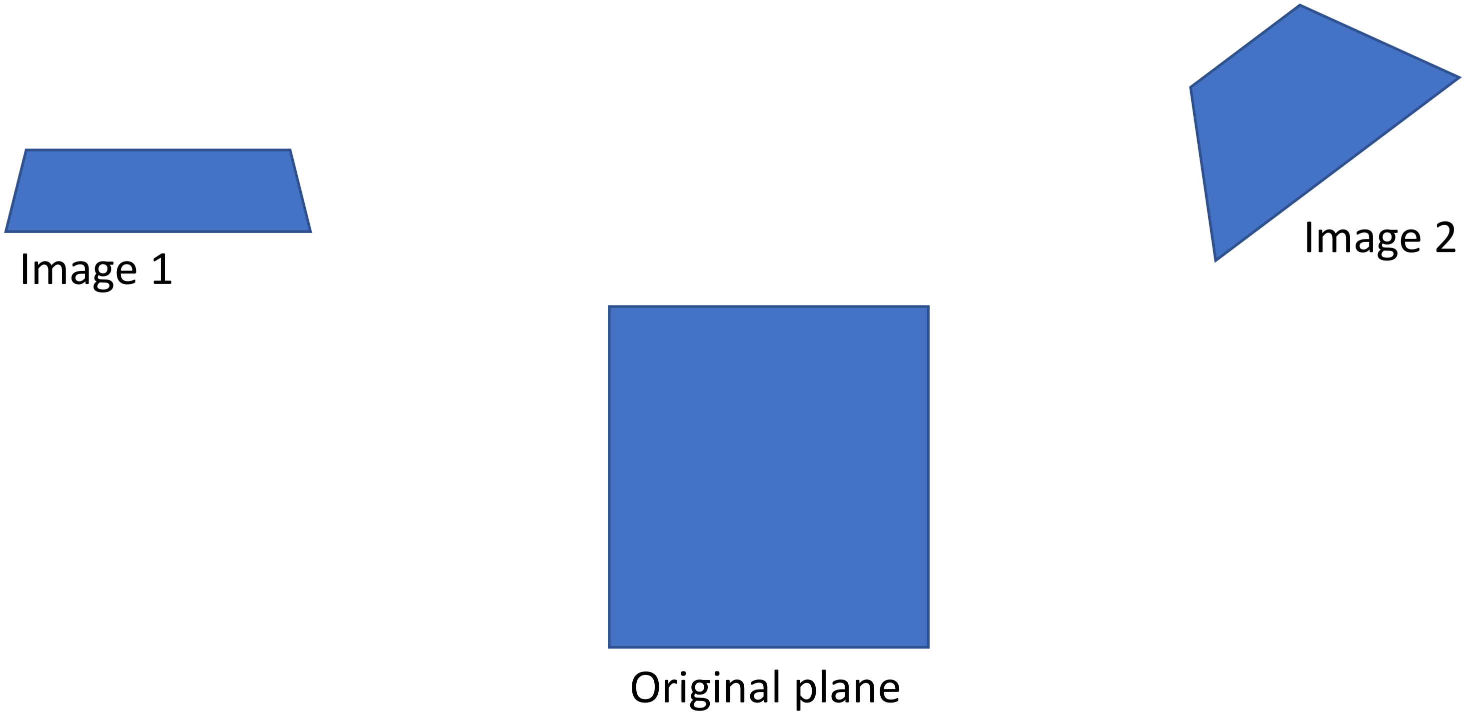
Fitting homographies

a

b

Homographies for image alignment

- A general mapping from one plane to another!
- Can also be used to align one photo of a plane to another photo of the same plane



Homographies for image alignment

- Can also be used to align one photo of a plane to another photo of the same plane

http://www.wired.com/gadgetlab/2010/07/camera-software-lets-you-see-into-the-past/

Image Alignment Algorithm

Given images A and B

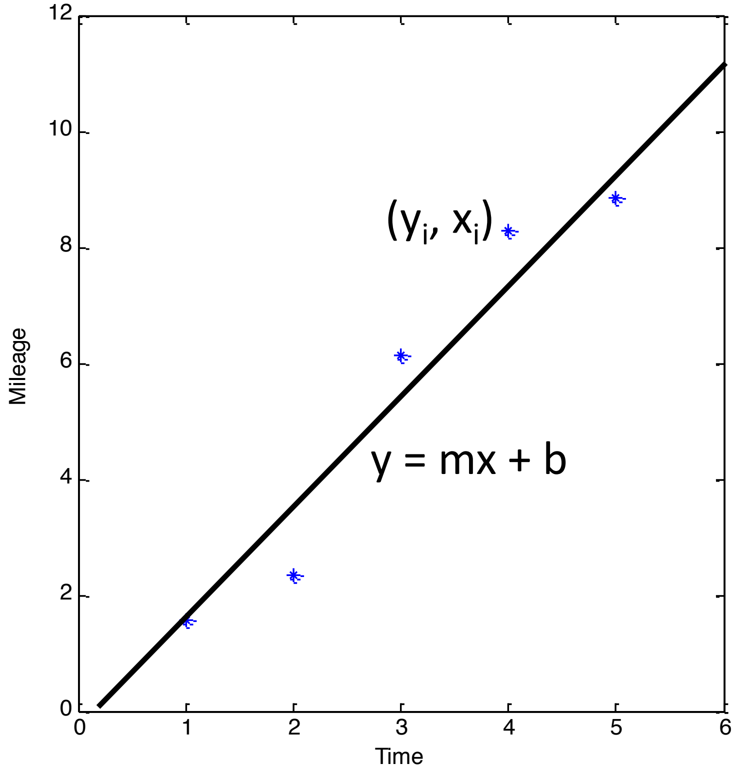
1. Compute image features for A and B
2. Match features between A and B
3. Compute homography between A and B

What could go wrong?

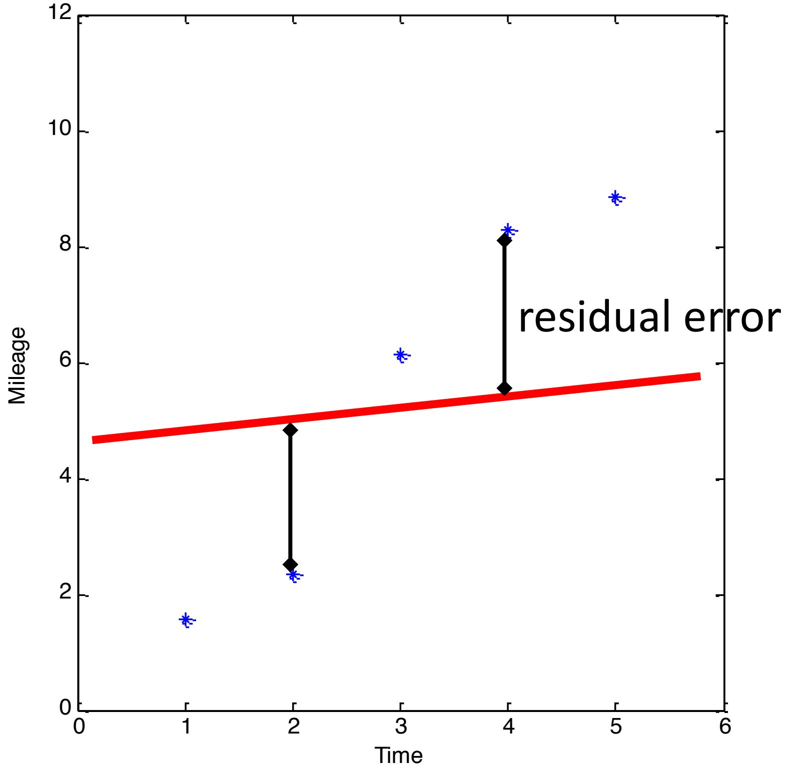
Fitting in general

- Fitting: find the parameters of a model that best fit the data
- Other examples:
  - least squares linear regression

Least squares: linear regression



Linear regression

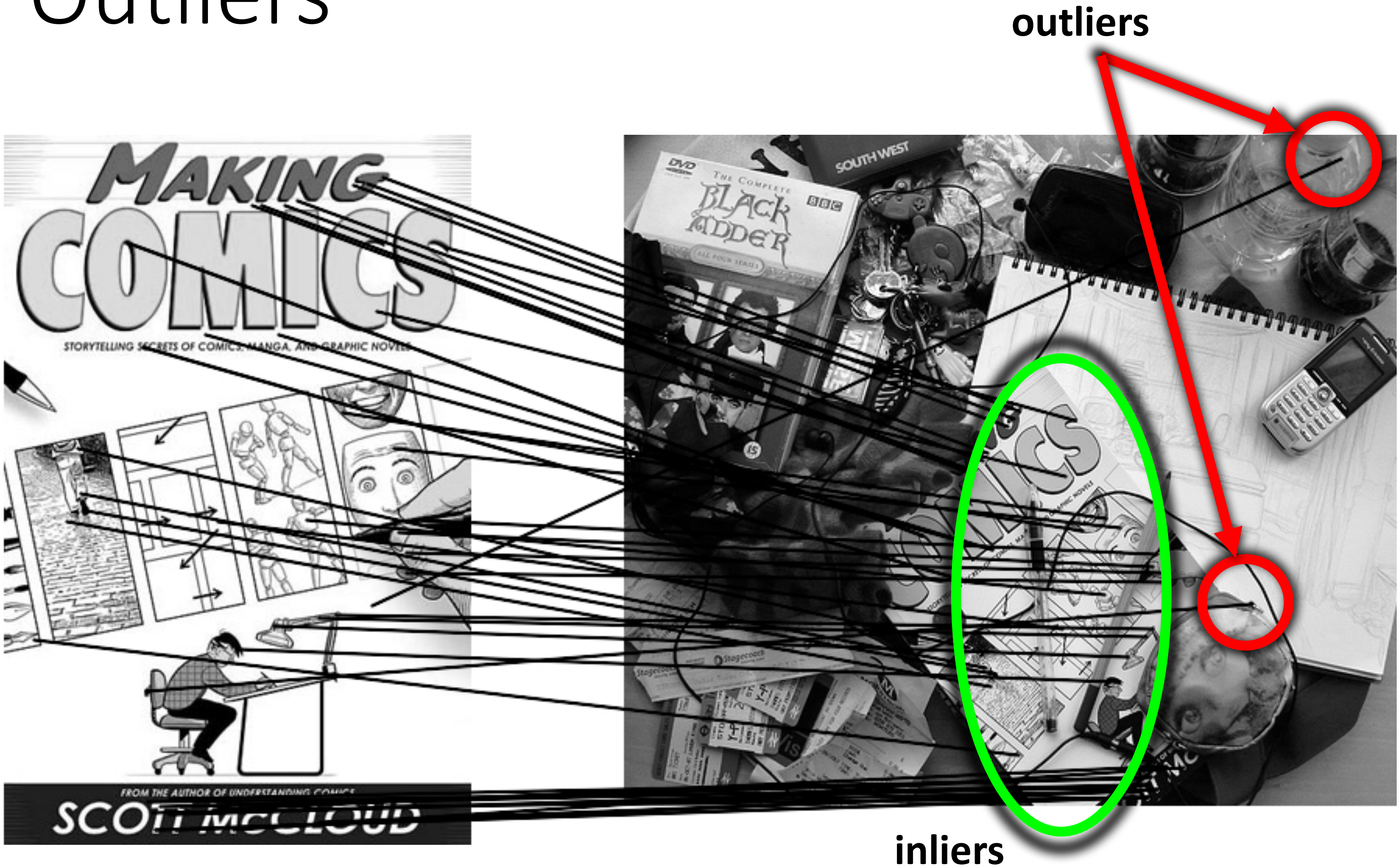


$$\mathrm{Cost}(m, b) = \sum_{i=1}^{n} |y_i - (mx_i + b)|^2$$

Linear regression

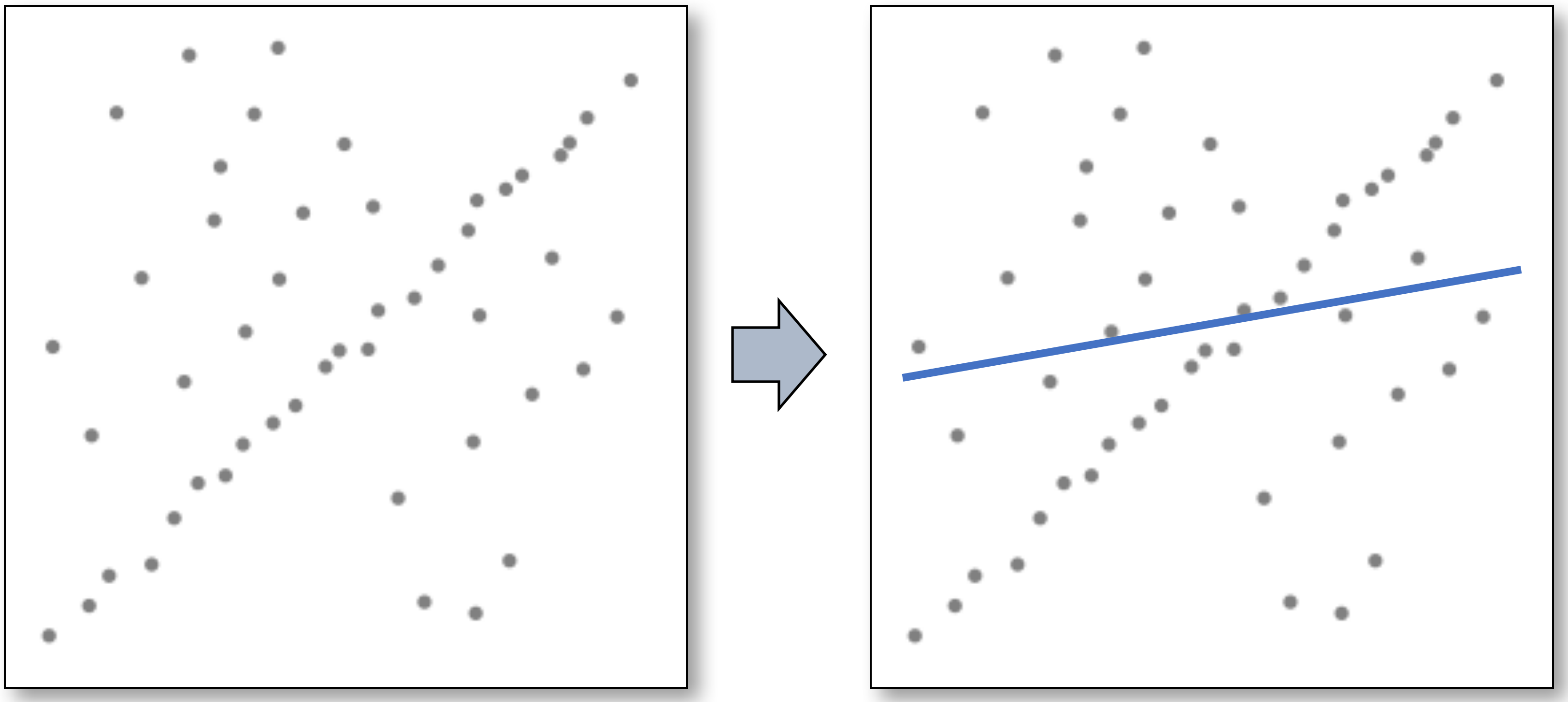
$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Outliers



outliers
MAKING COMICS
STORYTELLING SECRETS OF COMICS, MANGA, AND GRAPHIC NOVELS
FROM THE AUTHOR OF UNDERSTANDING COMICS
SCOTT McCLOUD
SOUTH WEST
DVD
THE COMPLETE BLACK ADDER
BBC
inliers

Robustness



Problem: Fit a line to these datapoints

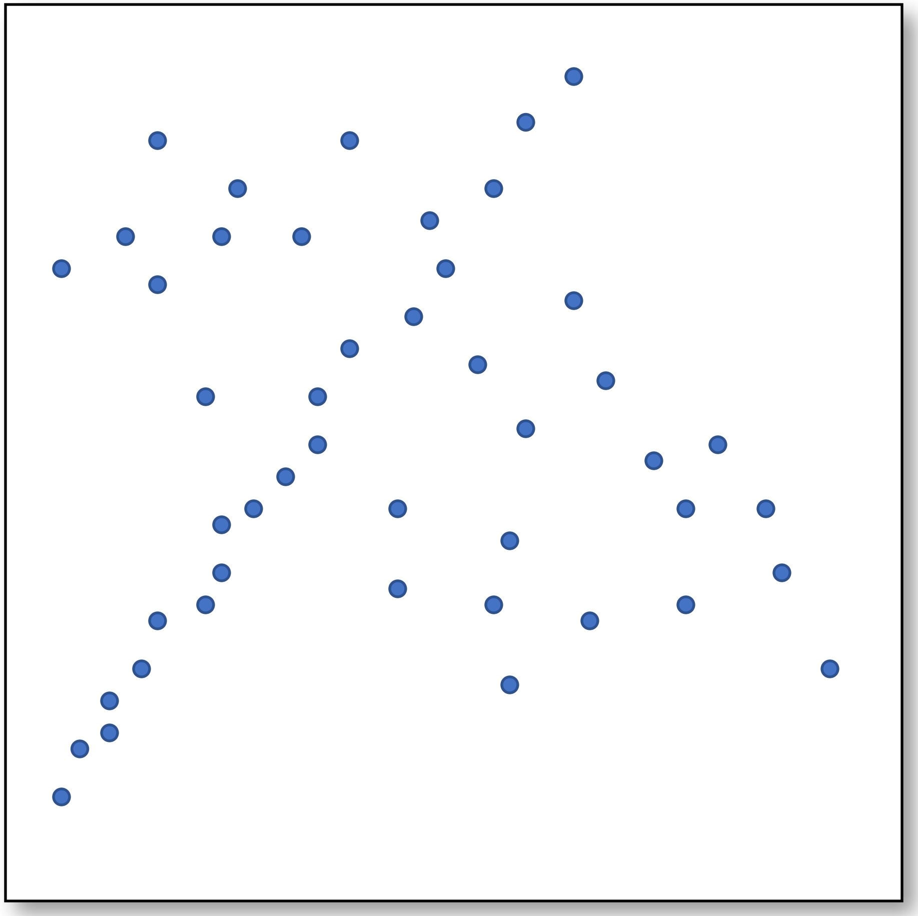
Least squares fit

Idea

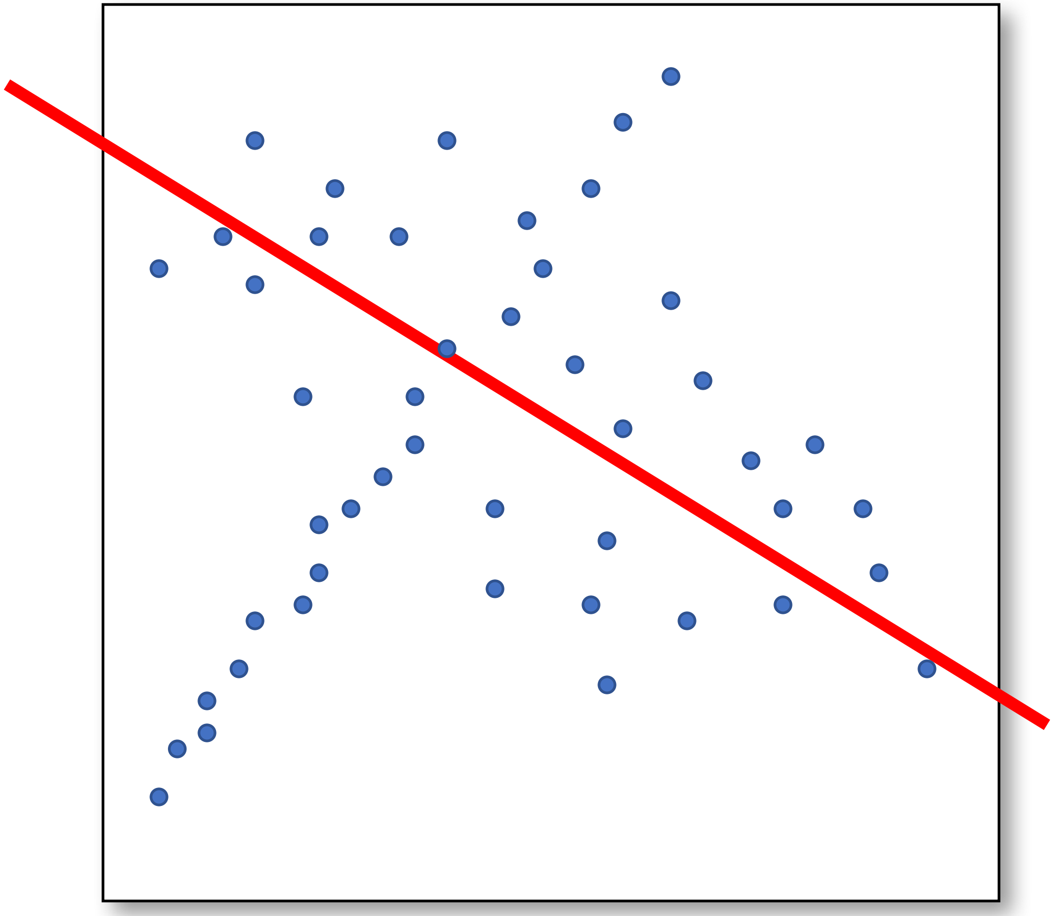
- Given a hypothesized line
- Count the number of points that “agree” with the line
  - “Agree” = within a small distance of the line
  - I.e., the **inliers** to that line

- For all possible lines, select the one with the largest number of inliers

Counting inliers

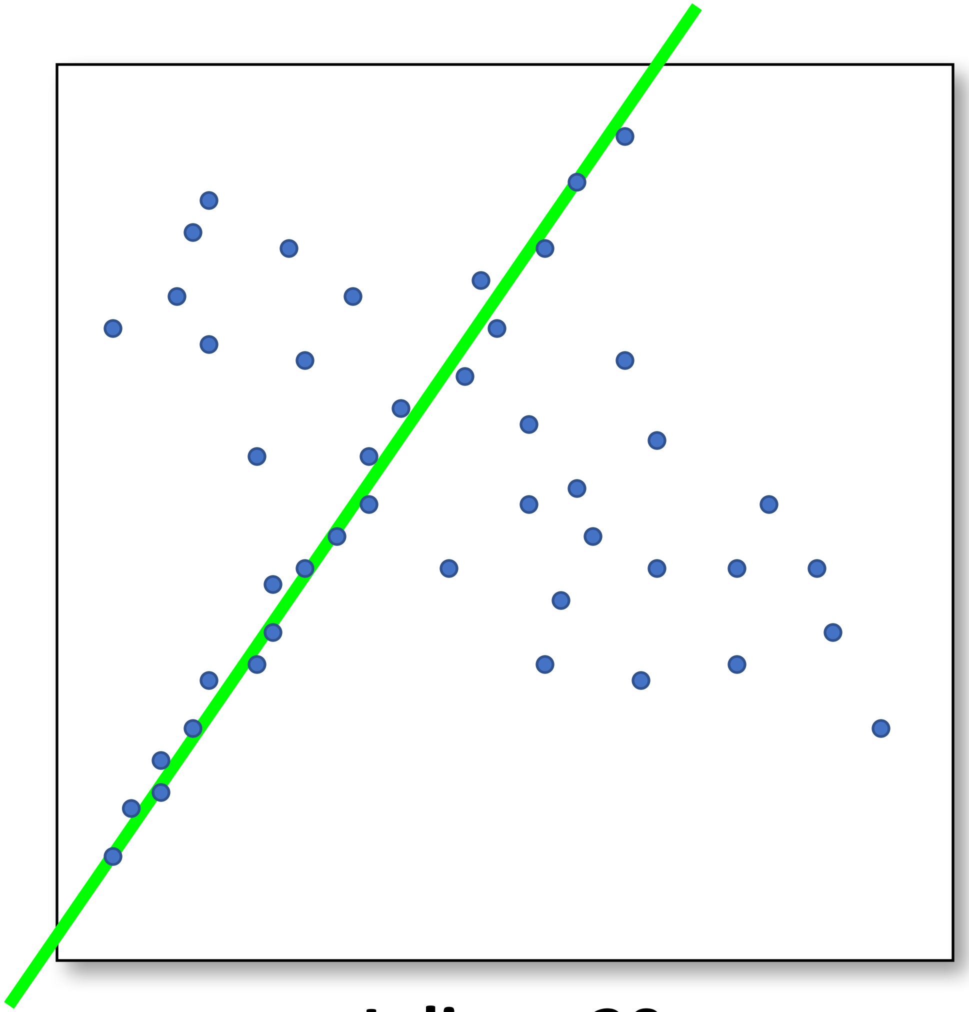


Counting inliers



Inliers: 3

Counting inliers



Inliers: 20

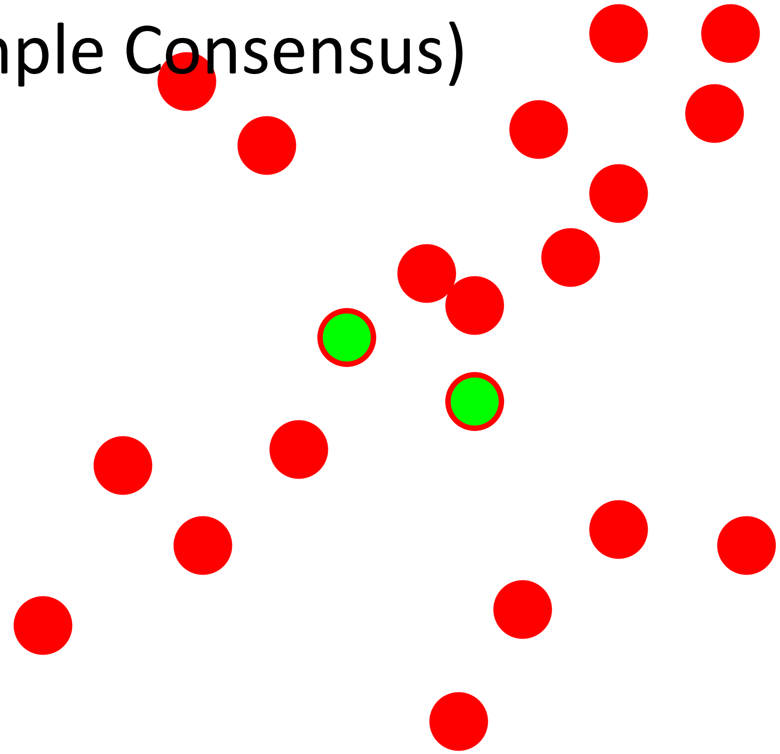
How do we find the best line?

- Unlike least-squares, no simple closed-form solution

- Hypothesize-and-test
  - Try out many lines, keep the best one
  - Which lines?

RANSAC (Random Sample Consensus)

Line fitting example



Algorithm:

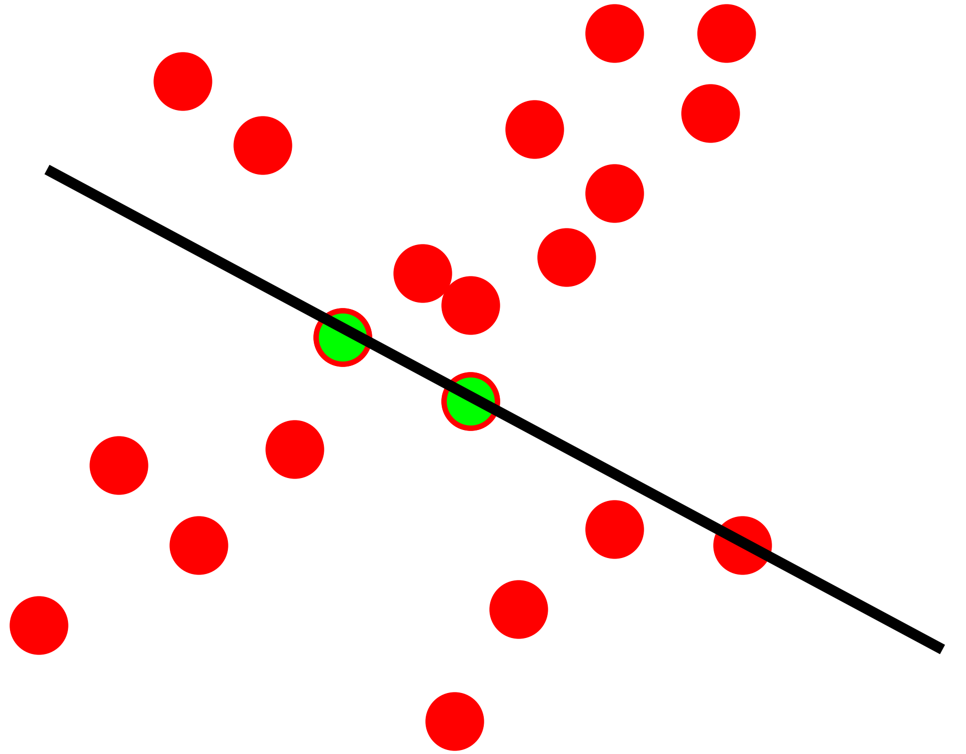
1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC

Line fitting example



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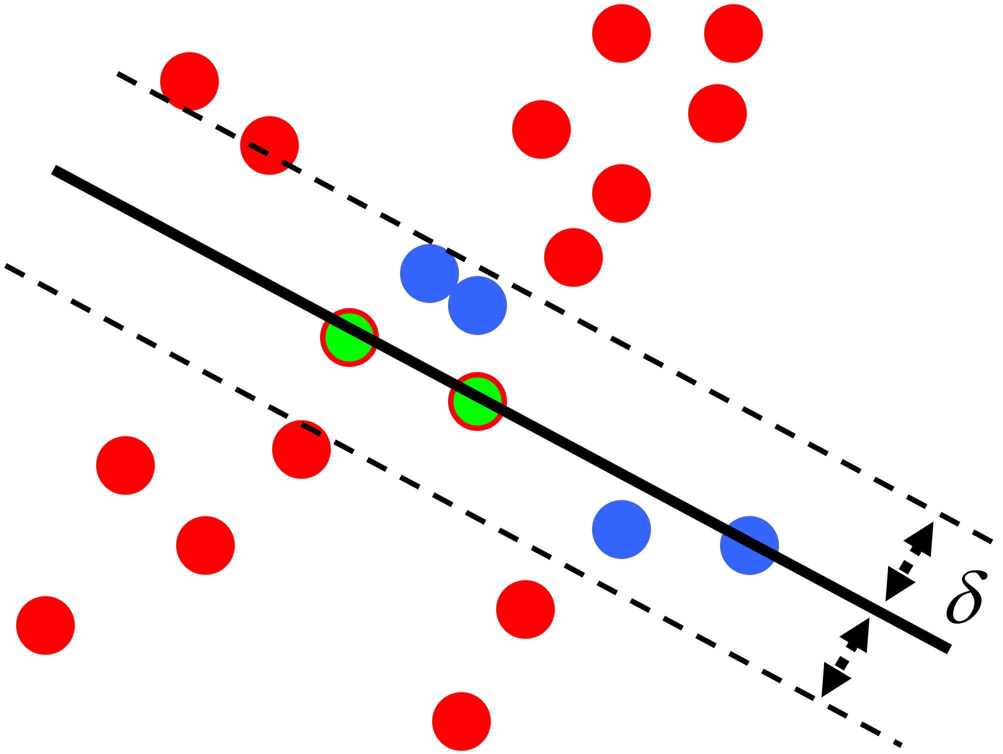
RANSAC

Line fitting example



$$N_I = 6$$

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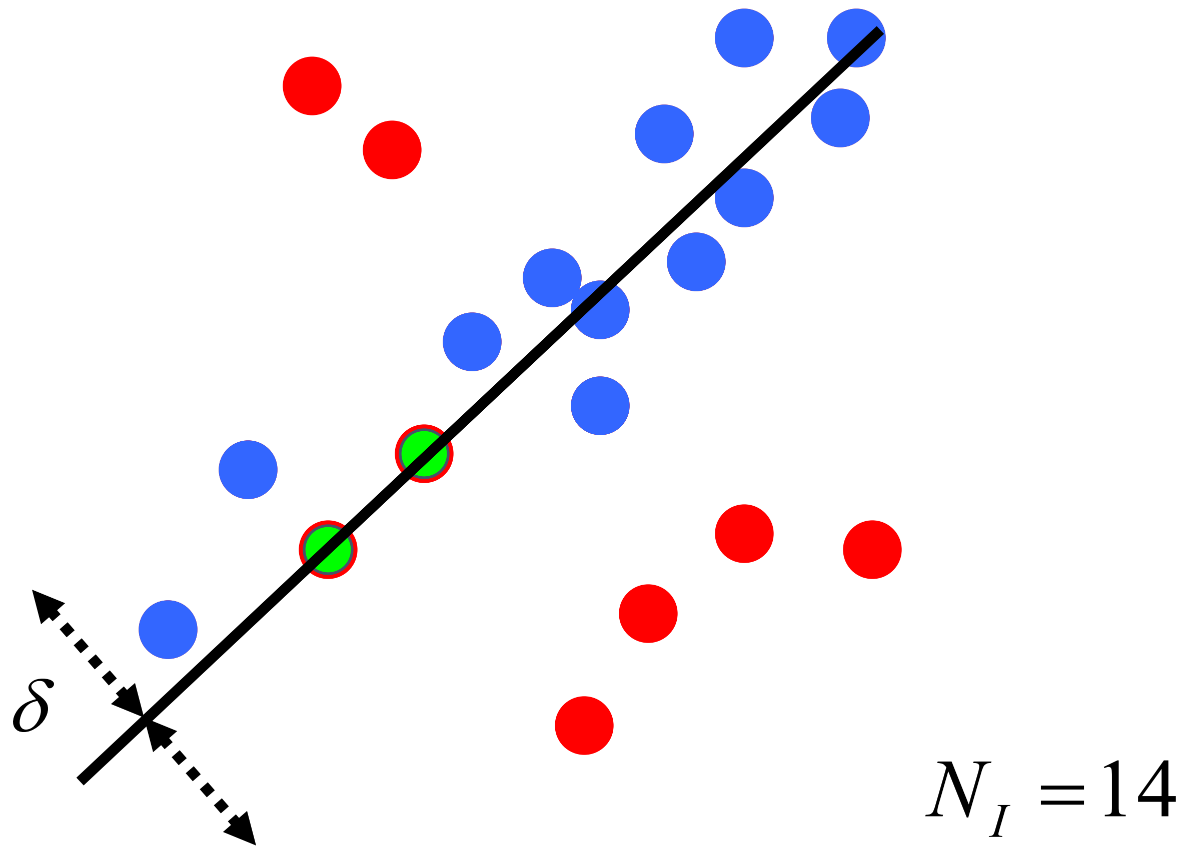
RANSAC



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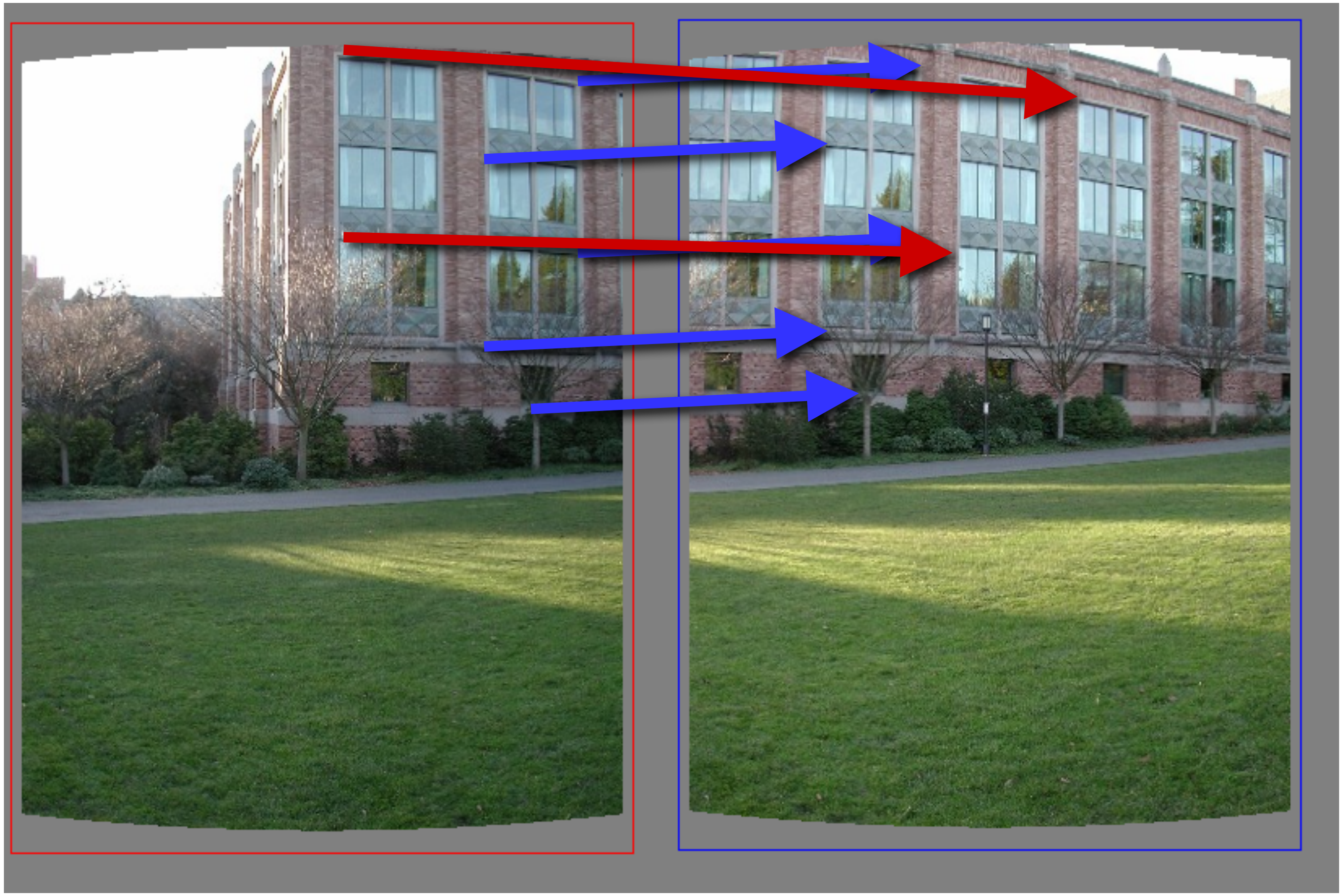
RANSAC

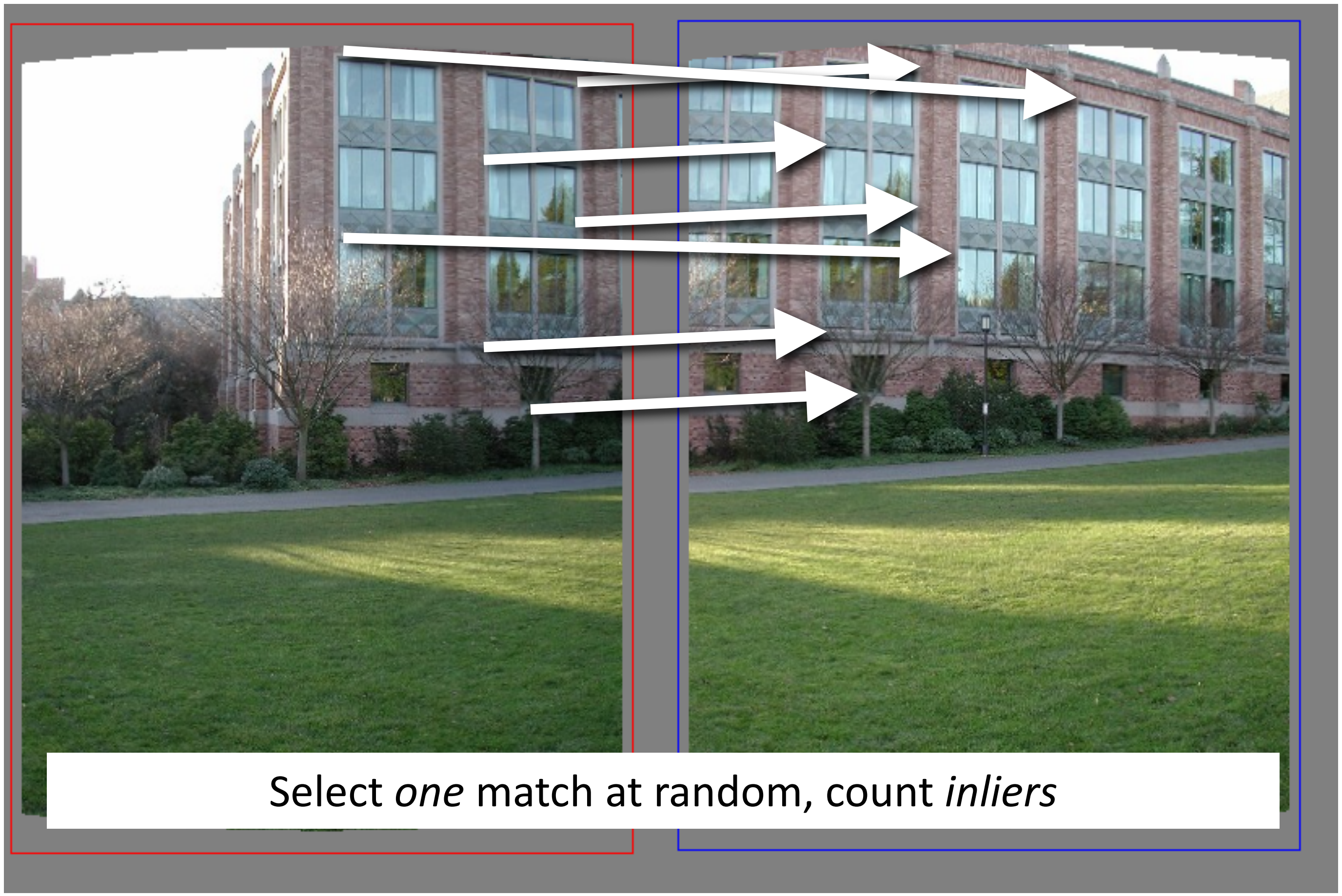
- Idea:
  - All the inliers will agree with each other on the translation vector; the (hopefully small) number of outliers will (hopefully) disagree with each other
    - RANSAC only has guarantees if there are < 50% outliers
  - "All good matches are alike; every bad match is bad in its own way."

– Tolstoy via Alyosha Efros

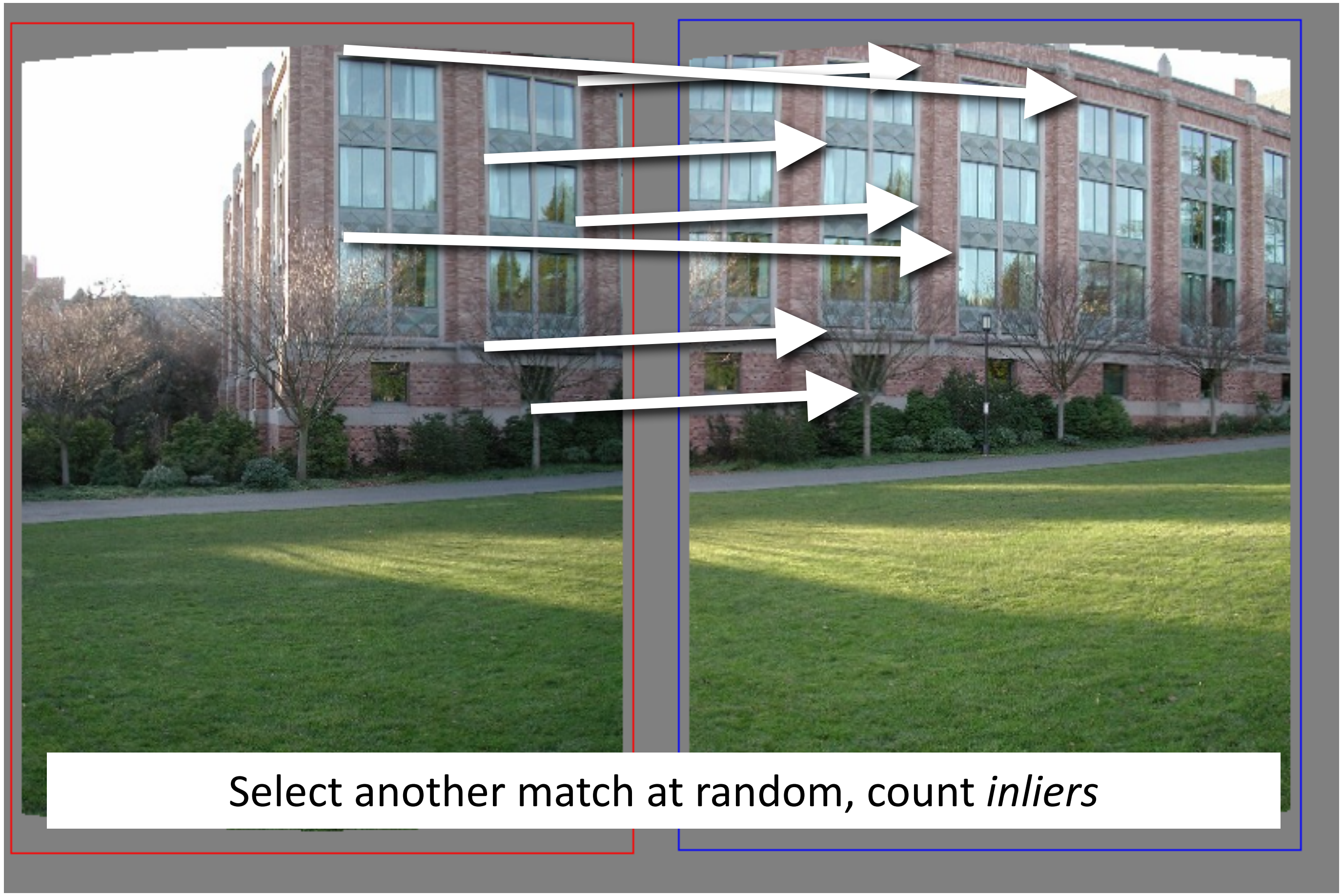
Translations



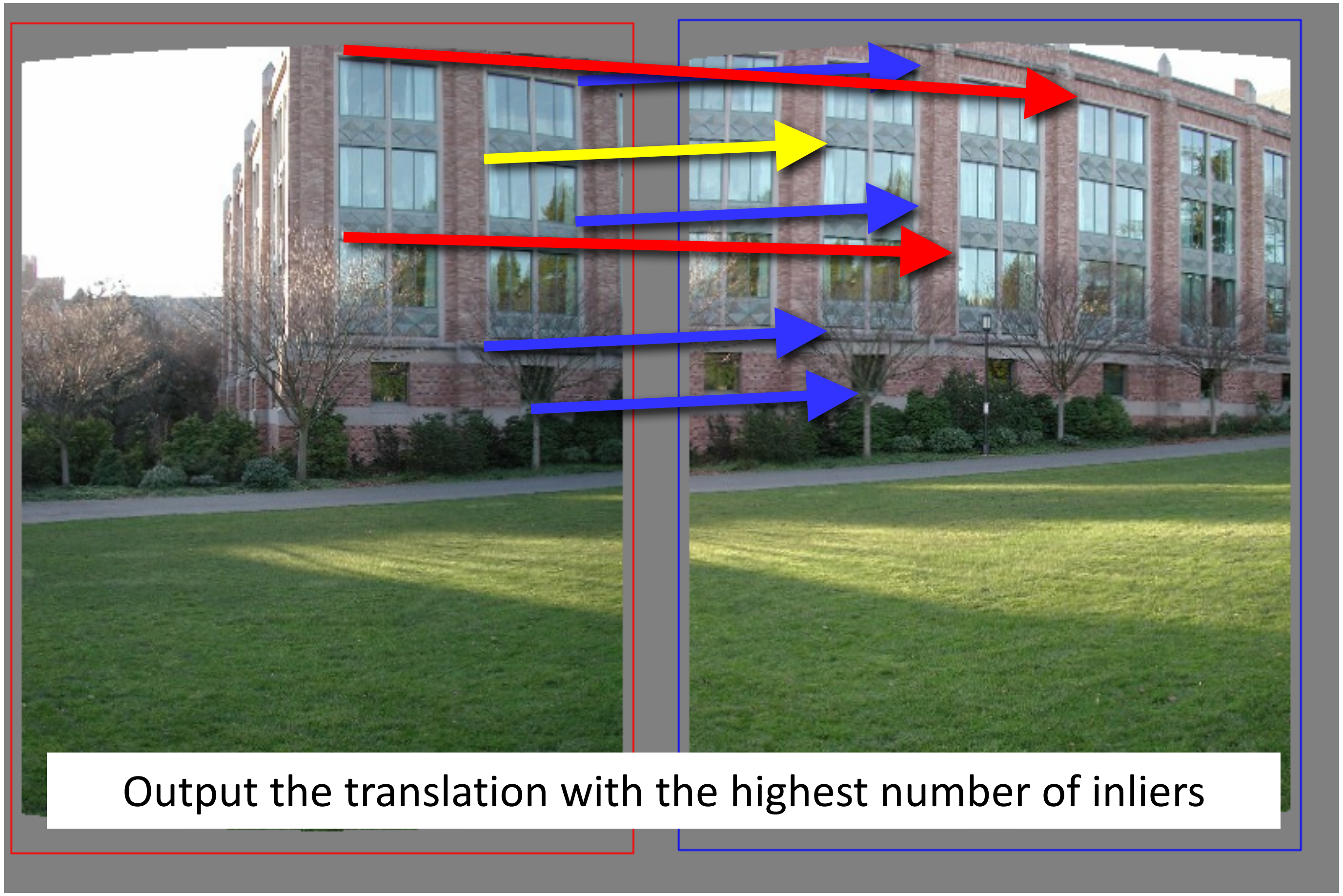
RAndom SAmple Consensus



RAndom SAmple Consensus

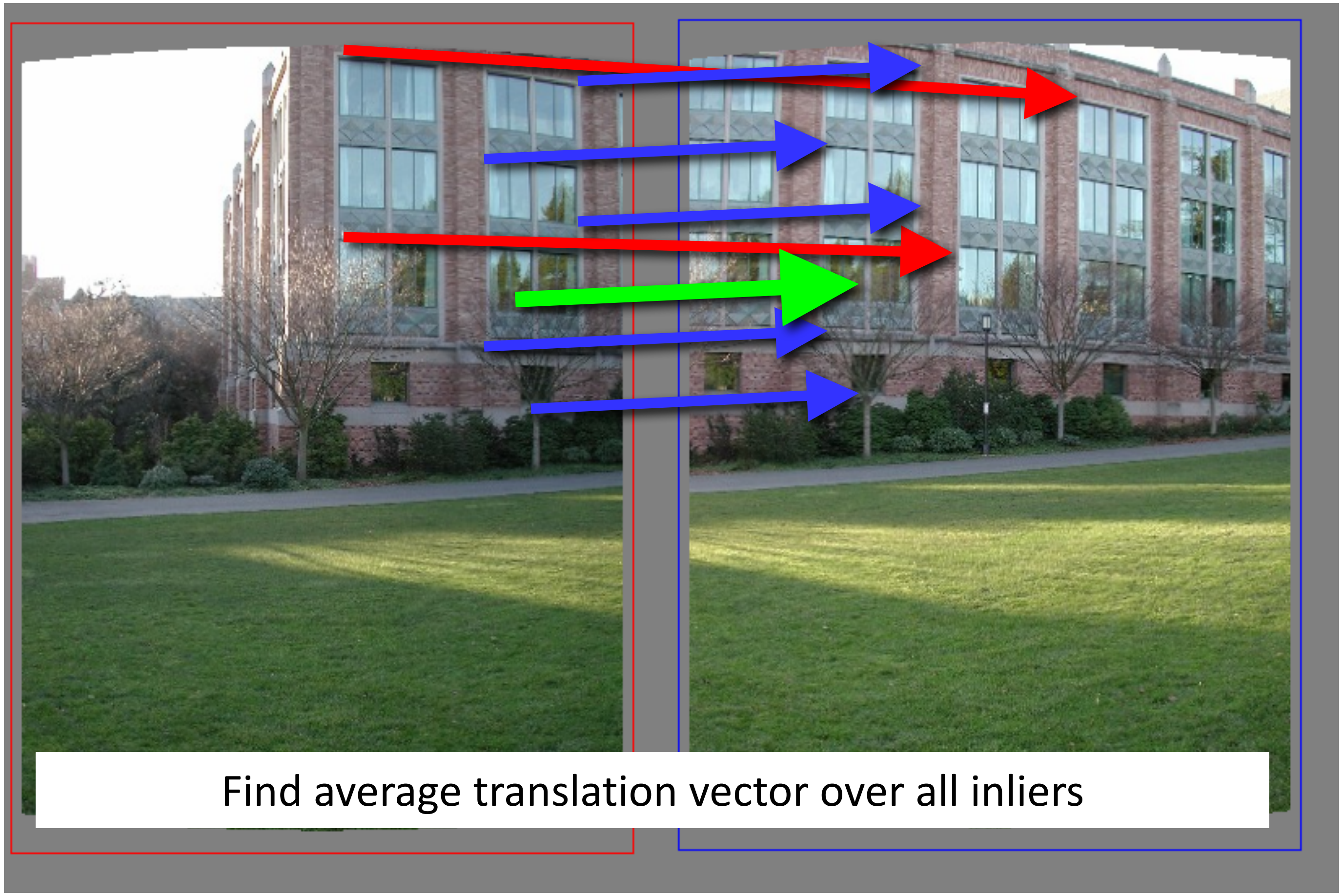


RAndom SAmple Consensus



Output the translation with the highest number of inliers

Final step: least squares fit



Find average translation vector over all inliers

RANSAC

- **Inlier threshold** related to the amount of noise we expect in inliers
  - Often model noise as Gaussian with some standard deviation (e.g., 3 pixels)
- **Number of rounds** related to the percentage of outliers we expect, and the probability of success we'd like to guarantee
  - Suppose there are 20% outliers, and we want to find the correct answer with 99% probability
  - How many rounds do we need?

How many rounds?

- If we have to choose *k* samples each time
  - with an inlier ratio p
  - and we want the right answer with probability *P*

	proportion of inliers *p*						
k	95%	90%	80%	75%	70%	60%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

$P = 0.99$

Source: M. Pollefeys

To ensure that the random sampling has a good chance of finding a true set of inliers, a sufficient number of trials S must be tried. Let p be the probability that any given correspondence is valid and P be the total probability of success after S trials. The likelihood in one trial that all k random samples are inliers is p^k. Therefore, the likelihood that S such trials will all fail is

$$1 - P = (1 - p^k)^S \tag{6.29}$$

and the required minimum number of trials is

$$S = \frac{\log(1 - P)}{\log(1 - p^k)}. \tag{6.30}$$

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2	2	3	5	6	7	11	17
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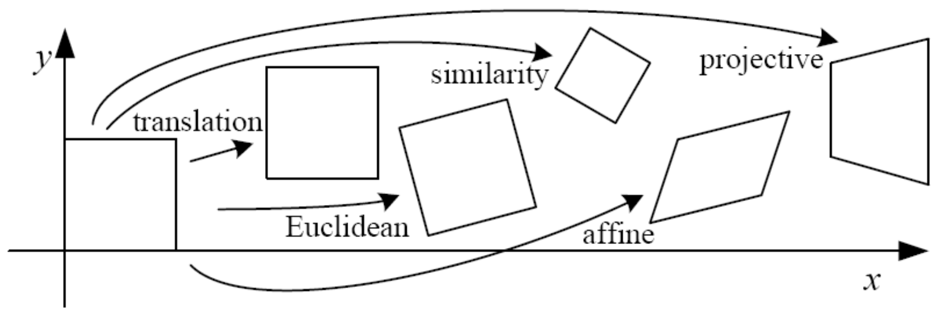
$P = 0.99$

How big is k?

- For alignment, depends on the motion model
  - Here, each sample is a correspondence (pair of matching points)



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation $+\cdots$	☐
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths $+\cdots$	◇
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles $+\cdots$	◇
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism $+\cdots$	▱
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	⏢

RANSAC pros and cons

- Pros
  - Simple and general
  - Applicable to many different problems
  - Often works well in practice
- Cons
  - Parameters to tune
  - Sometimes too many iterations are required
  - Can fail for extremely low inlier ratios

RANSAC

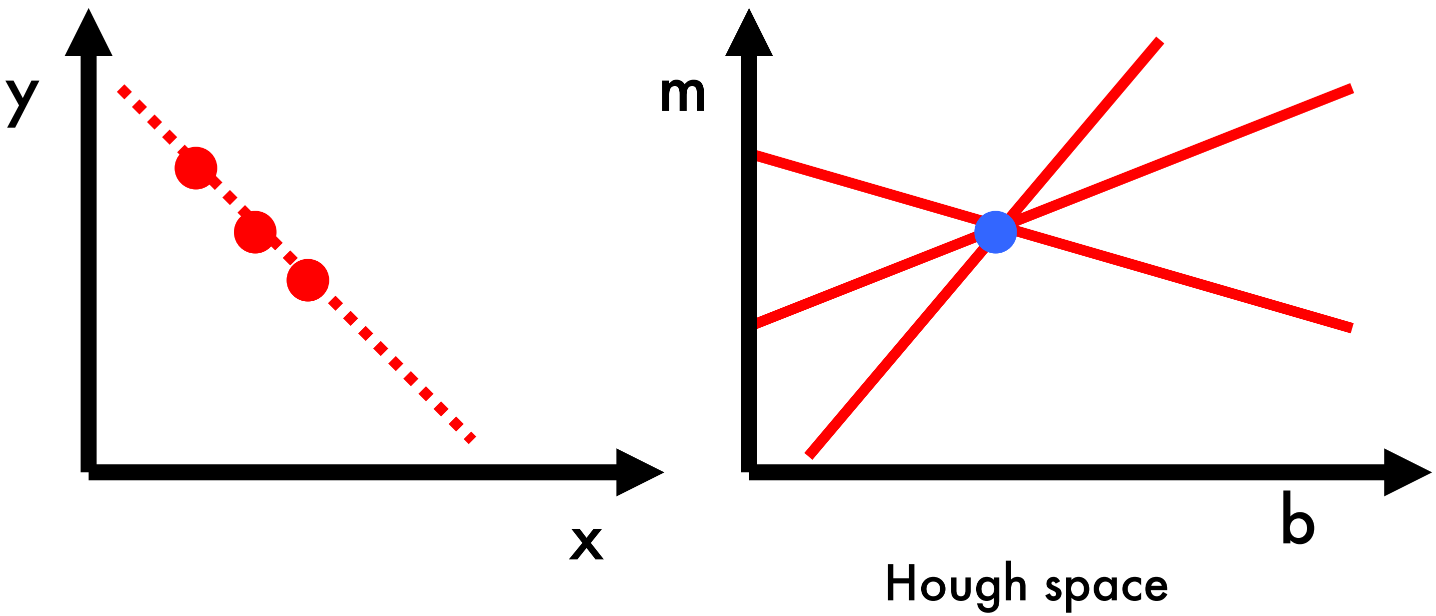
- An example of a “voting”-based fitting scheme
- Each hypothesis gets voted on by each data point, best hypothesis wins

- There are many other types of voting schemes
  - E.g., Hough transforms…

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Given a set of points, find the curve or line that explains the data points best



$$y = m\,x + b$$

Slide from S. Savarese

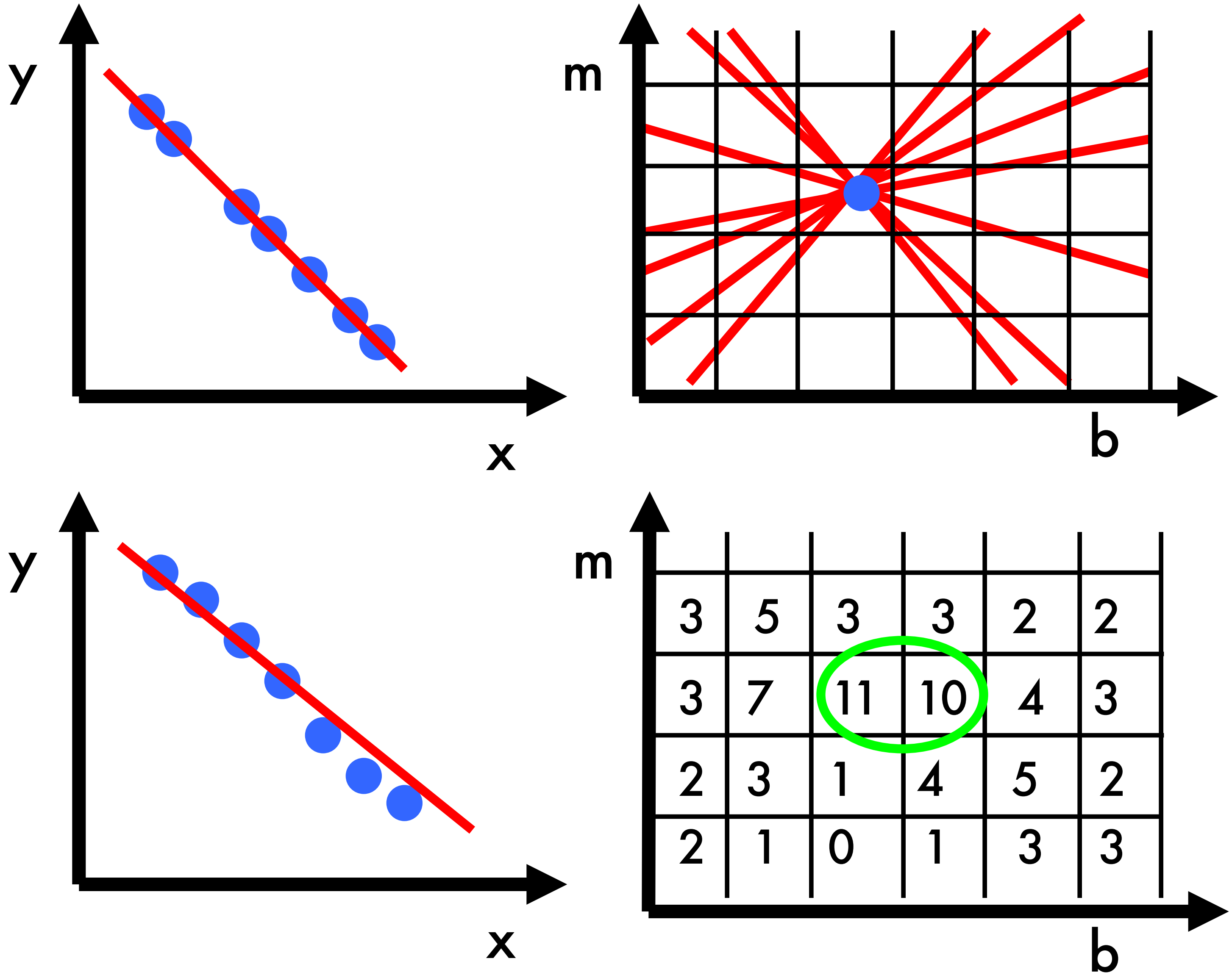
Hough Transform: Outline

1. Create a grid of parameter values

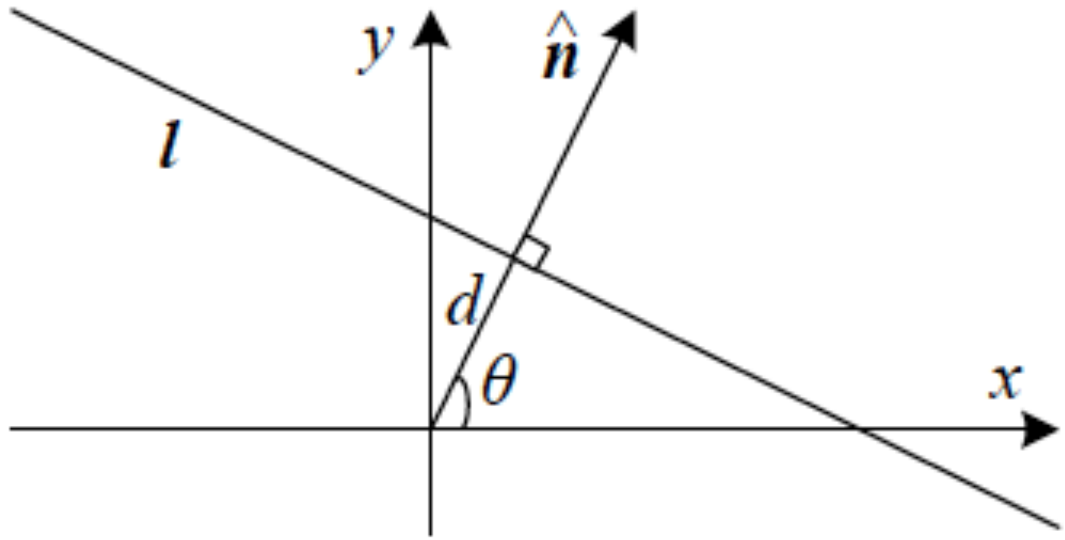
2. Each point votes for a set of parameters, incrementing those values in grid

3. Find maximum or local maxima in grid

Hough transform



Slide from S. Savarese



$$d = x cos\theta + y sin\theta$$

Hough transform