The correspondence problem

Why?

- Multiple images can give a clue about 3D structure

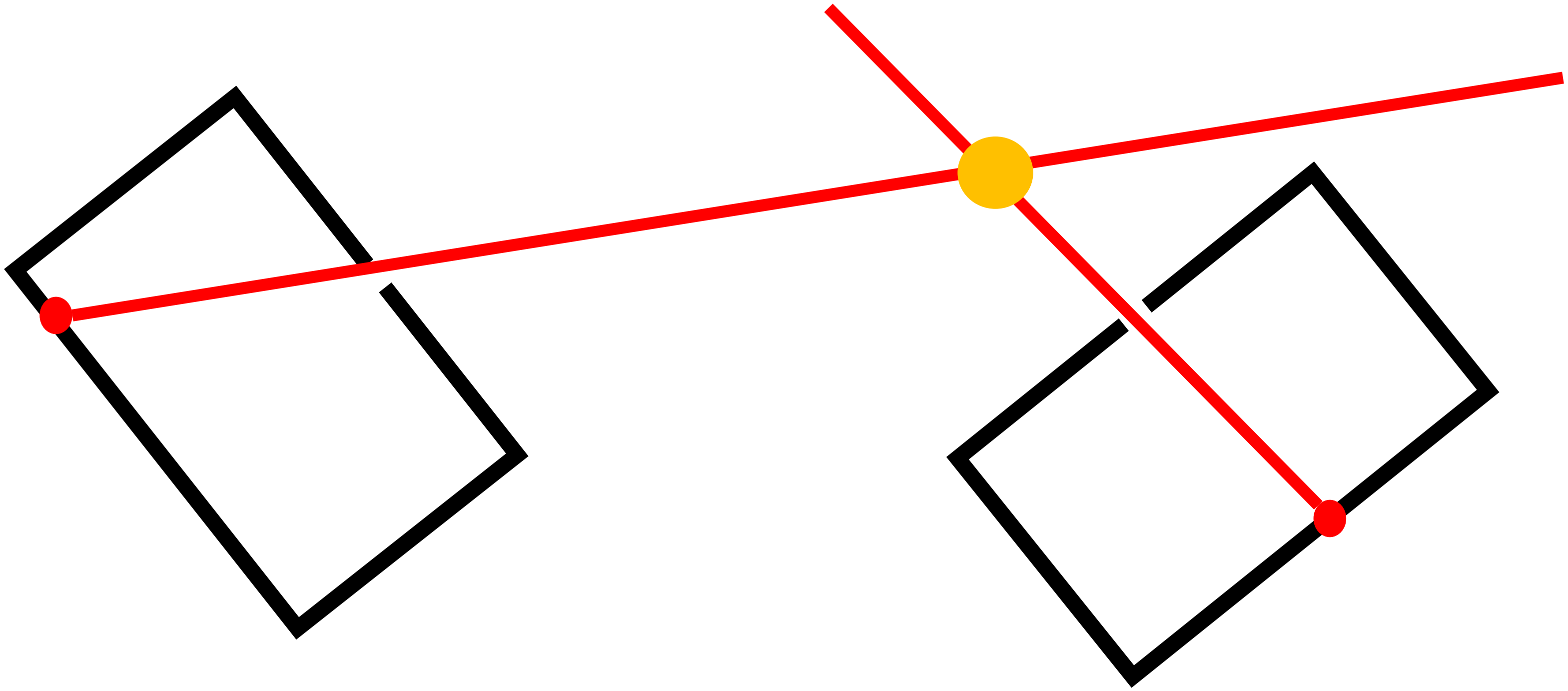
Why? Reconstruction

- Need to find which pixel in image 2 matches which in image 1 - the *correspondence* problem

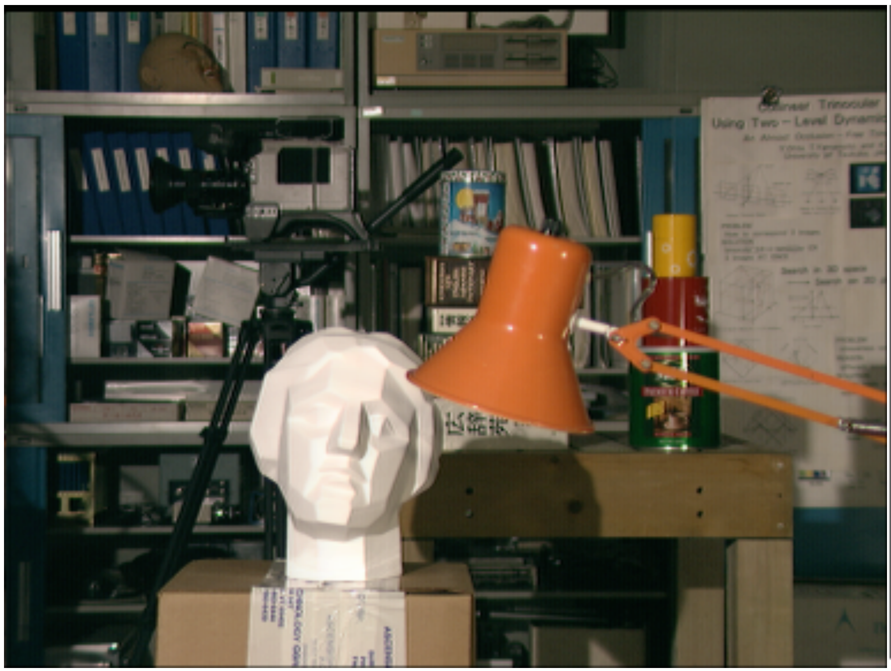
 

Reconstruction from correspondence

- Given known cameras, correspondence gives the location of 3D point (*Triangulation*)



Reconstruction from correspondence



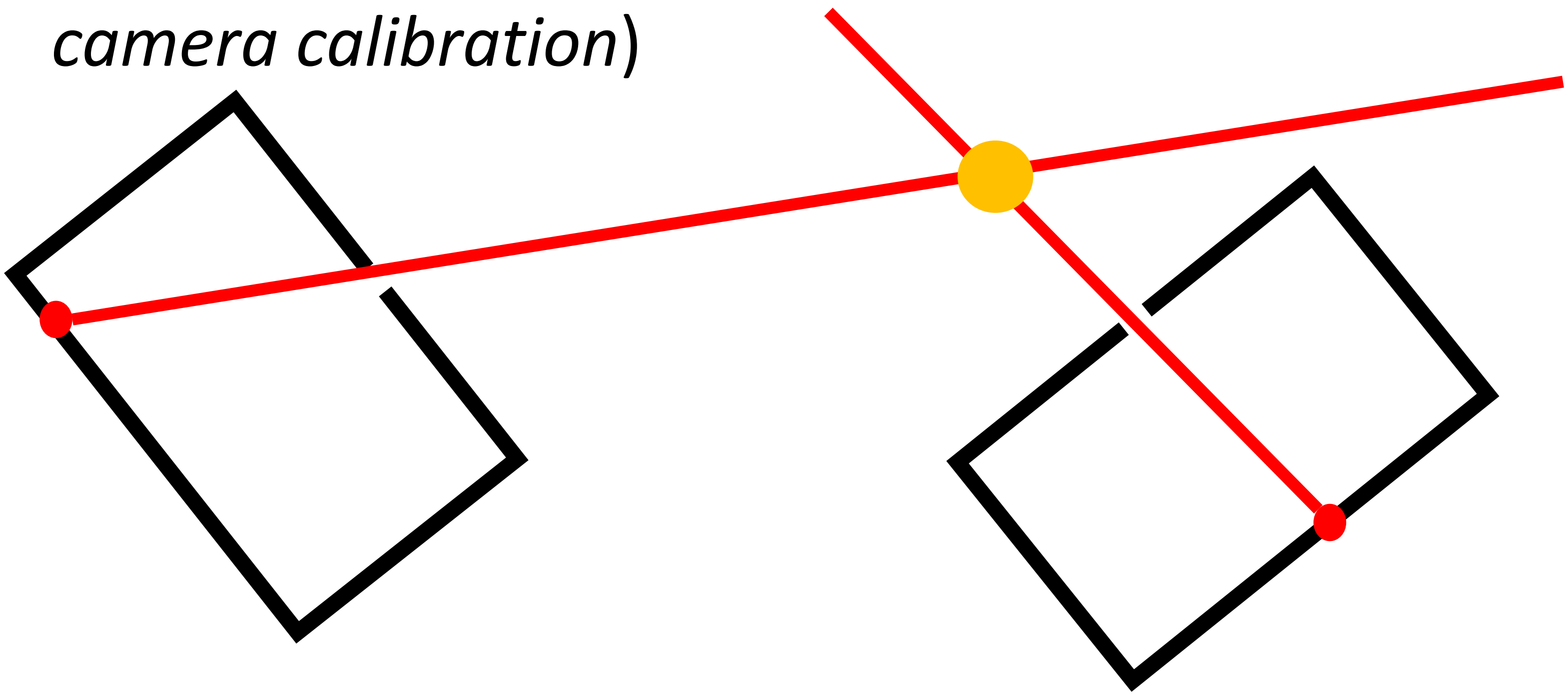
Reconstruction from correspondence

- Specific application: depth cameras

https://realsense.intel.com/stereo/
Microsoft Kinect

Reconstruction from correspondence - Pose estimation

- Given a 3D point, correspondence gives relationship between cameras (*Pose estimation / camera calibration*)



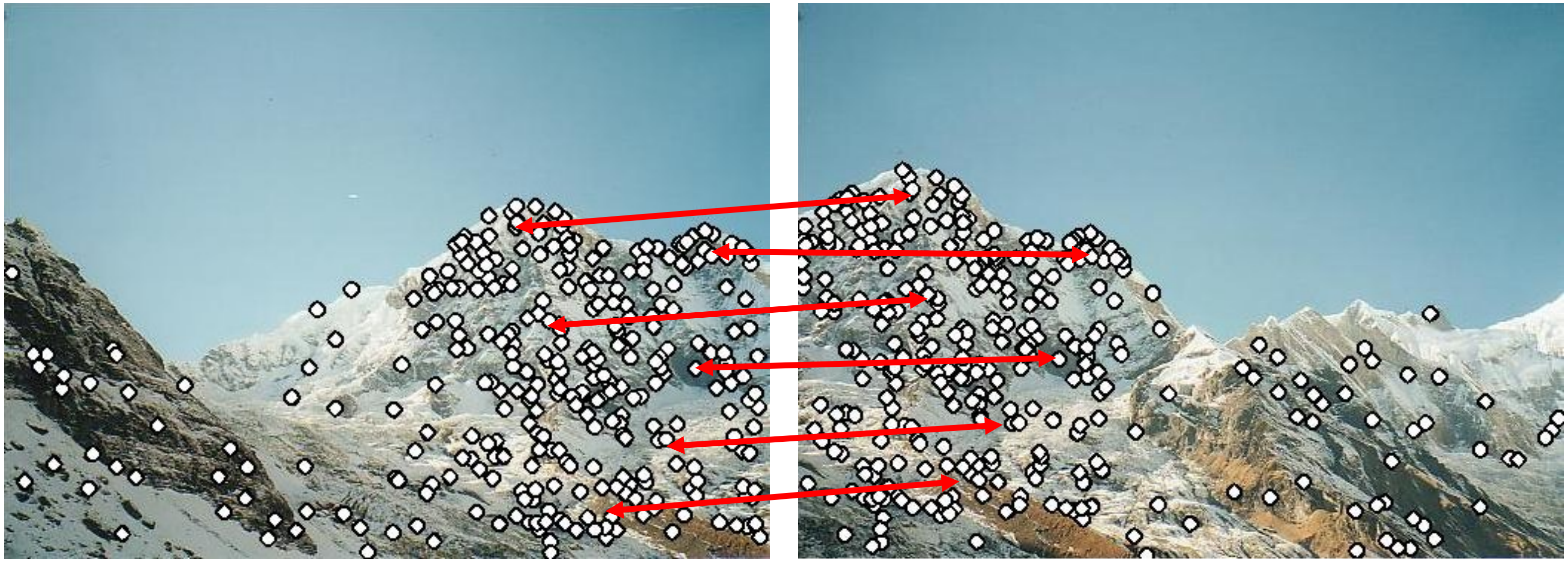
Pose-estimation

Pose-estimation / Camera calibration

- Specific application: panorama stitching
  - We have two images – how do we combine them?

Pose-estimation / Camera calibration

- Specific application: panorama stitching
  - We have two images – how do we combine them?



Step 1: extract correspondence

Pose-estimation / Camera calibration

- Specific application: panorama stitching
  - We have two images – how do we combine them?

Step 1: extract correspondence
Step 2: align images

Other applications of correspondence

- Recognition: Match image to product view

Lowe, IJCV 2004

Other applications of correspondence

- Image alignment
- Motion tracking
- Robot navigation

Correspondence can be challenging

Fei-Fei Li

Correspondence

by Diva Sian

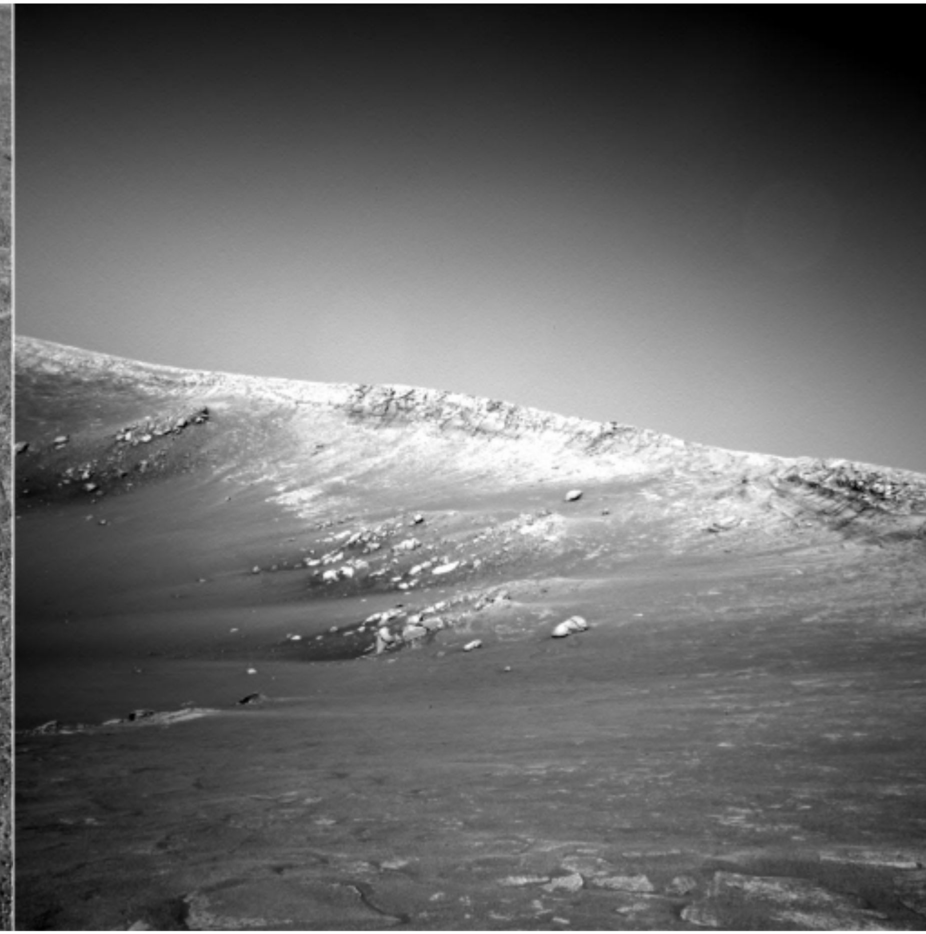
by swashford

Harder case

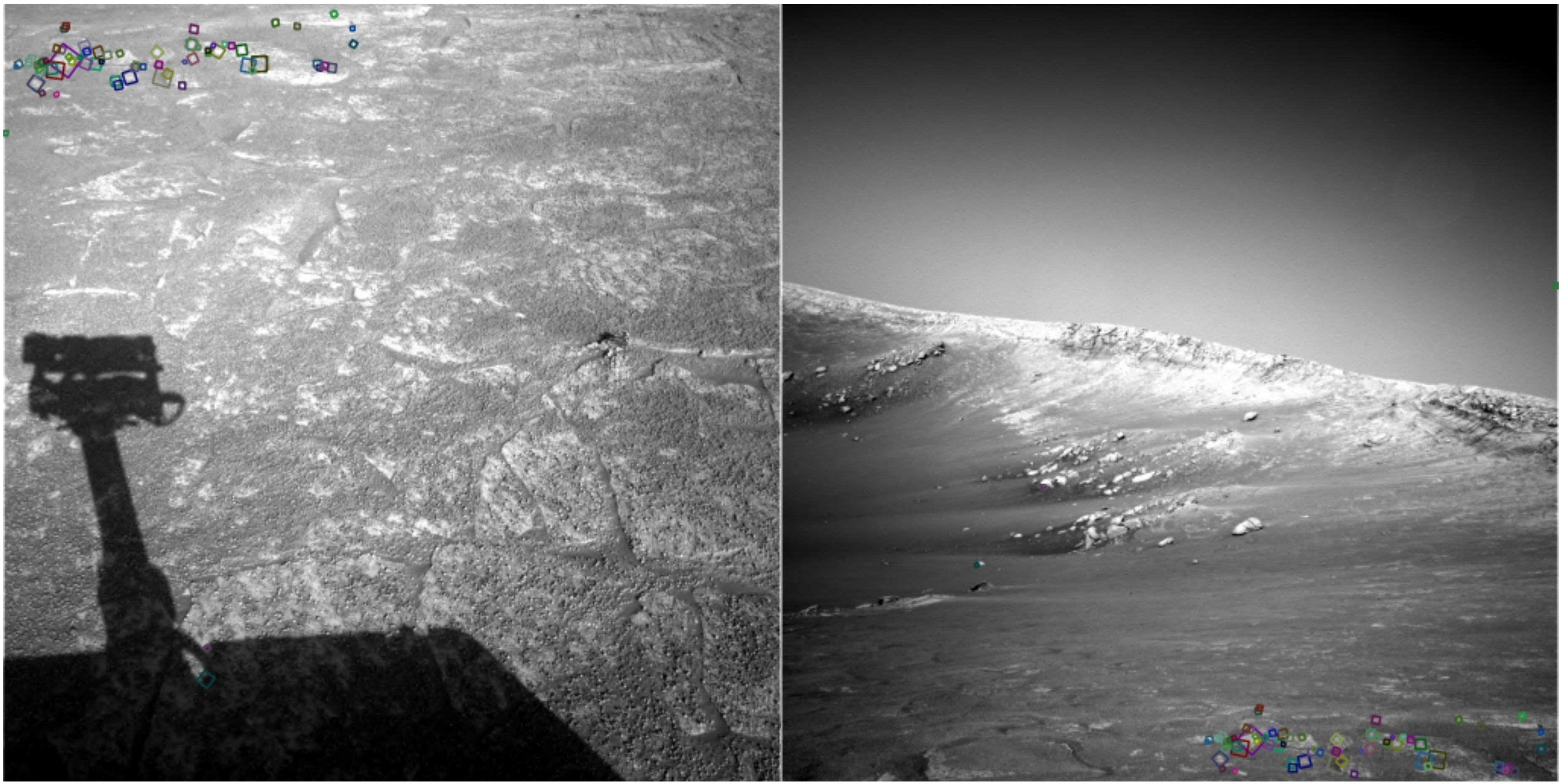
by Diva Sian

by scgbt

Harder still?



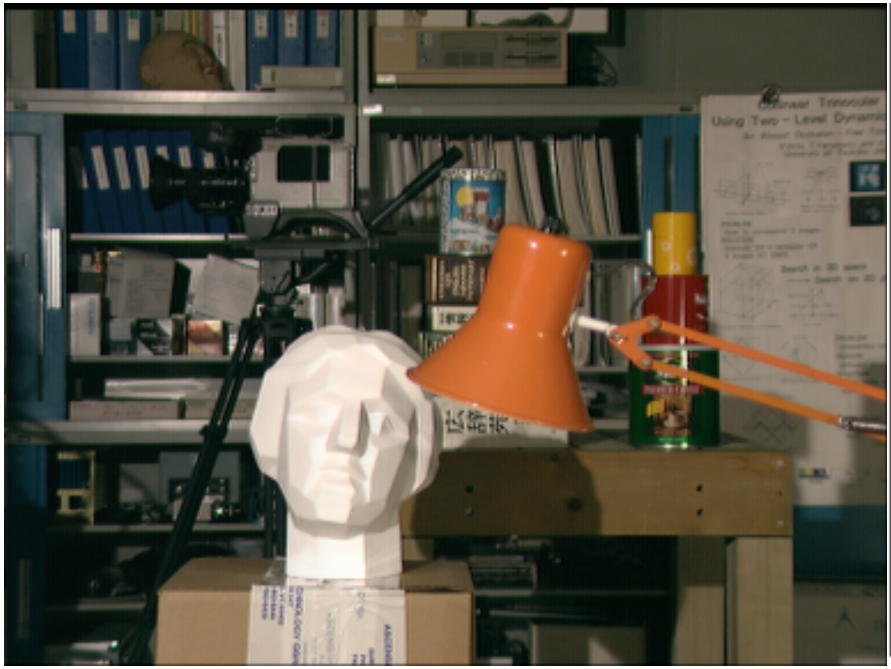
Answer below (look for tiny colored squares…)



NASA Mars Rover images
with SIFT feature matches

Dense correspondence

- Some applications demand correspondence for every pixel
  - For example dense 3D reconstruction



Sparse correspondence

- Sometimes a sparse set of correspondences are enough
  - E.g. estimating pose or camera relationships. Why?
  - Pose / camera relationships only consist of a small number of variables
  - Need only a little bit of information to recover it.

A general pipeline for correspondence

1. If sparse correspondences are enough, *choose points for which we will search for correspondences (feature points)*
2. For each point (or every pixel if dense correspondence), describe point using a *feature descriptor*
3. Find best matching descriptors across two images (*feature matching*)
4. Use feature matches to perform downstream task, e.g., pose estimation

A general pipeline for correspondence

1. If sparse correspondences are enough, *choose points for which we will search for correspondences (feature points)*
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Sparse correspondence

- Which pixels should be searching correspondence for?
    - *Feature points / keypoints*

What makes a good feature point?

Characteristics of good feature points

- Repeatability / invariance
  - The same feature point can be found in several images despite geometric and photometric transformations
- Saliency / distinctiveness
  - Each feature point is distinctive
  - Fewer ”false” matches

Goal: repeatability

- We want to detect (at least some of) the same points in both images.

No chance to find true matches!

- Yet we have to be able to run the detection procedure *independently* per image.

Kristen Grauman

Repeatability / invariance

- The feature detector should “fire” at consistent places in spite of rotation, translation etc.

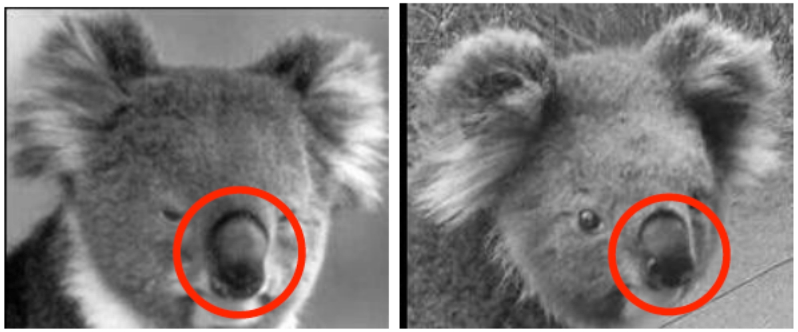
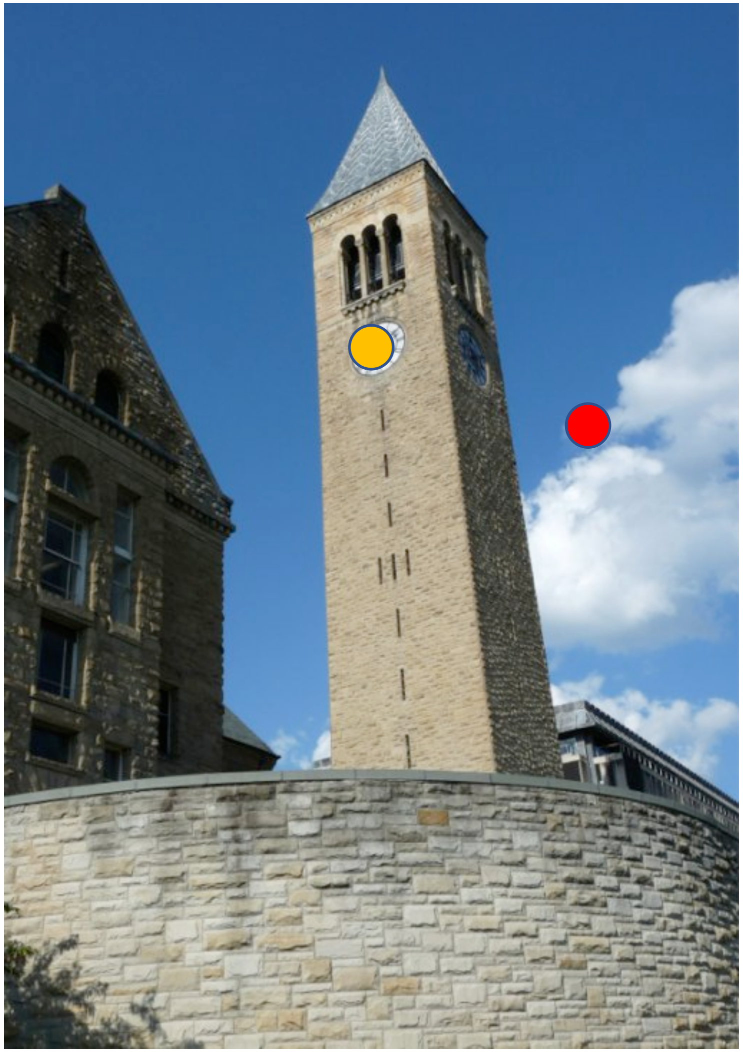


Image credit : L. Fei-Fei

Repeatability / invariance



Goal: distinctiveness

- The feature point should be distinctive enough that it is easy to match
  - Should *at least* be distinctive from other patches nearby

Where would you tell your friend to meet you?

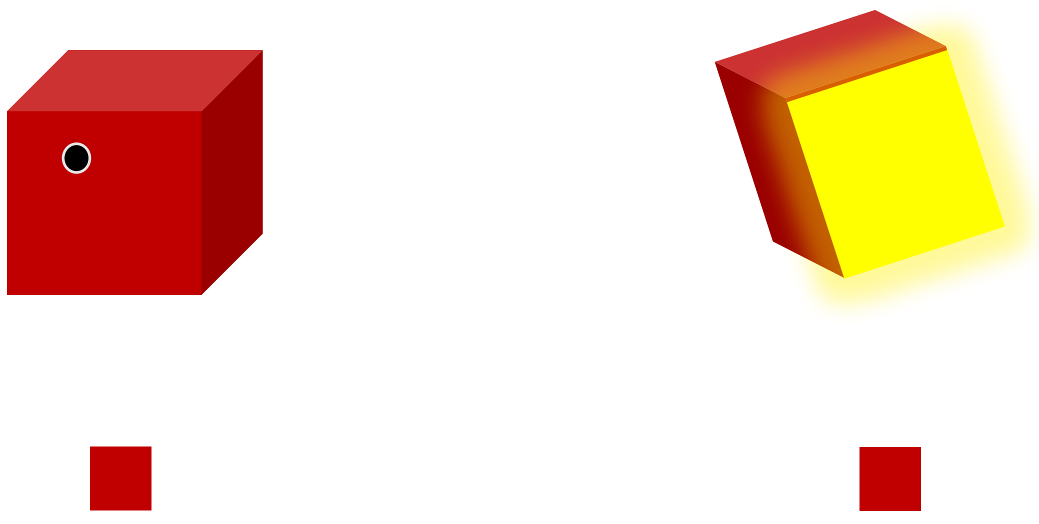
Where would you tell your friend to meet you?

Choosing distinctive interest points

- If you wanted to meet a friend would you say
  a) "Let's meet on campus."
  b) "Let's meet on Green street."
  c) "Let's meet at Green and Wright."
  - Corner detection

- Or if you were in a secluded area:
  a) "Let's meet in the Plains of Akbar."
  b) "Let's meet on the side of Mt. Doom."
  c) "Let's meet on top of Mt. Doom."
  - Blob (valley/peak) detection

The aperture problem



The aperture problem

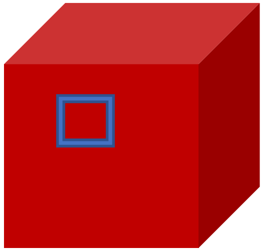
- Individual pixels are ambiguous
- Idea: Look at whole patches!

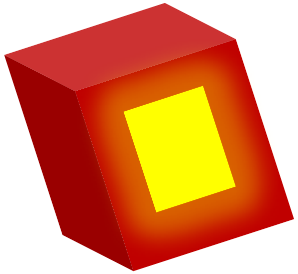
The aperture problem

- Individual pixels are ambiguous
- Idea: Look at whole patches!

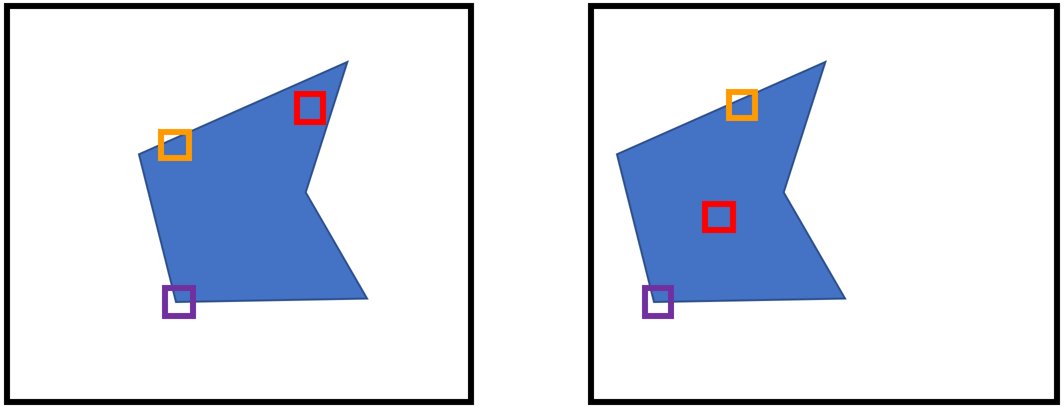
The aperture problem

- *Some local neighborhoods* are ambiguous



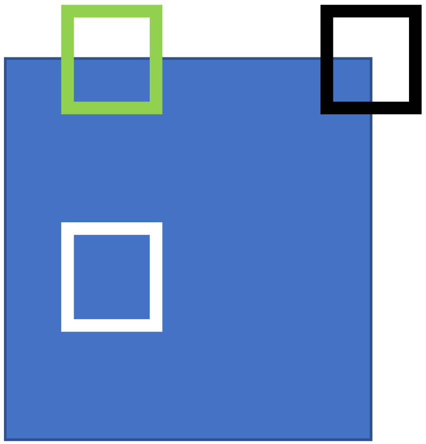


The aperture problem



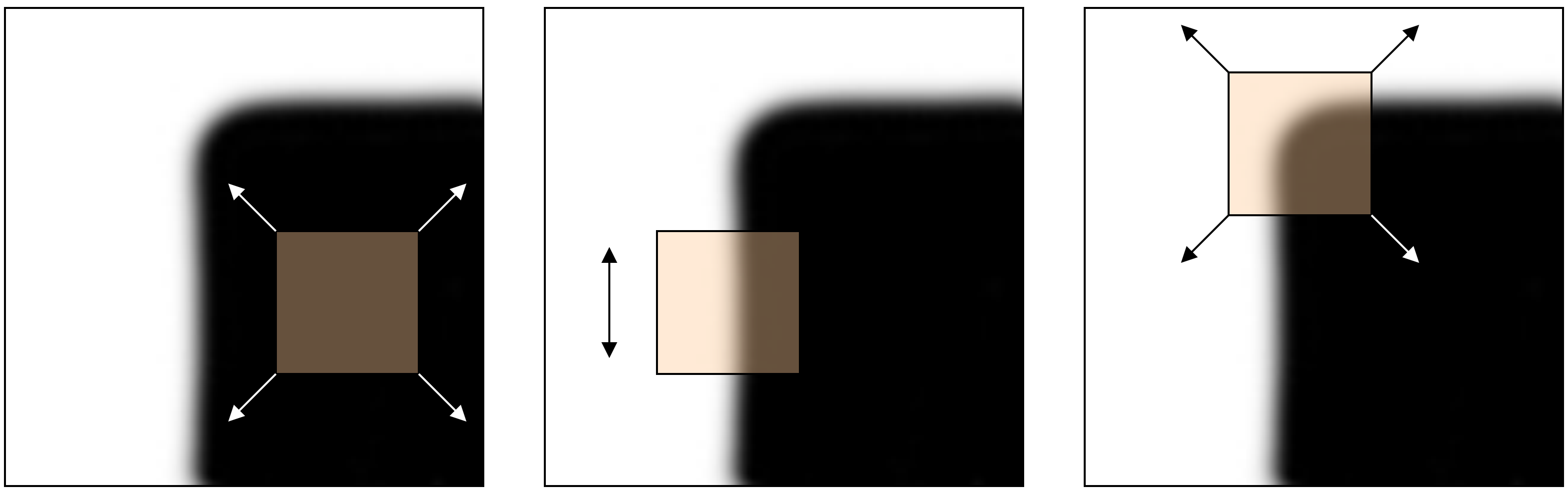
Corner detection

- Main idea: Translating window should cause large differences in patch appearance



Corner Detection: Basic Idea

- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions

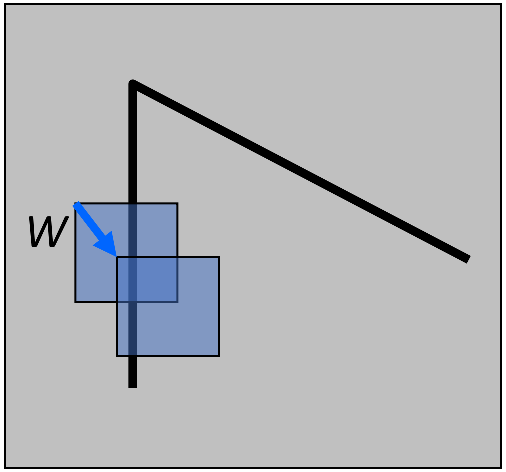
"edge":
no change along the edge direction

"corner":
significant change in all directions

Source: A. Efros

Corner detection the math

- Consider shifting the window *W* by (*u*,*v*)
  - how do the pixels in W change?
- Write pixels in window as a vector:



$$\phi_0 = [I(0,0), I(0,1), \ldots, I(n,n)]$$

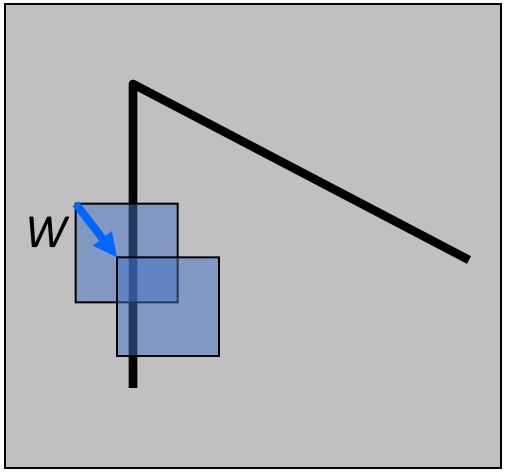
$$\phi_1 = [I(0+u, 0+v), I(0+u, 1+v), \ldots, I(n+u, n+v)]$$

$$E(u,v) = \|\phi_0 - \phi_1\|_2^2$$

Corner detection: the math

Consider shifting the window *W* by (*u,v*)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD "error" *E*(*u,v*):



$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u, y+v) - I(x,y) \right]^2$$

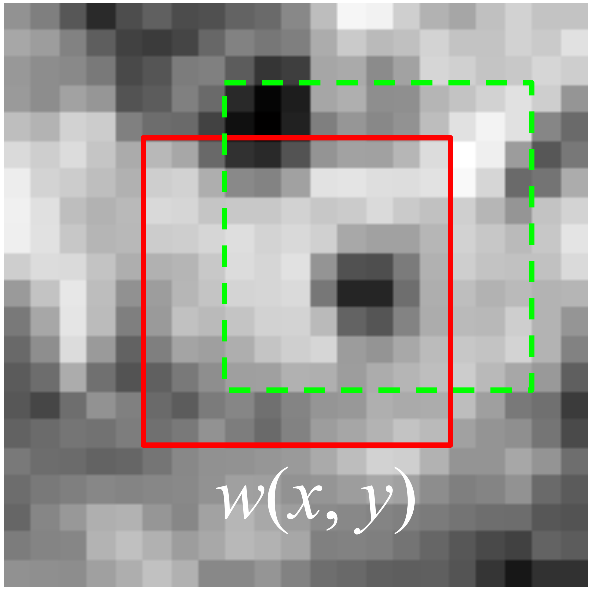
- We want E(u,v) to be *as high as possible for all u, v!*

Corner Detection: Mathematics

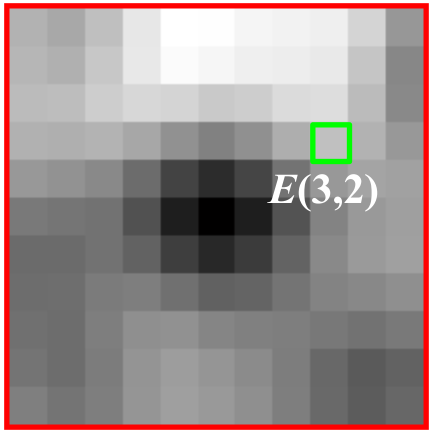
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

$I(x, y)$



$E(u, v)$



Corner Detection: Mathematics

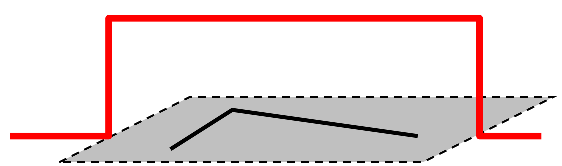
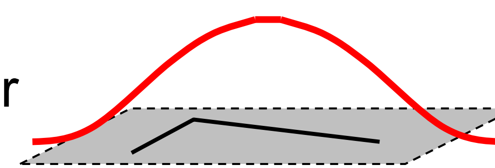
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window function

Shifted intensity

Intensity

Window function $w(x,y) =$  or 

1 in window, 0 outside

Gaussian

Source: R. Szeliski

Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

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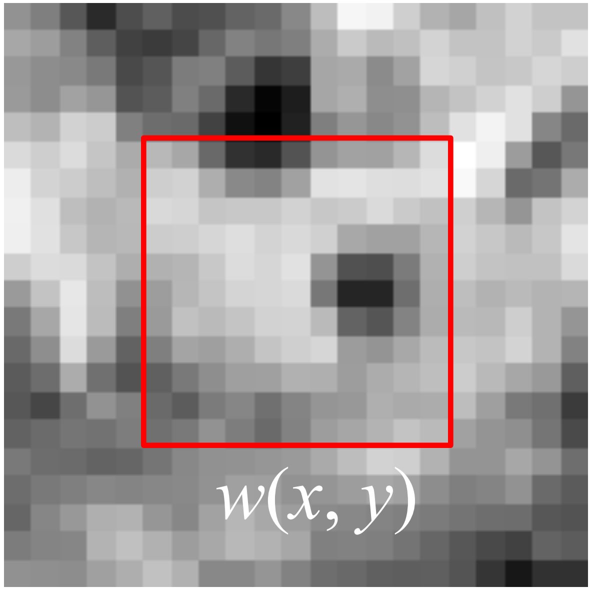
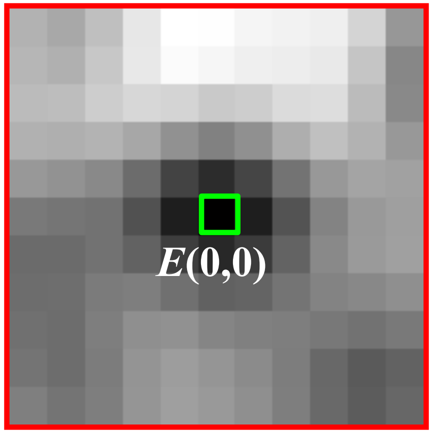


$E(u, v)$



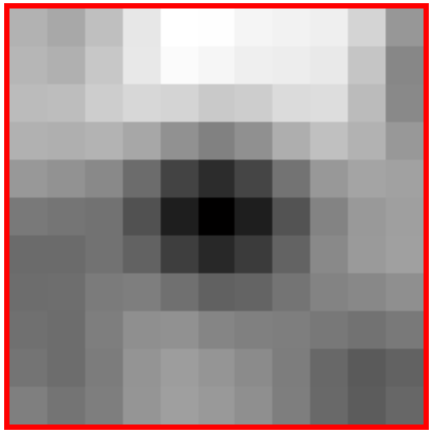
Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Small motion assumption

Taylor Series expansion of *I*:

$$I(x+u,y+v) = I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

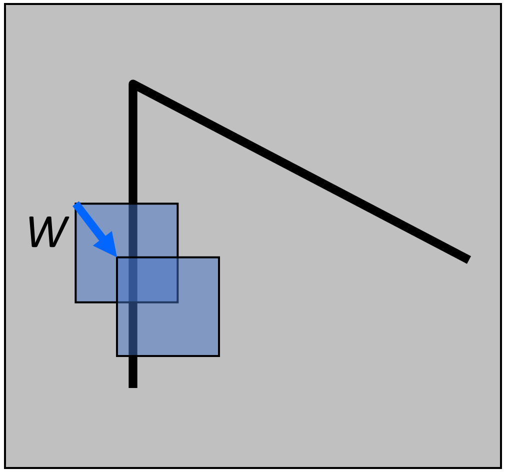
If the motion (u,v) is small, then first order approximation is good

$$\begin{aligned} I(x+u,y+v) &\approx I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \\ &\approx I(x,y) + [I_x \ I_y]\begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

shorthand: $I_x = \frac{\partial I}{\partial x}$

Plugging this into the formula on the previous slide…

Corner detection: the math

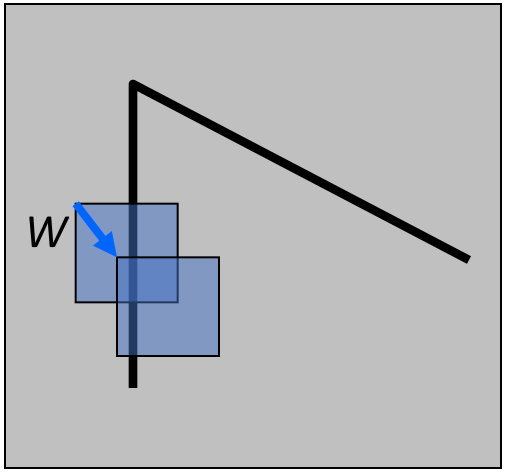


Consider shifting the window *W* by (*u,v*)

- define an SSD "error" *E(u,v)*:

$$
\begin{aligned}
E(u,v) &= \sum_{(x,y)\in W} \left[I(x+u,y+v) - I(x,y)\right]^2 \\
&\approx \sum_{(x,y)\in W} \left[I(x,y) + I_x u + I_y v - I(x,y)\right]^2 \\
&\approx \sum_{(x,y)\in W} \left[I_x u + I_y v\right]^2
\end{aligned}
$$

Corner detection: the math



Consider shifting the window W by (u,v)

- define an "error" $E(u,v)$:

$$E(u,v) \approx \sum_{(x,y)\in W} [I_x u + I_y v]^2$$

$$\approx Au^2 + 2Buv + Cv^2$$

$$A = \sum_{(x,y)\in W} I_x^2 \qquad B = \sum_{(x,y)\in W} I_x I_y \qquad C = \sum_{(x,y)\in W} I_y^2$$

- Thus, $E(u,v)$ is locally approximated as a quadratic error function

Interpreting the second moment matrix

Recall that we want E(u,v) to be as large as possible for all u,v

What does this mean in terms of M?

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

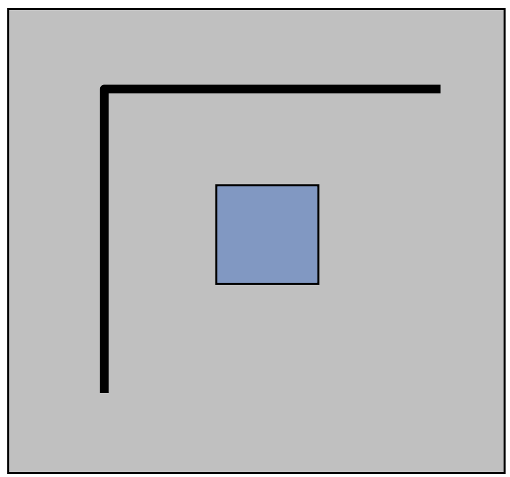
$$\underbrace{M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}}_{\text{Second moment matrix}}$$

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



$$M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$E(u,v) = 0 \quad \forall u, v$$

Flat patch: $I_x = 0$

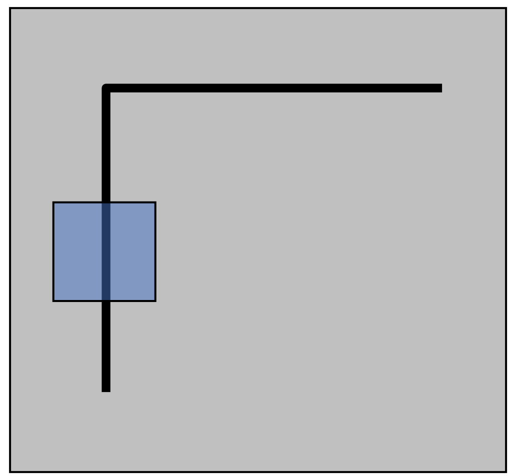
$I_y = 0$

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Vertical edge: $I_y = 0$

$$M = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$$

$$M \begin{bmatrix} 0 \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

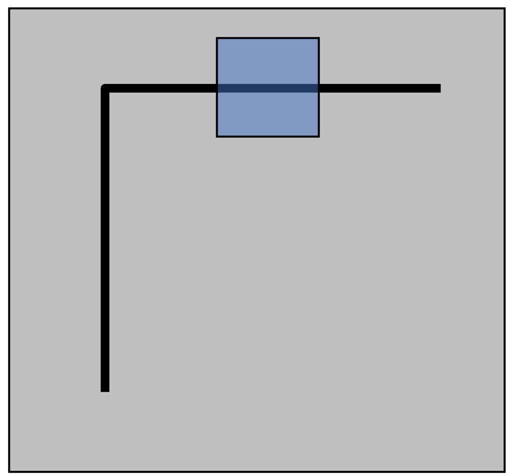
$$E(0,v) = 0 \quad \forall v$$

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



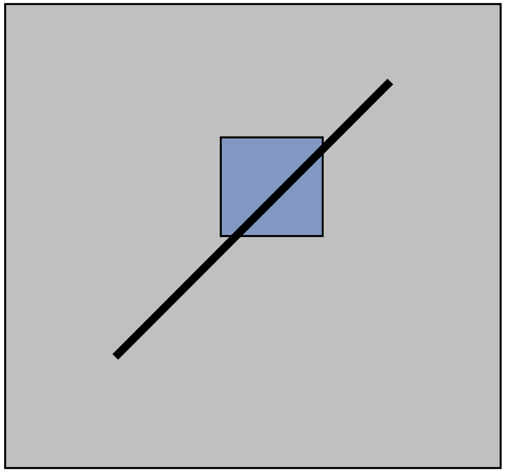
Horizontal edge: $I_x = 0$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$

$$M \begin{bmatrix} u \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$E(u,0) = 0 \ \forall u$$

What about edges in arbitrary orientation?



$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow E(u,v) = 0$$

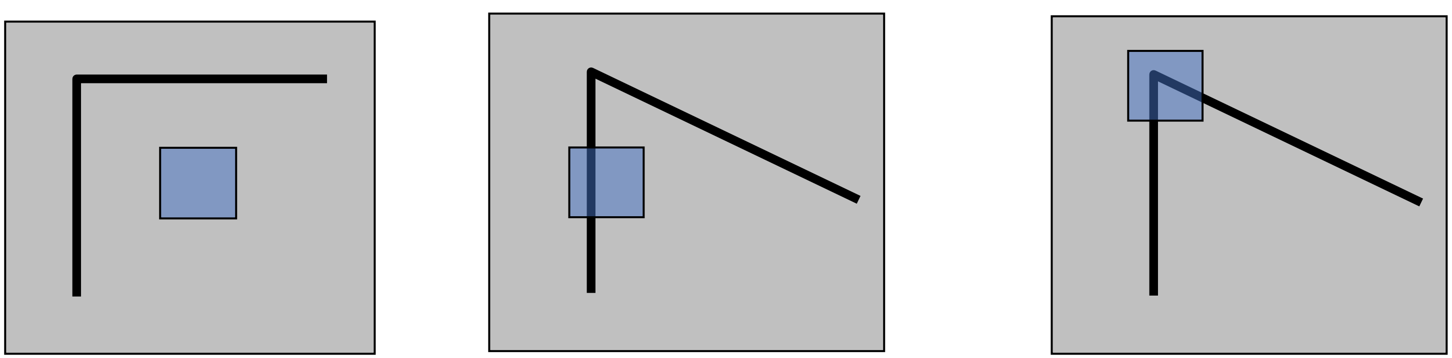
$$M \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Leftrightarrow E(u,v) = 0$$

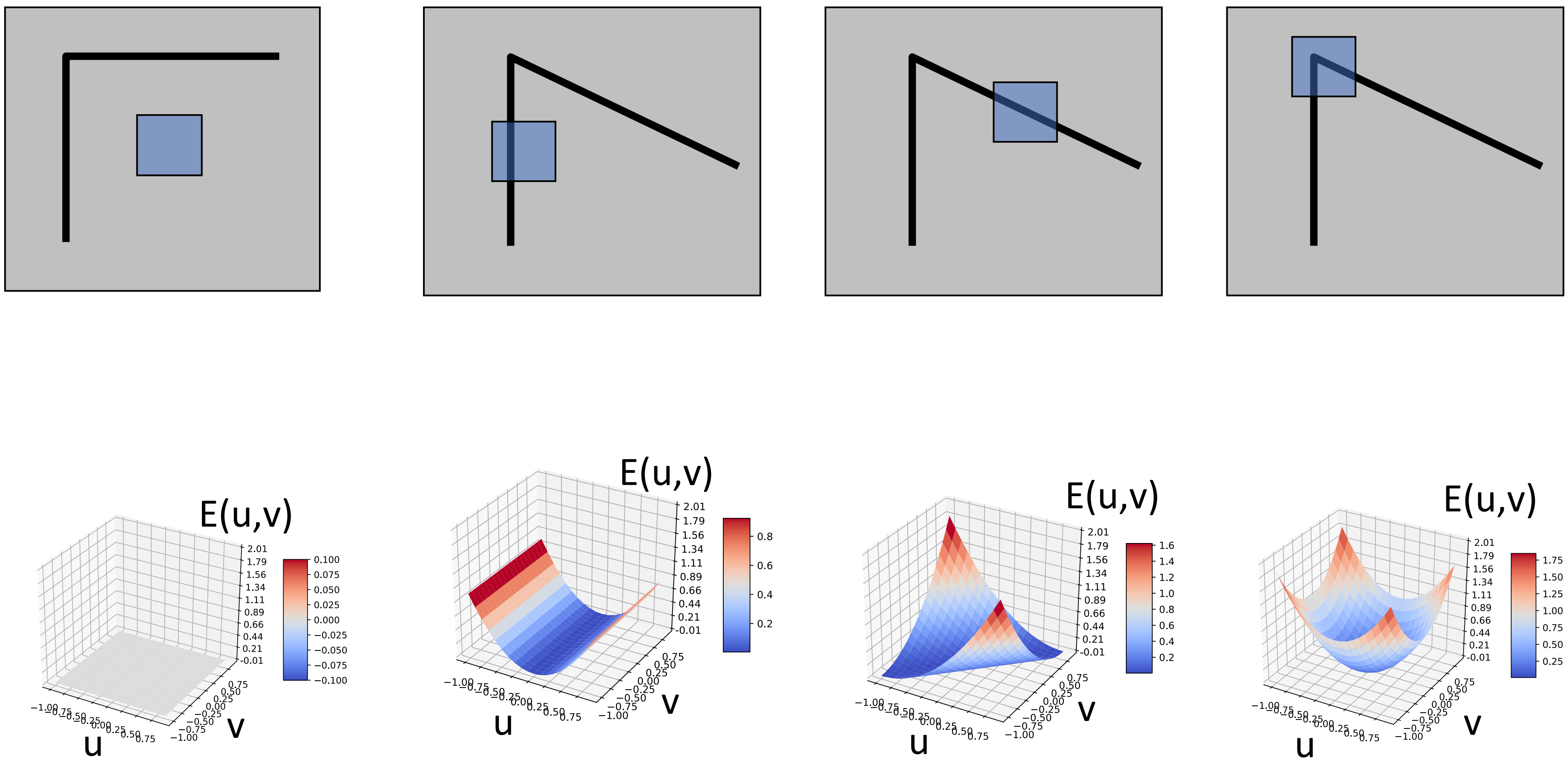
Solutions to Mx = 0 are directions for which E is 0: window can slide in this direction without changing appearance

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

Solutions to Mx = 0 are directions for which E is 0: window can slide in this direction without changing appearance

For corners, we want no such directions to exist



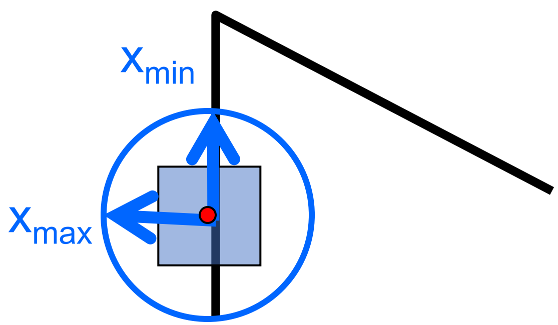

E(u,v)
u
v
E(u,v)
u
v
E(u,v)
u
v
E(u,v)
u
v

Eigenvalues and eigenvectors of M

- $Mx = 0 \Rightarrow Mx = \lambda x$: x is an eigenvector of M with eigenvalue 0
- M is 2 x 2, so it has 2 eigenvalues $(\lambda_{max}, \lambda_{min})$ with eigenvectors (x_{max}, x_{min})
- $E(x_{max}) = \ x_{max}^T M x_{max} = \lambda_{max} ||x_{max}||^2 = \lambda_{max}$ (eigenvectors have unit norm)
- $E(x_{min}) = x_{min}^T M x_{min} = \lambda_{min} ||x_{min}||^2 = \lambda_{min}$

Eigenvalues and eigenvectors of M

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$



M $x_{\max} = \lambda_{\max} x_{\max}$

M $x_{\min} = \lambda_{\min} x_{\min}$

Eigenvalues and eigenvectors of M

- Define shift directions with the smallest and largest change in error
- x_{max} = direction of largest increase in *E*
- λ_{max} = amount of increase in direction x_{max}
- x_{min} = direction of smallest increase in *E*
- λ_{min} = amount of increase in direction x_{min}

Interpreting the eigenvalues