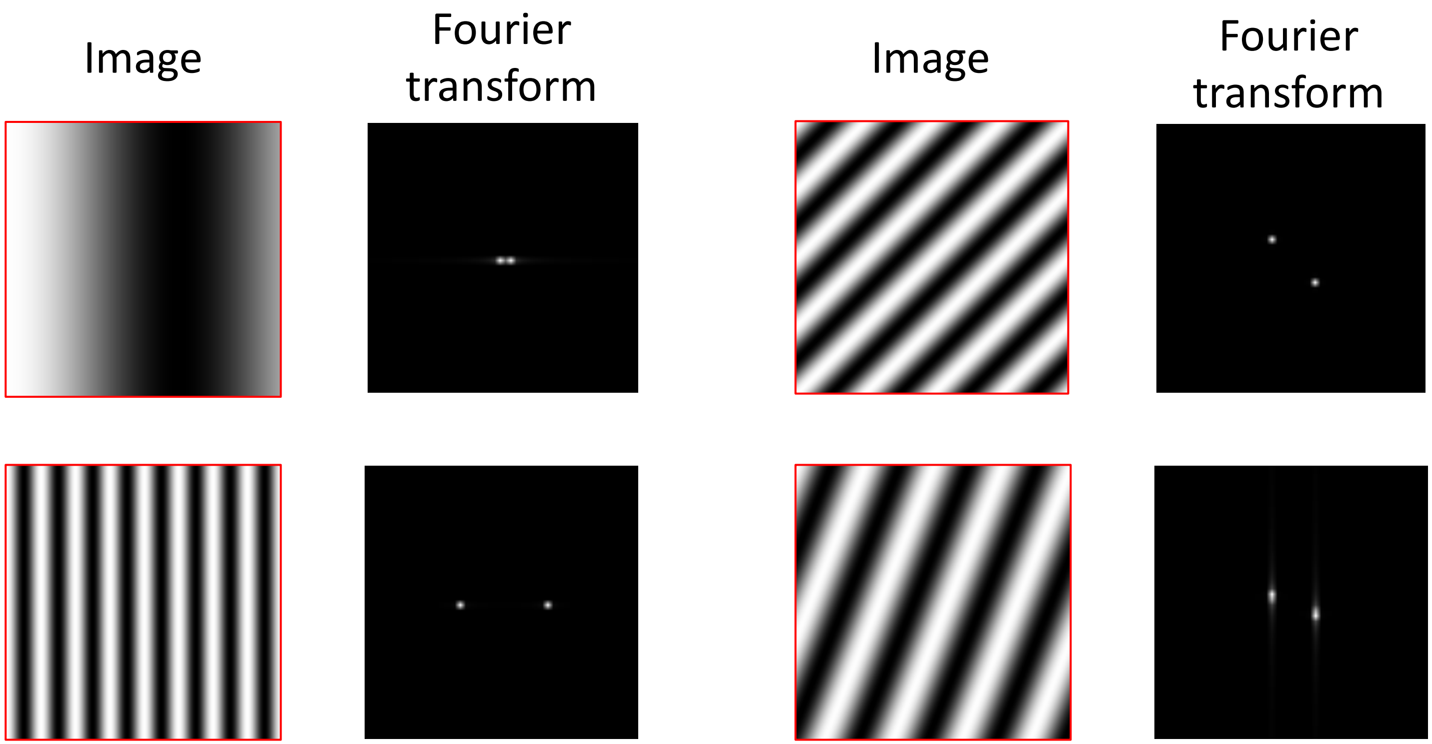
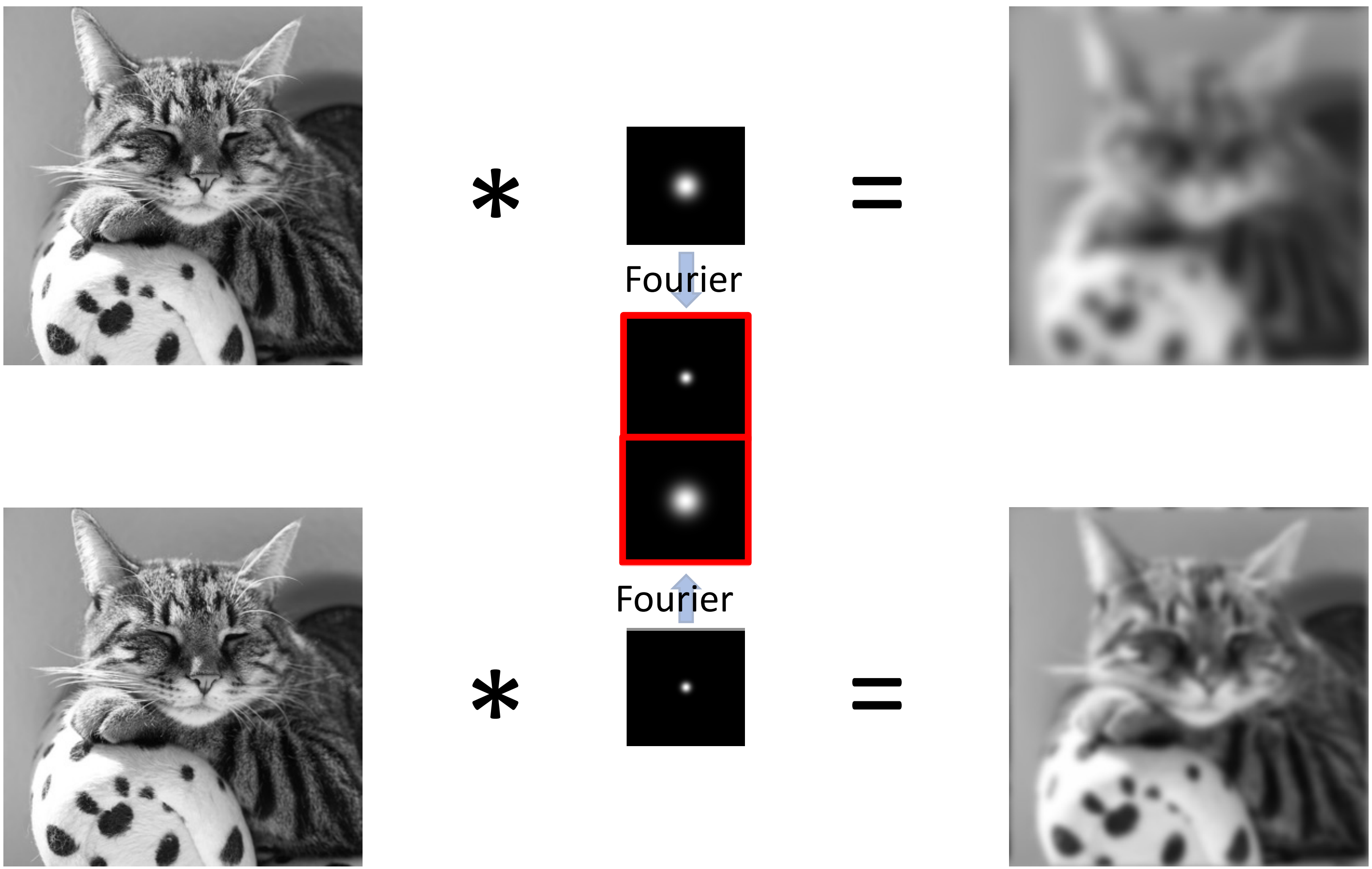
Fourier transforms and rescaling

Fourier transforms

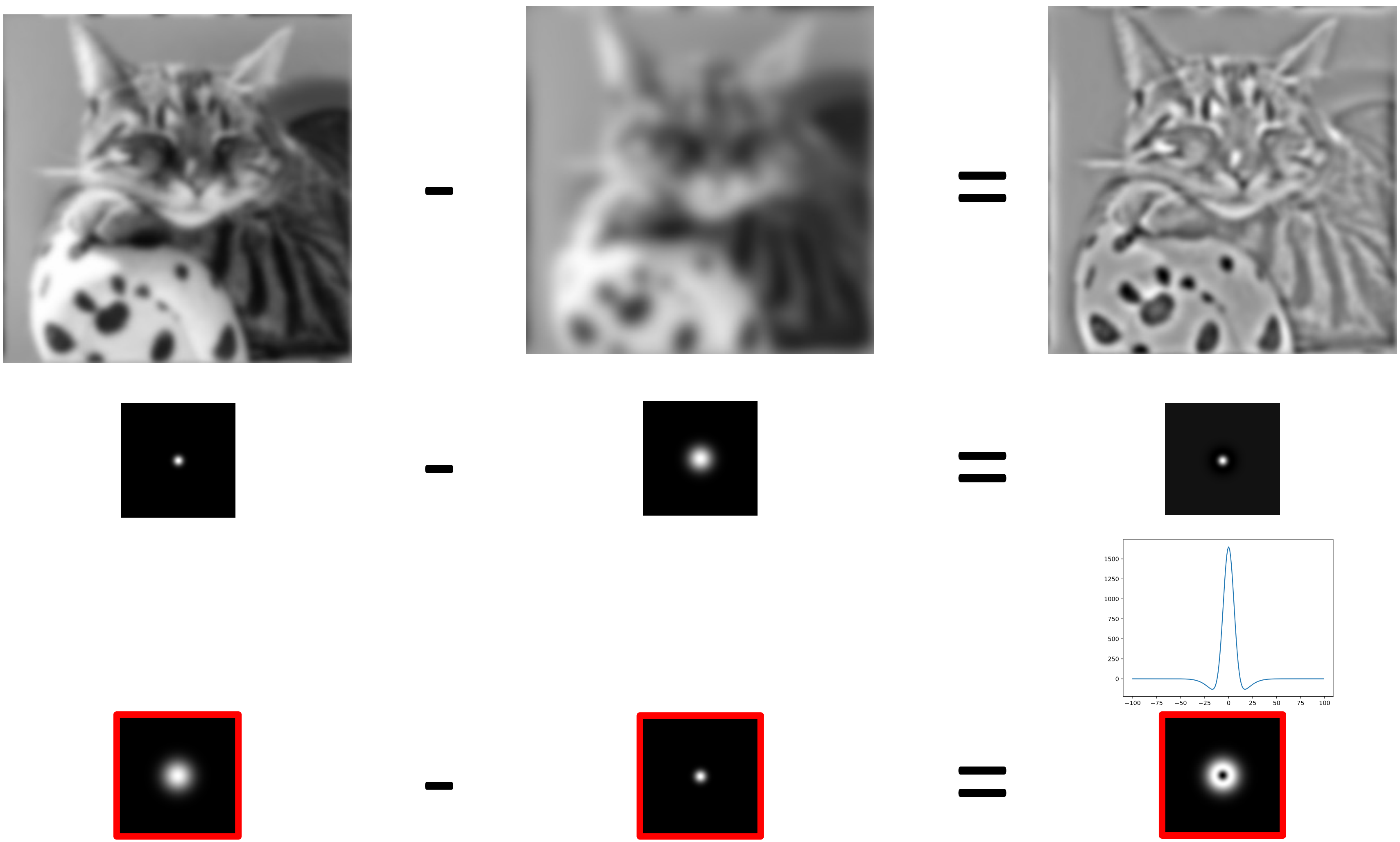


Low-pass filtering

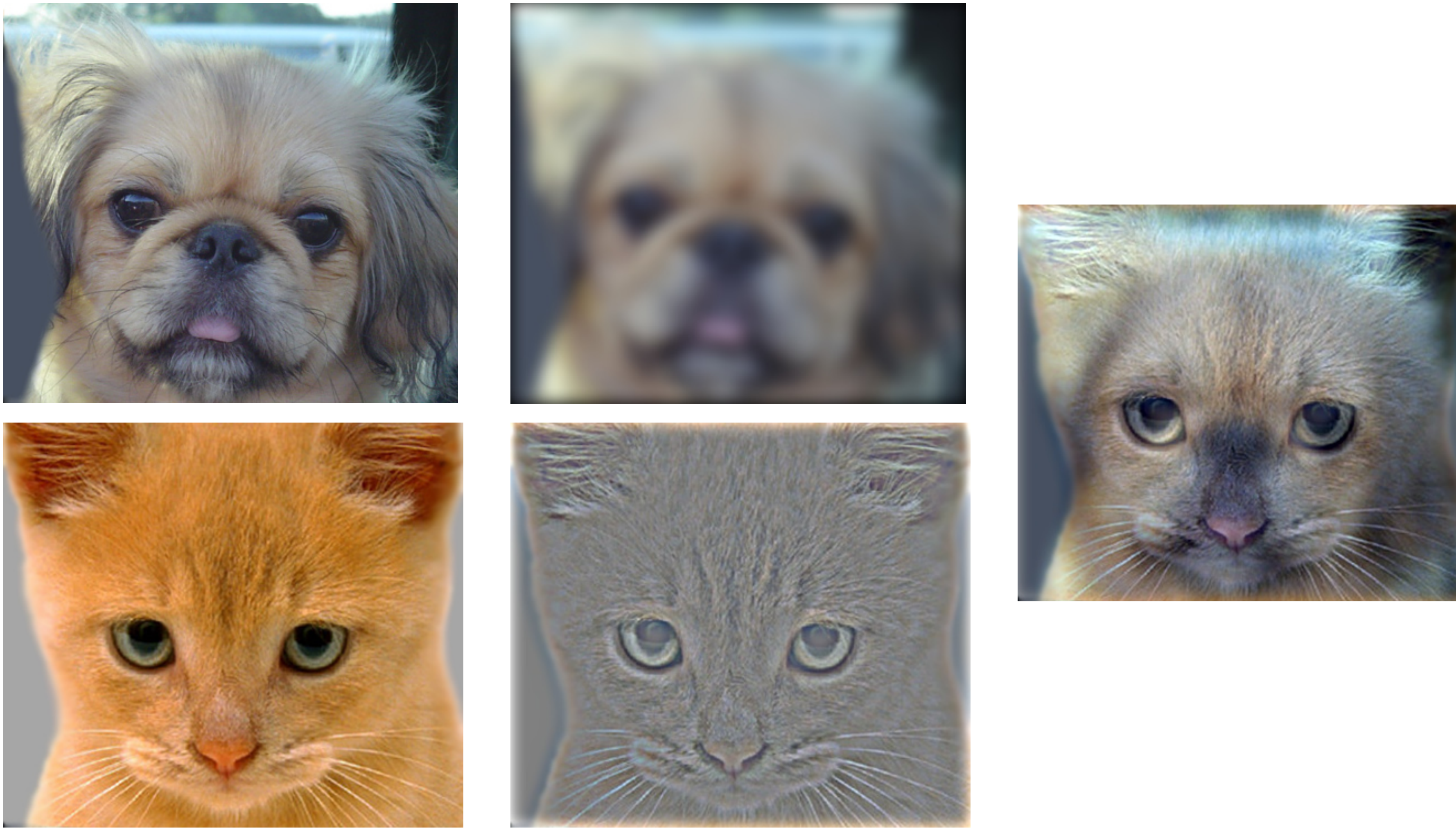


High-pass filtering

Band-pass filtering

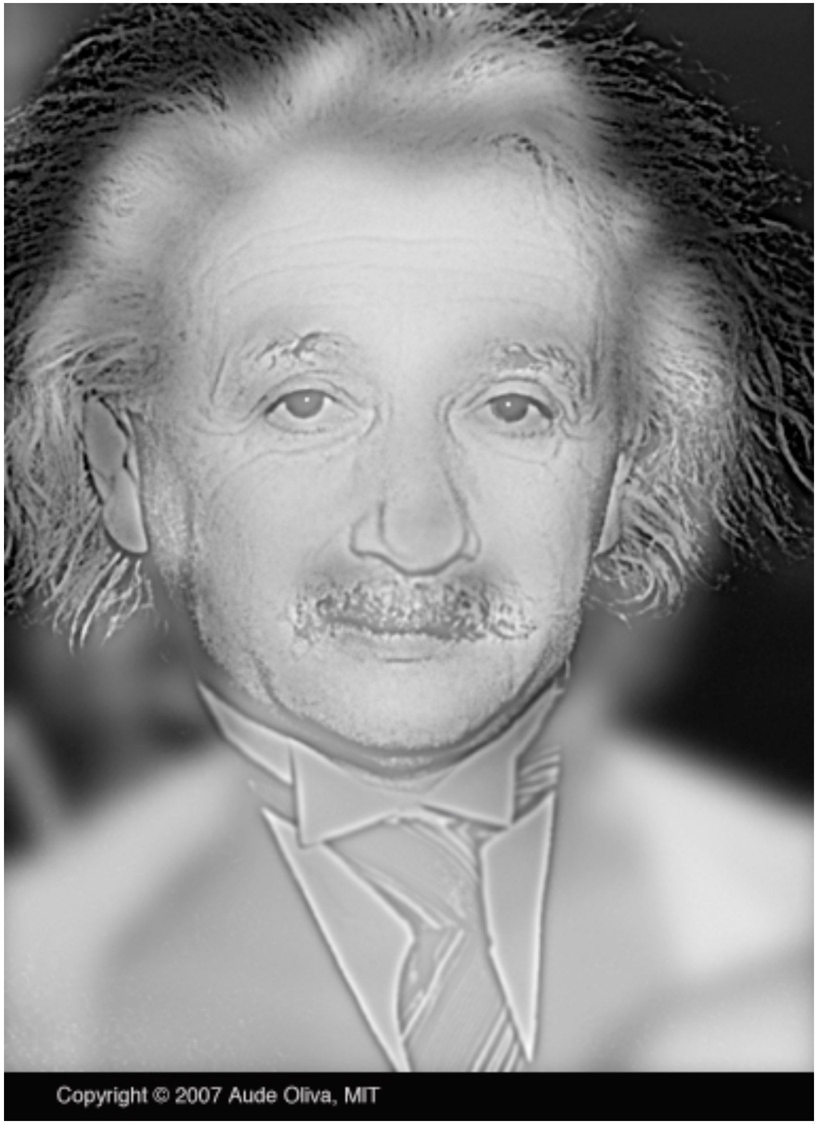


Hybrid images (PA1)



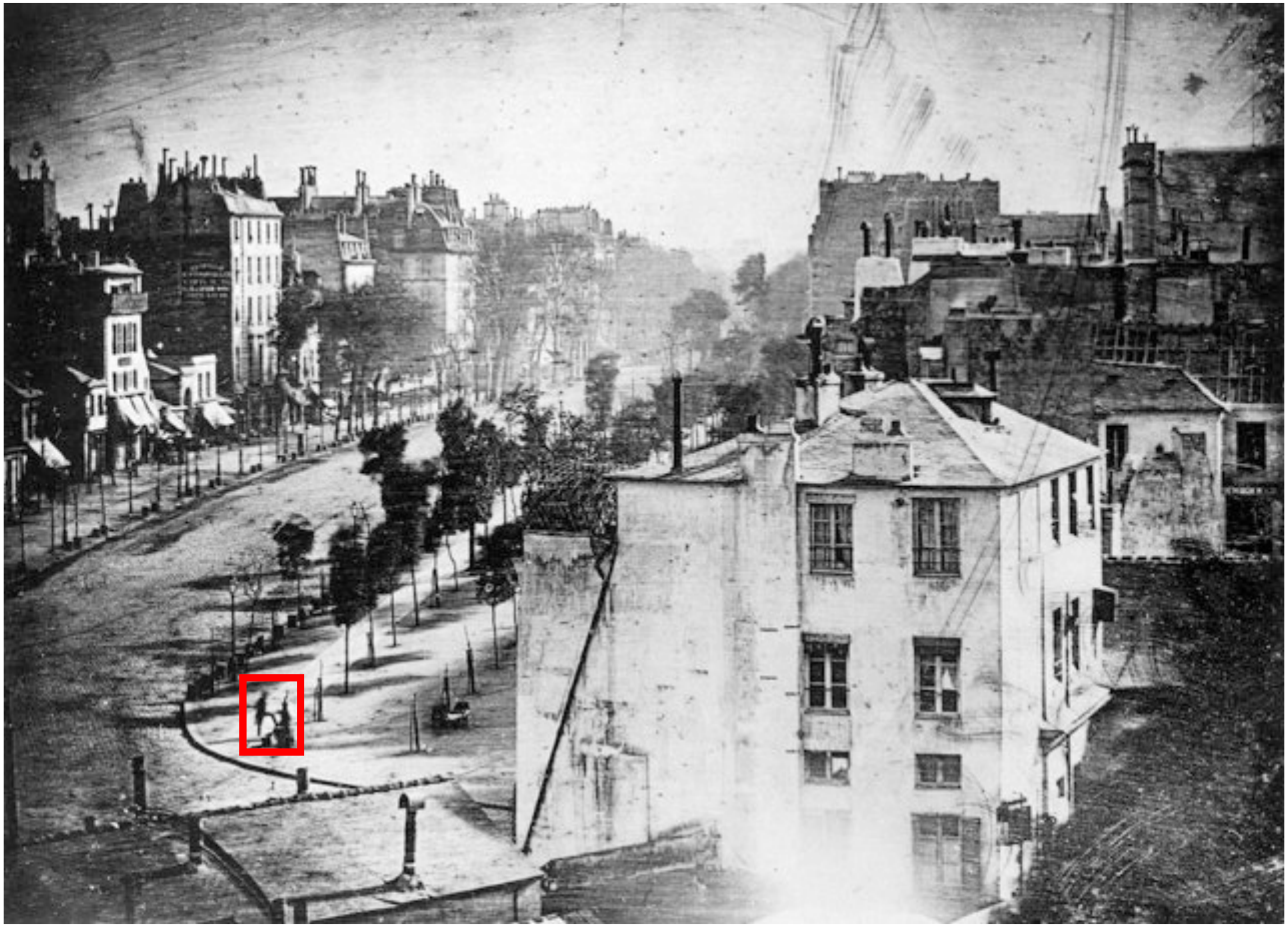
Hybrid images (PA1)

Hybrid images (PA1)



Resizing and resampling

Let’s enhance!



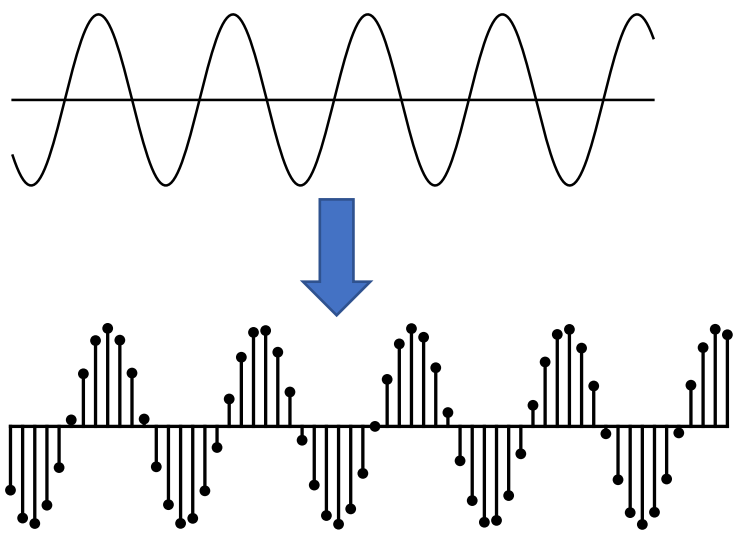
Louis Daguerre, 1838

Let’s enhance!

- When is enhancement possible?
- How can we model what happens when we upsample or downsample an image?
- Resizing up or down very common operation
  - Searching across scales
  - applications have different memory/quality tradeoffs

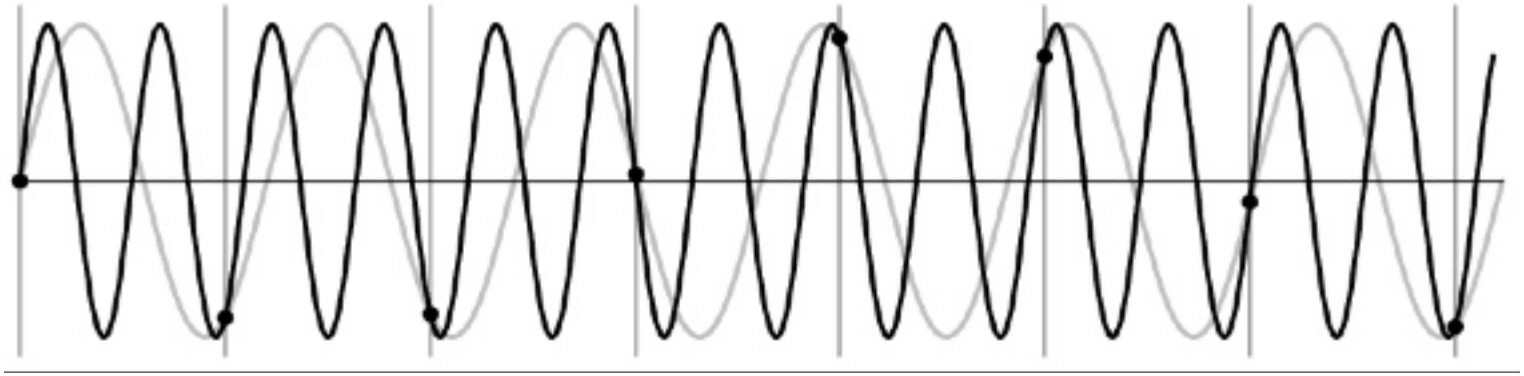
What is a (digital) image?

- True image is a function from R^2 to R
- Digital image is a sample from it
- 1D example:



- To enhance, we need to recover the original signal and sample again

Undersampling

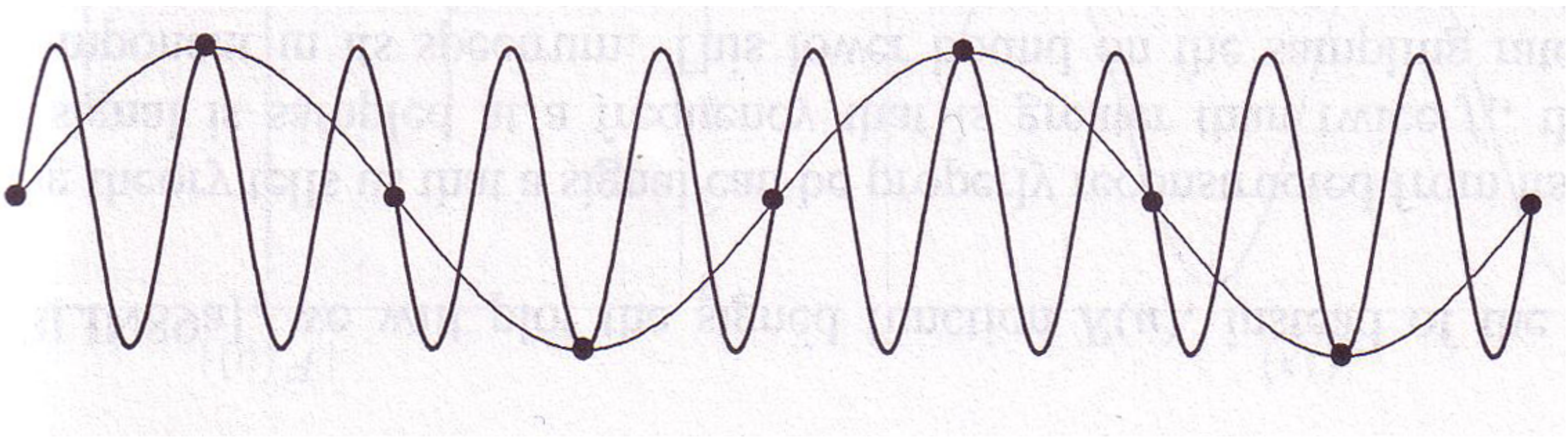


Undersampling

- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost
  - surprising result: indistinguishable from lower frequency
  - also was always indistinguishable from higher frequencies
  - *aliasing*: signals “traveling in disguise” as other frequencies

Aliasing

- When sampling is not adequate, impossible to distinguish between low and high frequency signal



Aliasing in time

Aliasing in time

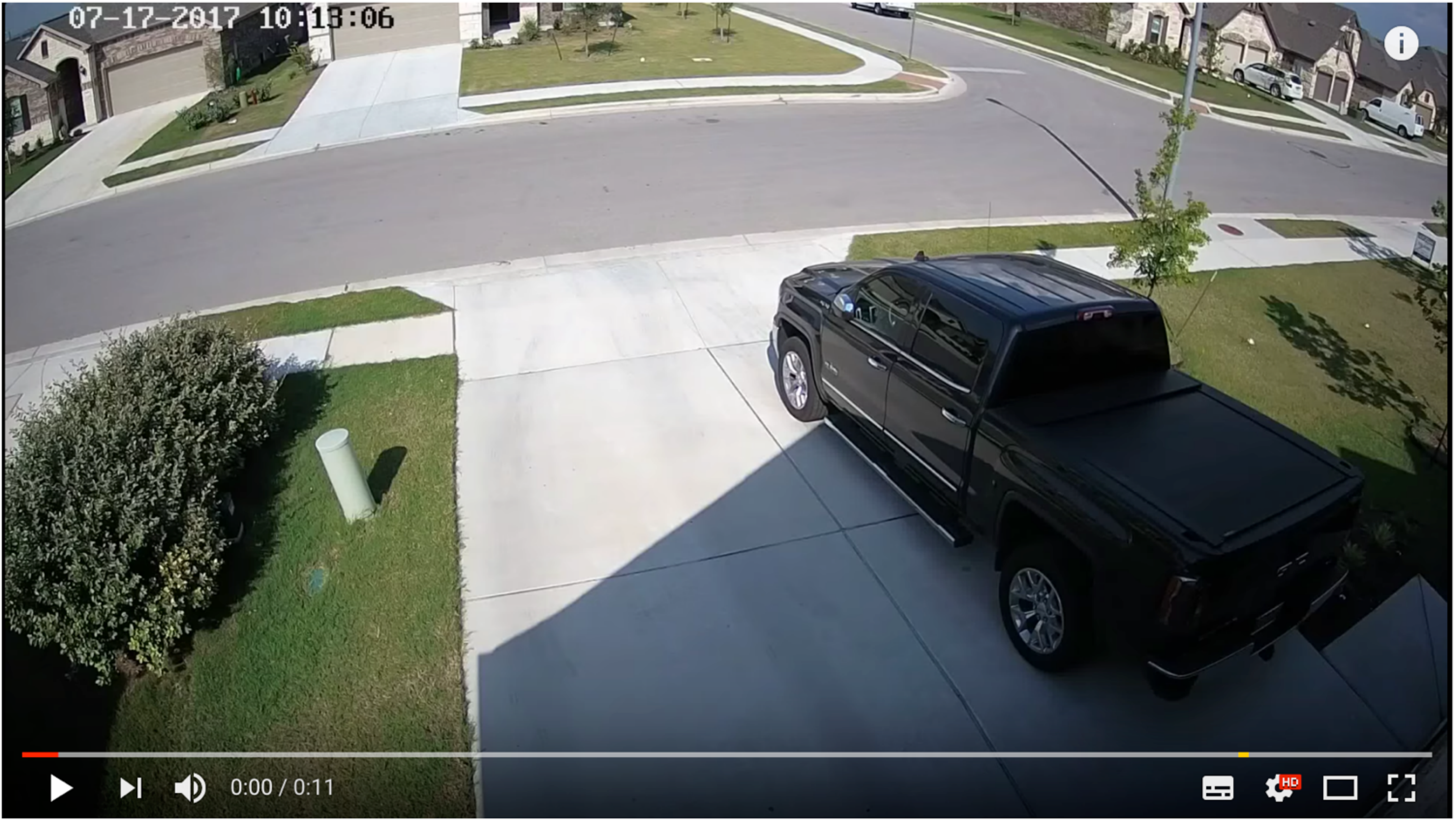


Image Scaling

What happens if we naively upsample?

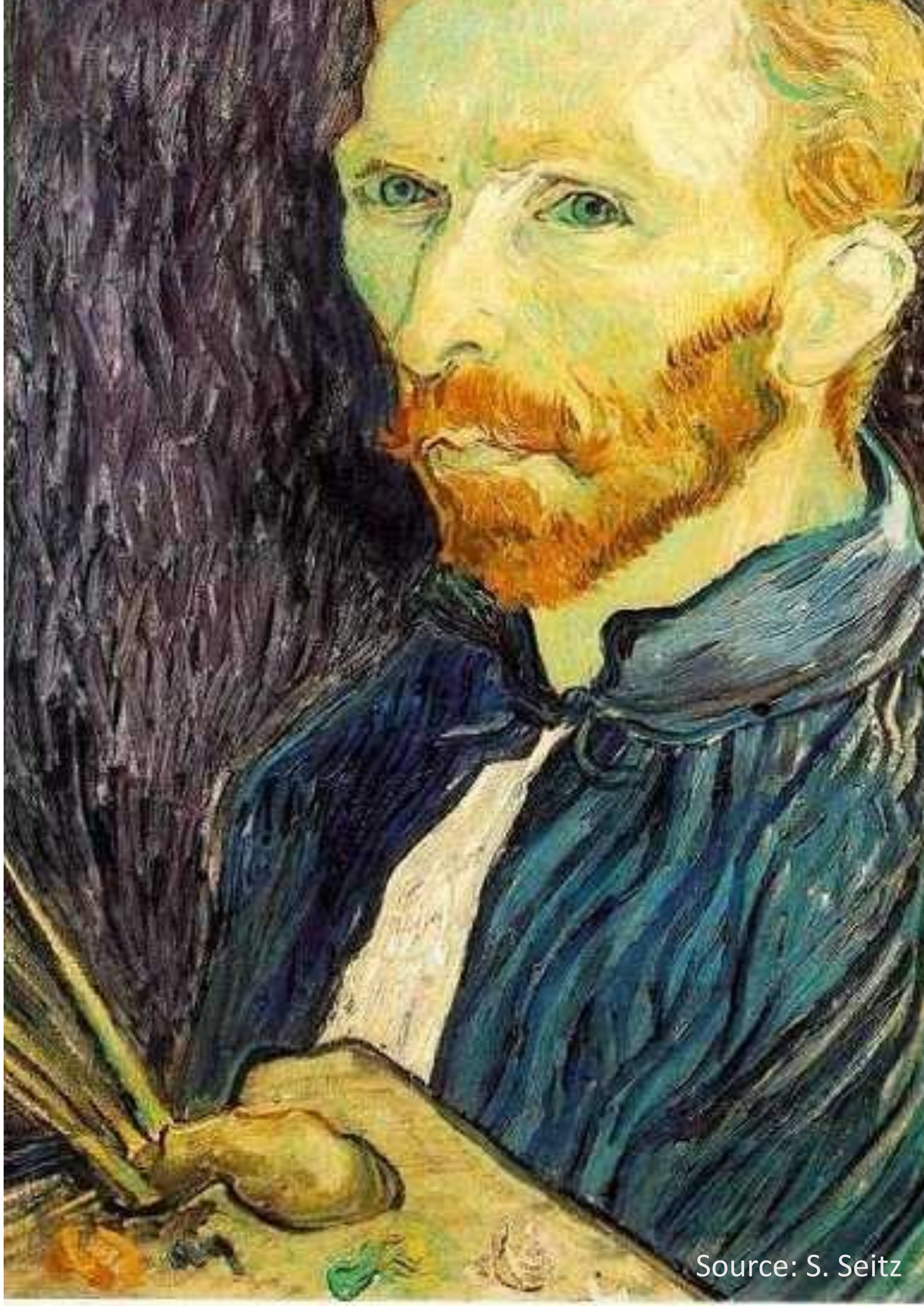
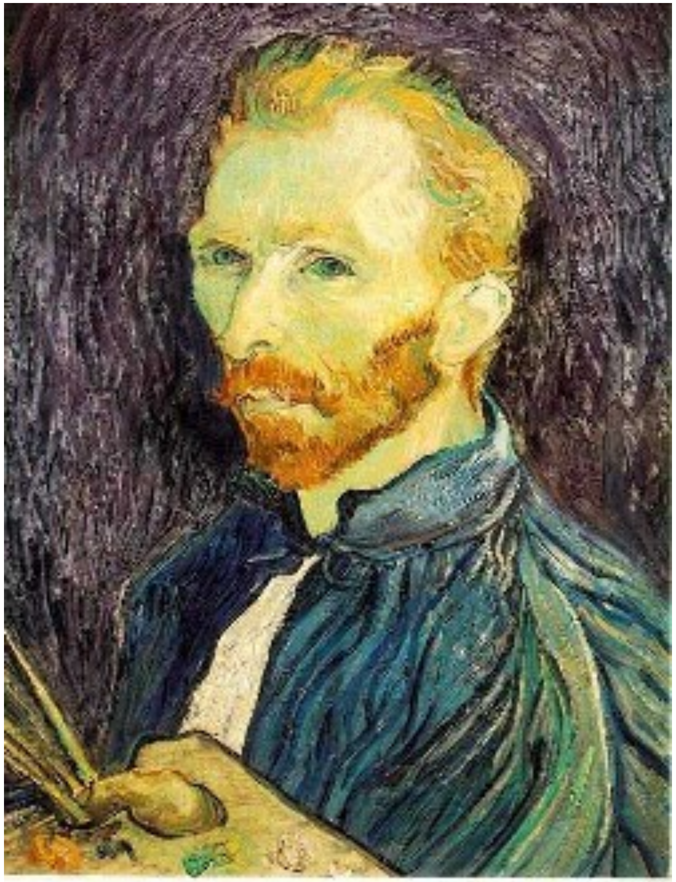


Image sub-sampling



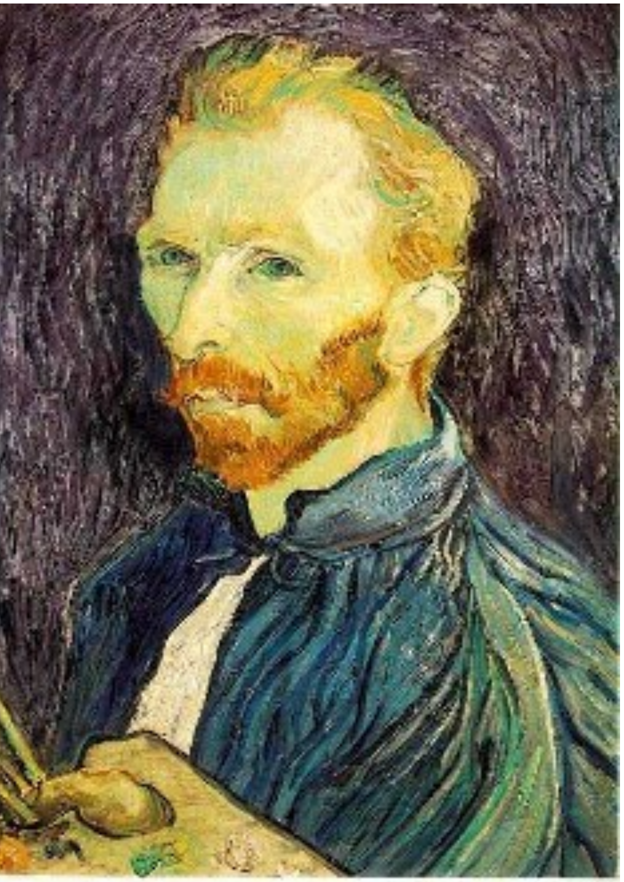
1/4

1/16

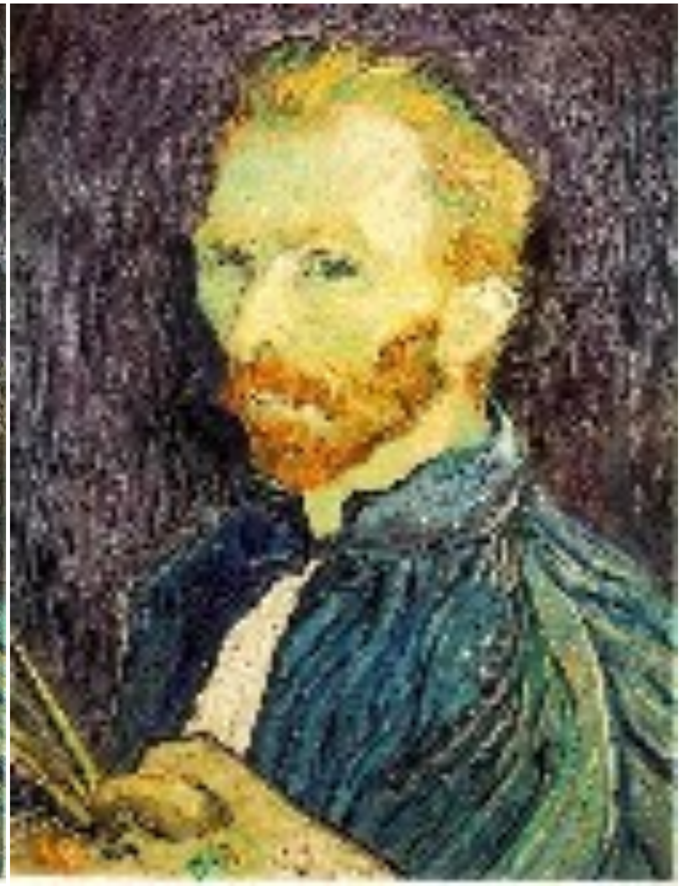
Throw away every other row and column to create a 1/2 size image
- called *image sub-sampling*

Source: S. Seitz

Image sub-sampling



1/2



1/4 (2x zoom)

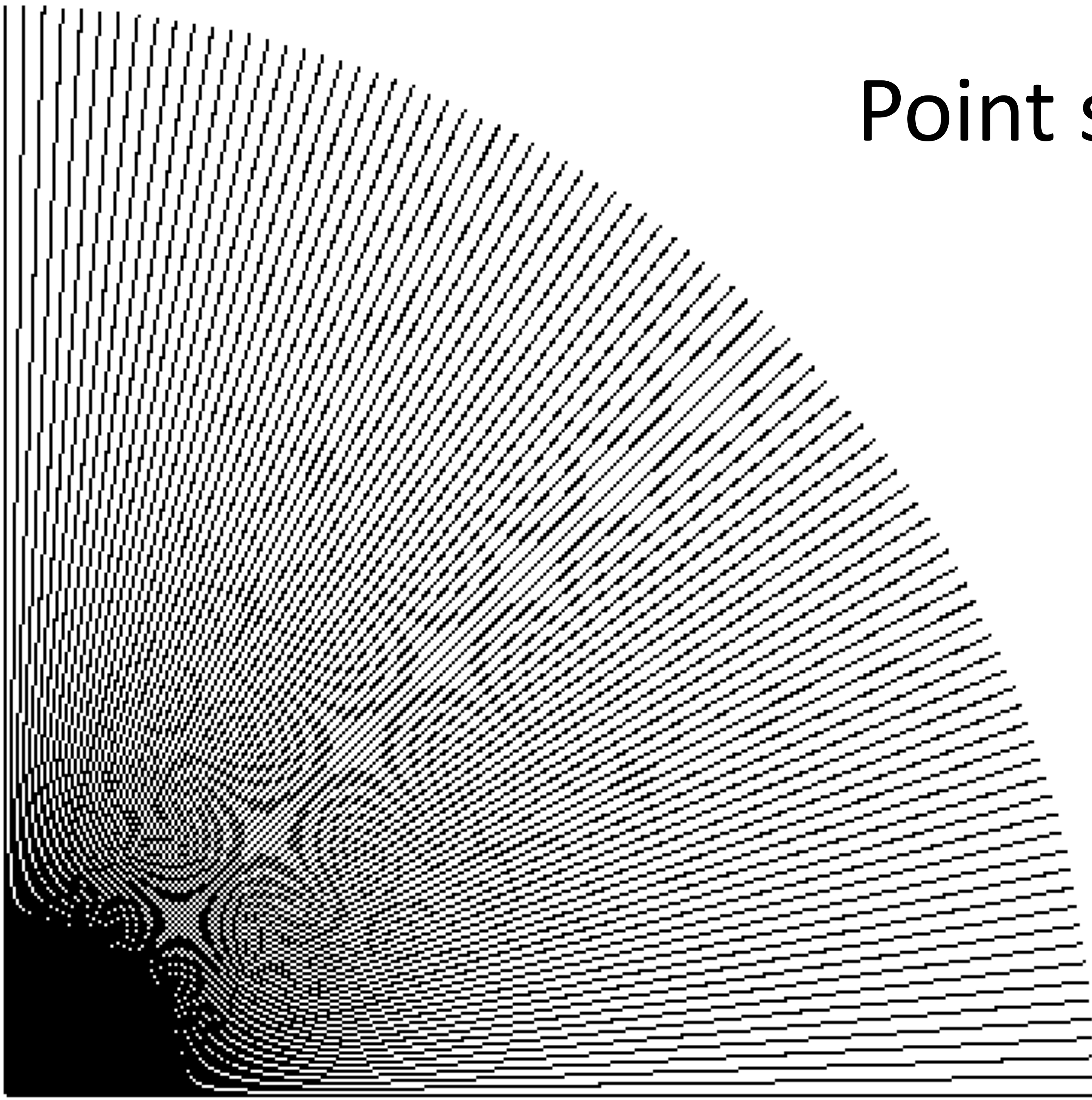
1/16 (4x zoom)

Why does this look so crufty? Aliasing!

Source: S. Seitz

Image sub-sampling

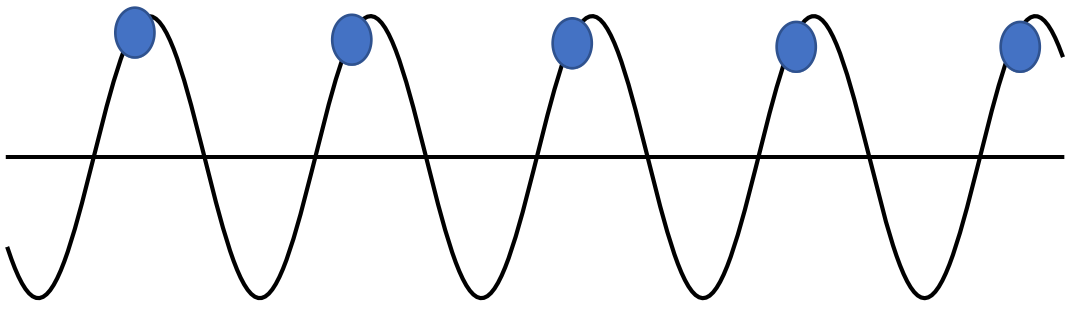
Source: F. Durand



Point sampling in action

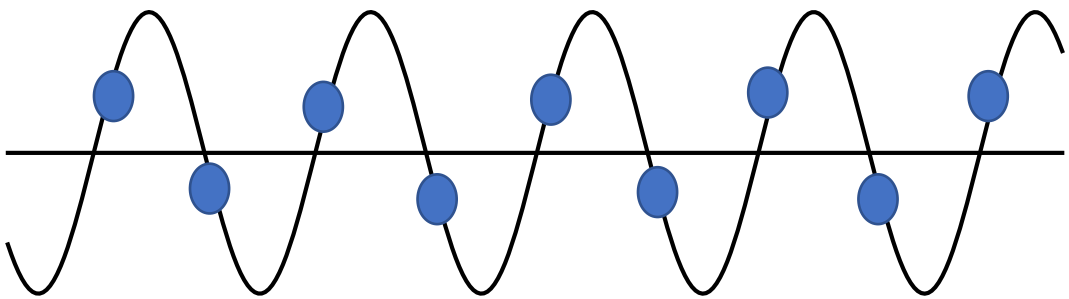
How many samples do we need?

- 1 sample per time period is too less:



How many samples do we need?

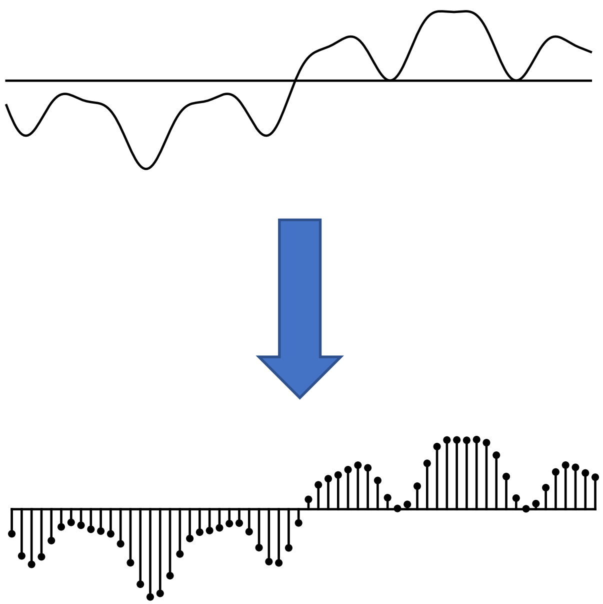
- 2 samples per time-period is enough



- Nyquist sampling theorem: Need to sample at least 2 times the frequency
- General signals? Need to sample at least 2 times the maximum frequency

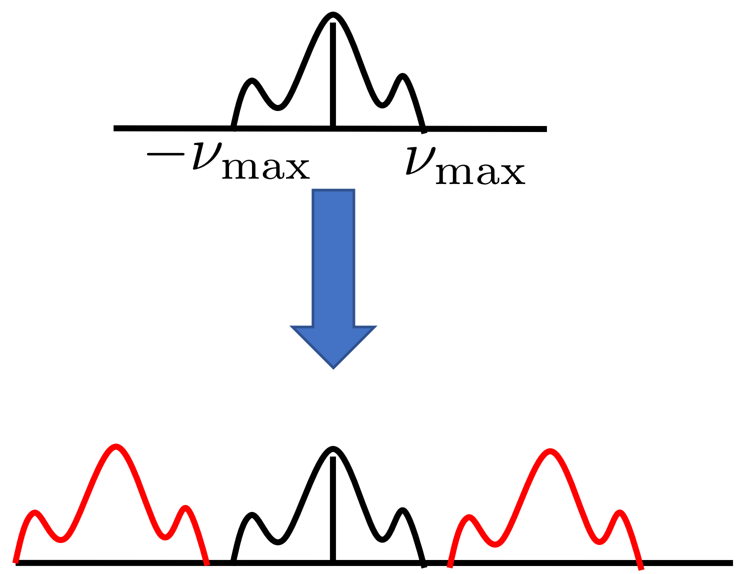
Nyquist sampling: why?

Spatial domain



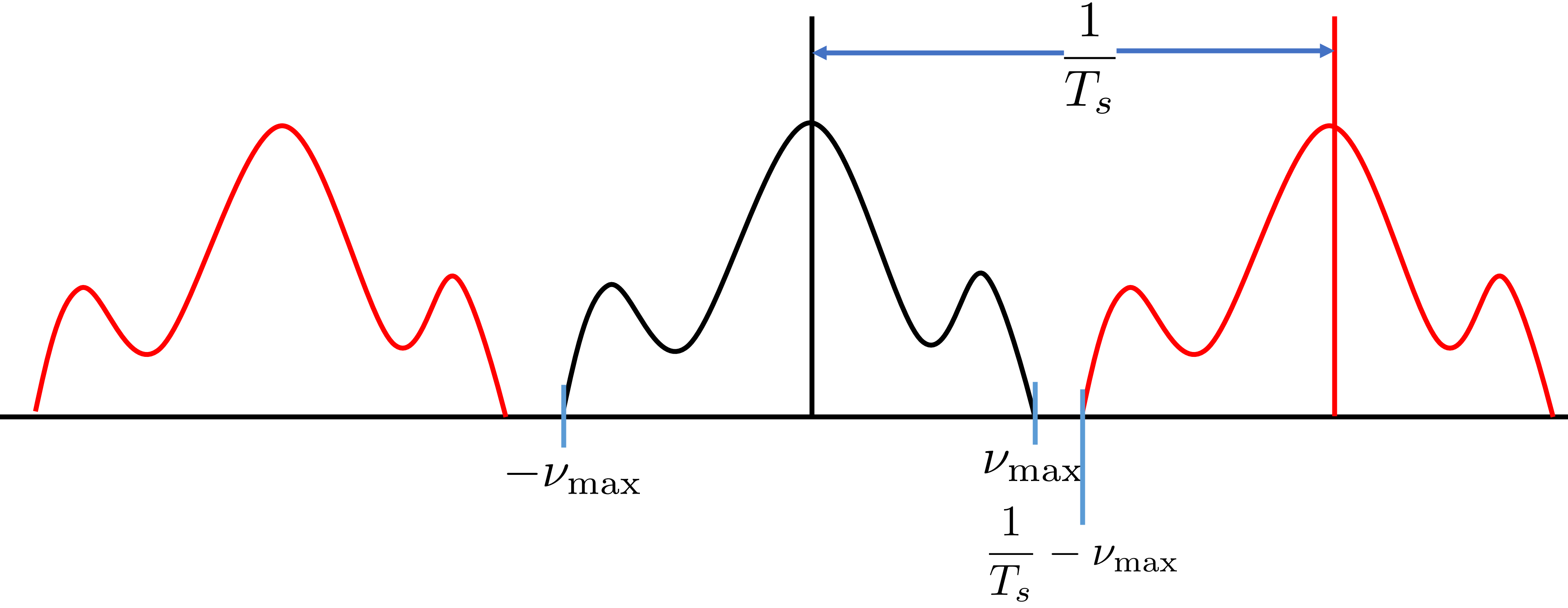
Sampling = Keep values at $t = kT_s$, make everything else 0

Frequency domain



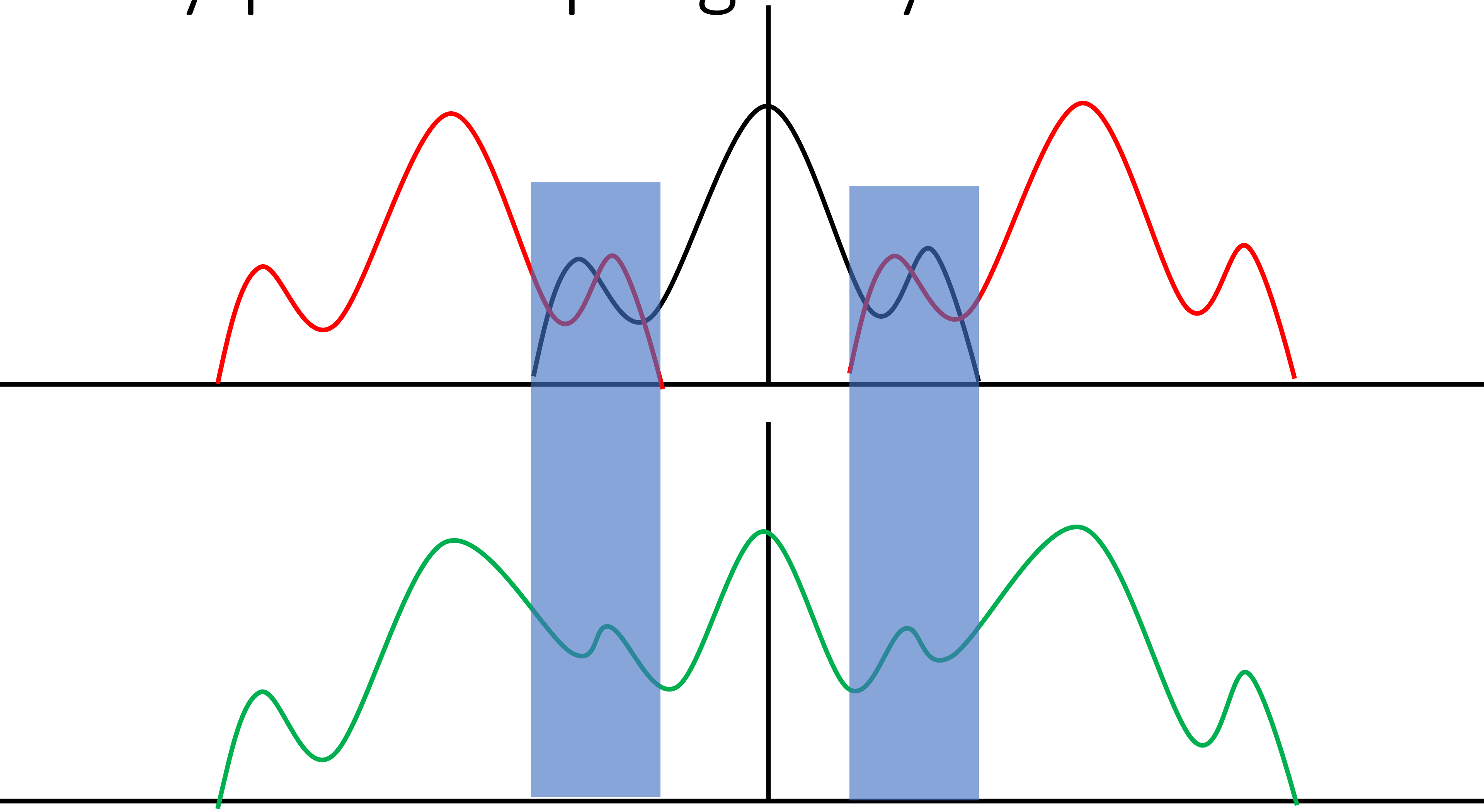
Sampling = Make frequency domain periodic with period $\nu = 1/T_s$ by making copies

Nyquist sampling: why?



$$\frac{1}{T_s} - \nu_{\max} > \nu_{\max} \Rightarrow \frac{1}{T_s} > 2\nu_{\max}$$

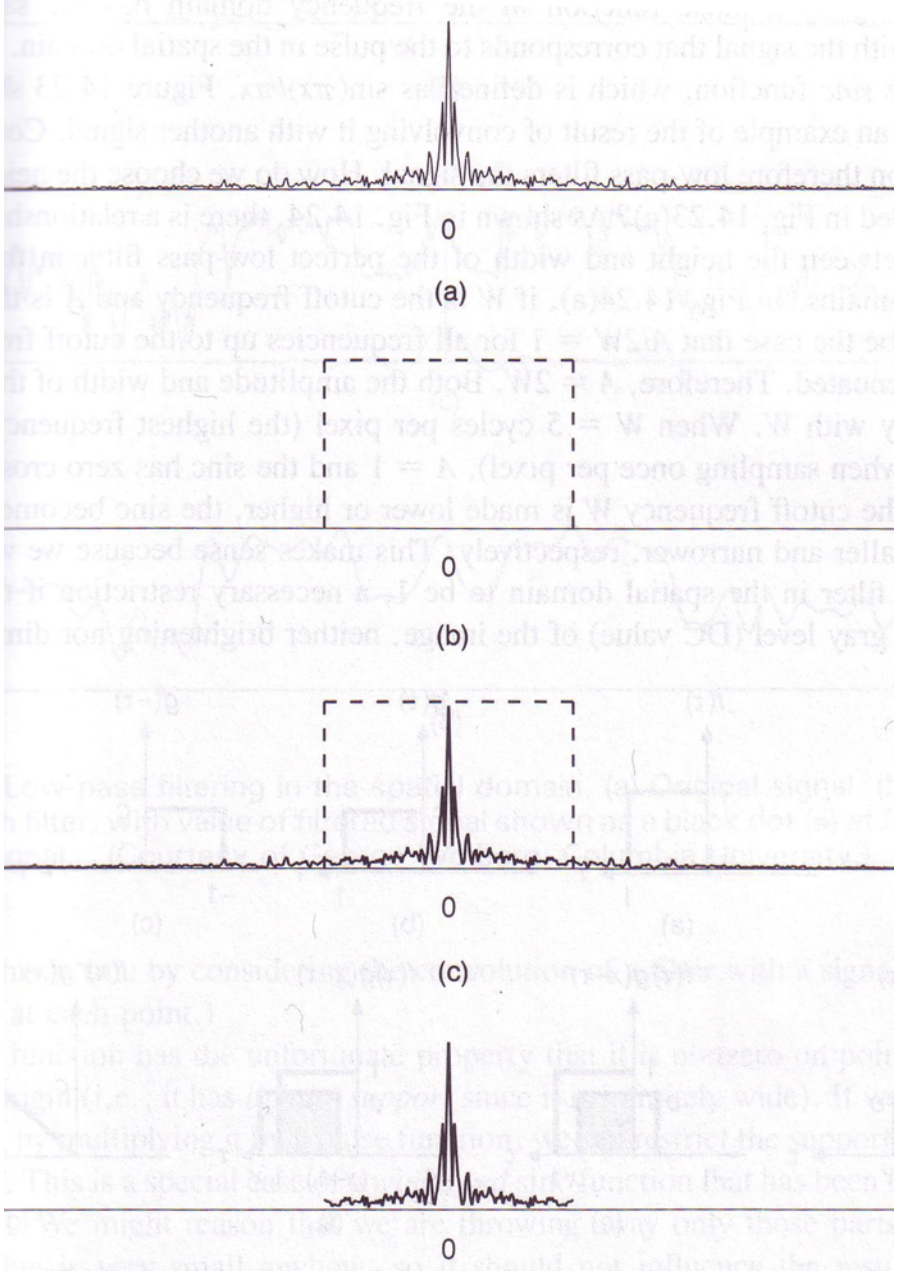
Nyquist sampling: why?



Aliasing and downsampling

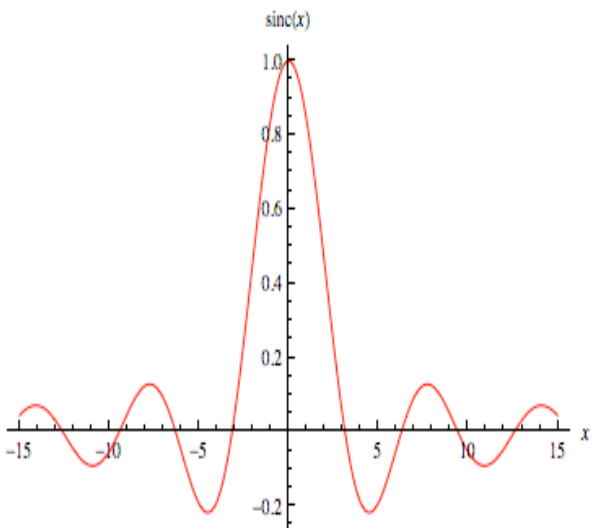
- Nyquist says must sample at at least twice maximum frequency
- When downsampling by a factor of two
  - Original image has frequencies that are too high
- How can we fix this?
- Eliminate them before sampling!
  - Convert to frequency space
  - Multiply with low-pass filter

Eliminating High Frequencies



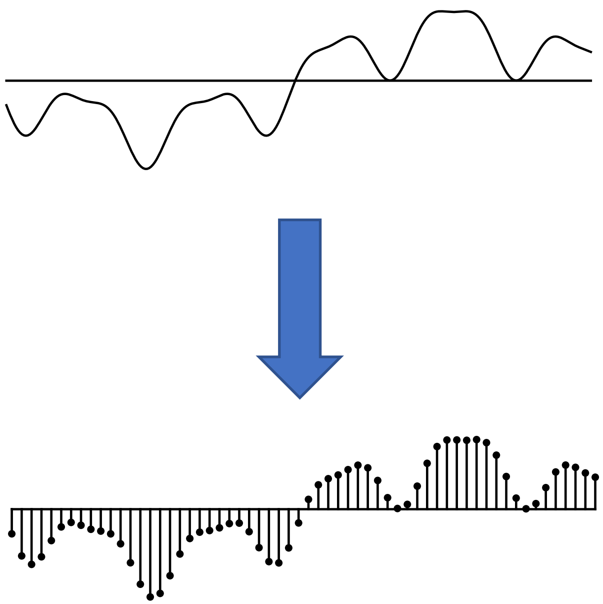
Process

- Can we do this in spatial domain?
  - Yes!
- Multiplication in frequency domain = convolution in spatial domain
- Box filter in frequency domain = sinc in spatial domain
- Multiplication with box filter in frequency domain = convolution with sinc filter in spatial domain



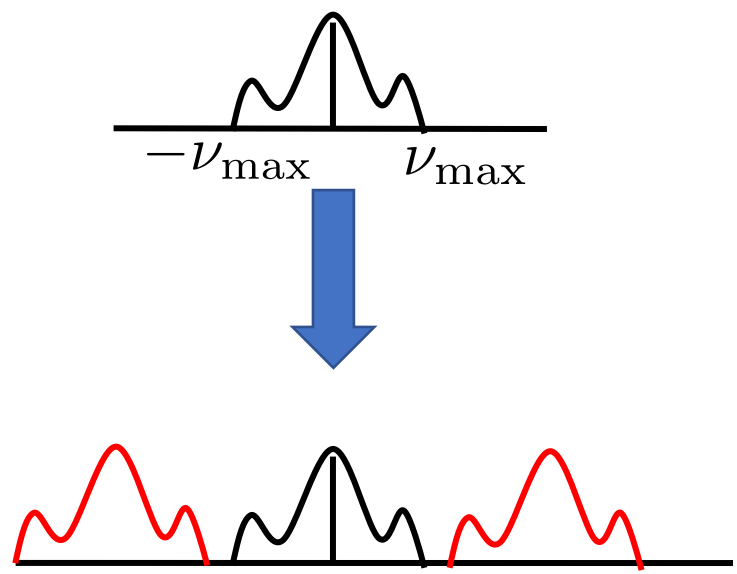
Reconstruction from samples

Spatial domain



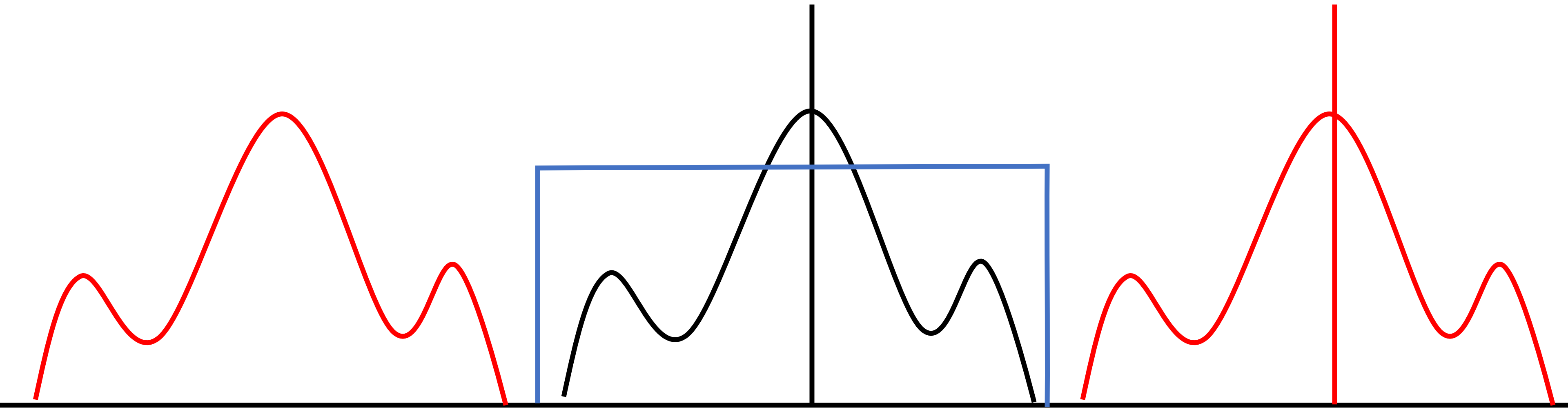
Sampling = Keep values at $t = kT_s$, make everything else 0

Frequency domain



Sampling = Make frequency domain periodic with period $\nu = 1/T_s$ by making copies

Reconstruction from samples



$$F_{recons}(\nu) = F_{sampled}(\nu)B(\nu)$$

Box filter in frequency space

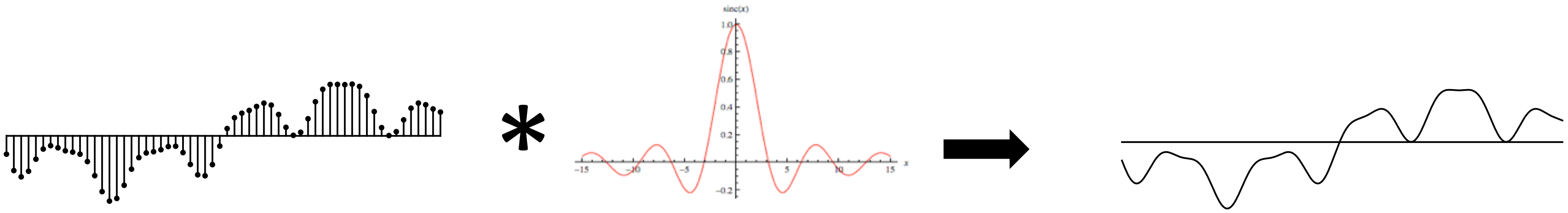
Reconstruction from samples

$$F_{recons}(\nu) = F_{sampled}(\nu)B(\nu)$$

- Multiplication in frequency domain = convolution in spatial domain
- Box filter in frequency domain = sinc filter in spatial domain
- Convolve sampled signal with sinc filter to reconstruct

Reconstruction from samples

- ”Sampled signal” is non-zero at sample points and 0 everywhere else
  - i.e., has holes



Recap: subsampling and reconstruction

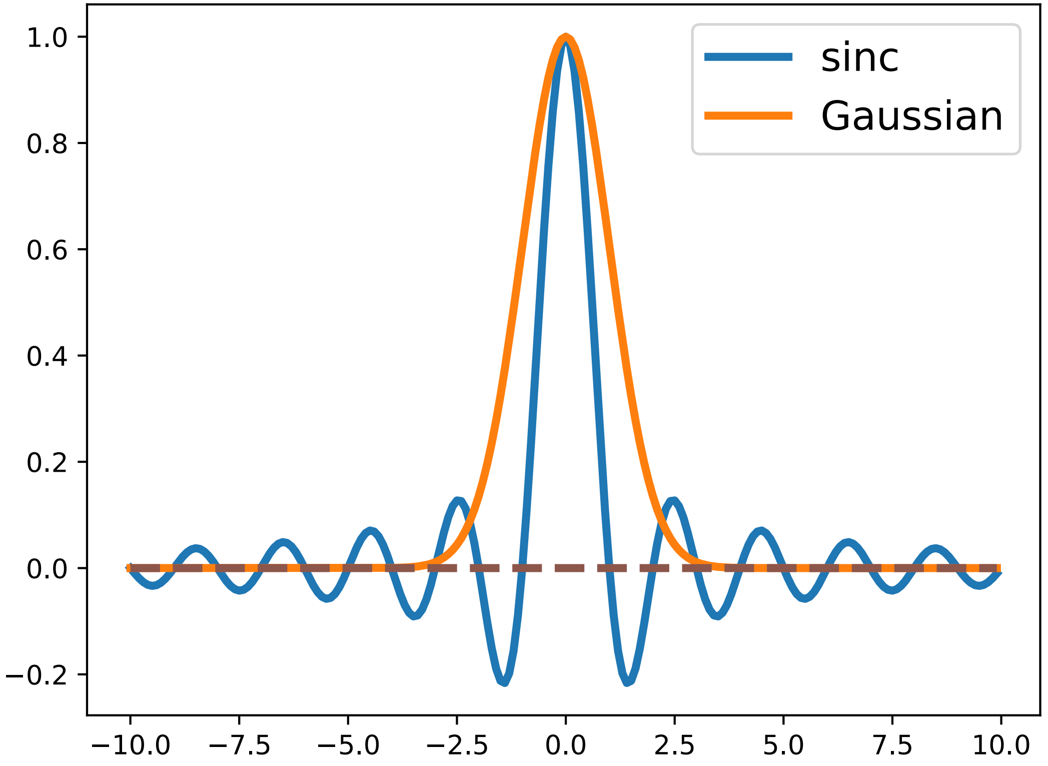
Subsampling

1. Convolve with sinc filter to eliminate high frequencies
2. Sample by picking only values at sample points

Reconstruction

1. Start with sampled signal (0 at non-sample points)
2. Convolve with sinc to reconstruct

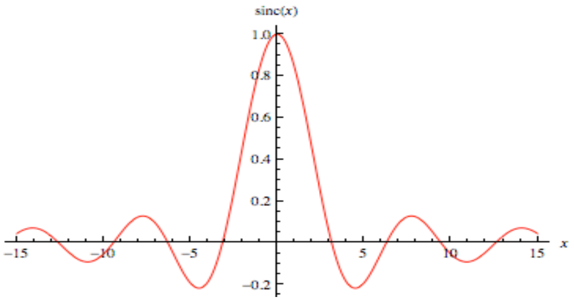
Sinc is annoying

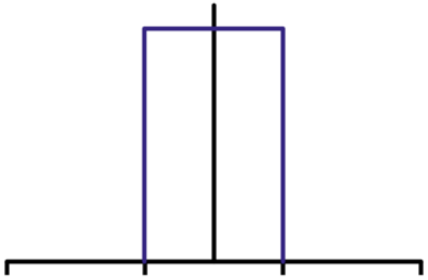


Sinc and Gaussian

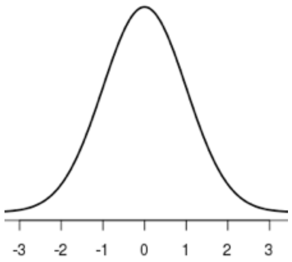
- Sinc is annoying: infinite spatial extent
- Use Gaussian instead!

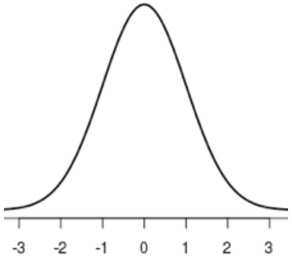
Sinc/box





Gaussian





Spatial domain

Frequency domain

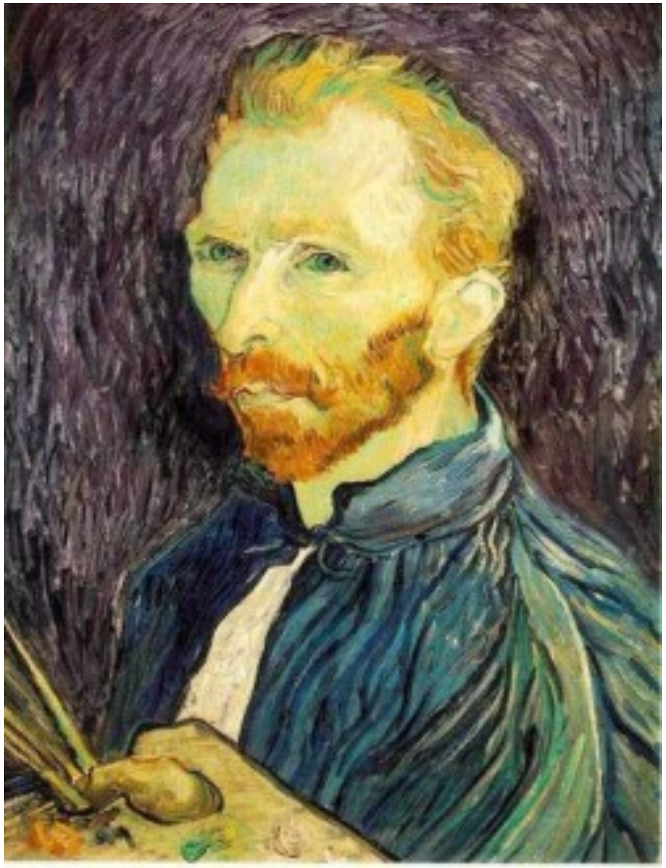
Subsampling images

- Step 1: Convolve with Gaussian to eliminate high frequencies
- Step 2: Drop unneeded pixels

Subsampling without removing high frequencies

Subsampling after removing high frequencies

Subsampling images correctly



Gaussian 1/2

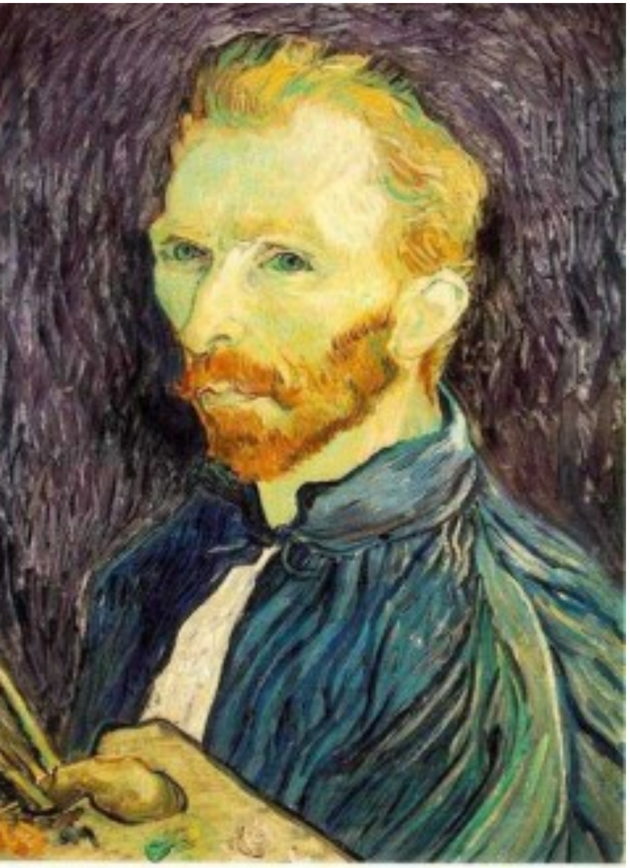
G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Source: S. Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

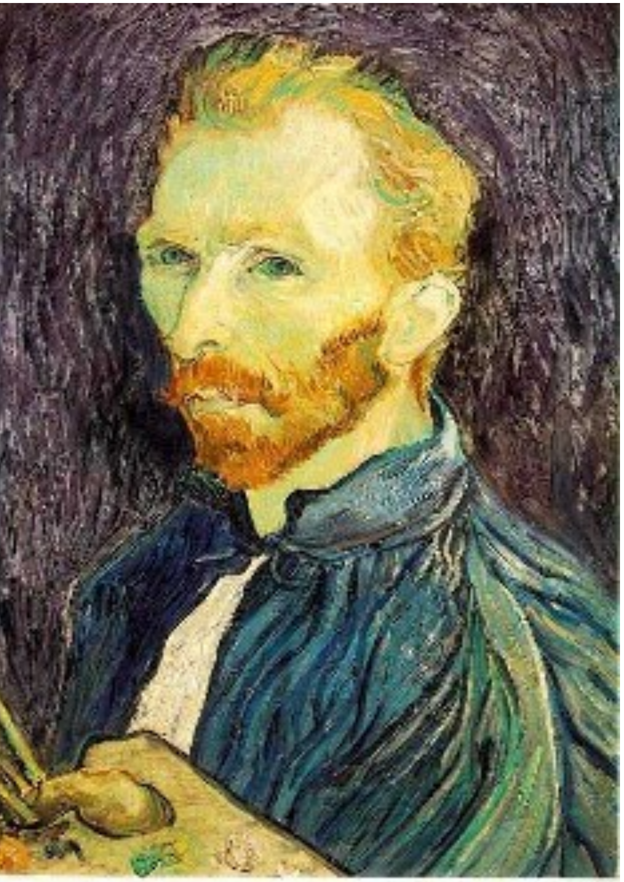
G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Source: S. Seitz

Compare with...



1/2

1/4 (2x zoom)

1/8 (4x zoom)

Source: S. Seitz

Upsampling images

Step 1: blow up to original size with 0's in between

Upsampling images

Step 2: Convolve with Gaussian

Take-away

- Subsampling causes aliasing
  - High frequencies masquerading as low frequencies
- Remove low frequencies by blurring!
  - Ideal: sinc
  - Common: Gaussian
- When upsampling, reconstruct missing values by convolution
  - Ideal: sinc
  - Common: Gaussian

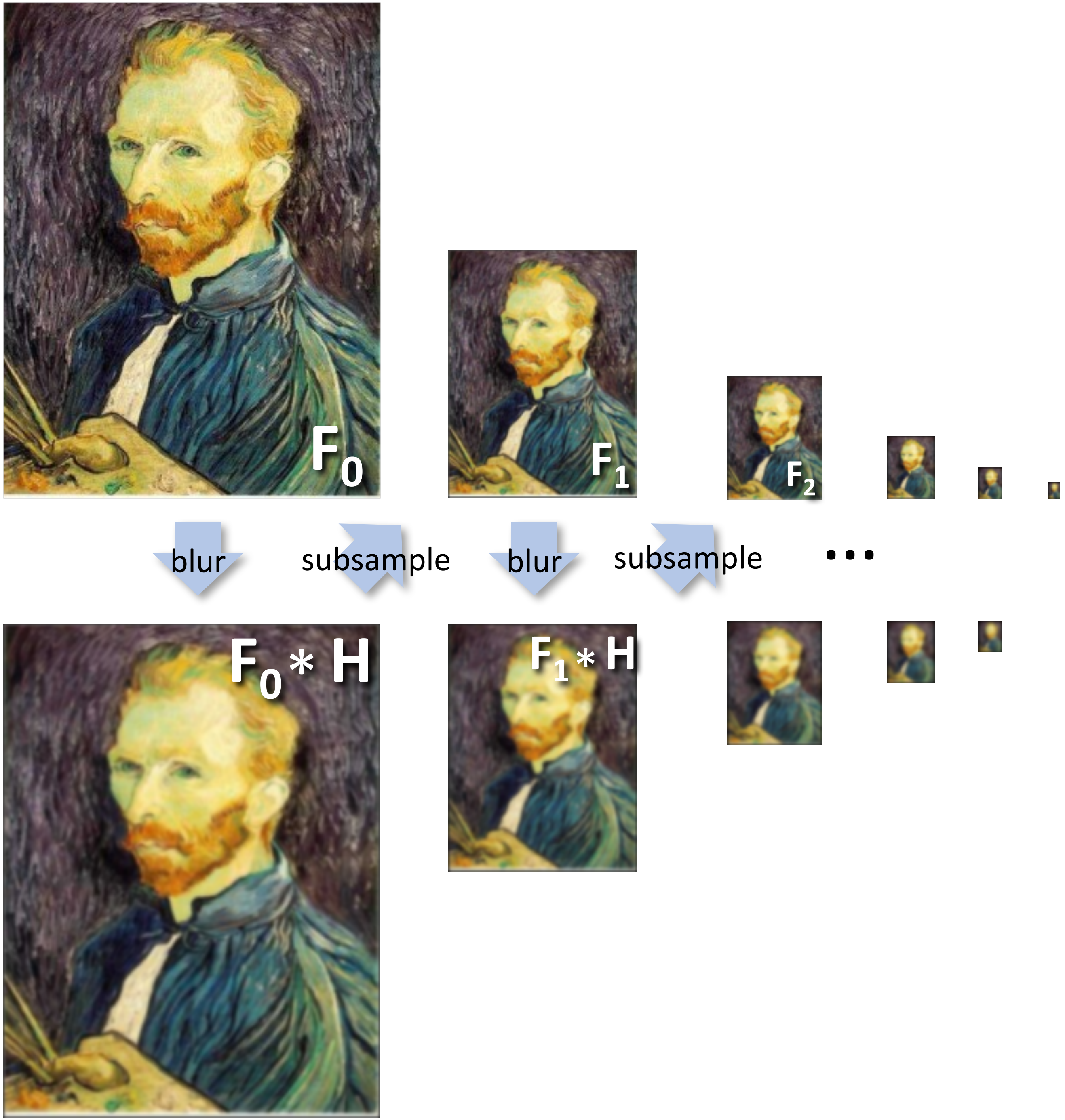
So… can we enhance?

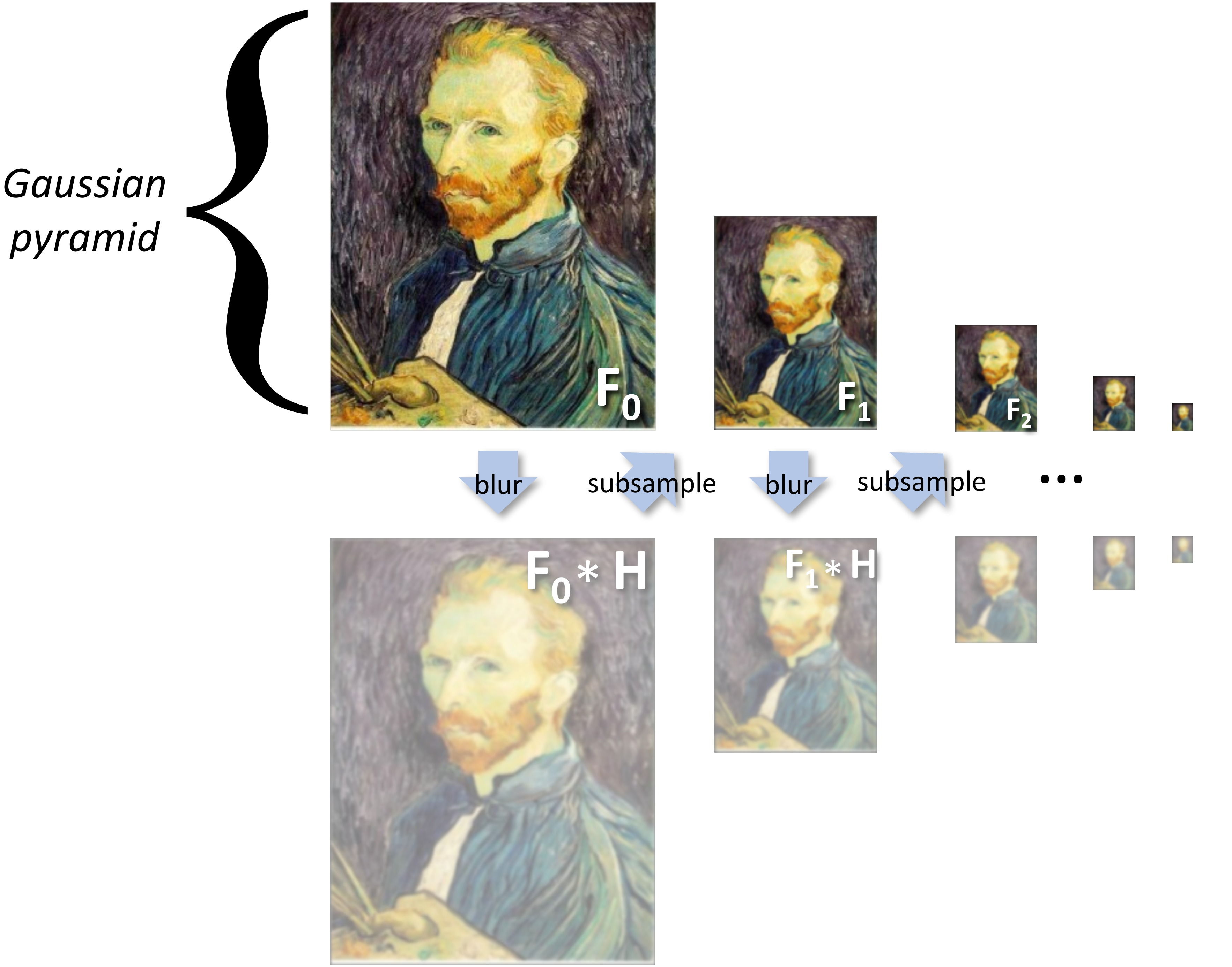
- Nyquist theorem limits frequencies we can reconstruct from subsampled image
- Can only reconstruct max sampling frequency/2
- Sorry CSI!

Pyramids

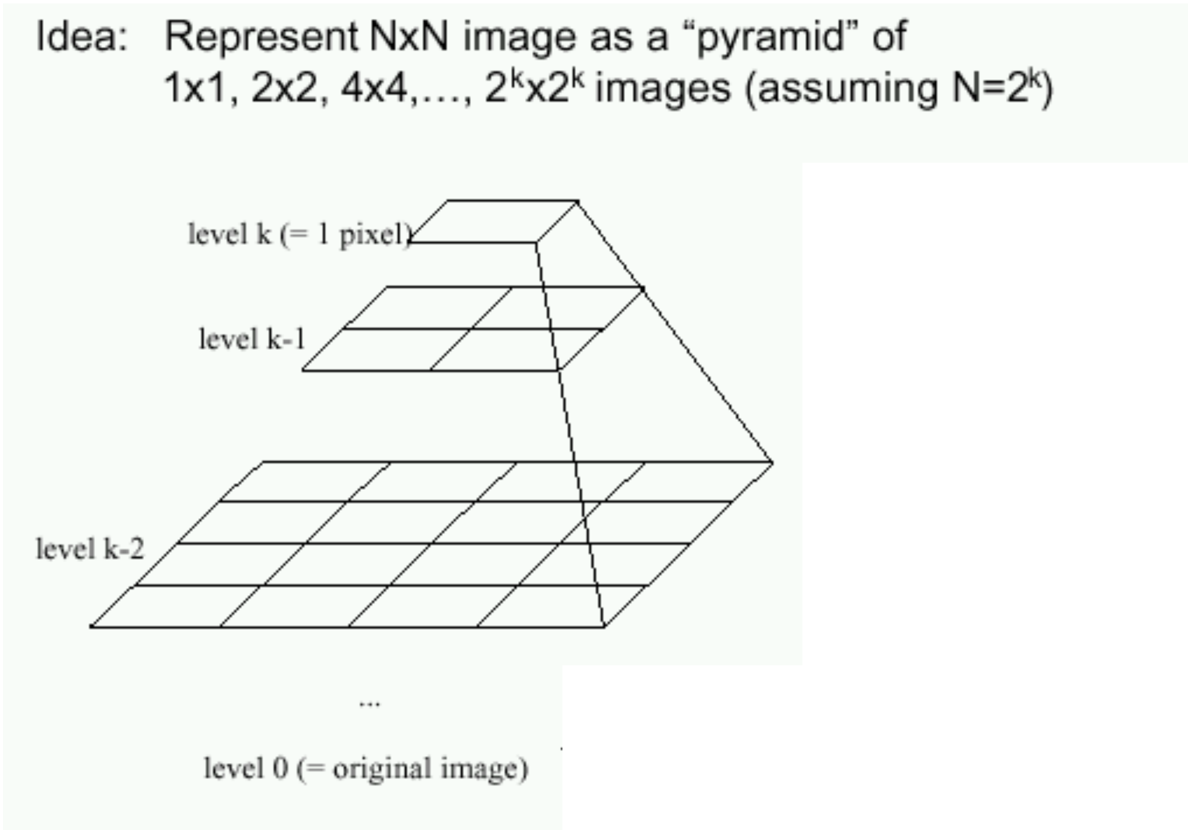
Gaussian pre-filtering

- Solution: filter the image, *then* subsample




Gaussian pyramid
F0
F1
F2
blur
subsample
blur
subsample
...
F0 * H
F1 * H

Gaussian pyramids [Burt and Adelson, 1983]

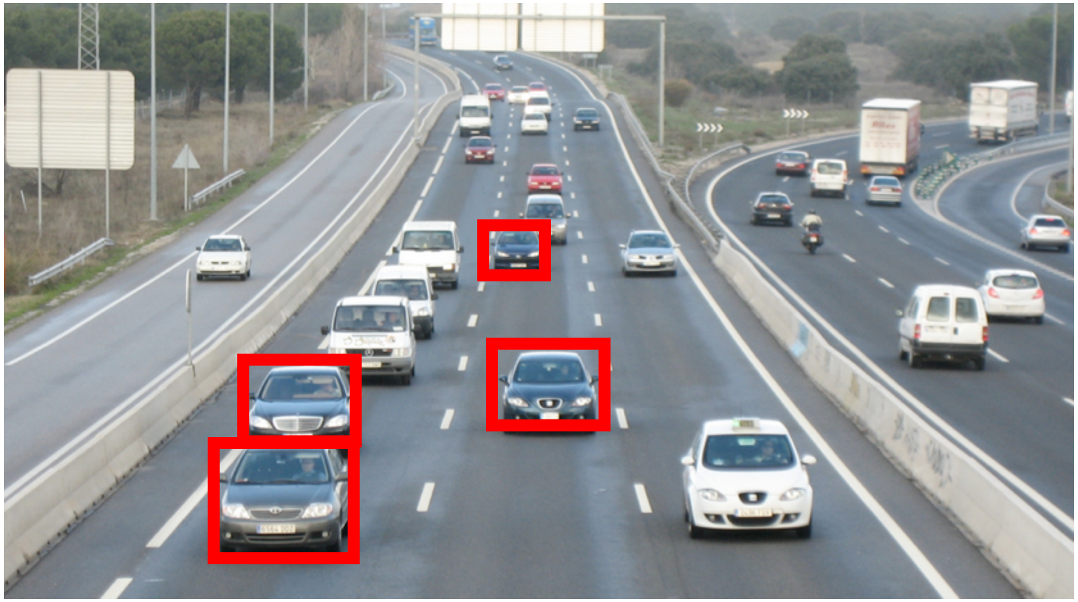


- In computer graphics, a *mip map* [Williams, 1983]

Gaussian Pyramids have all sorts of applications in computer vision

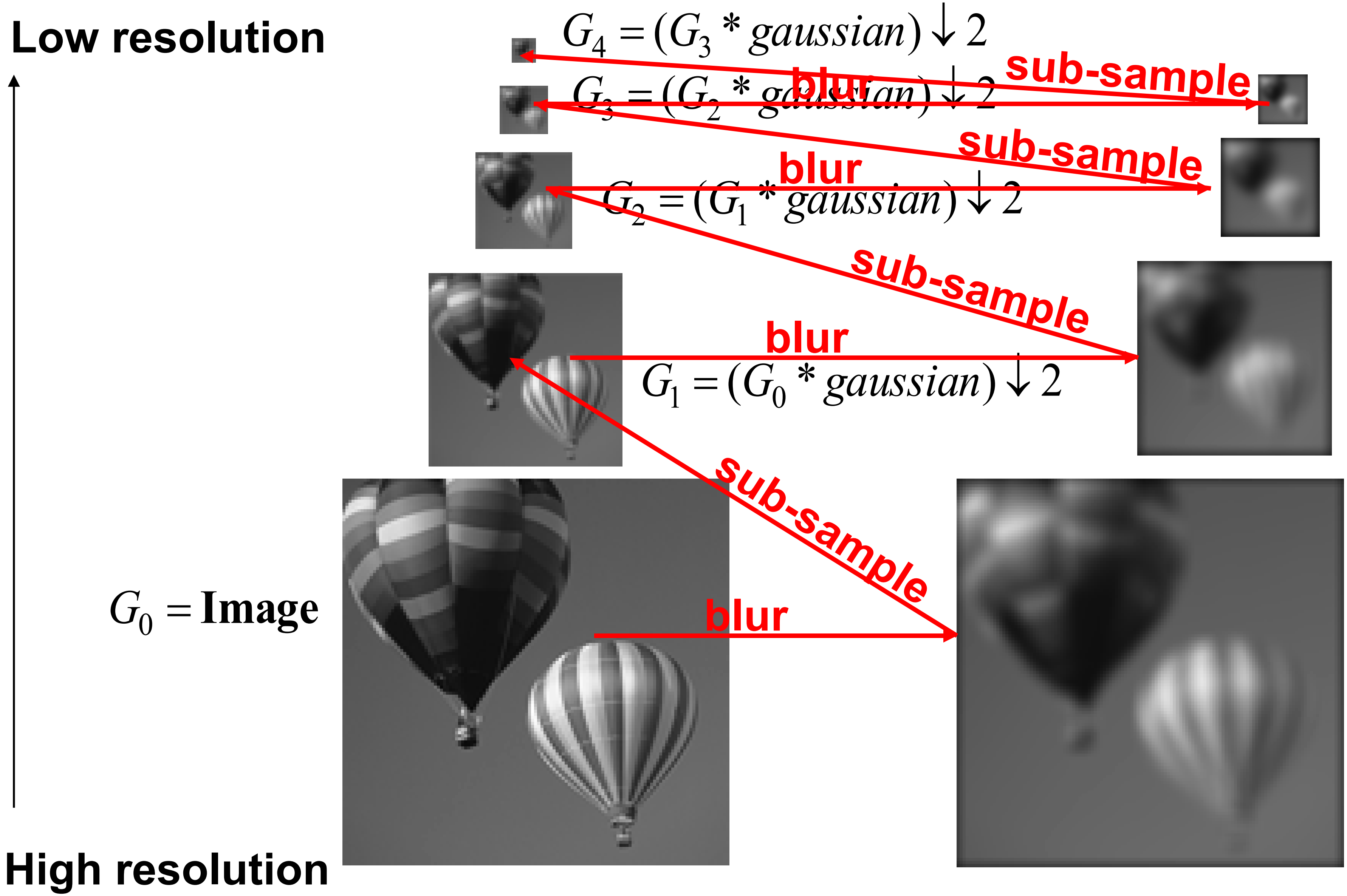
Source: S. Seitz

Gaussian pyramids - Searching over scales



Gaussian pyramids - Searching over scales

The Gaussian Pyramid



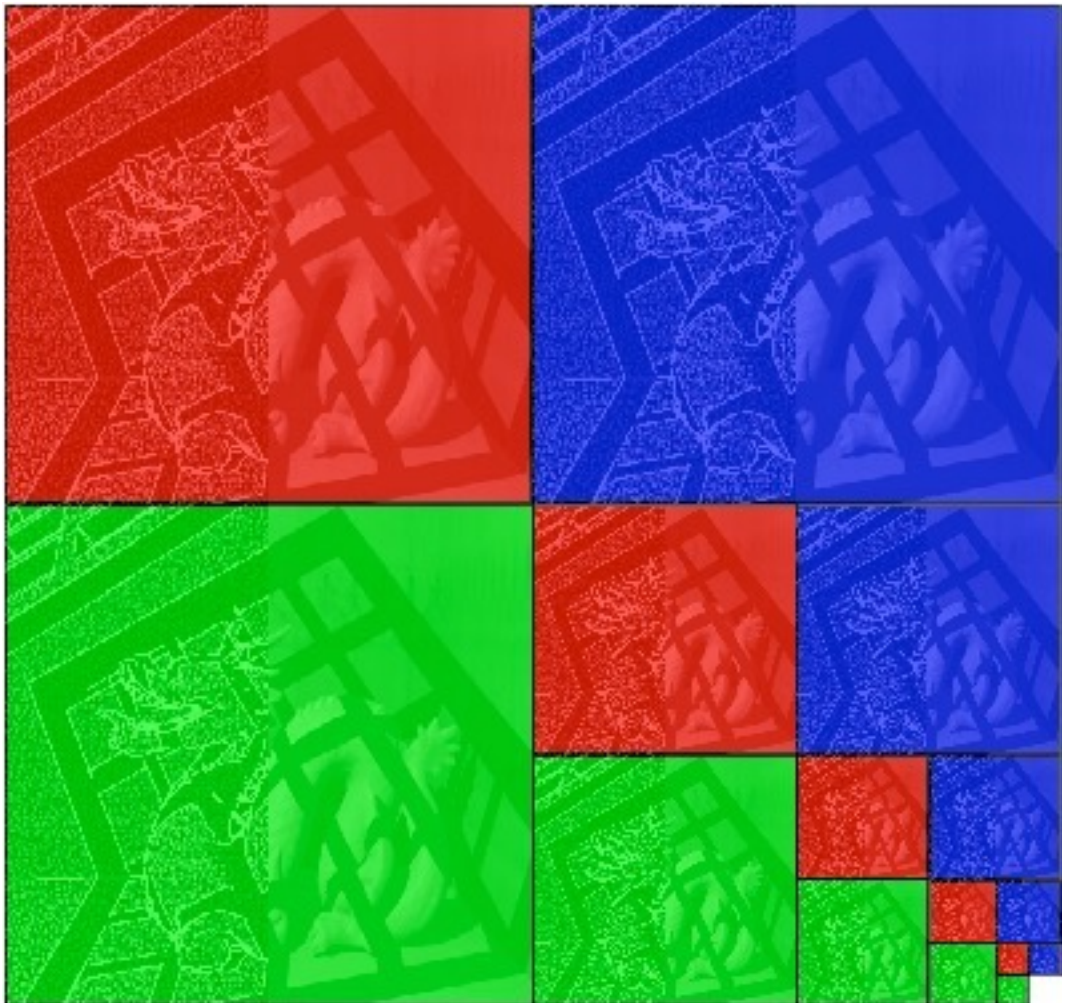
Gaussian pyramid and stack

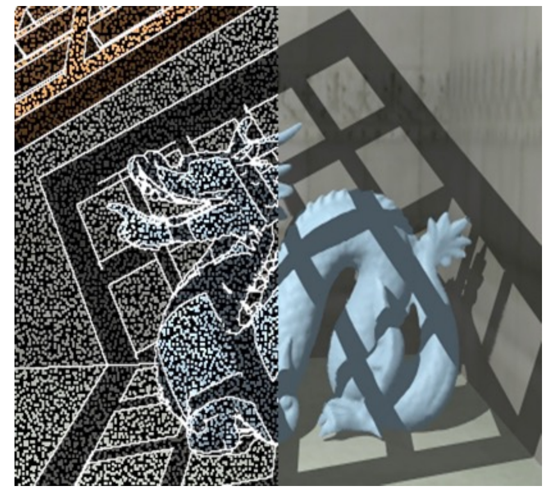
512 256 128 64 32 16 8

Source: Forsyth

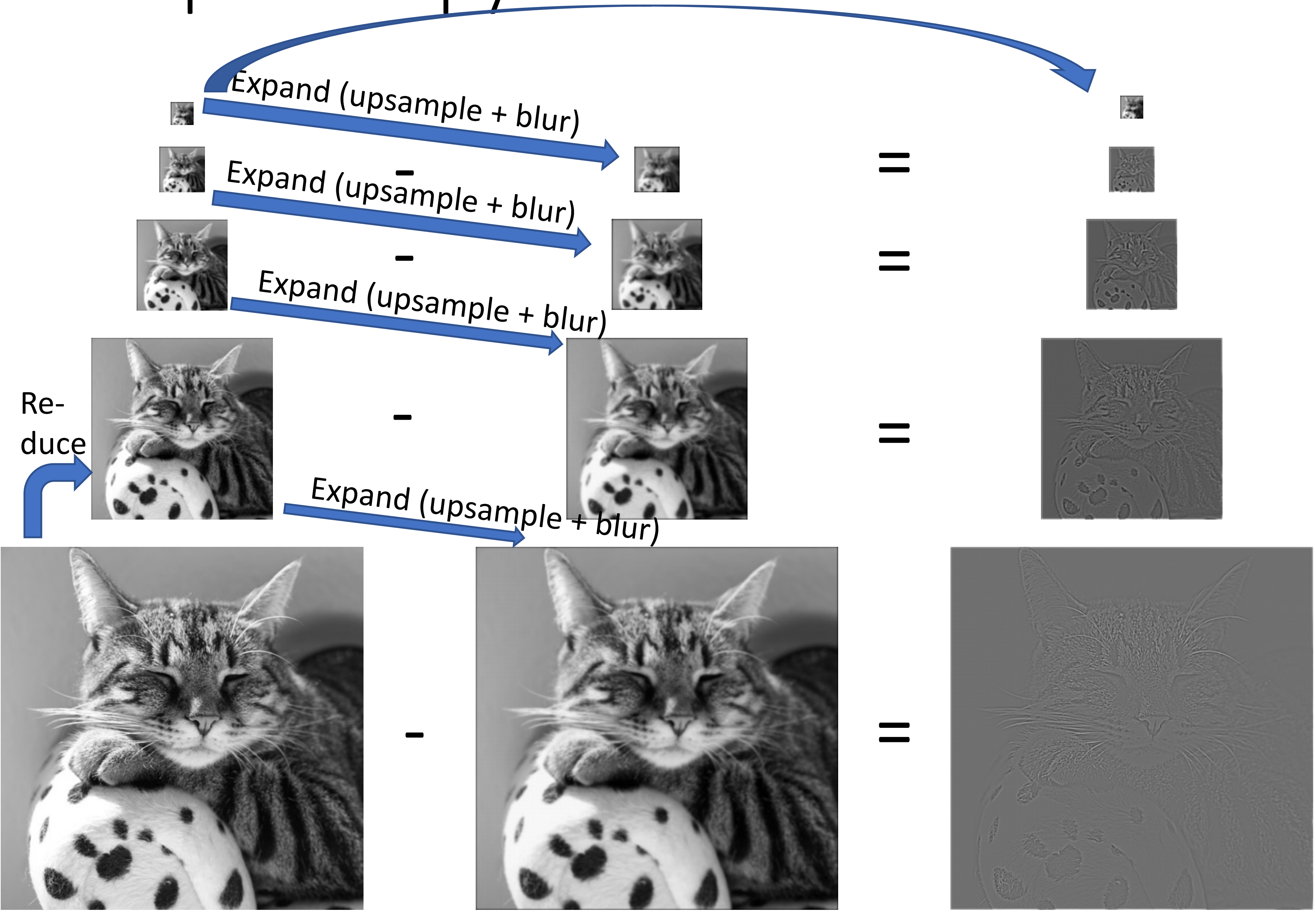
Memory Usage

- What is the size of the pyramid?





Laplacian pyramid



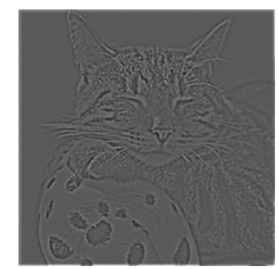
Laplacian pyramid

$L_4 = G_4 =$

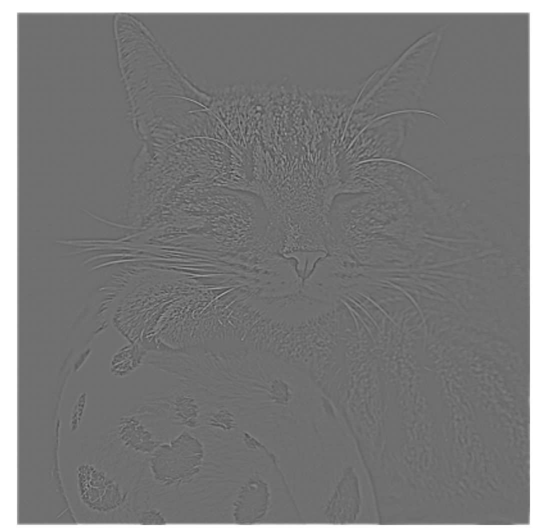
$L_3 = G_3$ - expand$(G_4) =$

$L_2 = G_2$ - expand$(G_3) =$

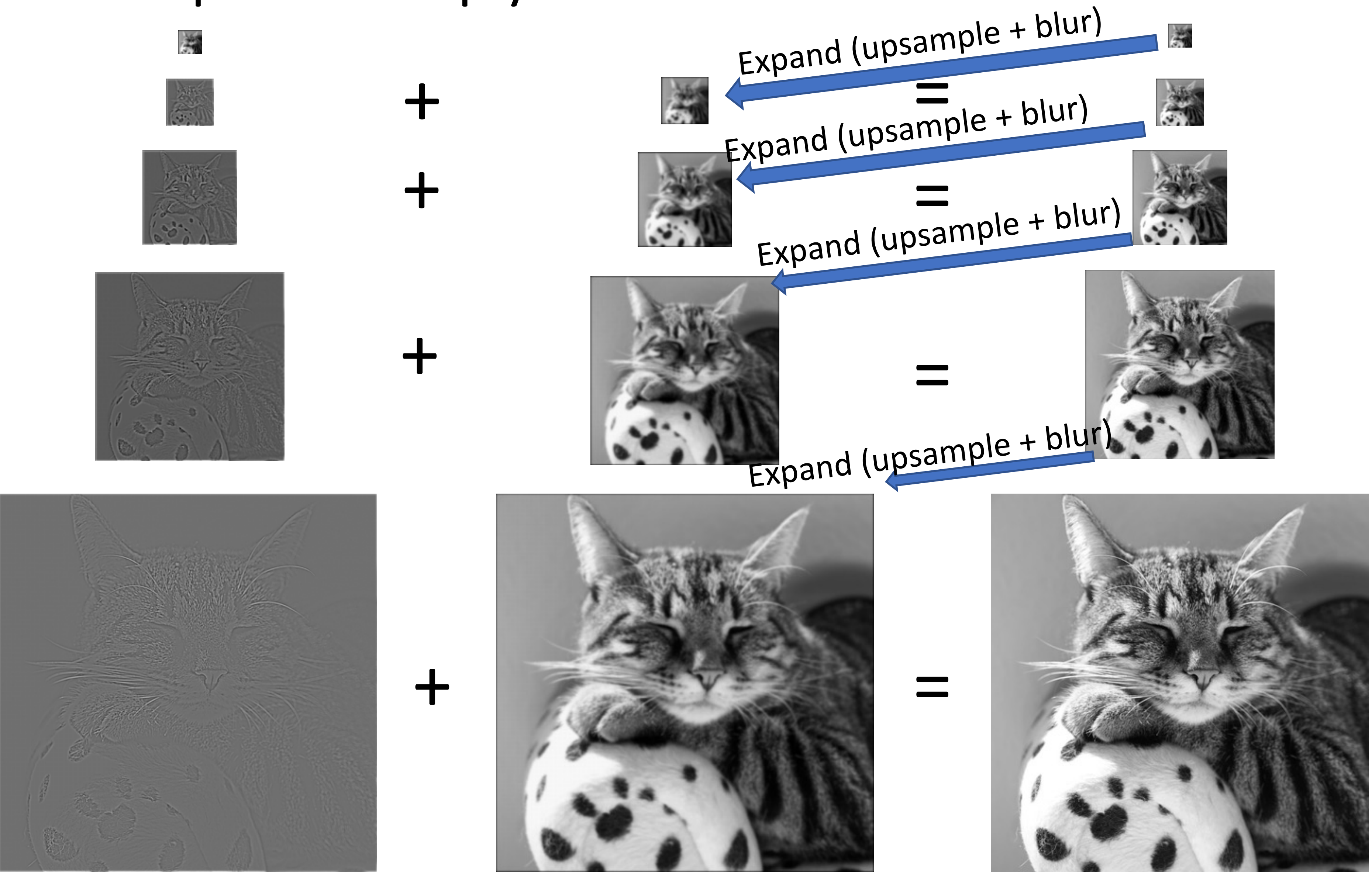
$L_1 = G_1$ - expand$(G_2) =$



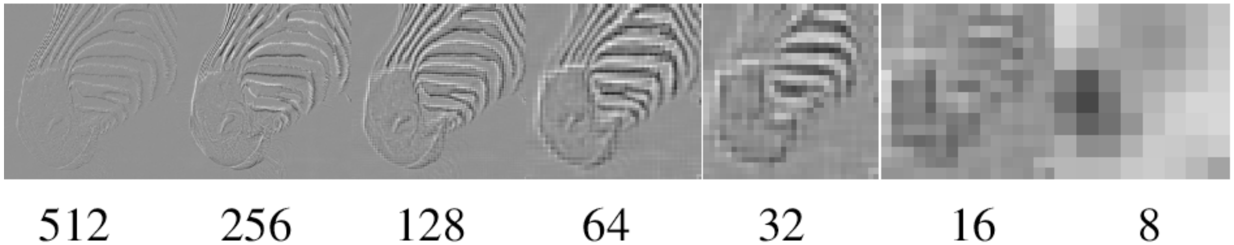
$L_0 = G_0$ - expand$(G_1) =$

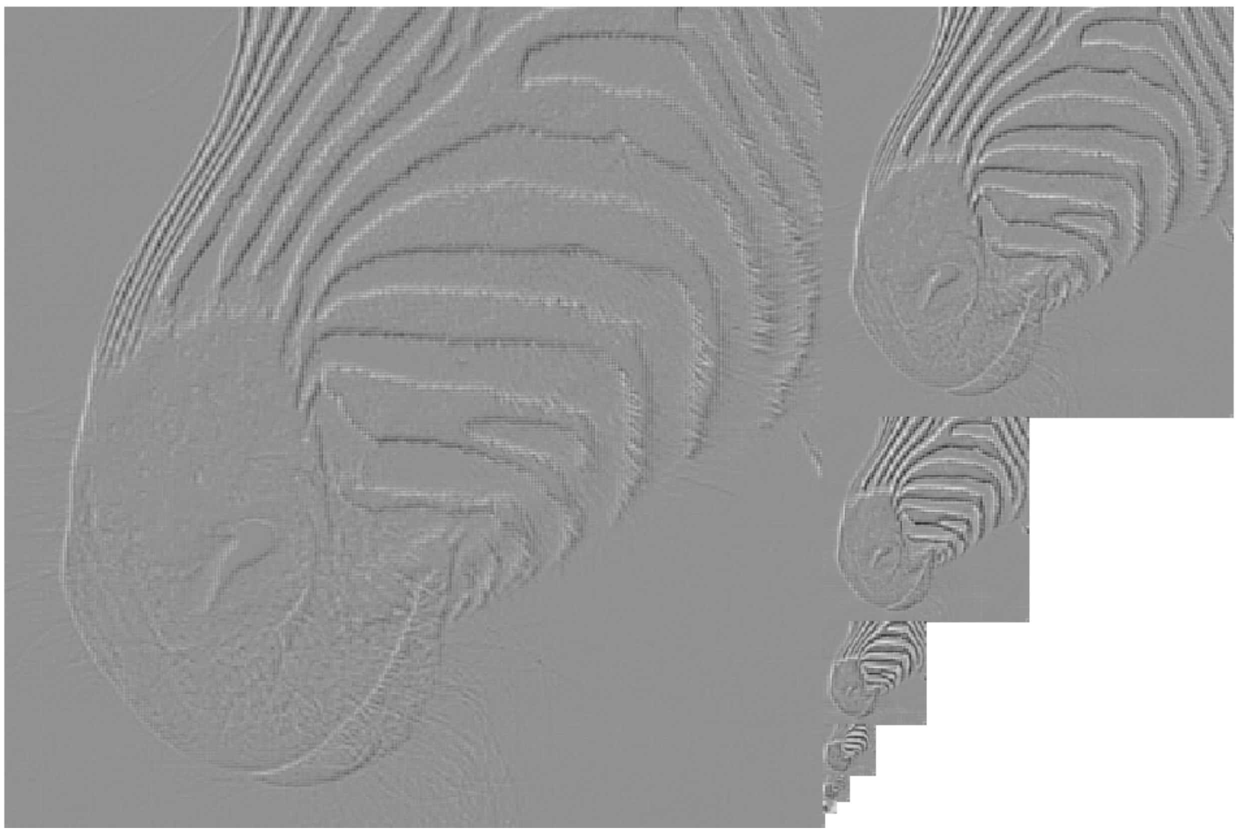


Reconstructing the image from a Laplacian pyramid



Laplacian pyramid





Source: Forsyth