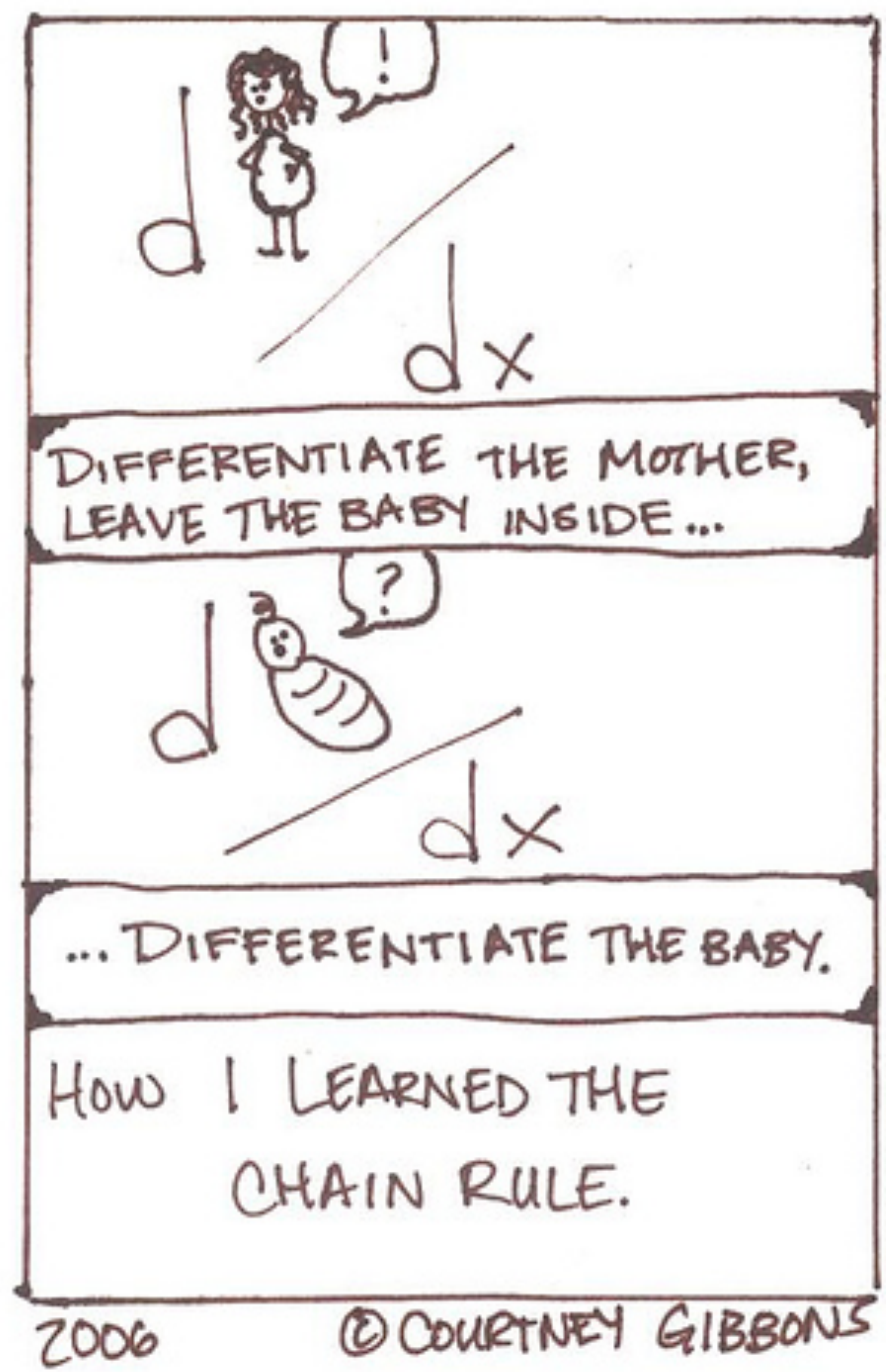
Lecture 36: Backprop and ConvNets

CS 4670
Sean Bell



http://brownsharpie.courtneygibbons.org/?p=90

Helping the Blind

(posted yesterday)

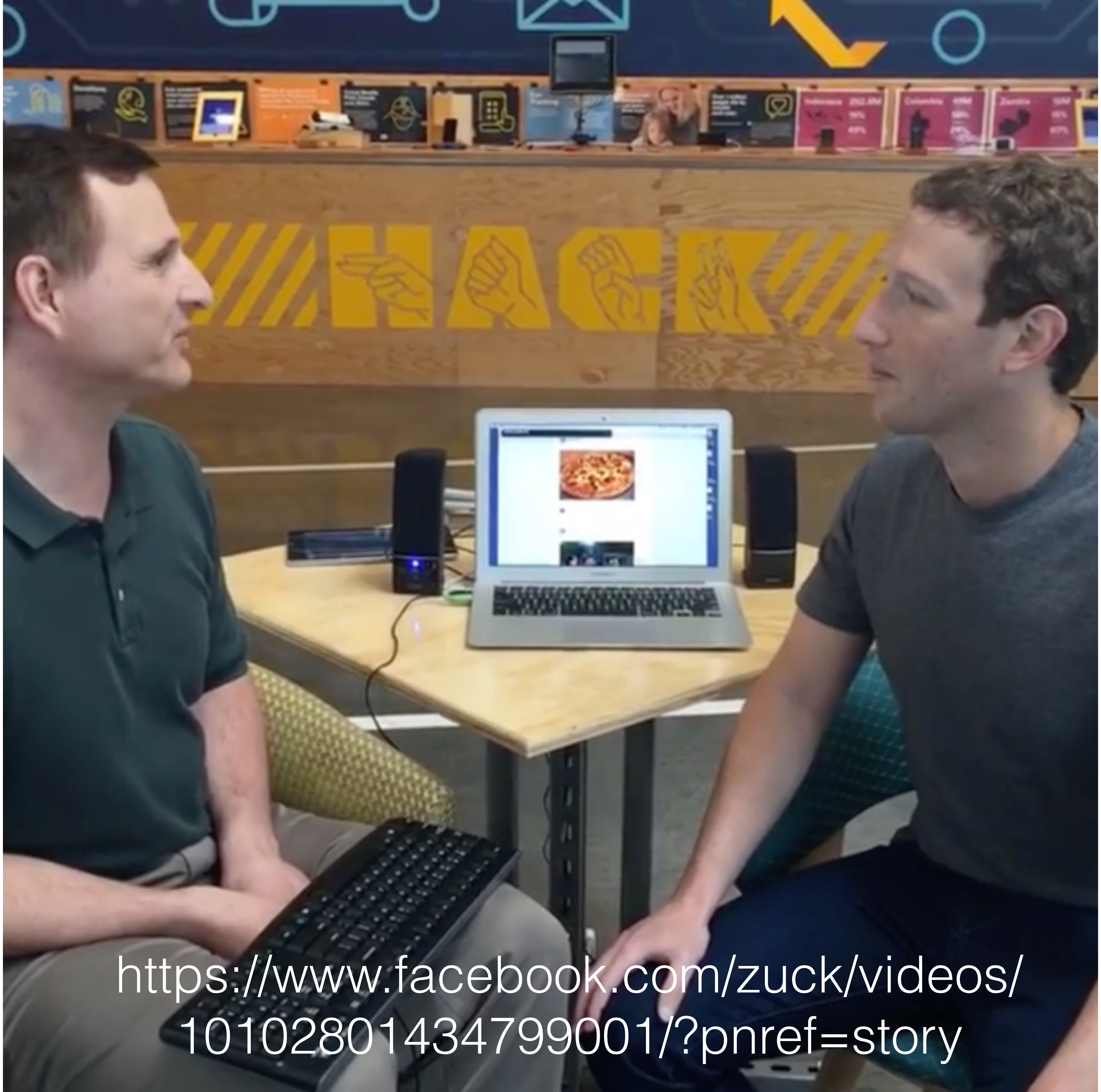
https://www.facebook.com/zuck/videos/10102801434799001/?pnref=story

Helping the Blind

(posted yesterday)

"sunday night splurge"

https://www.facebook.com/zuck/videos/10102801434799001/?pnref=story



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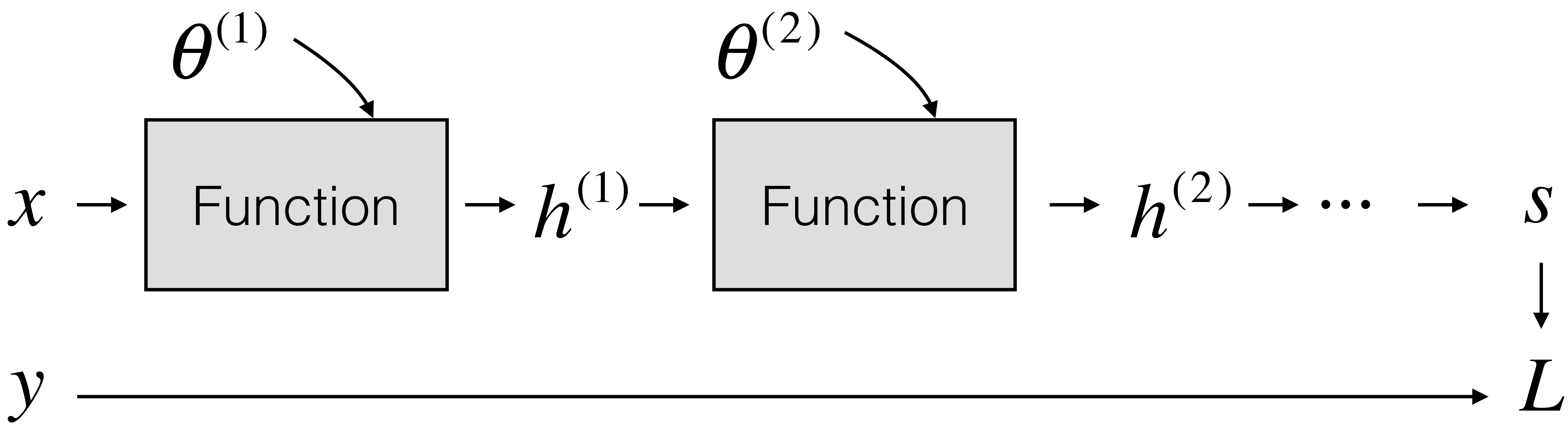
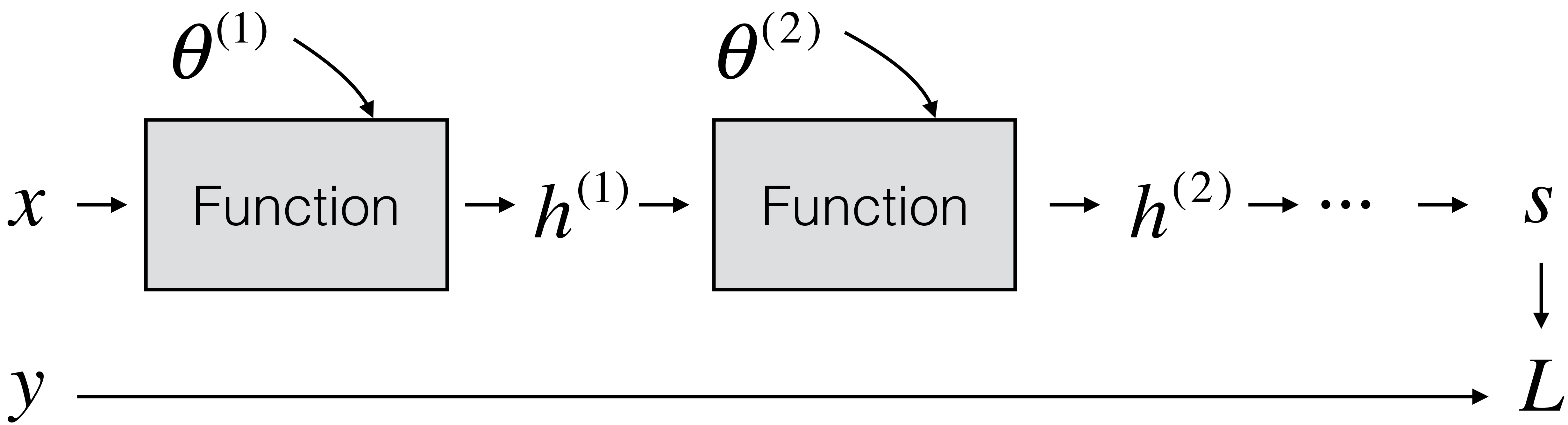
Review: Setup

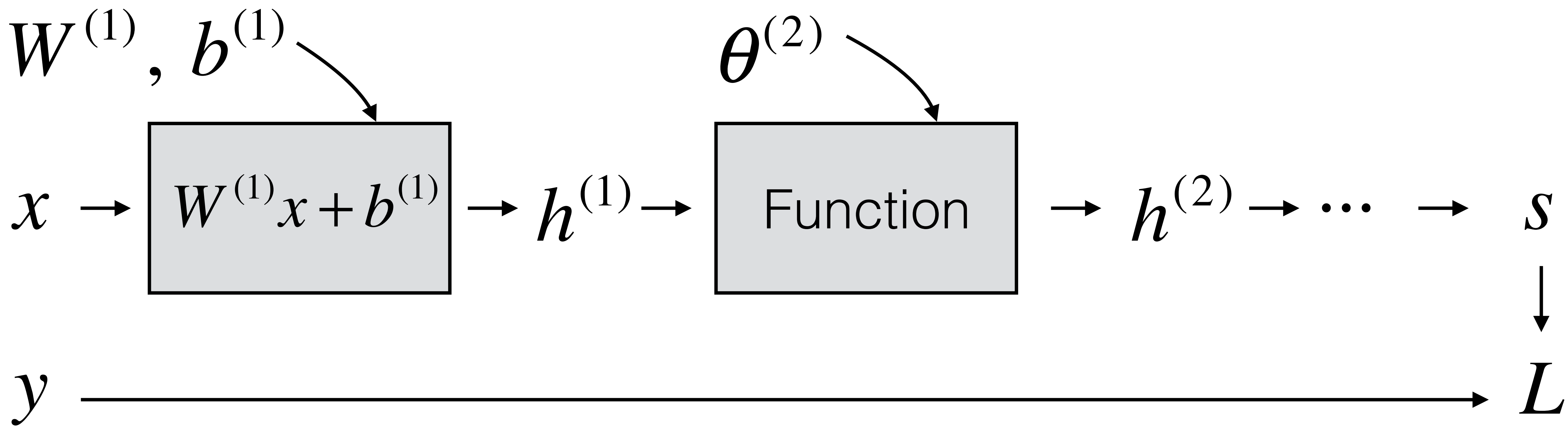


Review: Setup



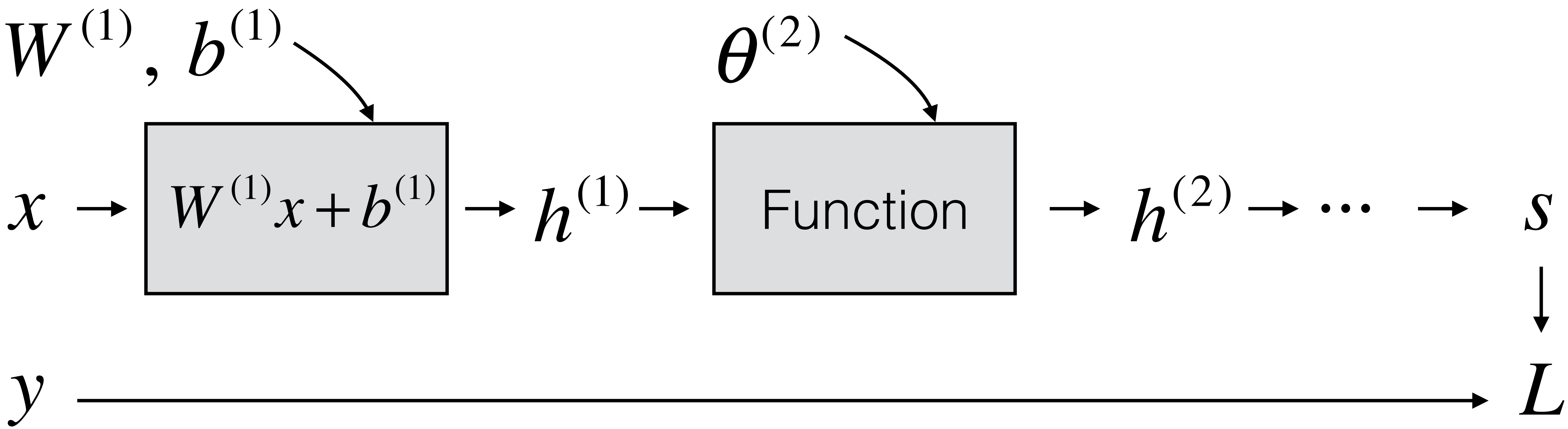
- **Goal:** Find a value for parameters ($\theta^{(1)}$, $\theta^{(2)}$, …), so that the loss (L) is small

Review: Setup

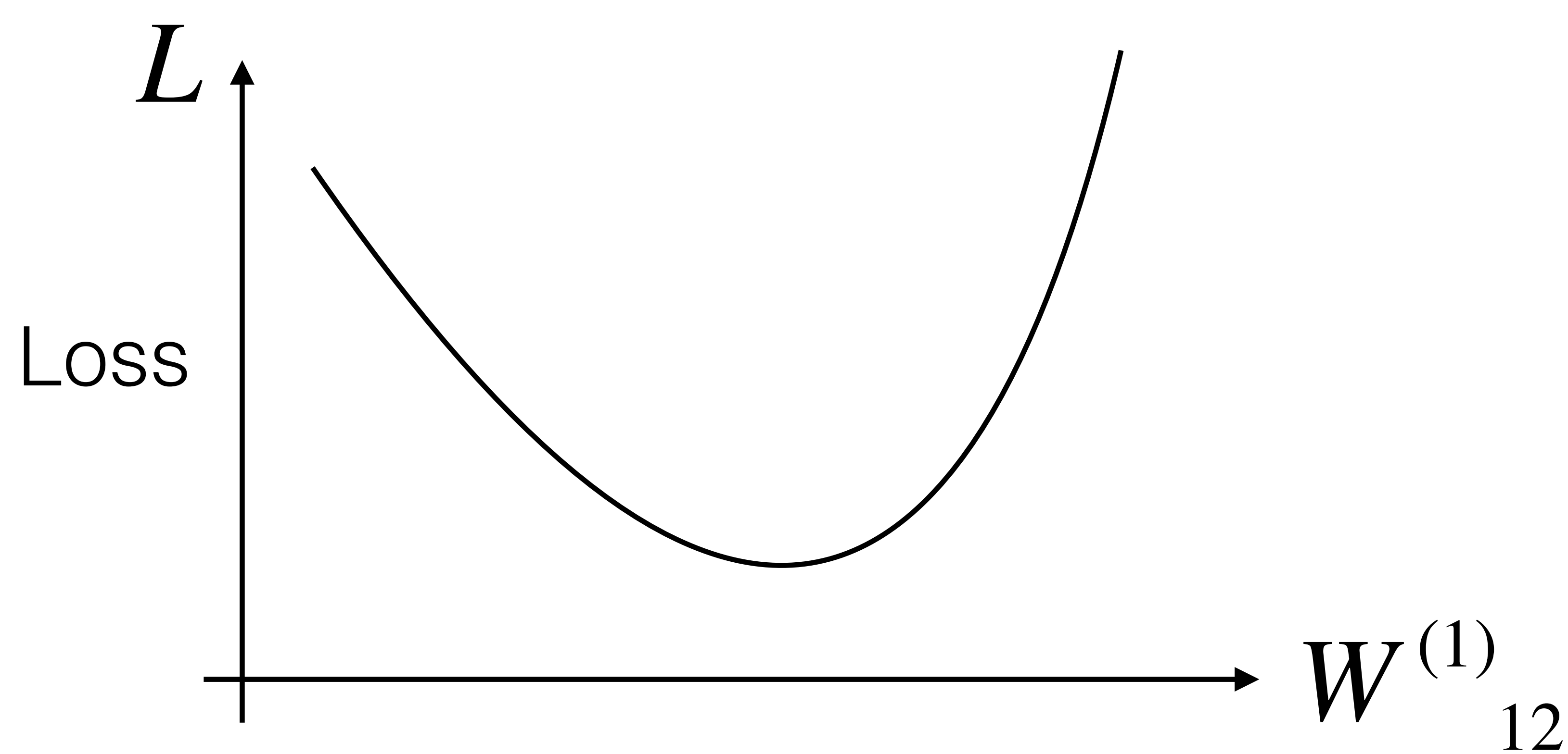


Toy Example:

Review: Setup

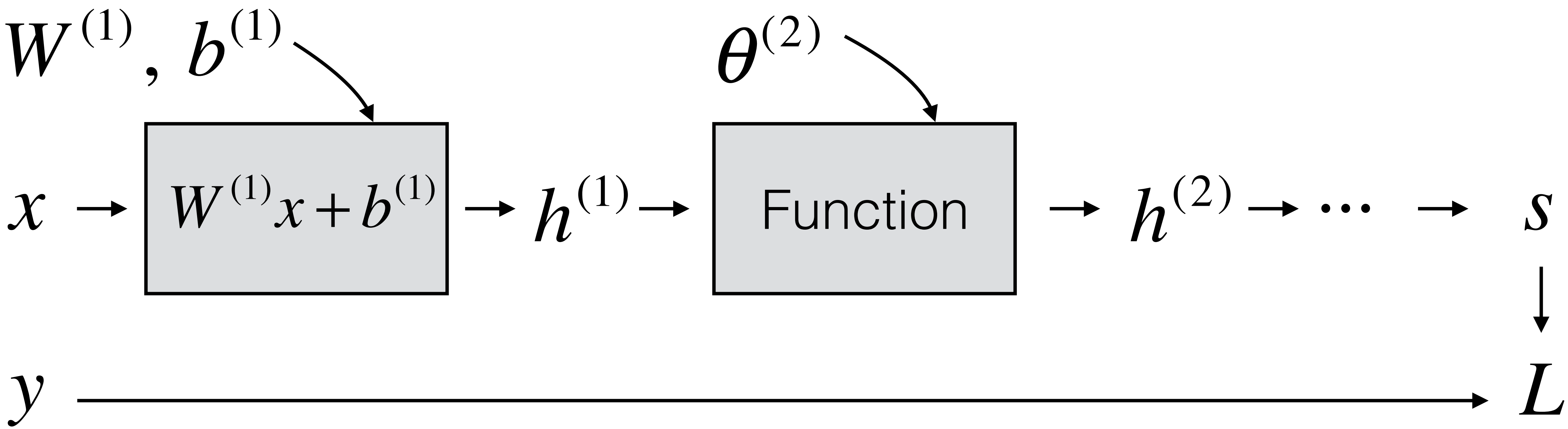


Toy Example:

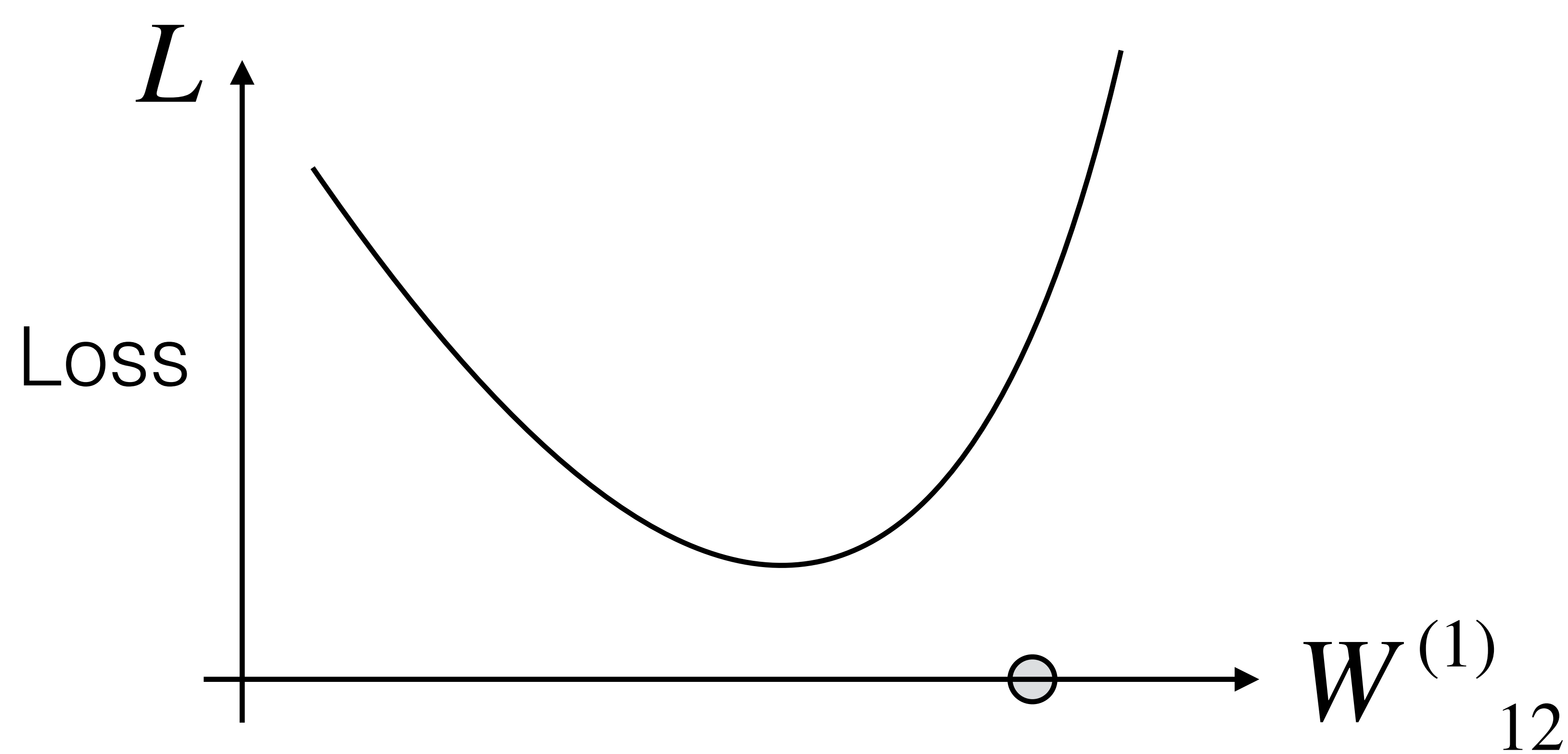


A weight somewhere in the network

Review: Setup



Toy Example:



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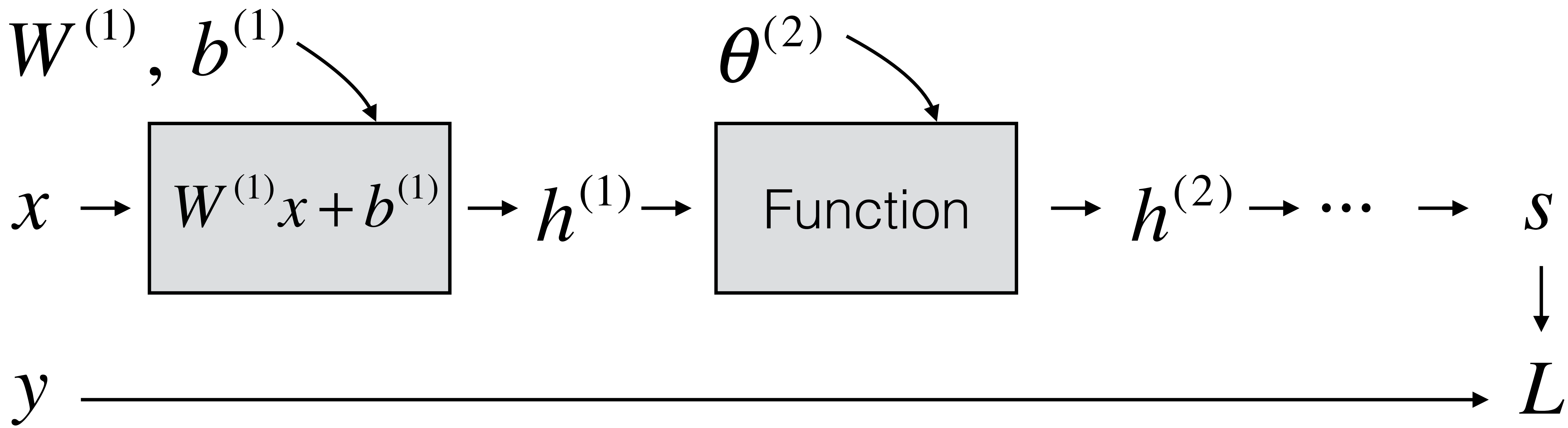
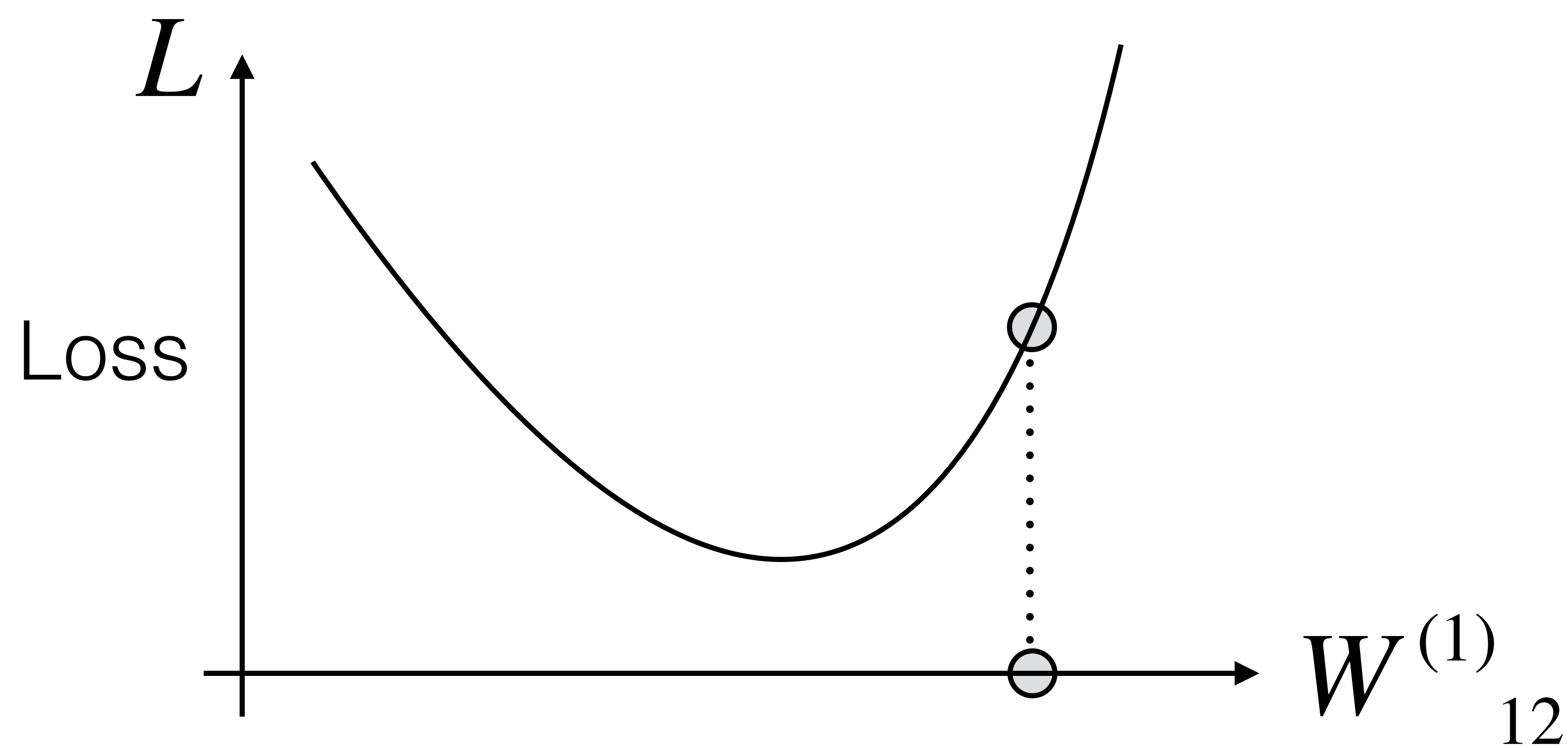


Toy Example:



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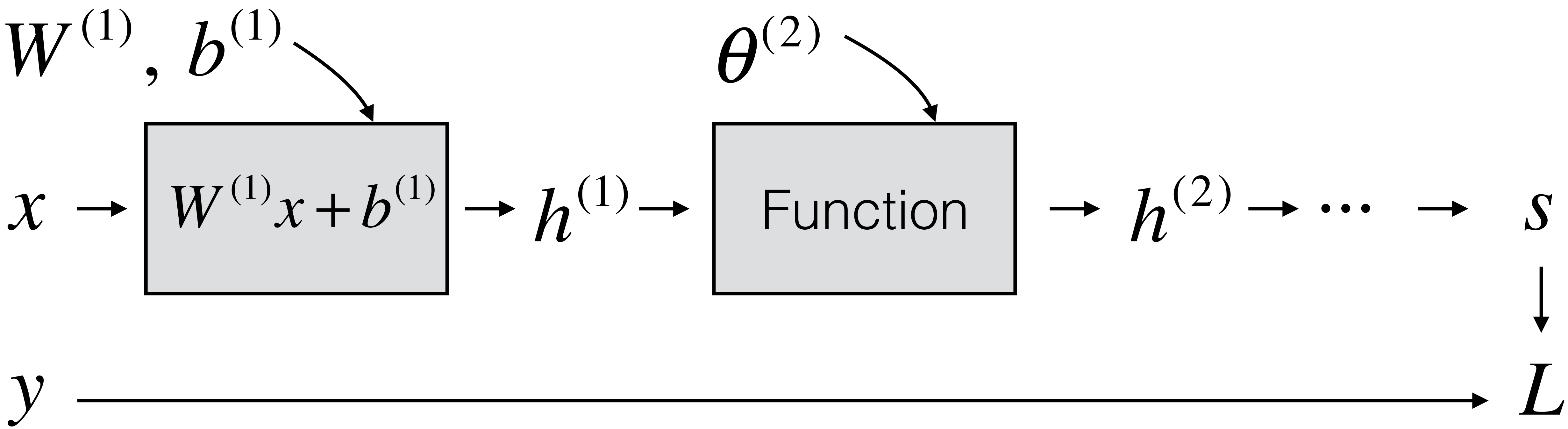
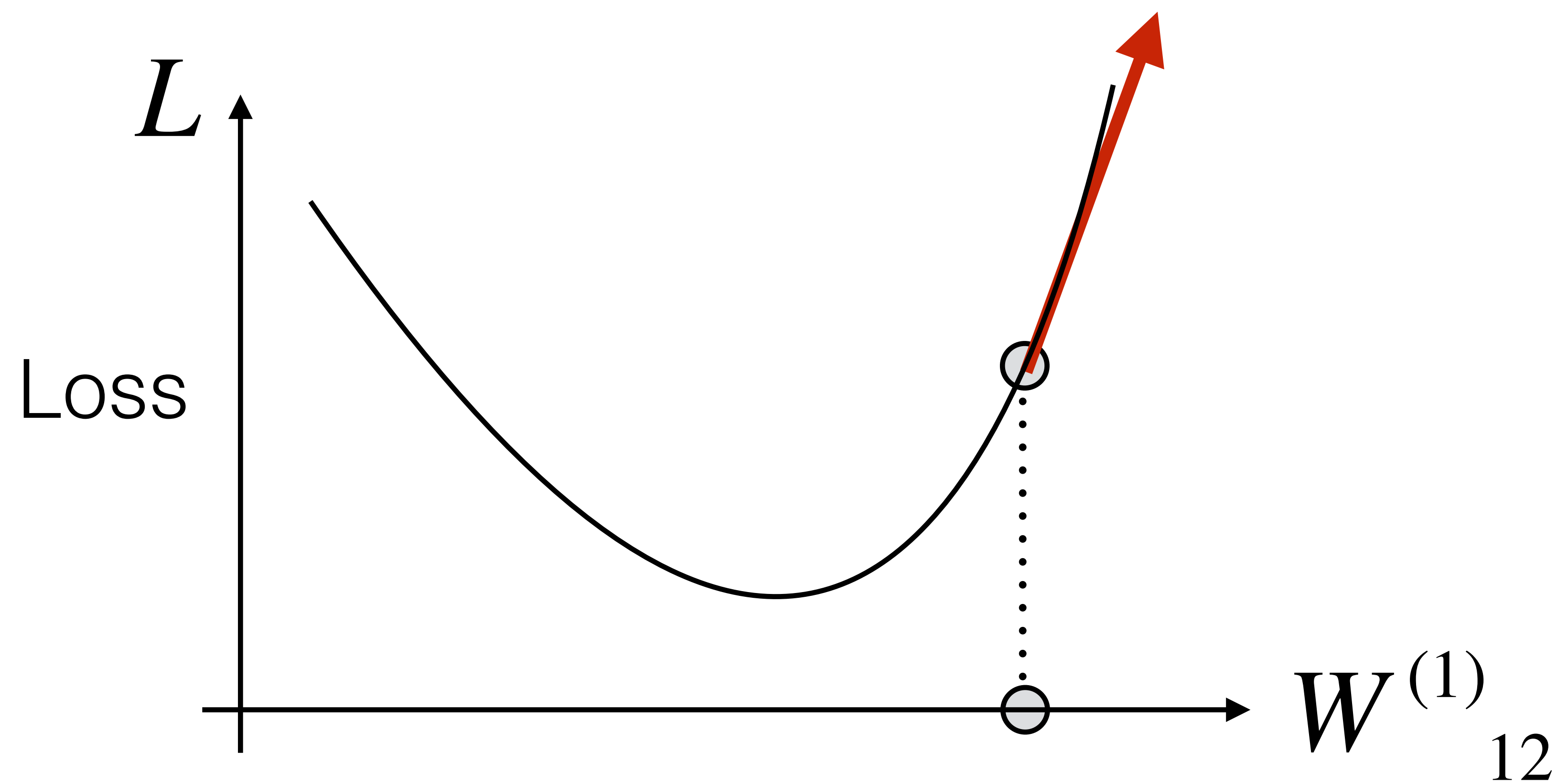
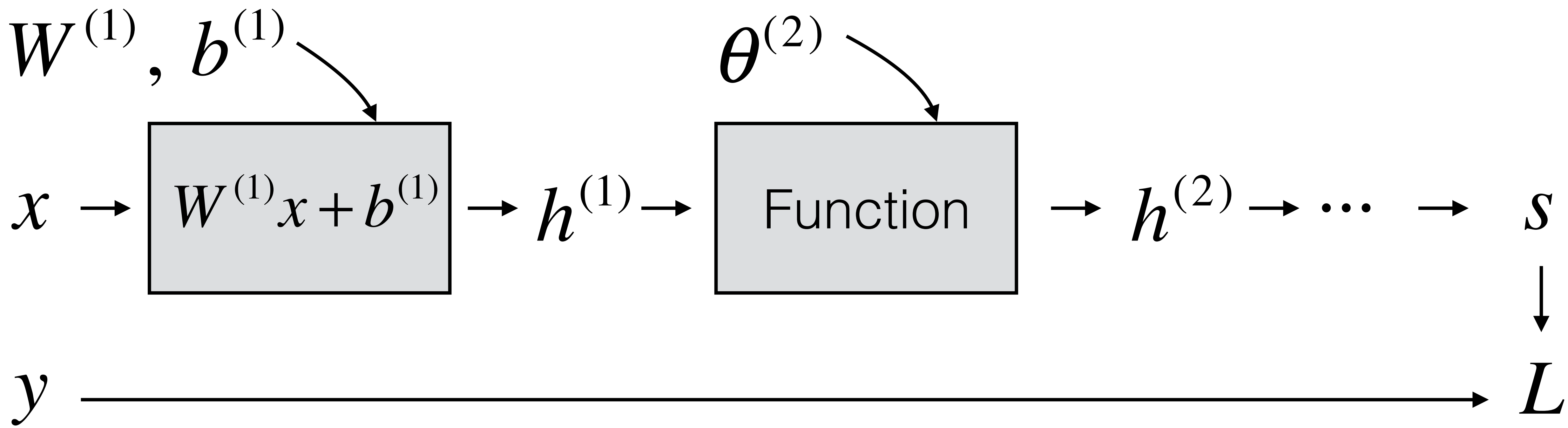


Toy Example:

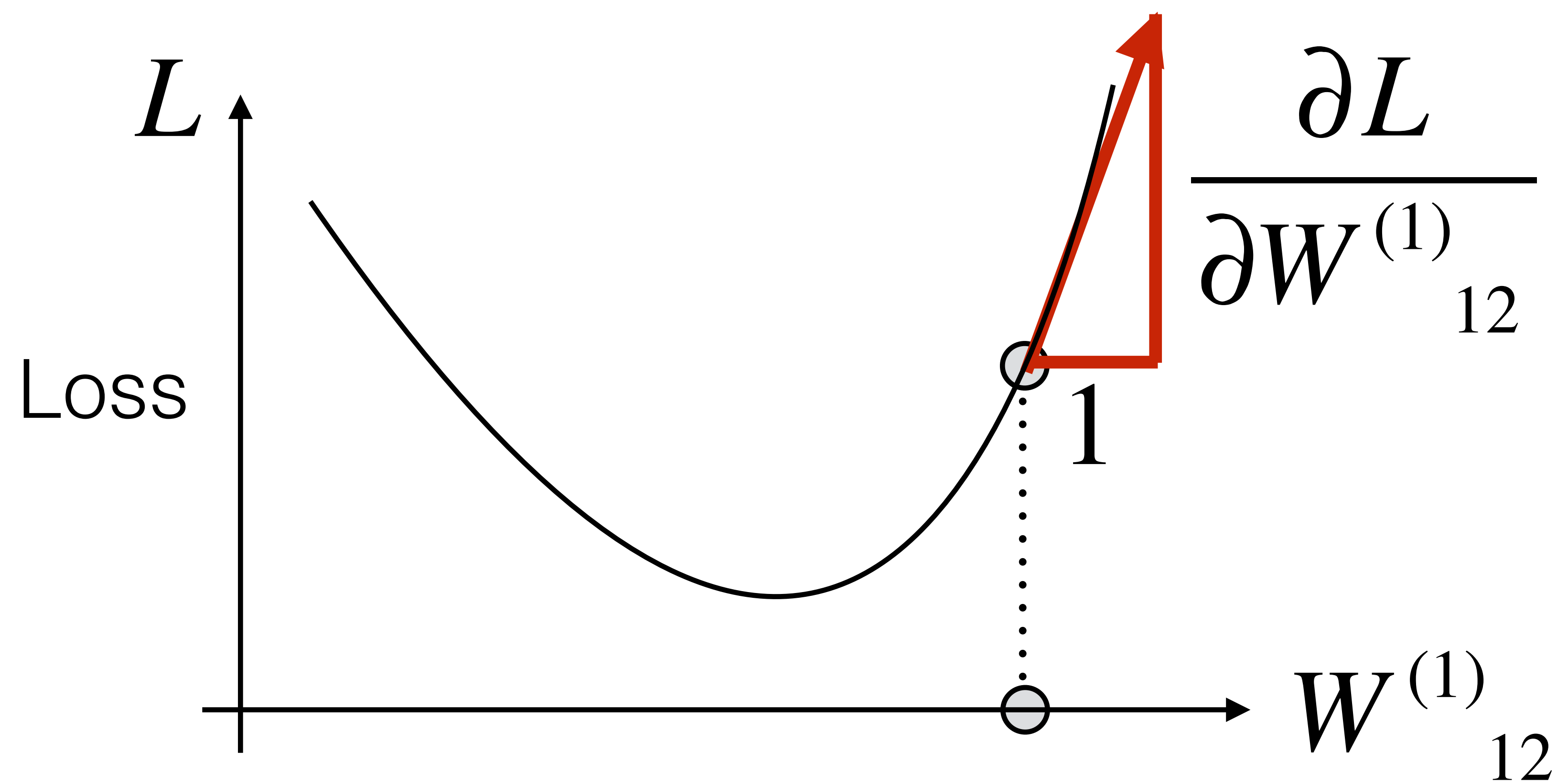


A weight somewhere in the network

Review: Setup

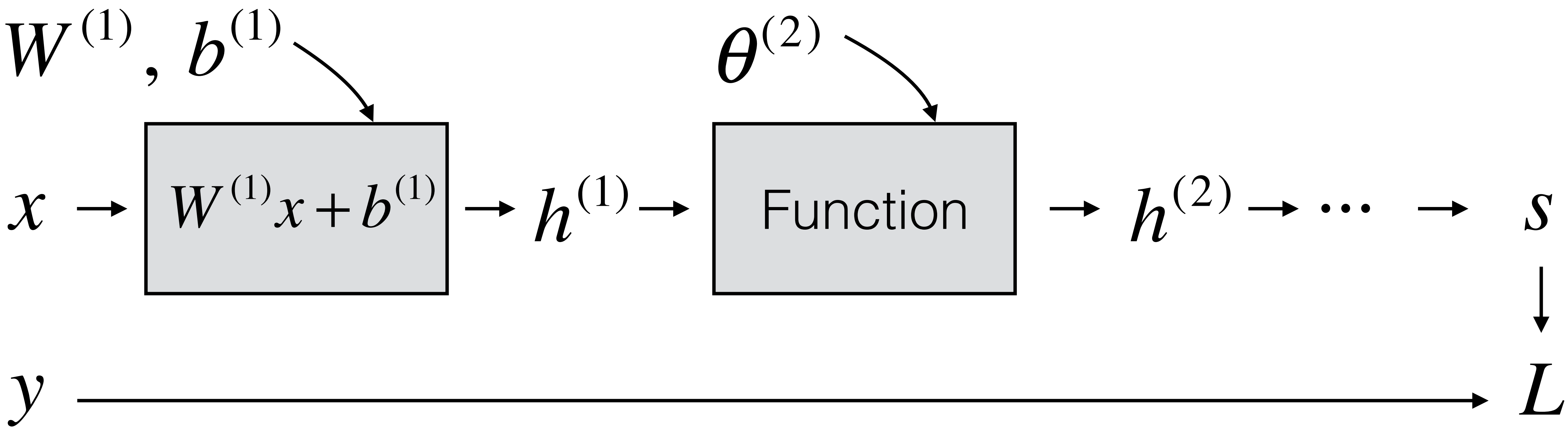


Toy Example:

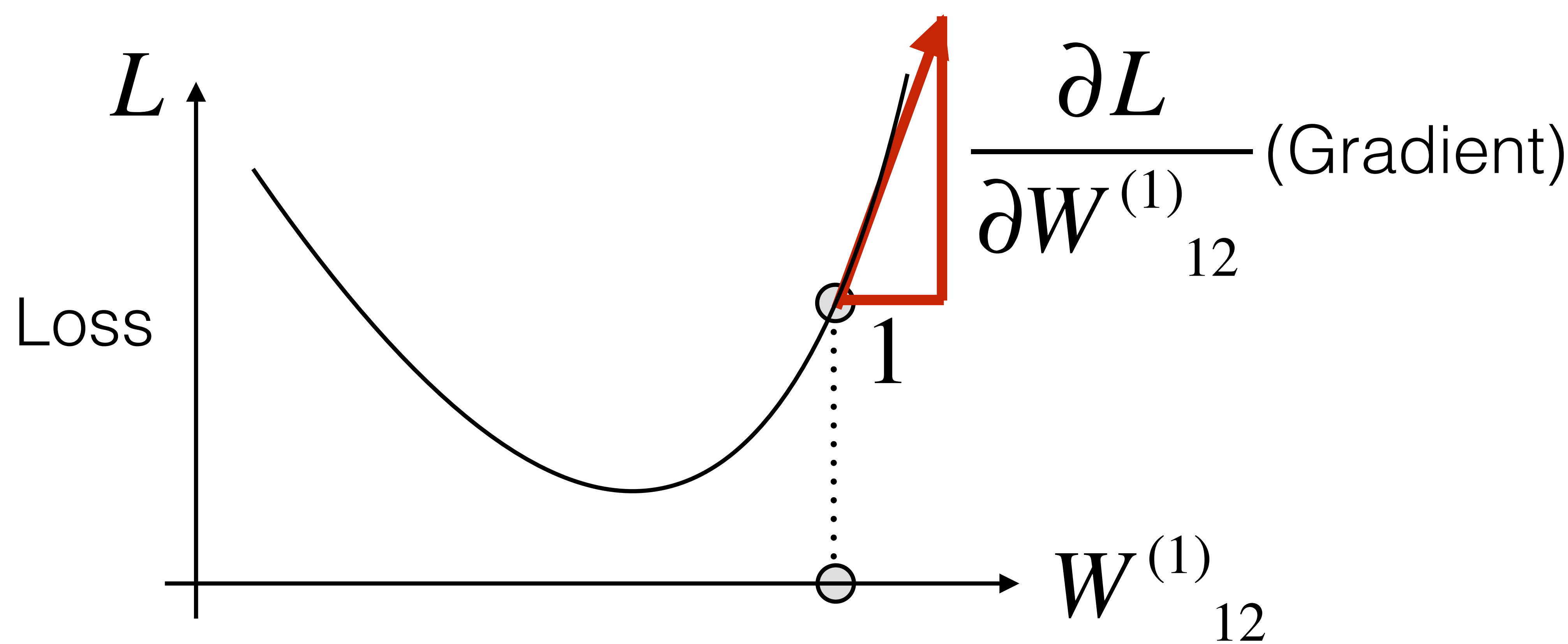


A weight somewhere in the network

Review: Setup

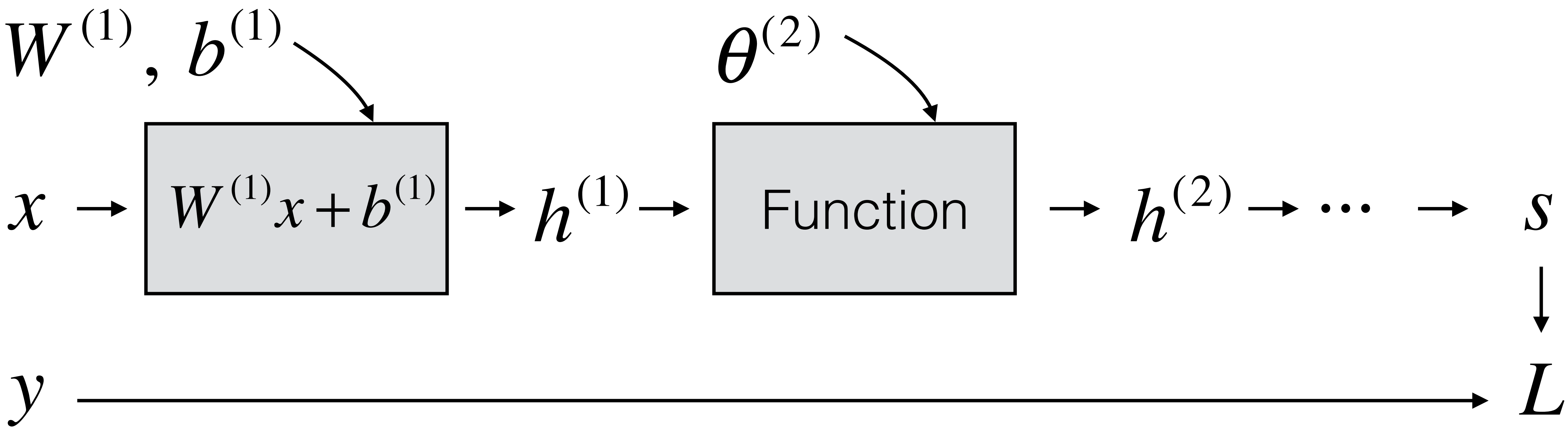


Toy Example:



A weight somewhere in the network

Review: Setup



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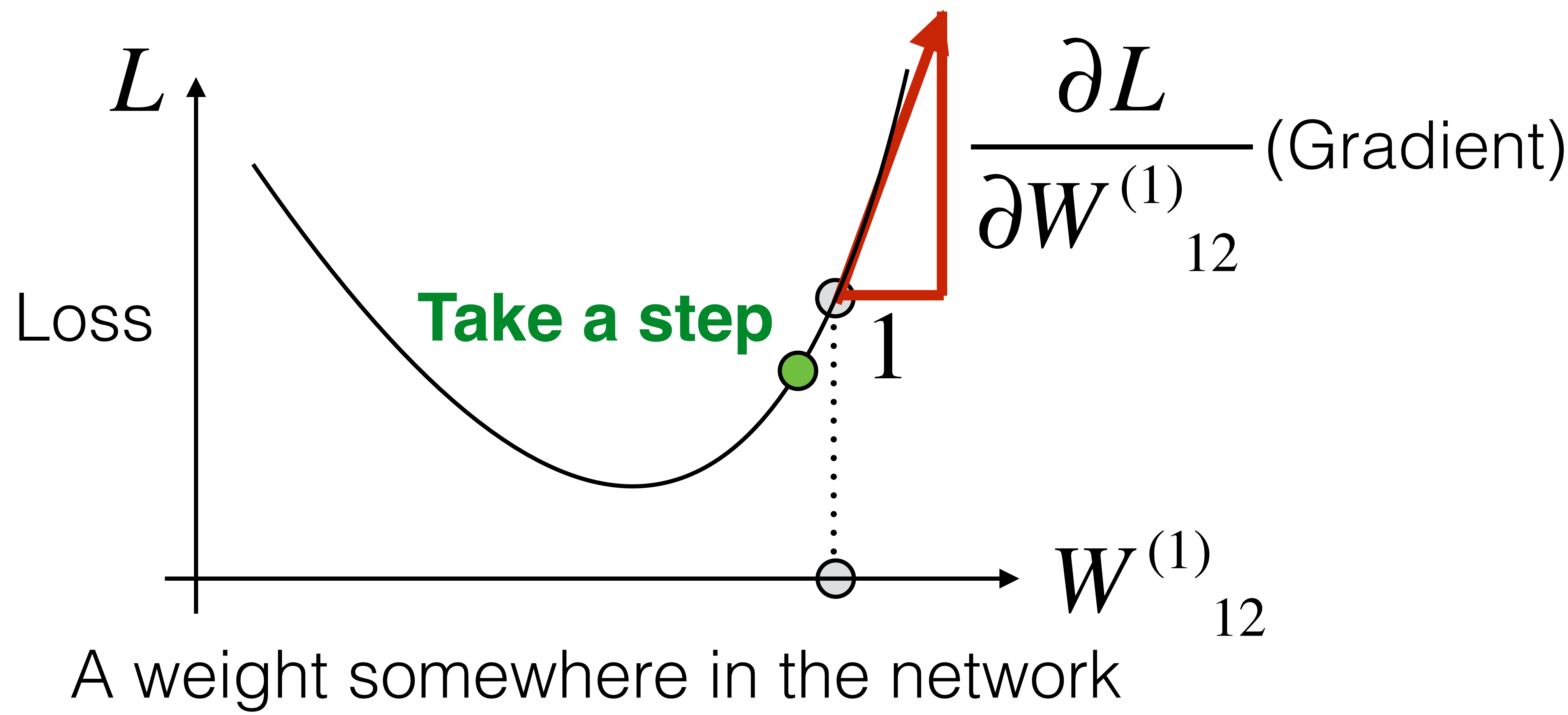

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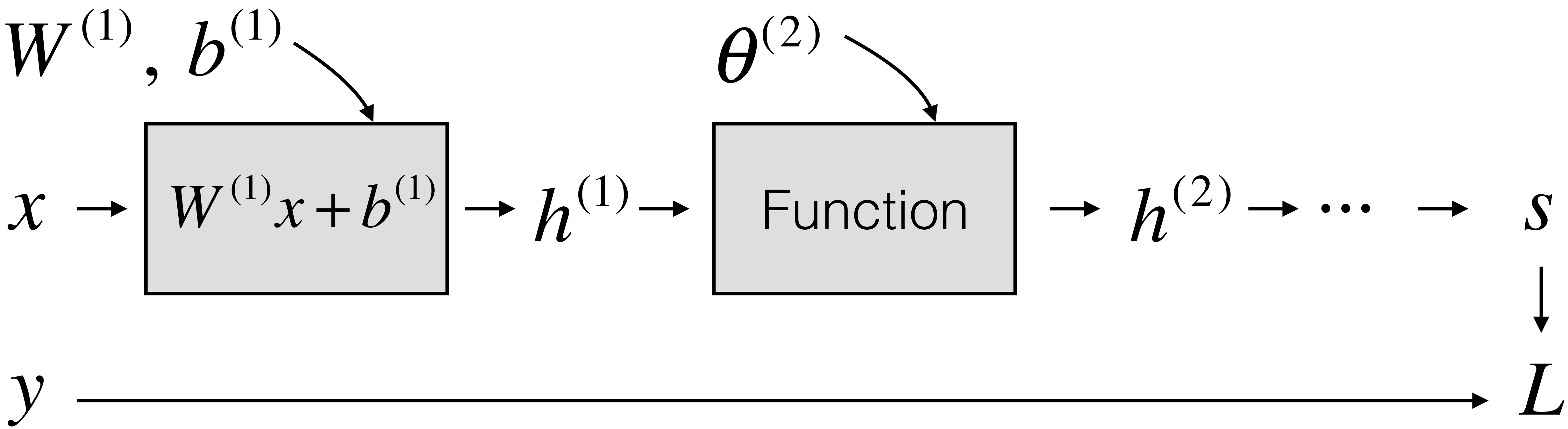
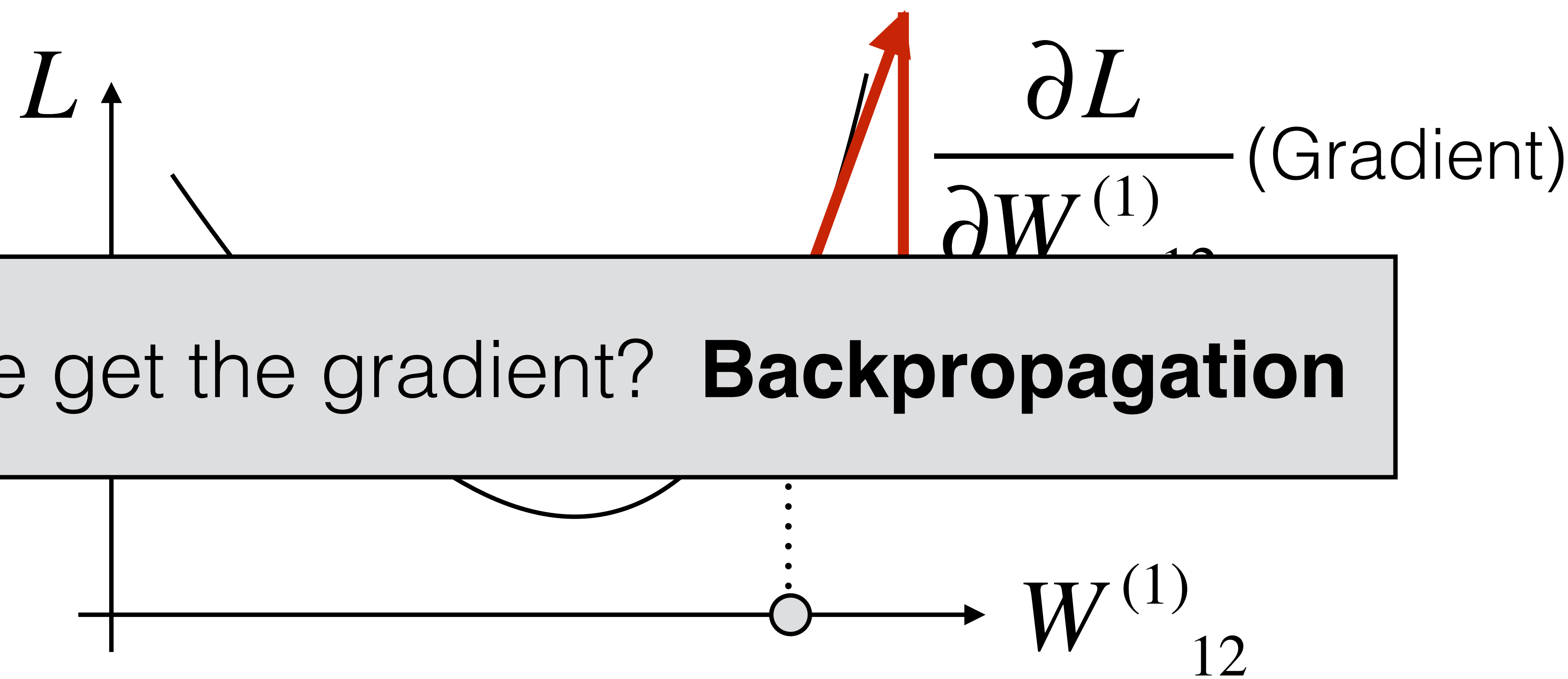


Toy Example:



How do we get the gradient? **Backpropagation**

A weight somewhere in the network

Backprop

It’s just the chain rule

Backpropagation

[Rumelhart, Hinton, Williams. Nature 1986]

Learning representations by back-propagating errors

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* Institute for Cognitive Science, C-015, University of California, San Diego, La Jolla, California 92093, USA
† Department of Computer Science, Carnegie-Mellon University, Pittsburgh, Philadelphia 15213, USA

We describe a new learning procedure, back-propagation, for networks of neurone-like units. The procedure repeatedly adjusts the weights of the connections in the network so as to minimize a measure of the difference between the actual output vector of the net and the desired output vector. As a result of the weight adjustments, internal 'hidden' units which are not part of the input or output come to represent important features of the task domain, and the regularities in the task are captured by the interactions of these units. The ability to create useful new features distinguishes back-propagation from earlier, simpler methods such as the perceptron-convergence procedure[1].

There have been many attempts to design self-organizing neural networks. The aim is to find a powerful synaptic modification rule that will allow an arbitrarily connected neural network to develop an internal structure that is appropriate for

more difficult when we introduce hidden units whose actual or desired states are not specified by the task. (In perceptrons, there are 'feature analysers' between the input and output that are not true hidden units because their input connections are fixed by hand, so their states are completely determined by the input vector: they do not learn representations.) The learning procedure must decide under what circumstances the hidden units should be active in order to help achieve the desired input-output behaviour. This amounts to deciding what these units should represent. We demonstrate that a general purpose and relatively simple procedure is powerful enough to construct appropriate internal representations.

The simplest form of the learning procedure is for layered networks which have a layer of input units at the bottom; any number of intermediate layers; and a layer of output units at the top. Connections within a layer or from higher to lower layers are forbidden, but connections can skip intermediate layers. An input vector is presented to the network by setting the states of the input units. Then the states of the units in each layer are determined by applying equations (1) and (2) to the connections coming from lower layers. All units within a layer have their states set in parallel, but different layers have their states set sequentially, starting at the bottom and working upwards until the states of the output units are determined.

The total input, x_j, to unit j is a linear function of the outputs, y_i, of the units that are connected to j and of the weights, w_{ji}, on these connections

$$x_j = \sum_i y_i w_{ji} \qquad (1)$$

Chain rule recap

I hope everyone remembers the chain rule:

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial x}$$

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$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial h} \frac{\partial h}{\partial x}$$

Forward propagation: $x \longrightarrow h \longrightarrow \cdots$

Backward propagation: $\frac{\partial L}{\partial x} \longleftarrow \frac{\partial L}{\partial h} \longleftarrow \cdots$

Chain rule recap

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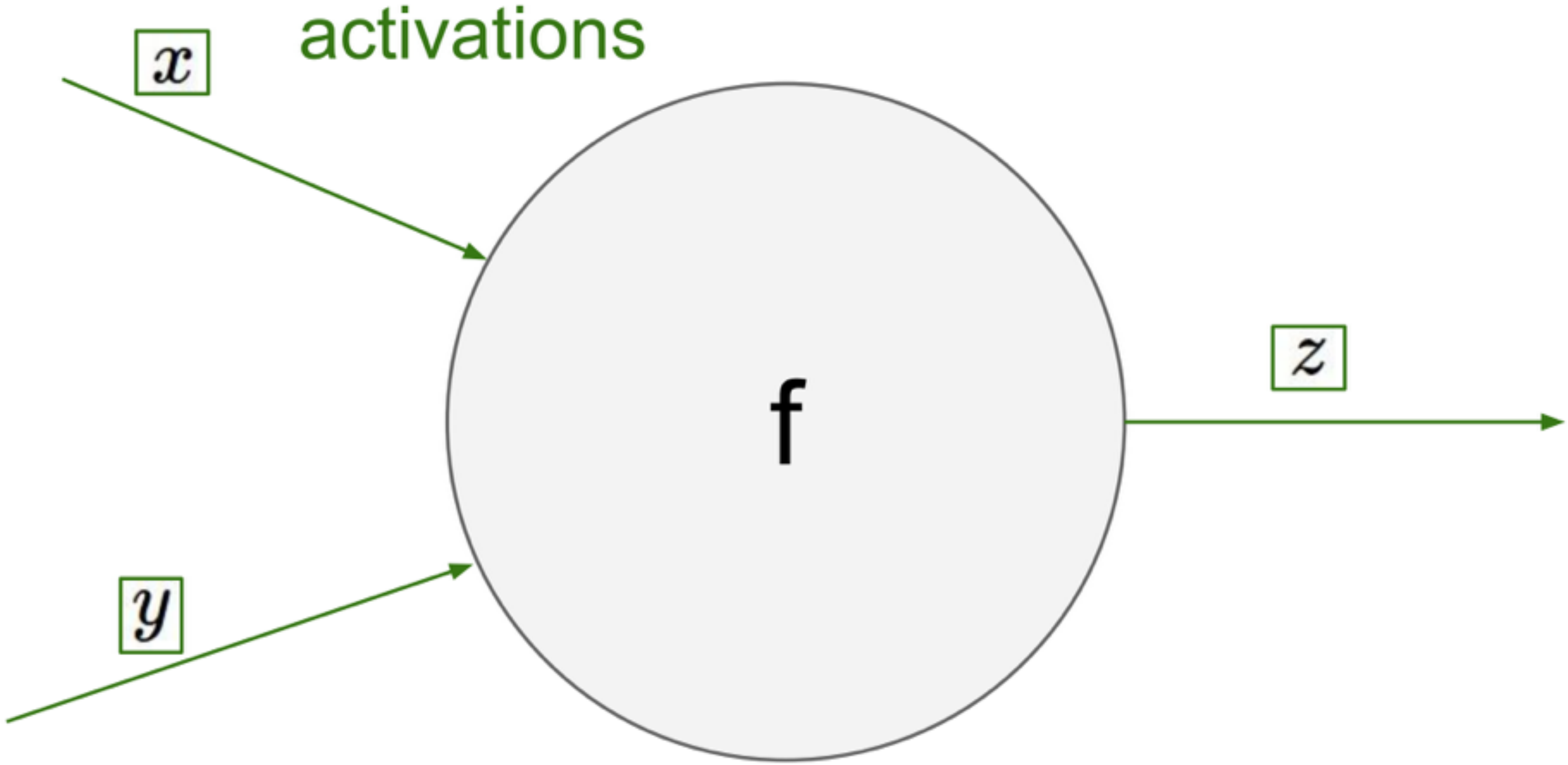
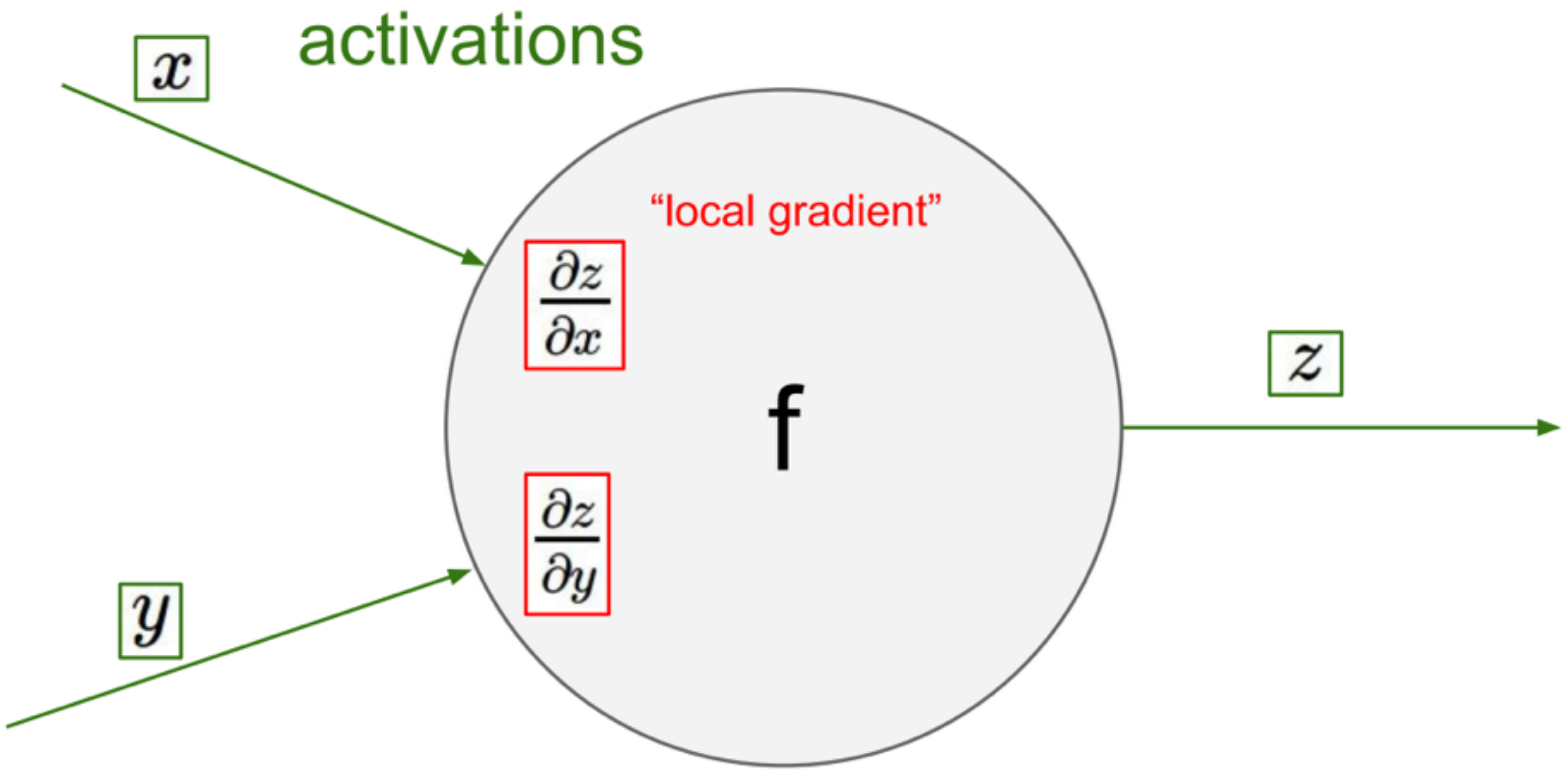
Forward propagation: $x \longrightarrow h \longrightarrow \cdots$

Backward propagation: $\frac{\partial L}{\partial x} \longleftarrow \frac{\partial L}{\partial h} \longleftarrow \cdots$

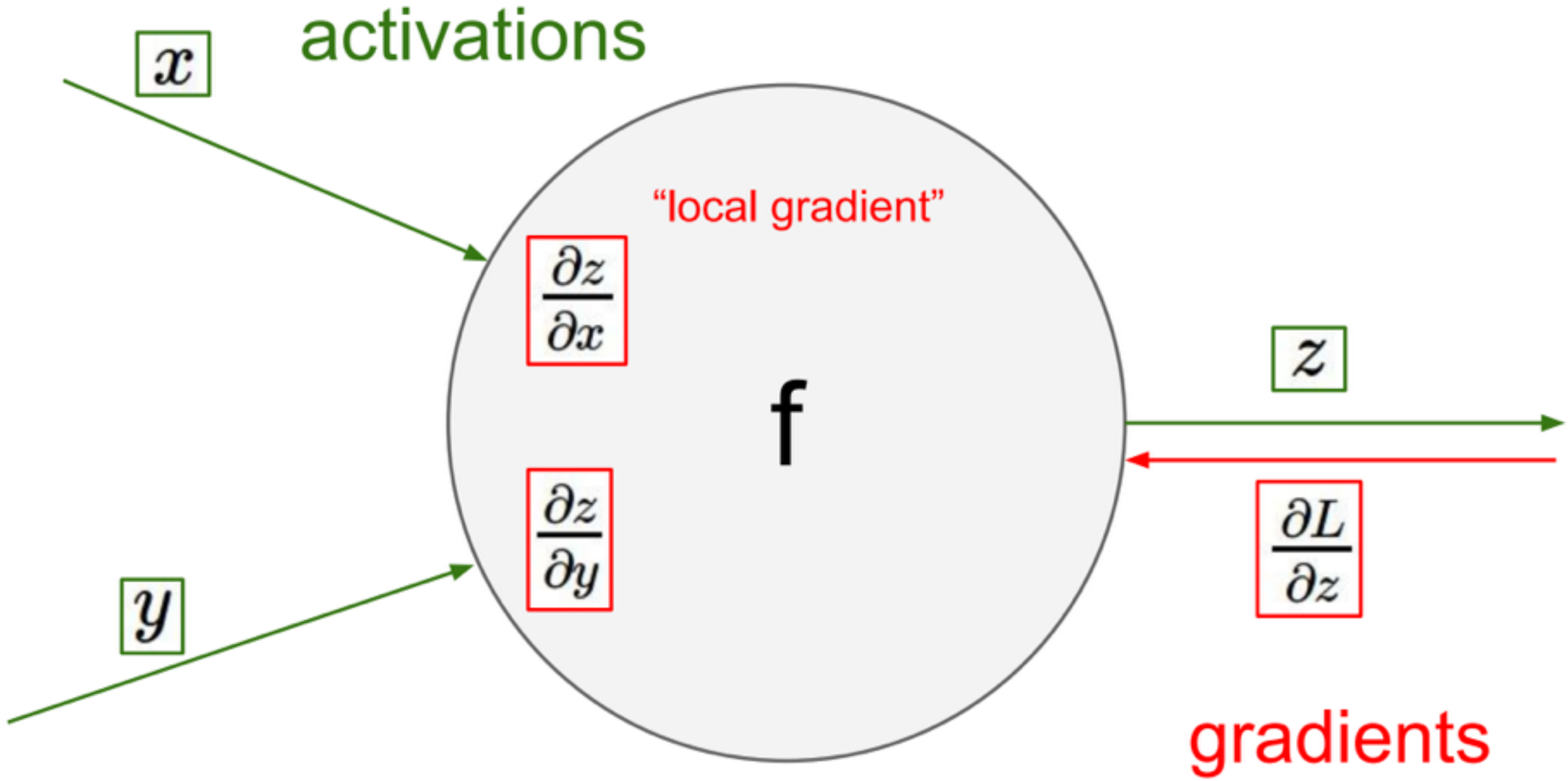
(extends easily to multi-dimensional x and y)



Slide from Karpathy 2016



Slide from Karpathy 2016



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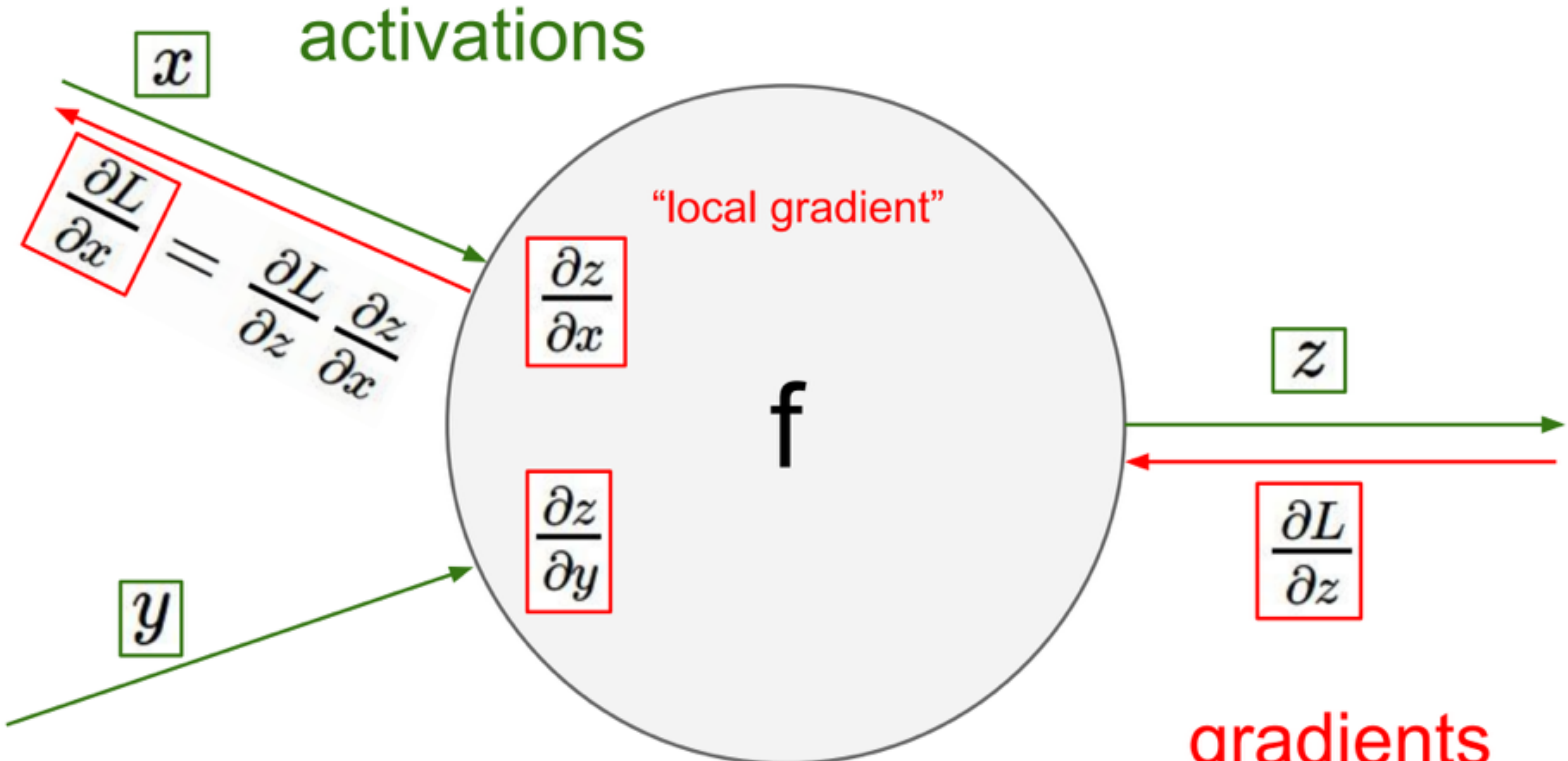

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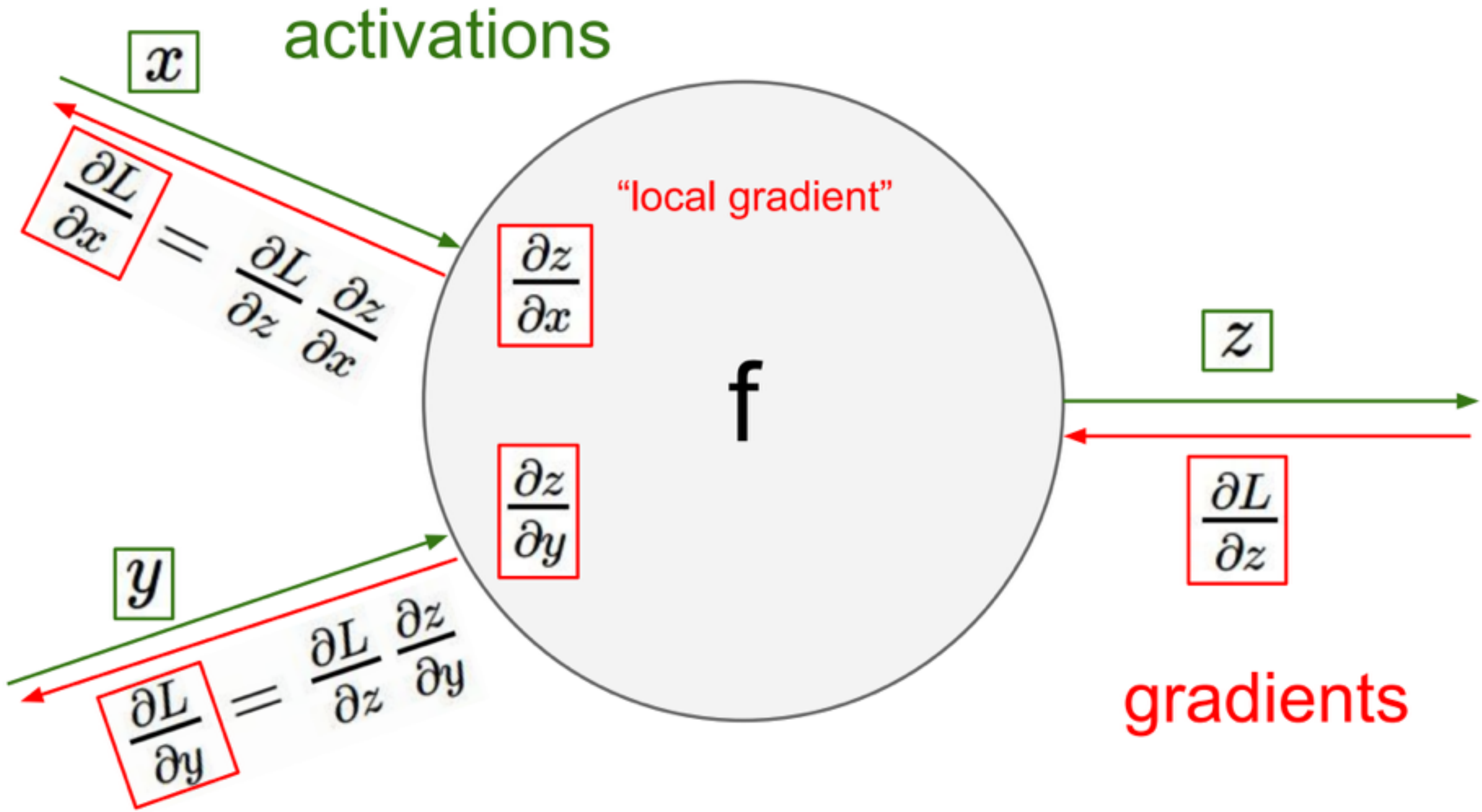

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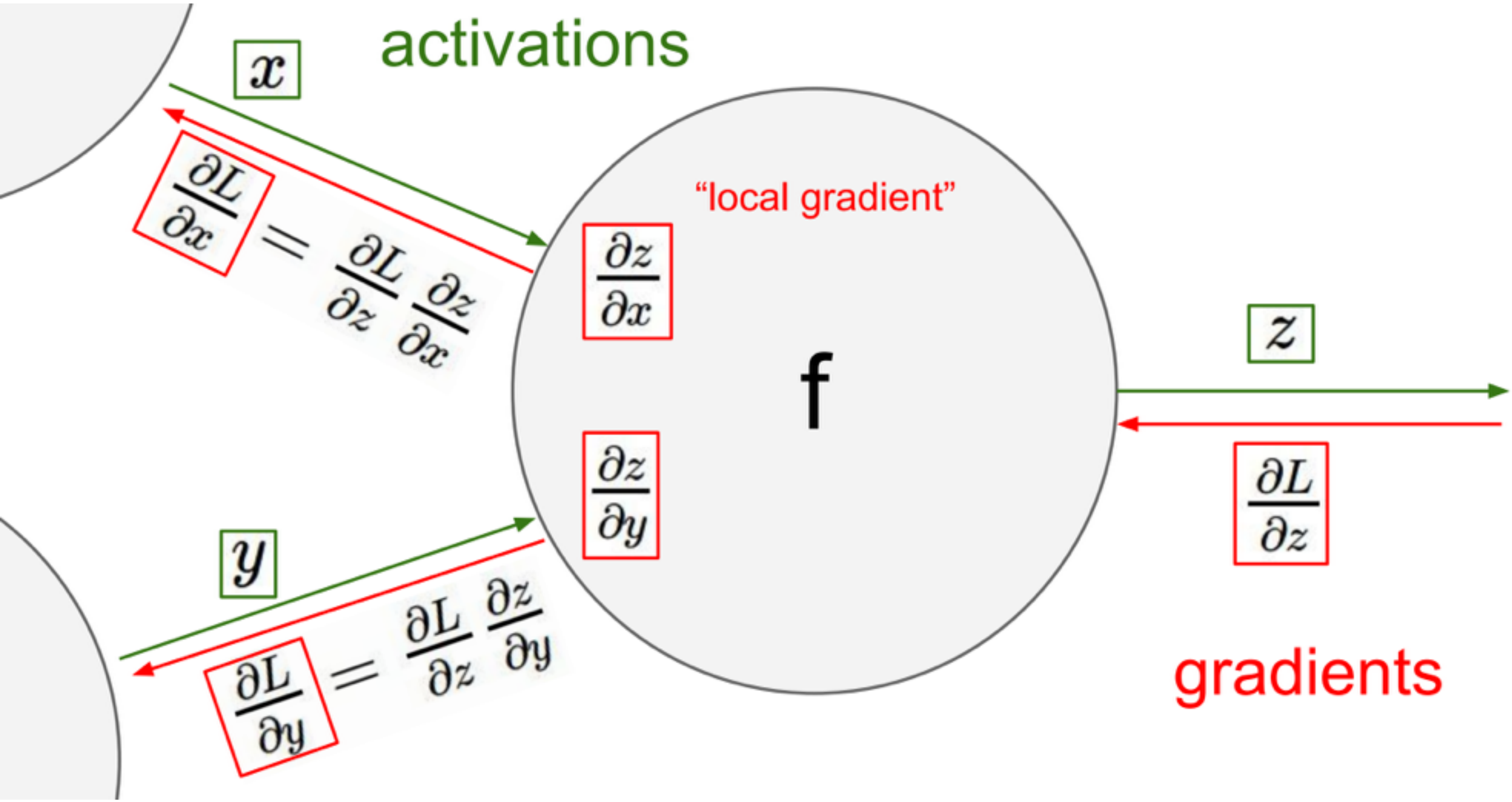
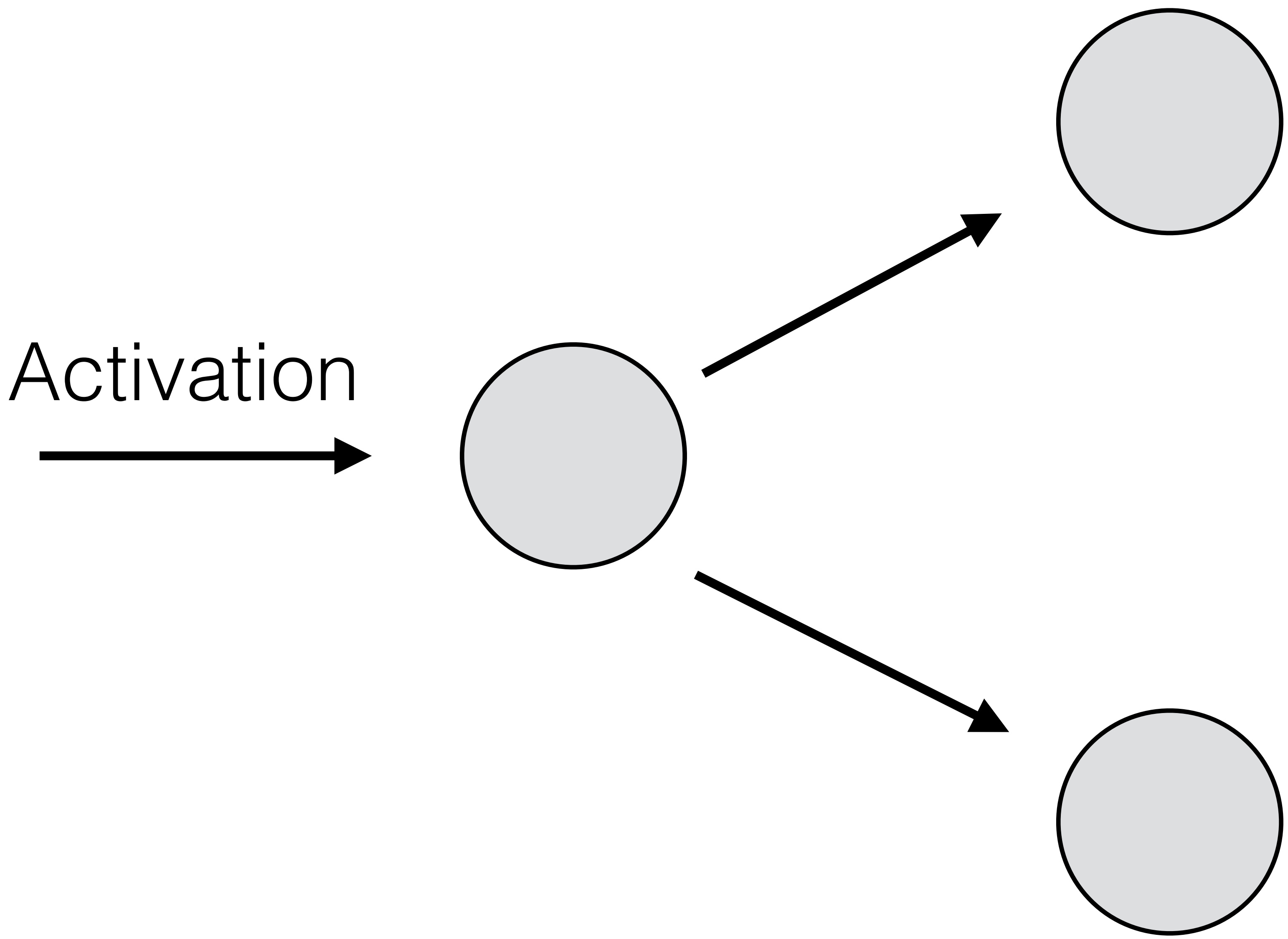
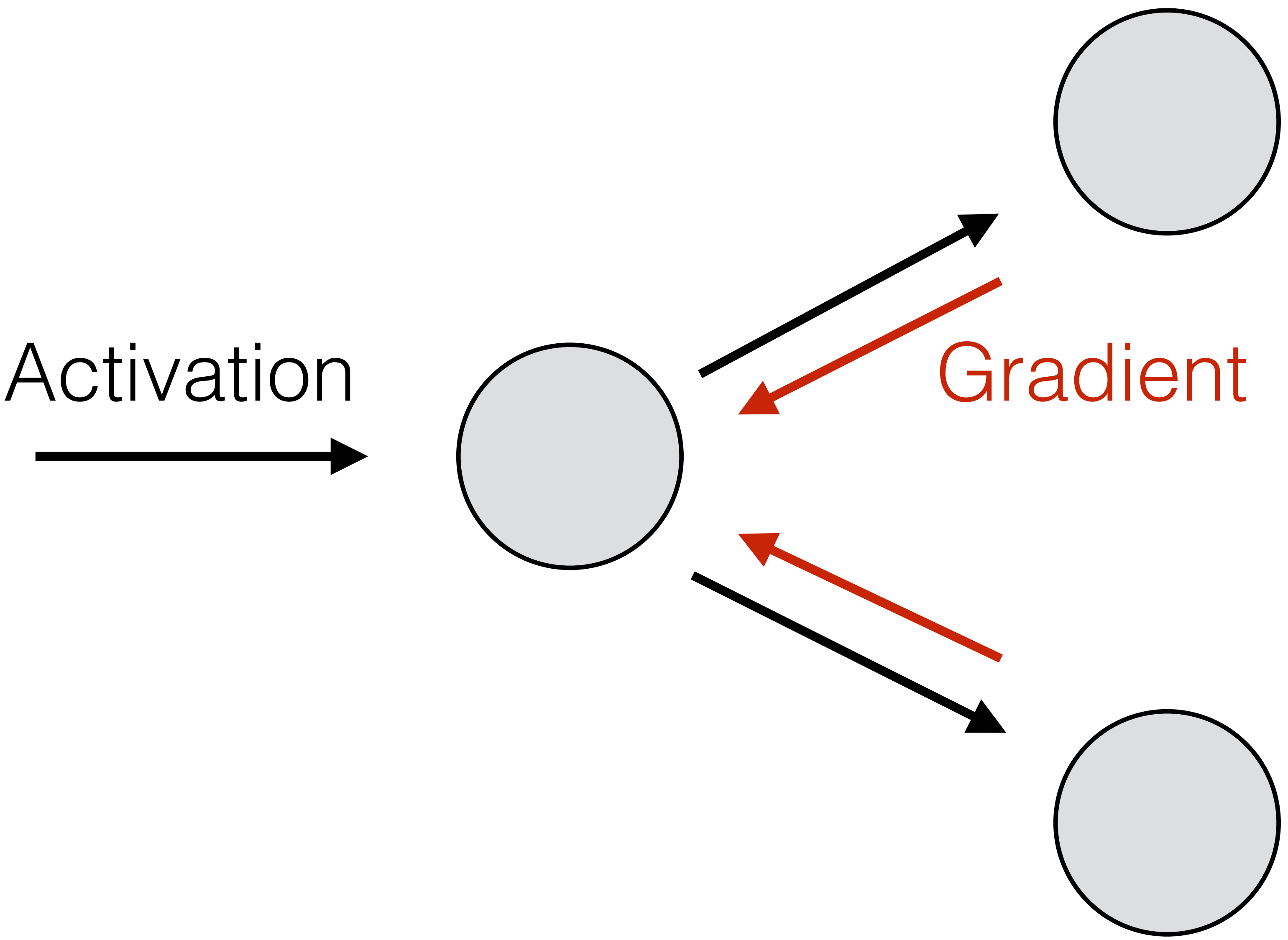


Slide from Karpathy 2016

Gradients add at branches



Gradients add at branches



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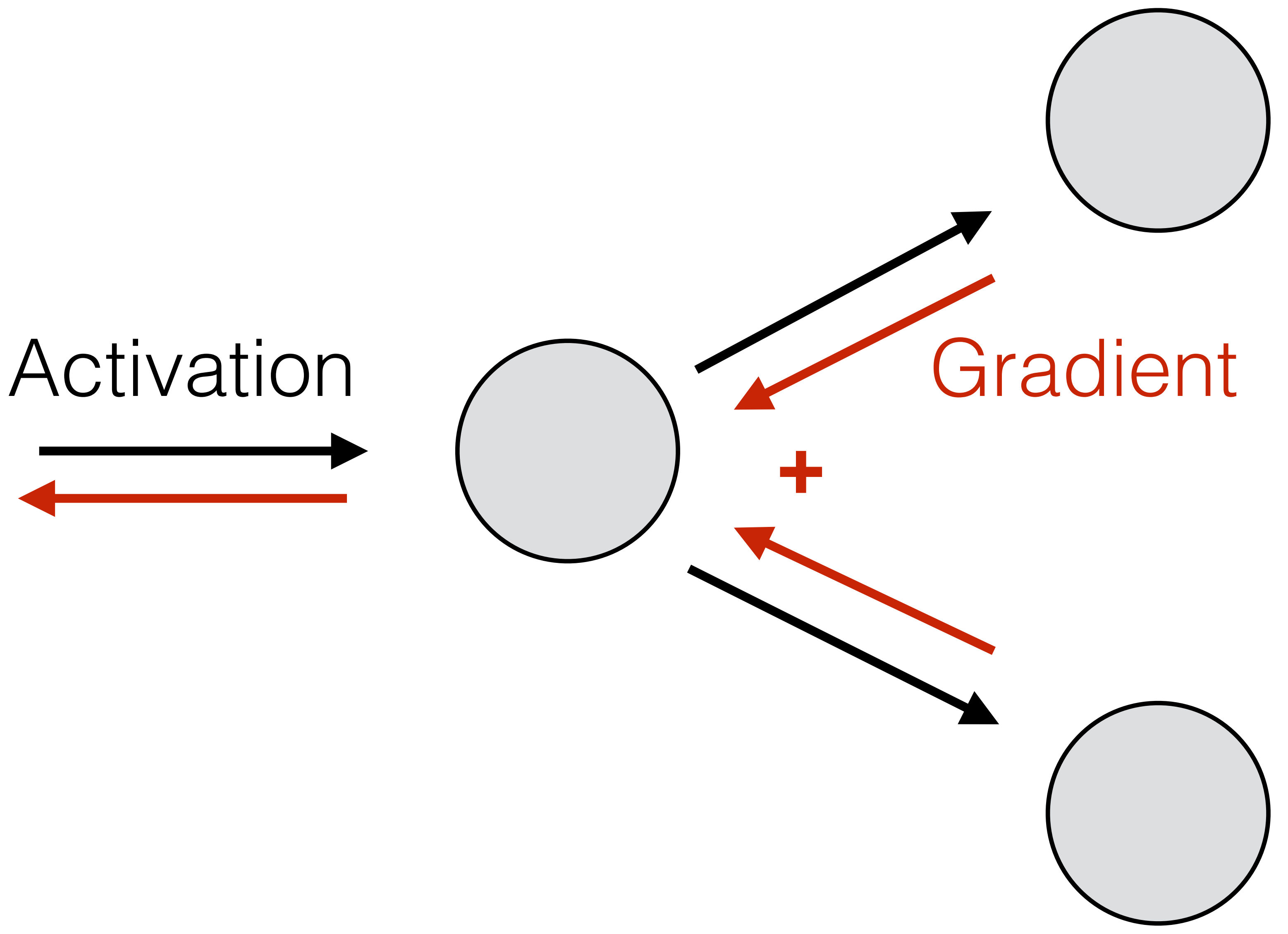
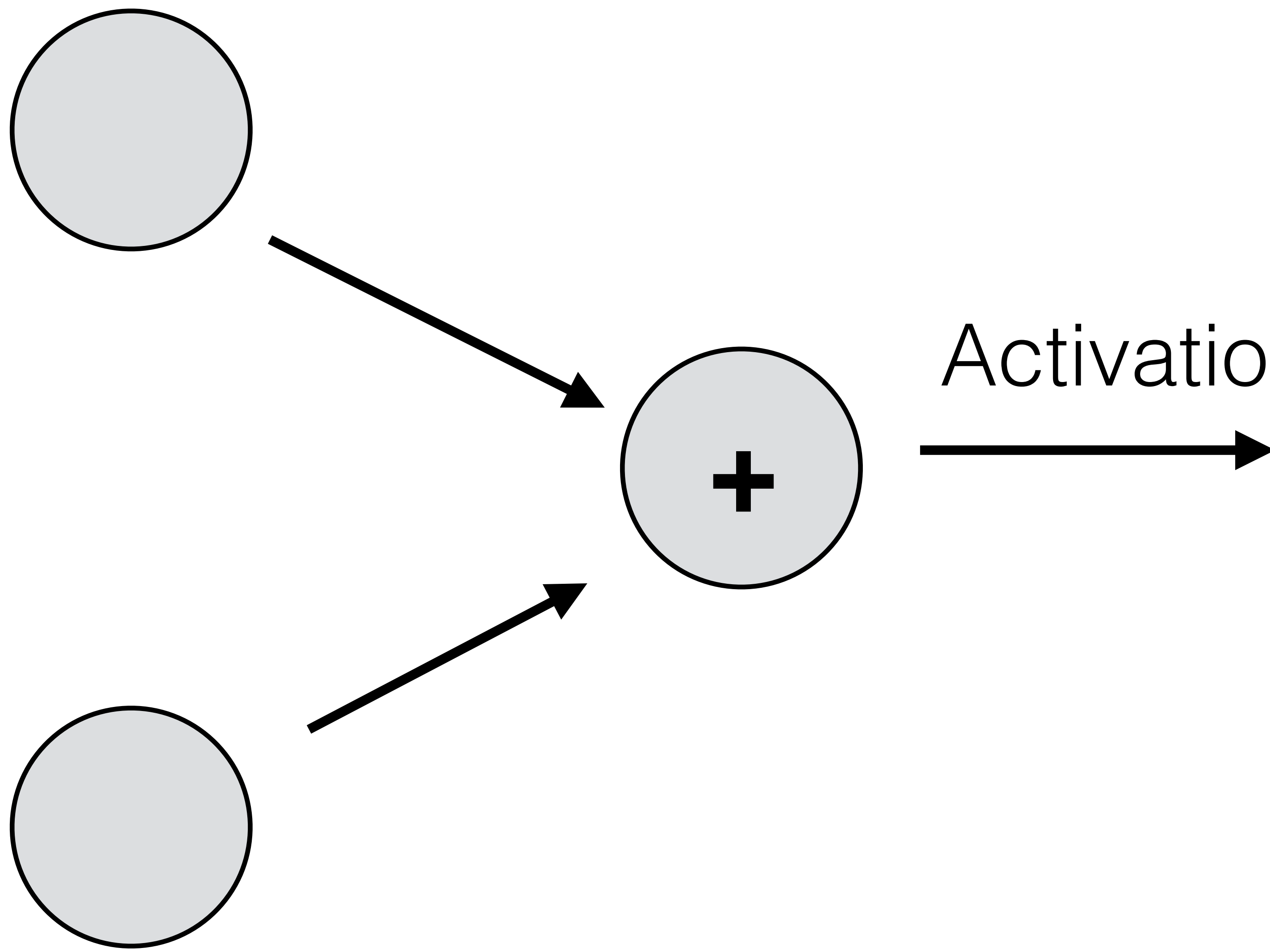


Gradients copy through sums



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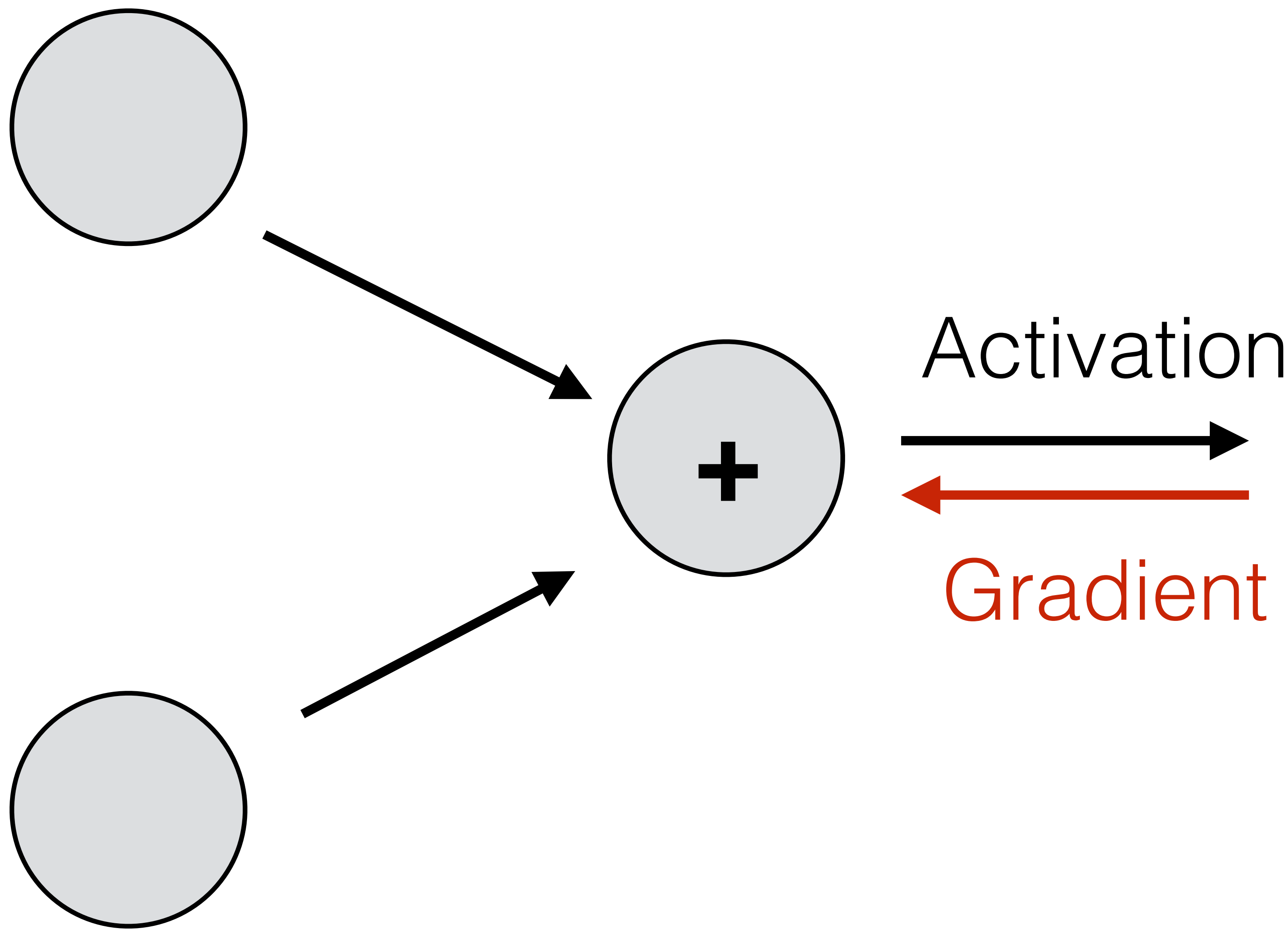

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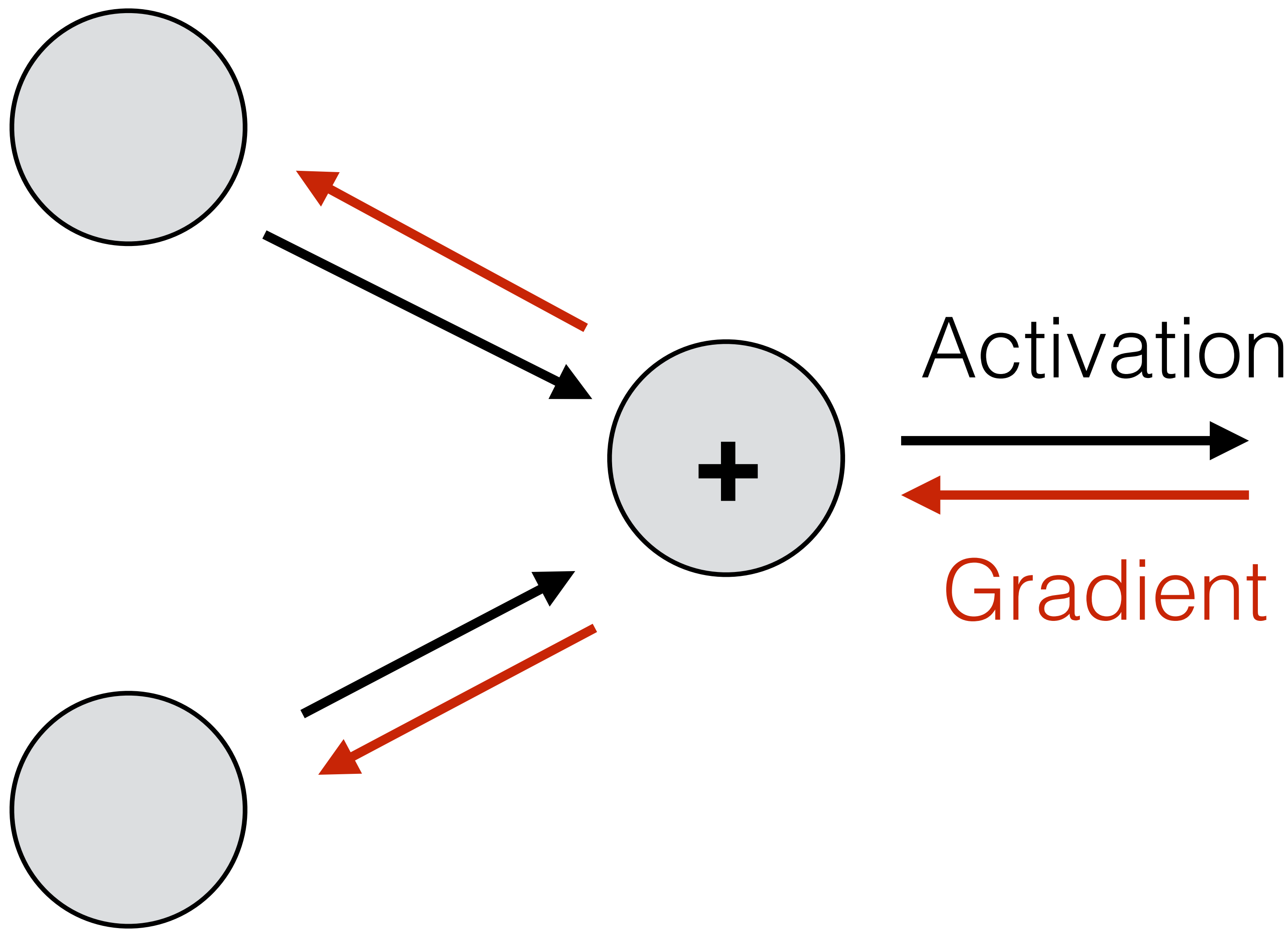
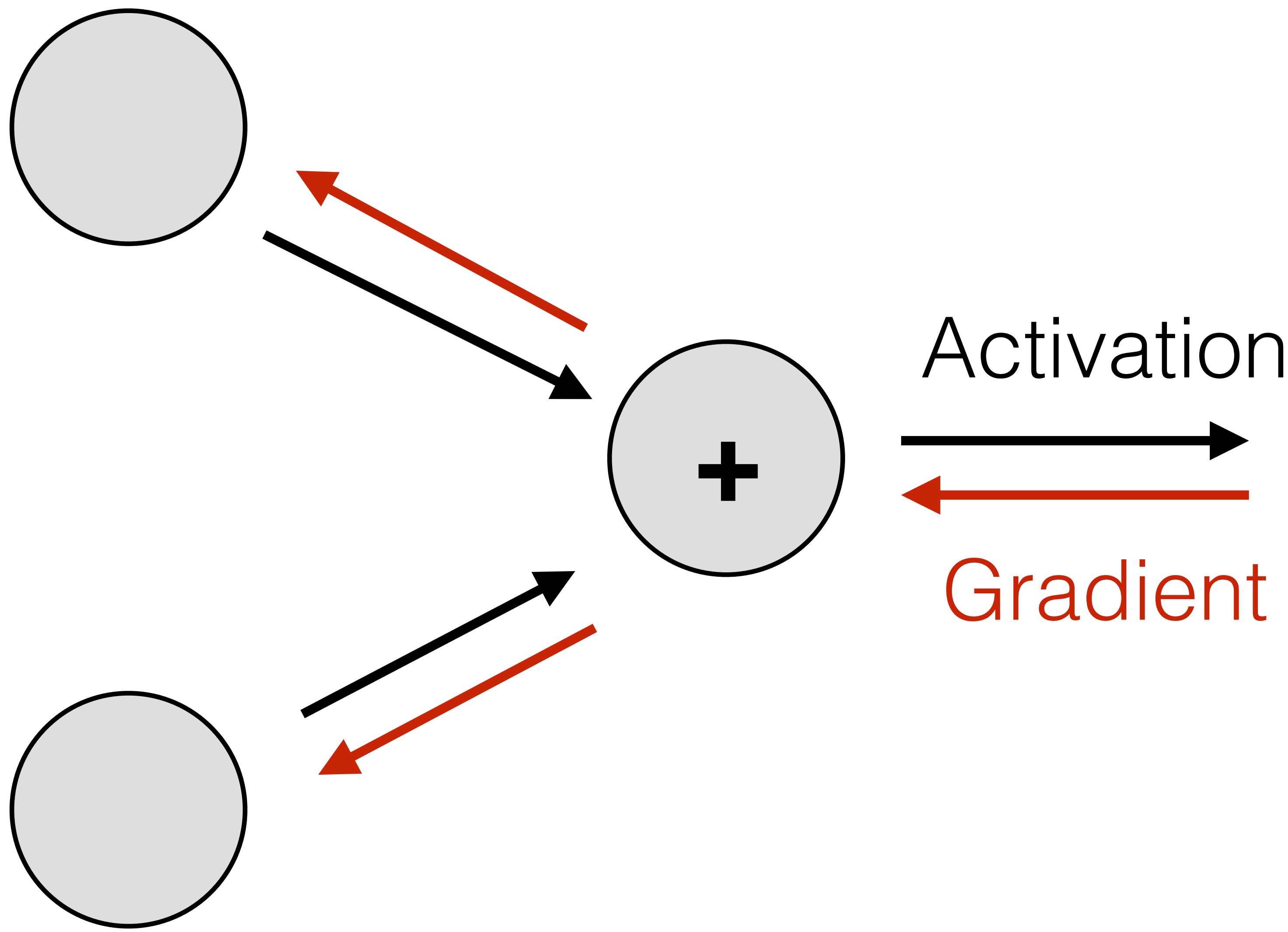
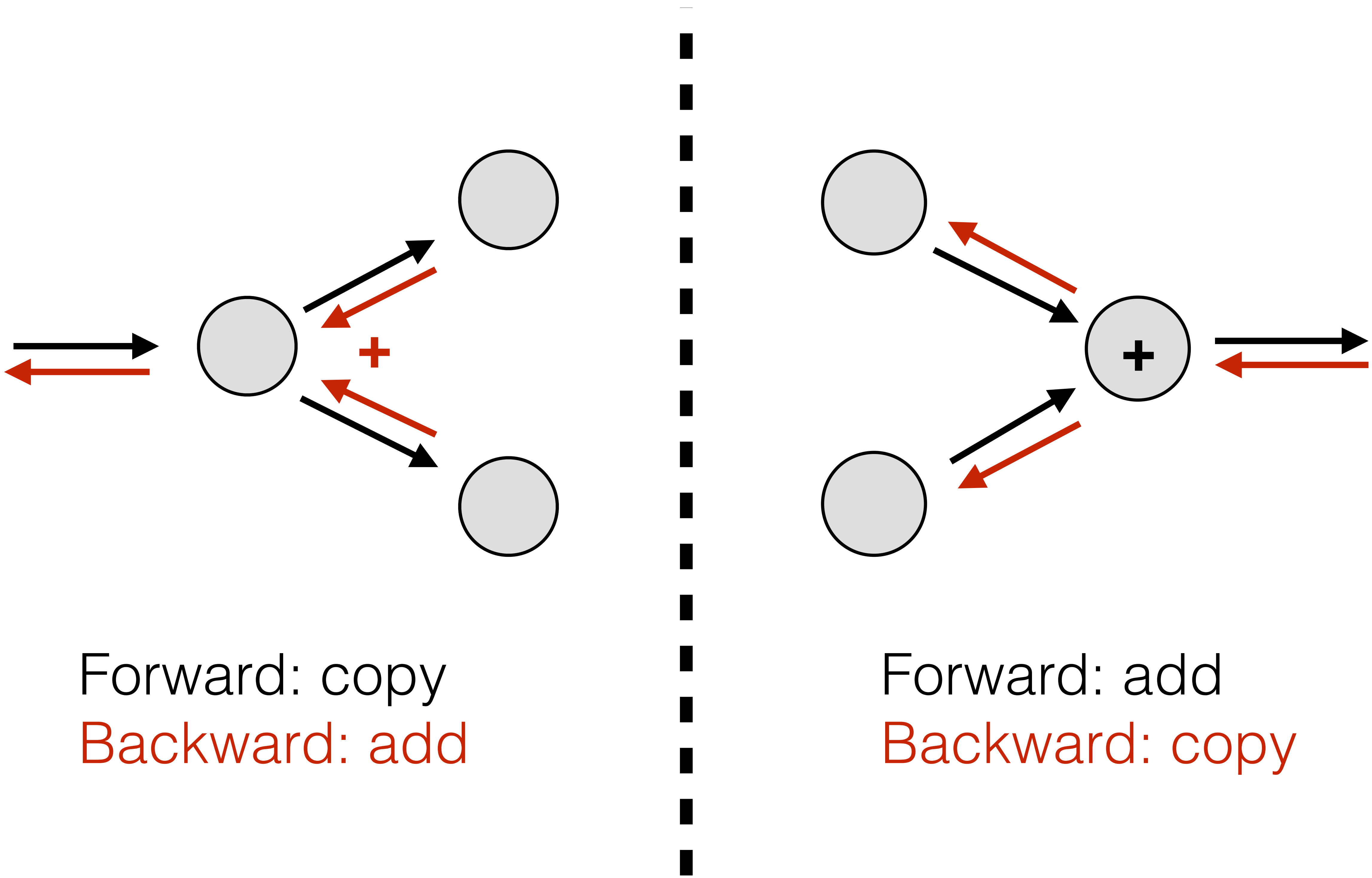


Gradients copy through sums

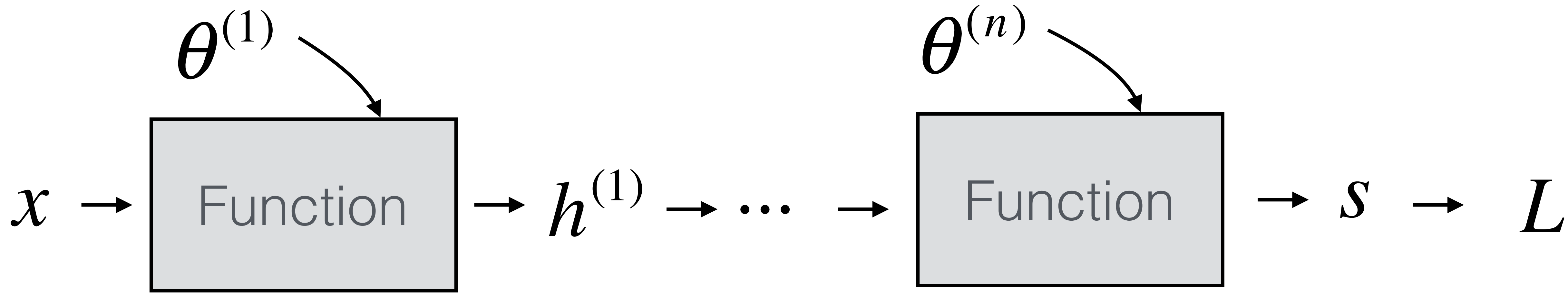


The gradient flows through both branches at “full strength”

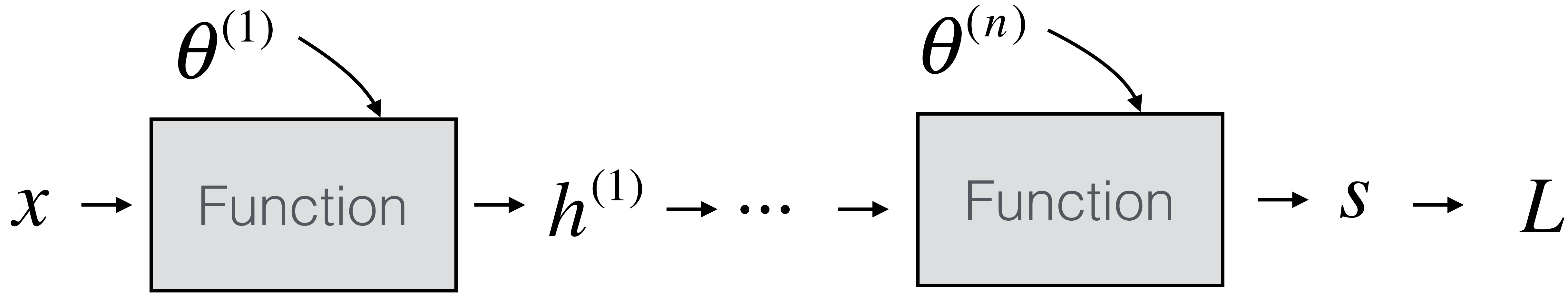
Symmetry between forward and backward



Forward Propagation:

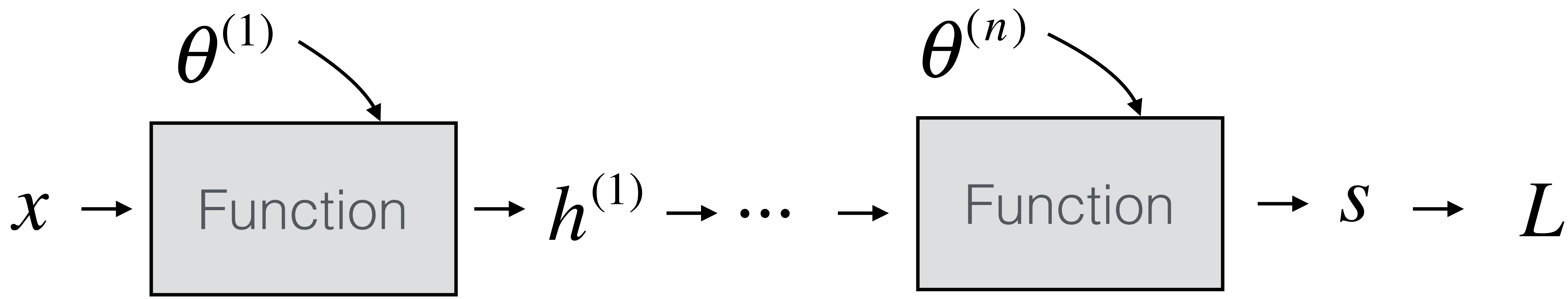


Forward Propagation:



Backward Propagation:

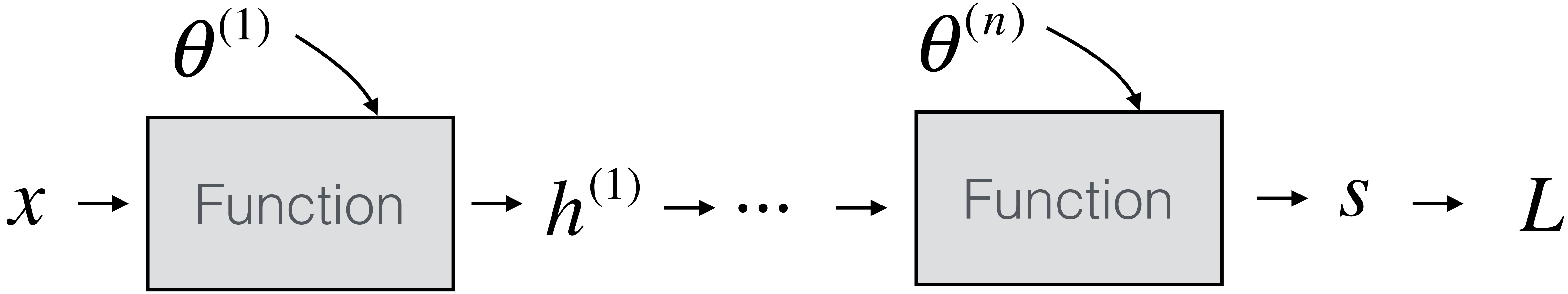
Forward Propagation:



Backward Propagation:

L

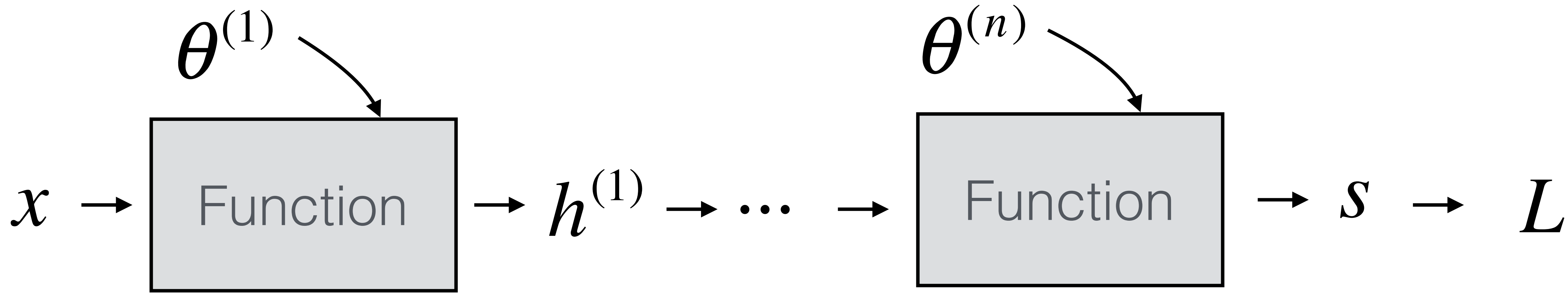
Forward Propagation:



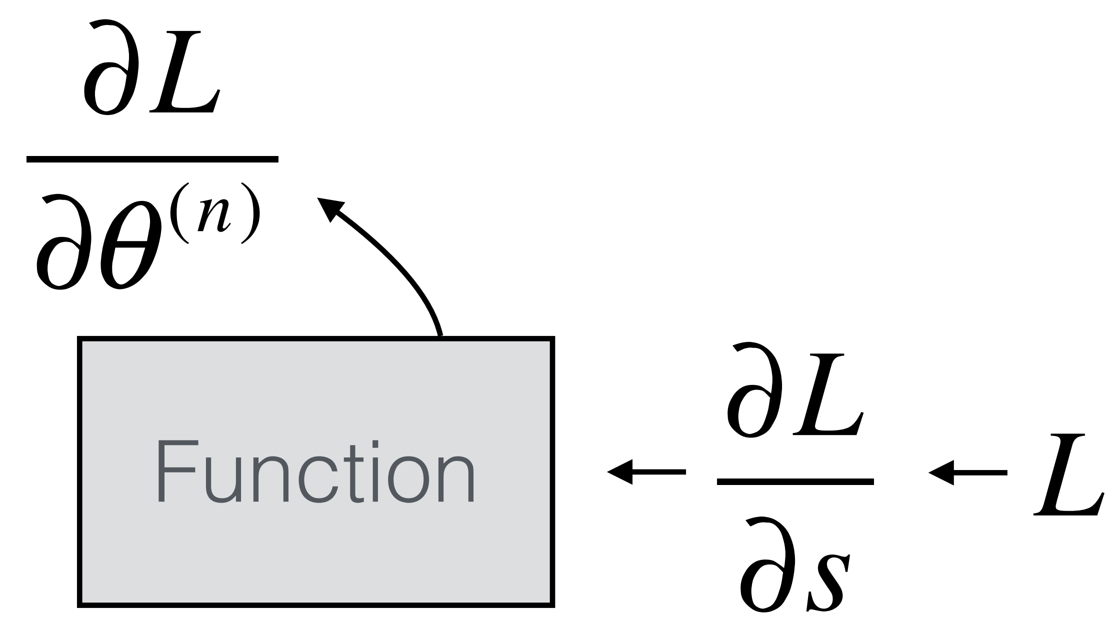
Backward Propagation:

$$\frac{\partial L}{\partial s} \leftarrow L$$

Forward Propagation:



Backward Propagation:



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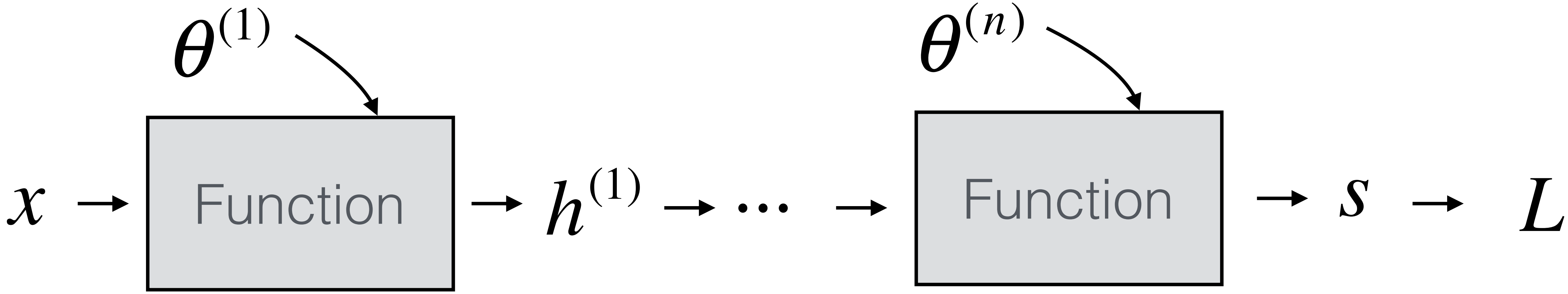

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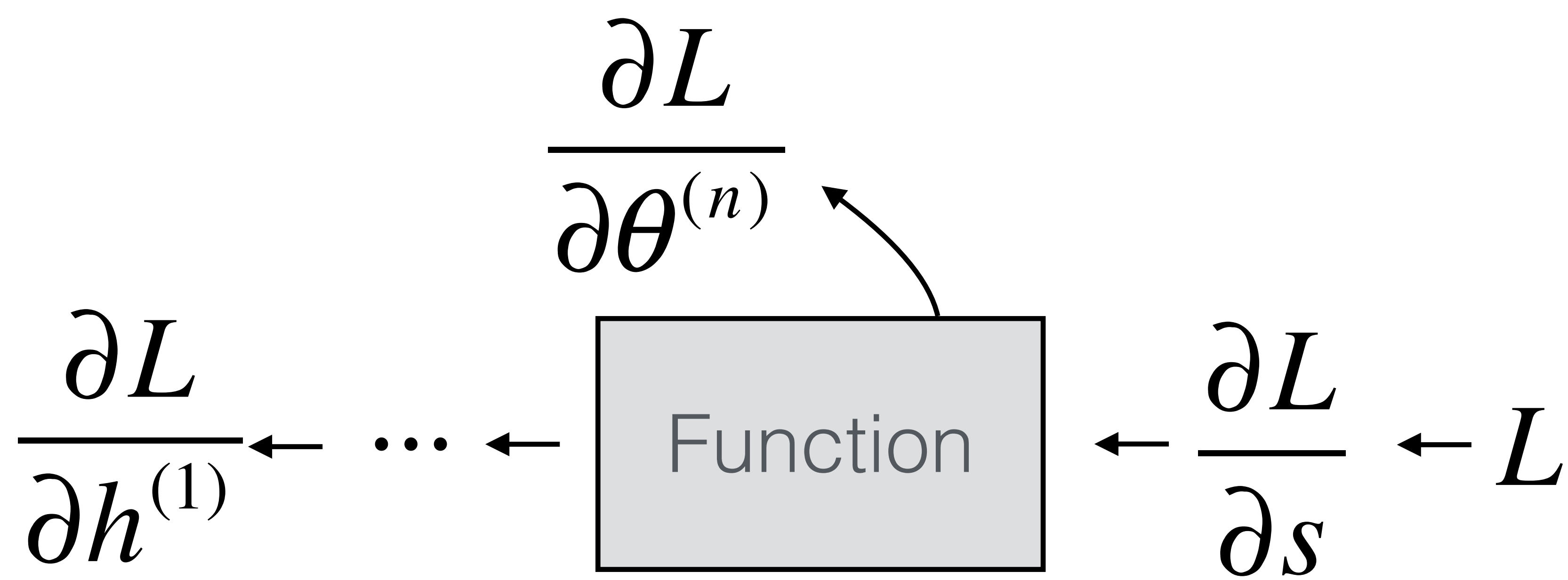

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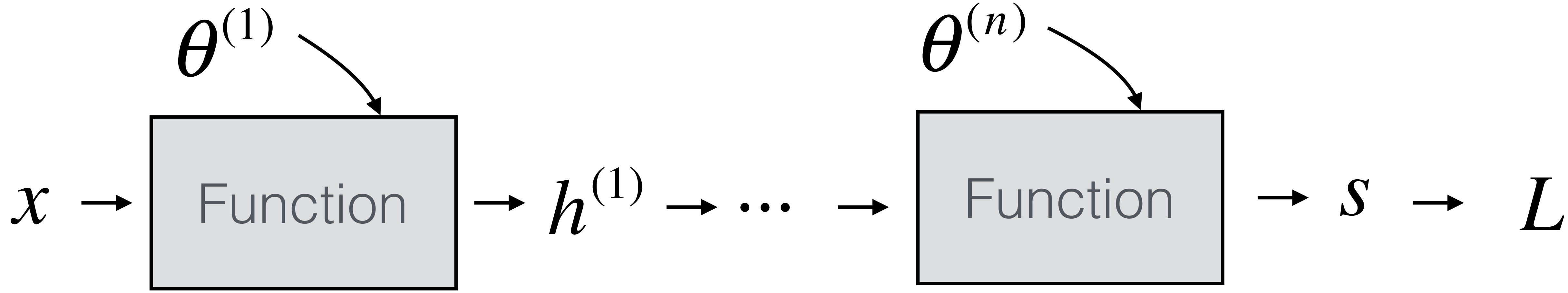

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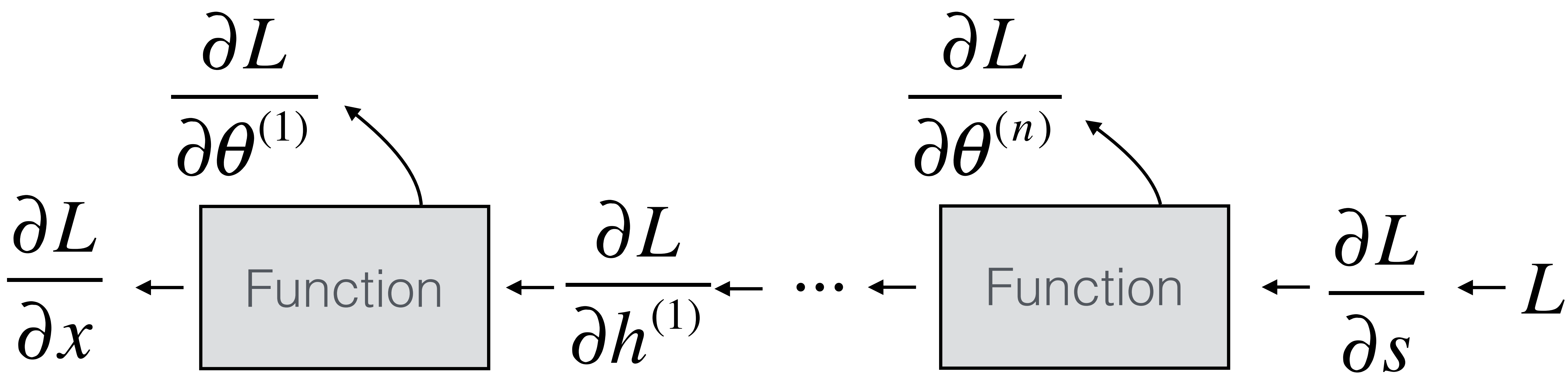
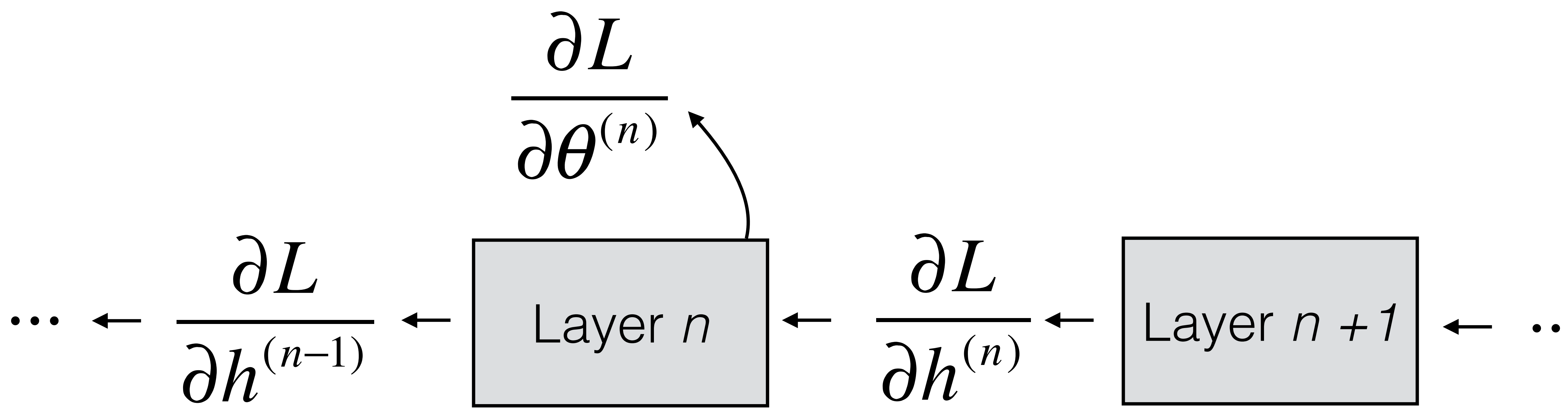
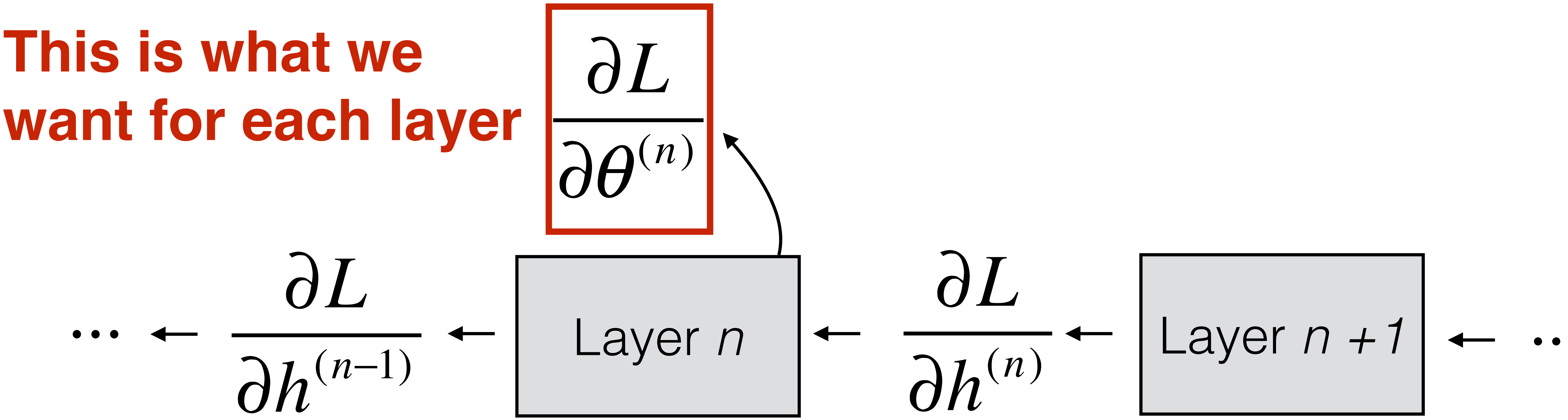


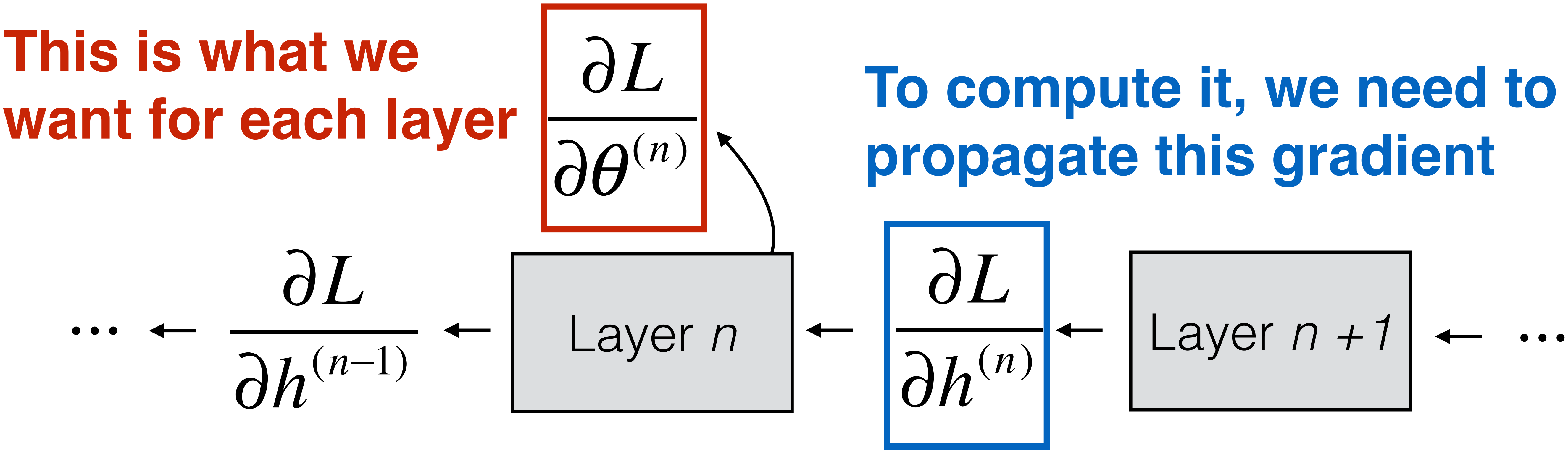
What to do for each layer

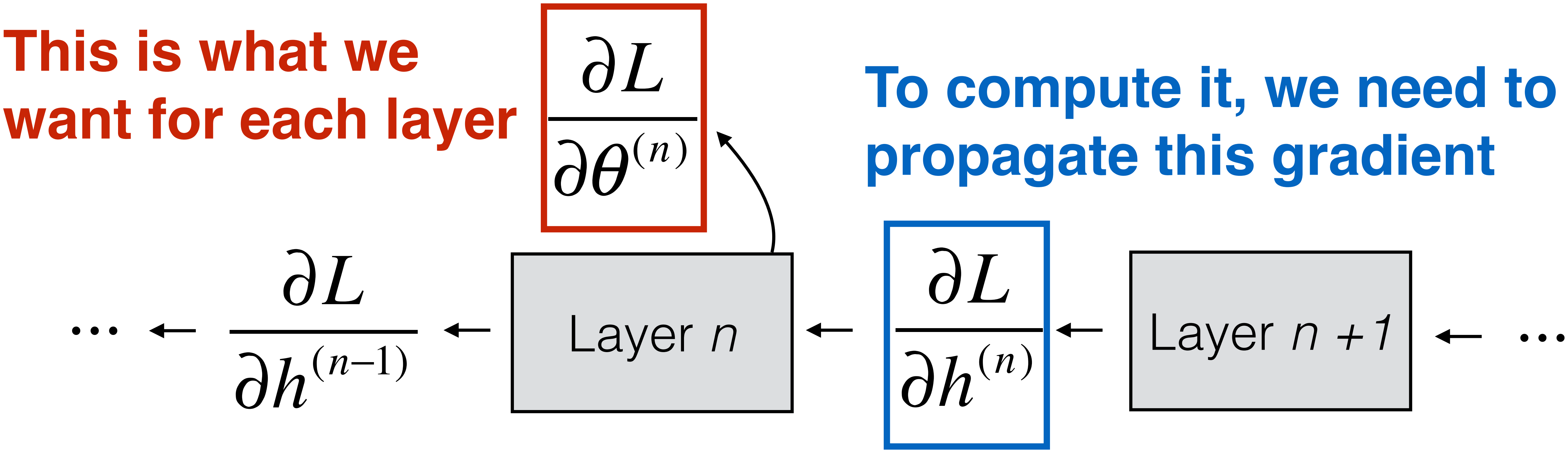


$\frac{\partial L}{\partial \theta^{(n)}}$
...
$\frac{\partial L}{\partial h^{(n-1)}}$
Layer n
$\frac{\partial L}{\partial h^{(n)}}$
Layer $n+1$
...

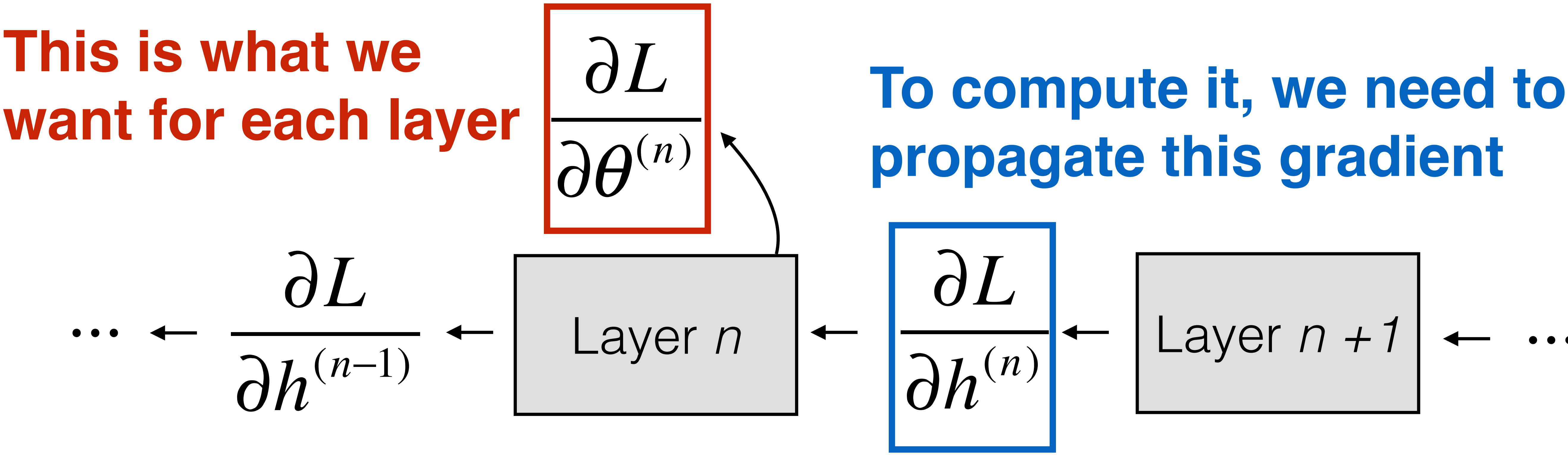


This is what we want for each layer
$\frac{\partial L}{\partial \theta^{(n)}}$
$\cdots \leftarrow \frac{\partial L}{\partial h^{(n-1)}} \leftarrow$
Layer n
$\leftarrow \frac{\partial L}{\partial h^{(n)}} \leftarrow$
Layer $n+1$
$\leftarrow \cdots$


This is what we want for each layer
$\frac{\partial L}{\partial \theta^{(n)}}$
To compute it, we need to propagate this gradient
$\frac{\partial L}{\partial h^{(n-1)}}$
Layer n
$\frac{\partial L}{\partial h^{(n)}}$
Layer $n+1$



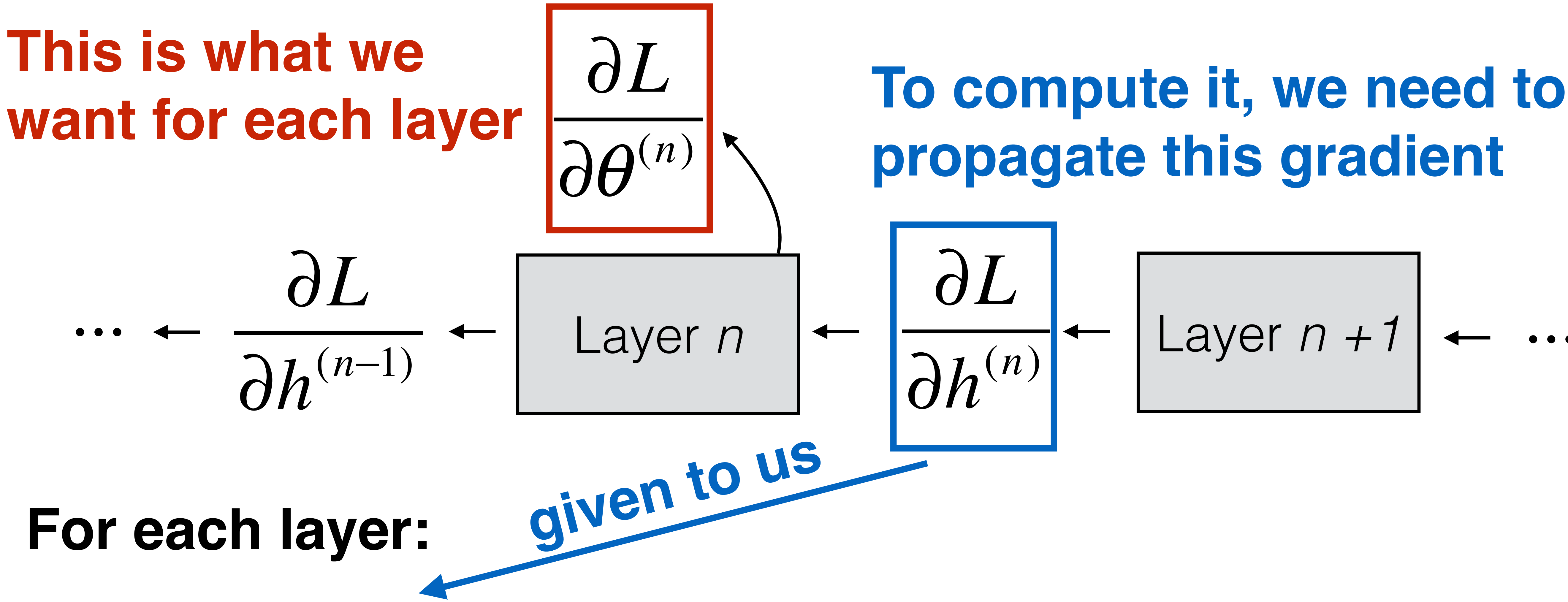
For each layer:



For each layer:

$$\boxed{\frac{\partial L}{\partial \theta^{(n)}}} = \frac{\partial L}{\partial h^{(n)}} \cdot \frac{\partial h^{(n)}}{\partial \theta^{(n)}}$$

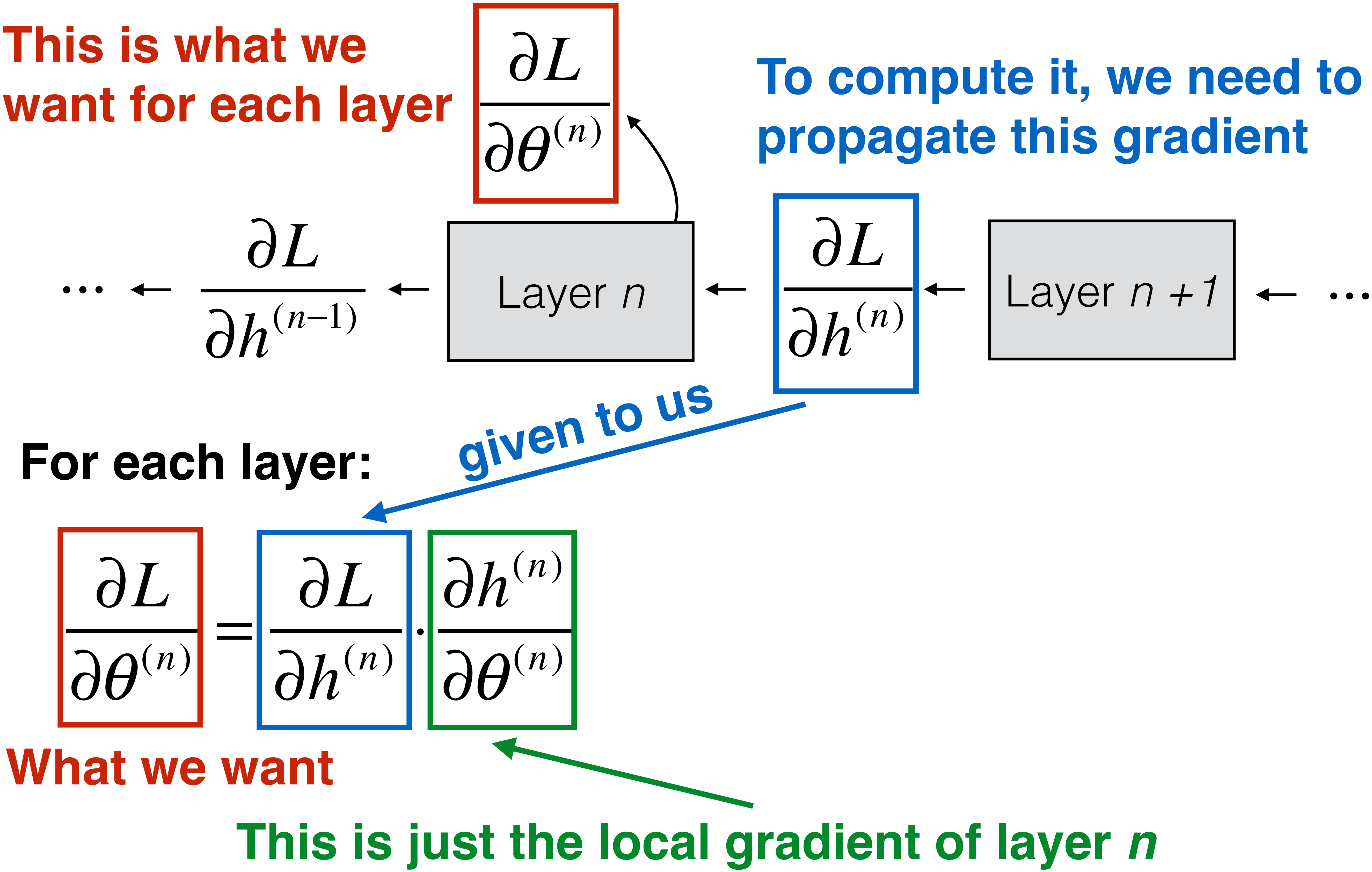
What we want



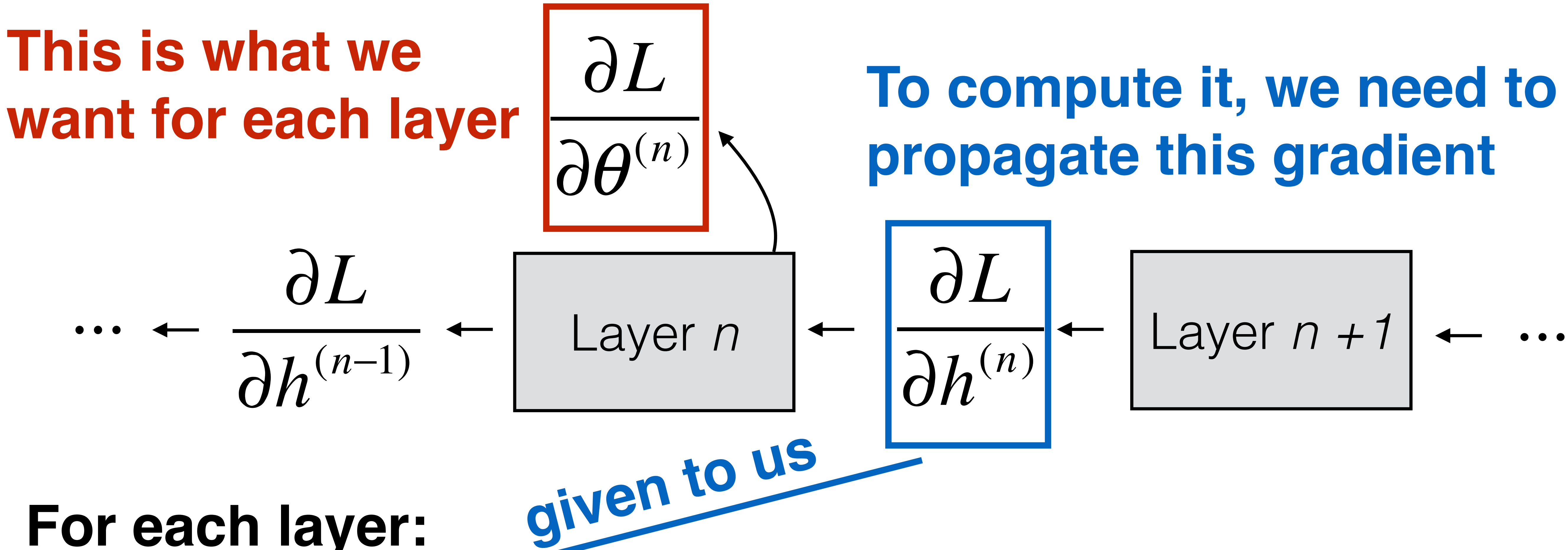
For each layer:

$$\frac{\partial L}{\partial \theta^{(n)}} = \frac{\partial L}{\partial h^{(n)}} \cdot \frac{\partial h^{(n)}}{\partial \theta^{(n)}}$$

What we want



This is what we want for each layer
$\frac{\partial L}{\partial \theta^{(n)}}$
To compute it, we need to propagate this gradient
$\cdots \leftarrow \frac{\partial L}{\partial h^{(n-1)}} \leftarrow$ Layer n $\leftarrow \frac{\partial L}{\partial h^{(n)}} \leftarrow$ Layer $n+1$ $\leftarrow \cdots$
given to us
For each layer:
$\frac{\partial L}{\partial \theta^{(n)}} = \frac{\partial L}{\partial h^{(n)}} \cdot \frac{\partial h^{(n)}}{\partial \theta^{(n)}}$
What we want
This is just the local gradient of layer *n*



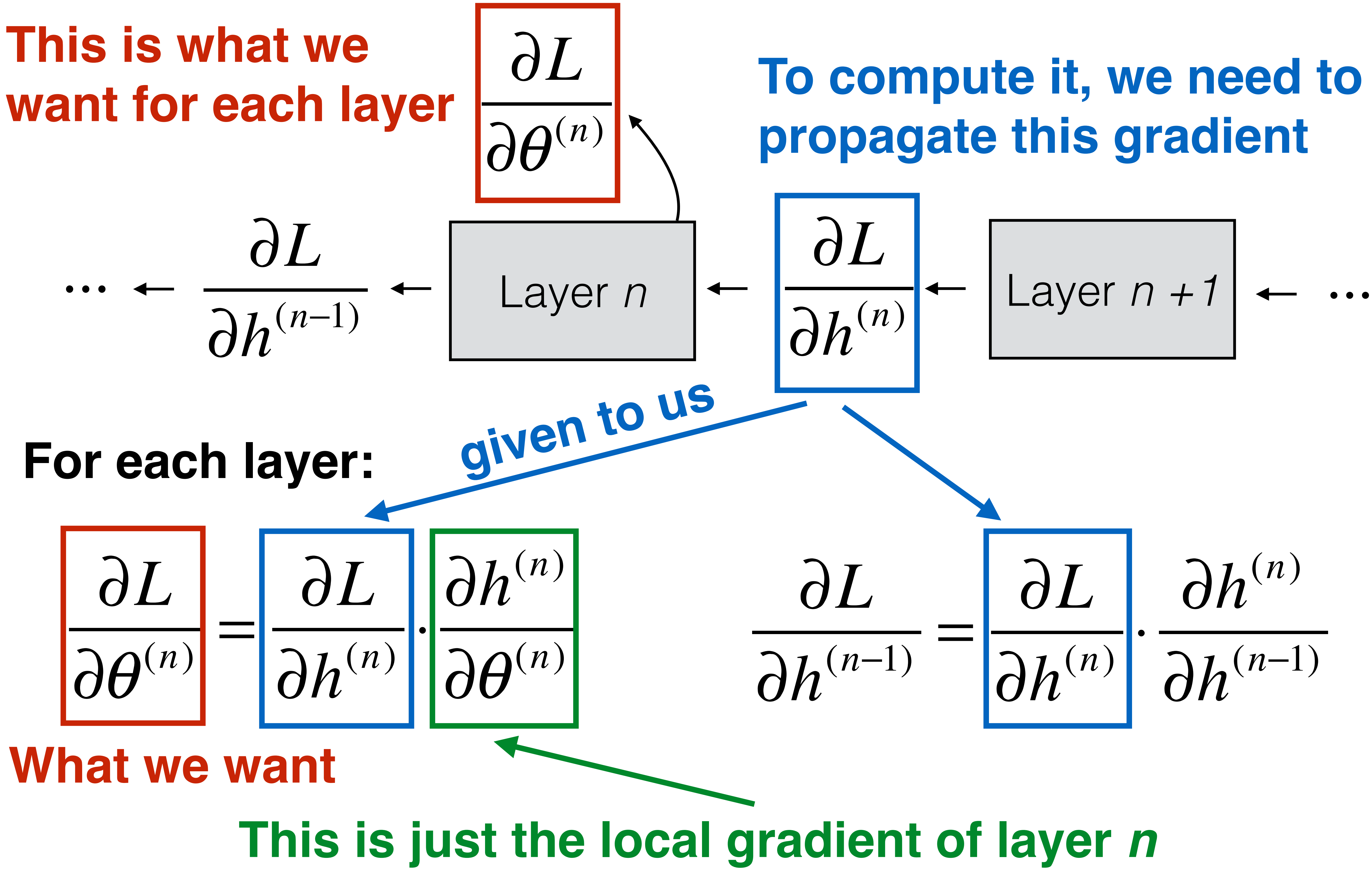
For each layer:

$$\frac{\partial L}{\partial \theta^{(n)}} = \frac{\partial L}{\partial h^{(n)}} \cdot \frac{\partial h^{(n)}}{\partial \theta^{(n)}}$$

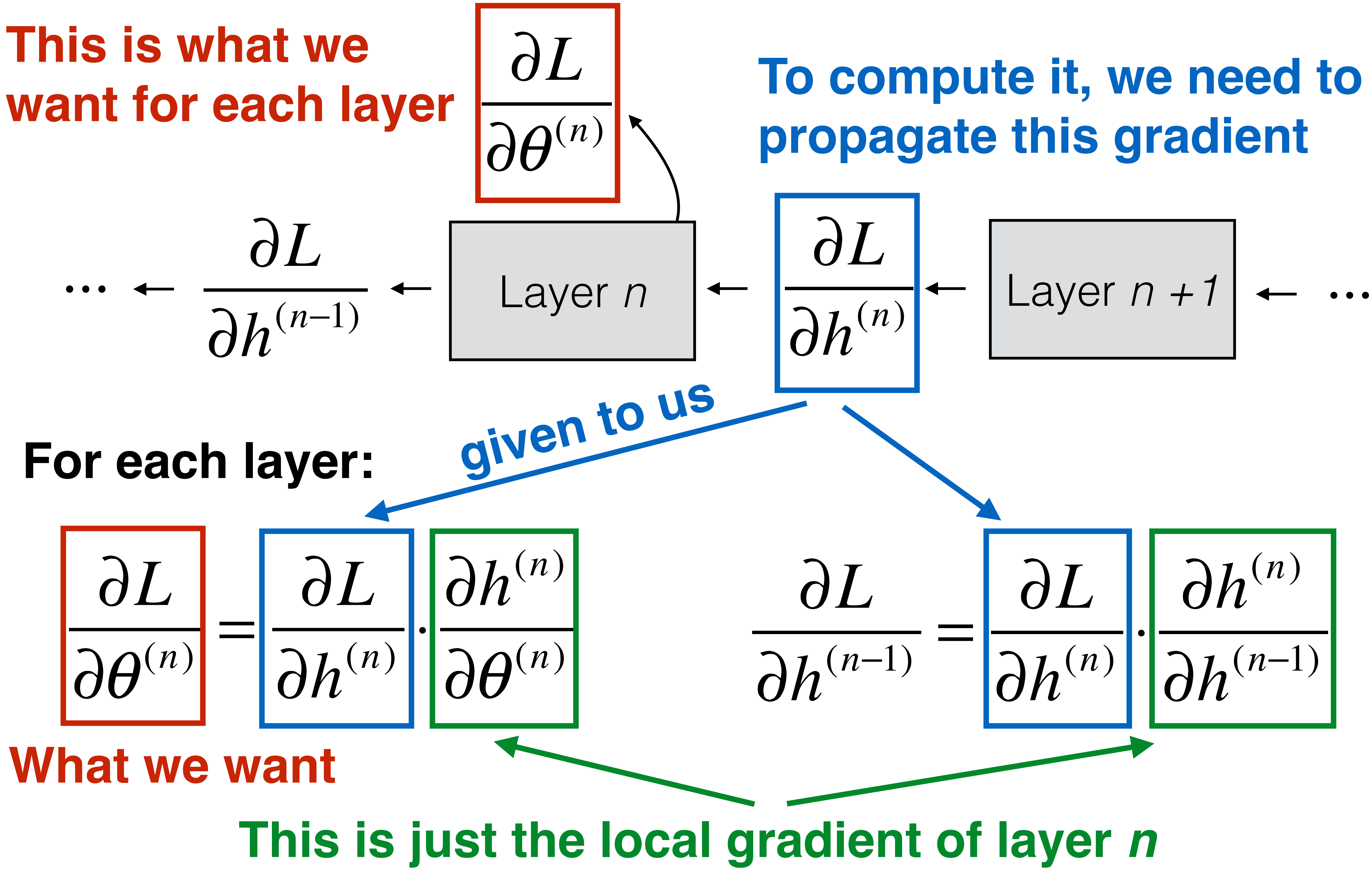
What we want

This is just the local gradient of layer *n*

$$\frac{\partial L}{\partial h^{(n-1)}} = \frac{\partial L}{\partial h^{(n)}} \cdot \frac{\partial h^{(n)}}{\partial h^{(n-1)}}$$



This is what we want for each layer
$\frac{\partial L}{\partial \theta^{(n)}}$
To compute it, we need to propagate this gradient
$\cdots \leftarrow \frac{\partial L}{\partial h^{(n-1)}} \leftarrow$ Layer n $\leftarrow \frac{\partial L}{\partial h^{(n)}} \leftarrow$ Layer $n+1$ $\leftarrow \cdots$
given to us
For each layer:
$\frac{\partial L}{\partial \theta^{(n)}} = \frac{\partial L}{\partial h^{(n)}} \cdot \frac{\partial h^{(n)}}{\partial \theta^{(n)}}$
$\frac{\partial L}{\partial h^{(n-1)}} = \frac{\partial L}{\partial h^{(n)}} \cdot \frac{\partial h^{(n)}}{\partial h^{(n-1)}}$
What we want
This is just the local gradient of layer n


This is what we want for each layer
$\frac{\partial L}{\partial \theta^{(n)}}$
To compute it, we need to propagate this gradient
$\cdots \leftarrow \frac{\partial L}{\partial h^{(n-1)}} \leftarrow$ Layer n $\leftarrow \frac{\partial L}{\partial h^{(n)}} \leftarrow$ Layer $n+1$ $\leftarrow \cdots$
given to us
For each layer:
$\frac{\partial L}{\partial \theta^{(n)}} = \frac{\partial L}{\partial h^{(n)}} \cdot \frac{\partial h^{(n)}}{\partial \theta^{(n)}}$
$\frac{\partial L}{\partial h^{(n-1)}} = \frac{\partial L}{\partial h^{(n)}} \cdot \frac{\partial h^{(n)}}{\partial h^{(n-1)}}$
What we want
This is just the local gradient of layer *n*

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For each layer, we compute:

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(Received during backprop) (Can compute immediately)

30s cat picture break

http://stylonica.com/cat-pictures/

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x, h 3D arrays

Examples

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(for a single input)

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- Derive it by hand, and check it numerically

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This also works:

```
def forward(X):
    return X - np.mean(X, axis=1, keepdims=True)
```

Example: Mean Subtraction

(for a single input)

The backward pass is easy:

```
def backward(dh):
    return forward(dh)
```

(Remember they're usually not the same)

Example: Euclidean Loss

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- Euclidean loss layer:

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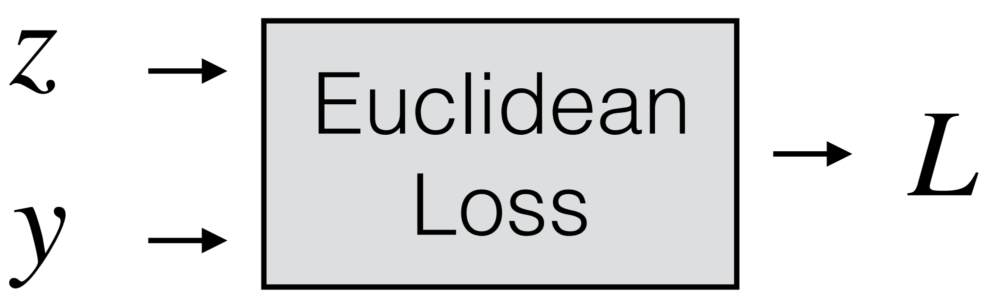

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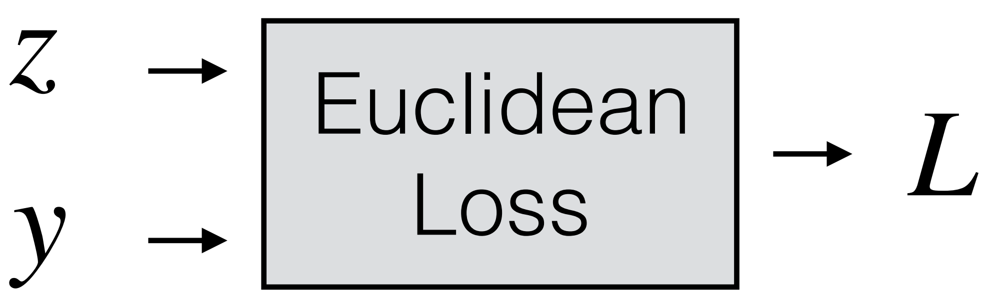

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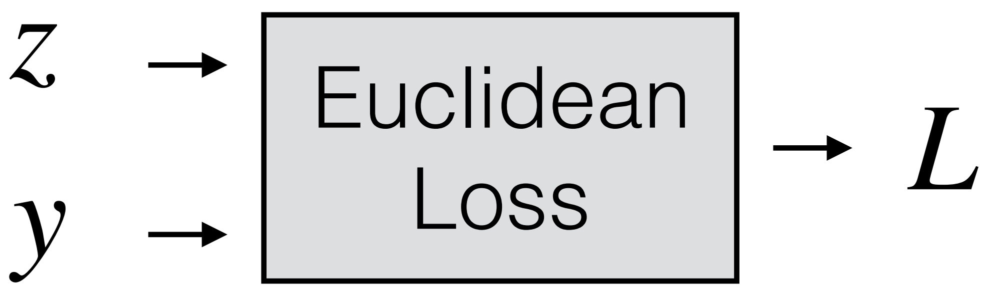

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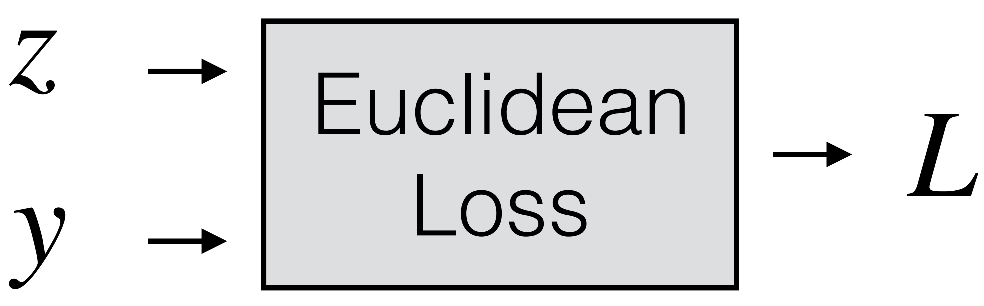

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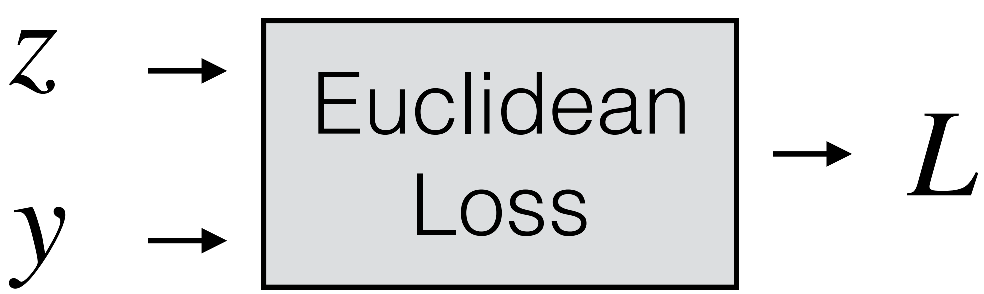

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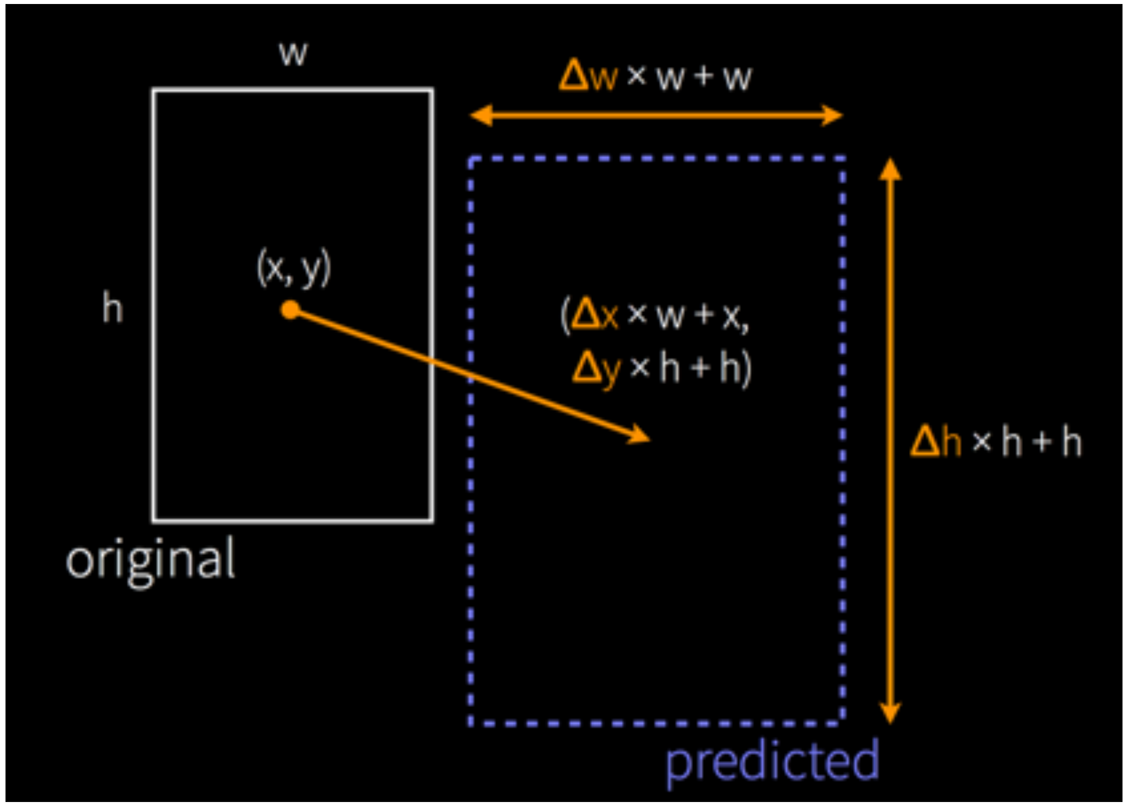

Bounding box regression from the R-CNN object detector [Girshick 2014]

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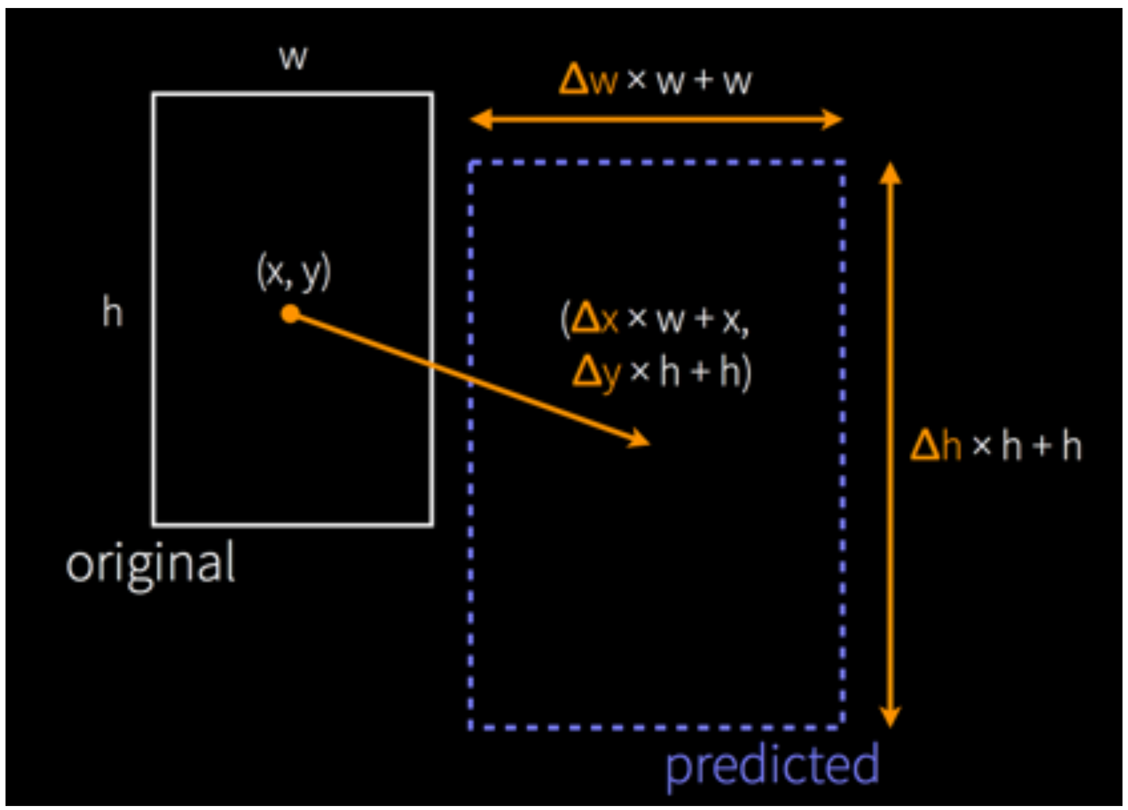

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- **Note:** Can be unstable and other losses often work better. Alternatives: L1 distance (instead of L2), discretizing into category bins and using softmax

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(note that this is with respect to Li, not L)

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(You should be able to derive this)

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It’s a loss function for predicting categories?

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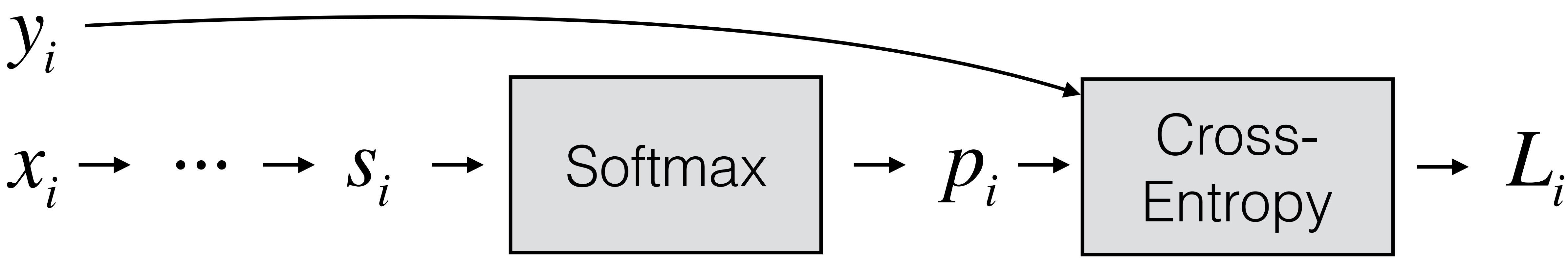

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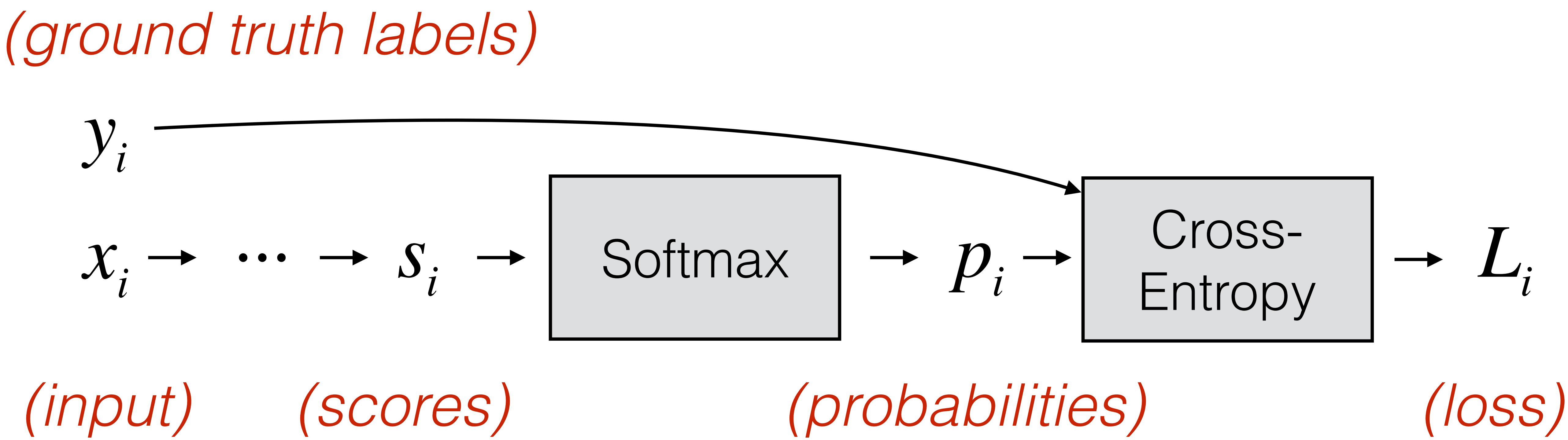

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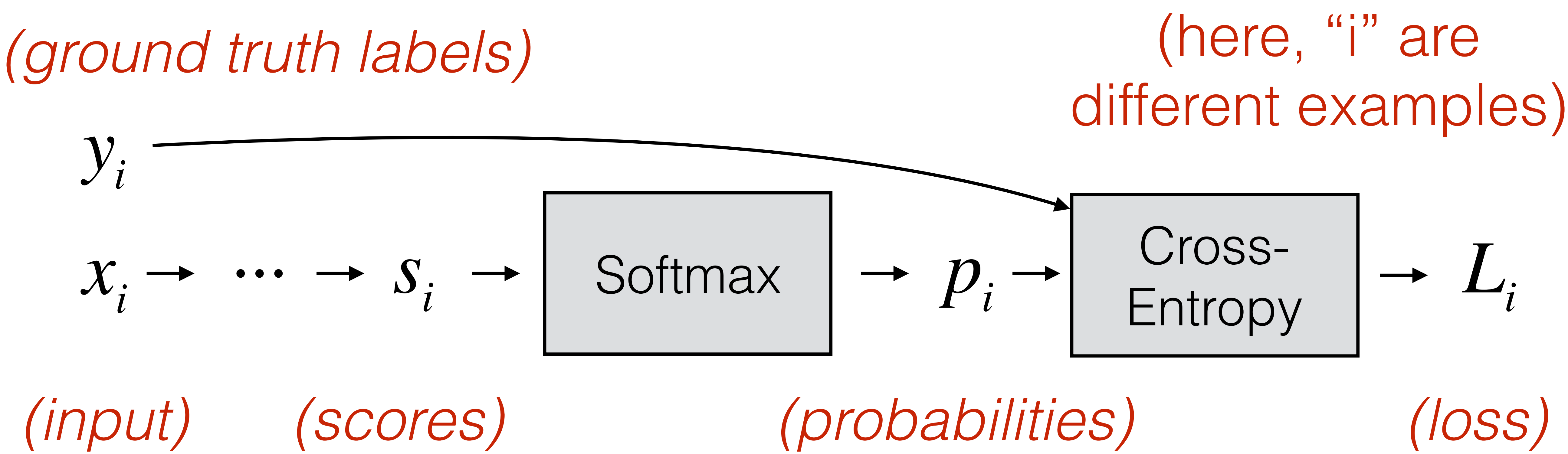

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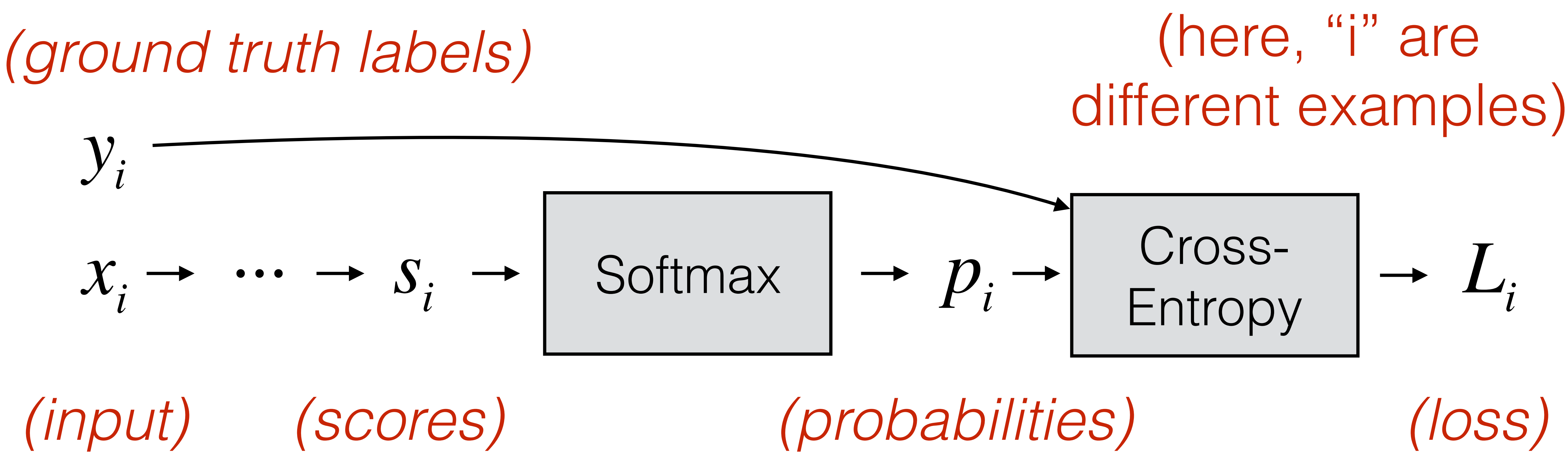

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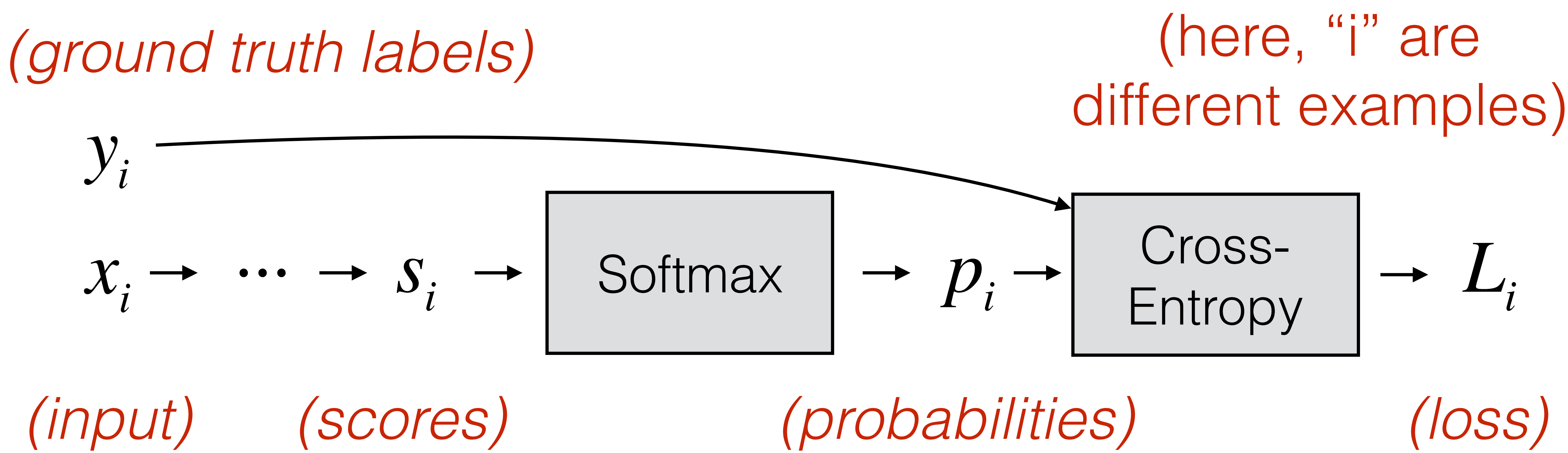

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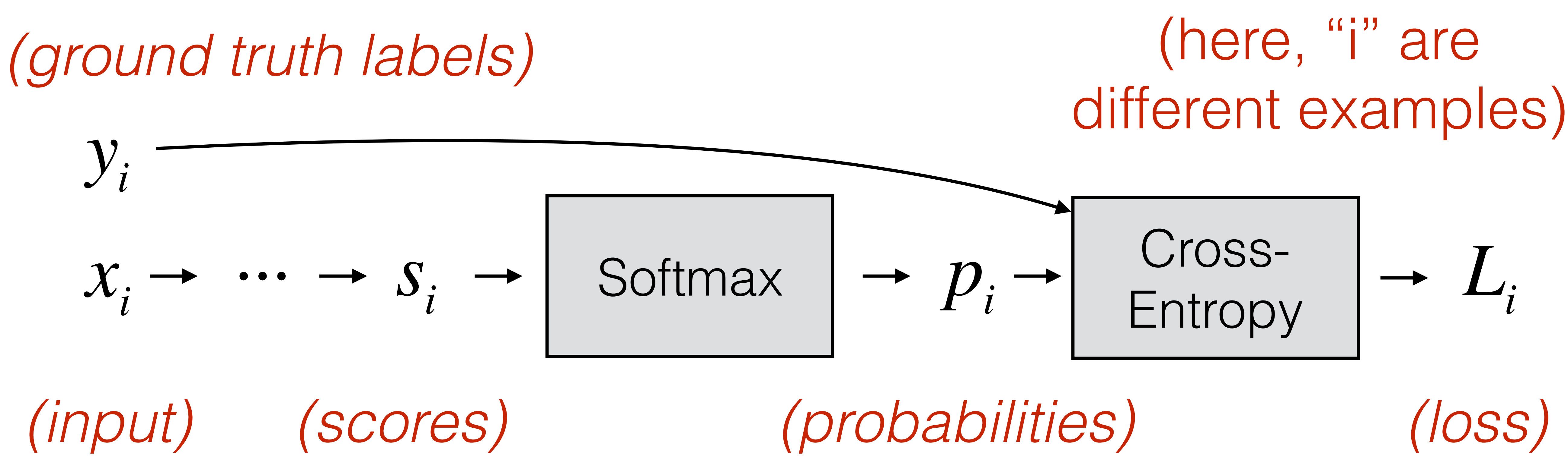

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(Avg. over examples)

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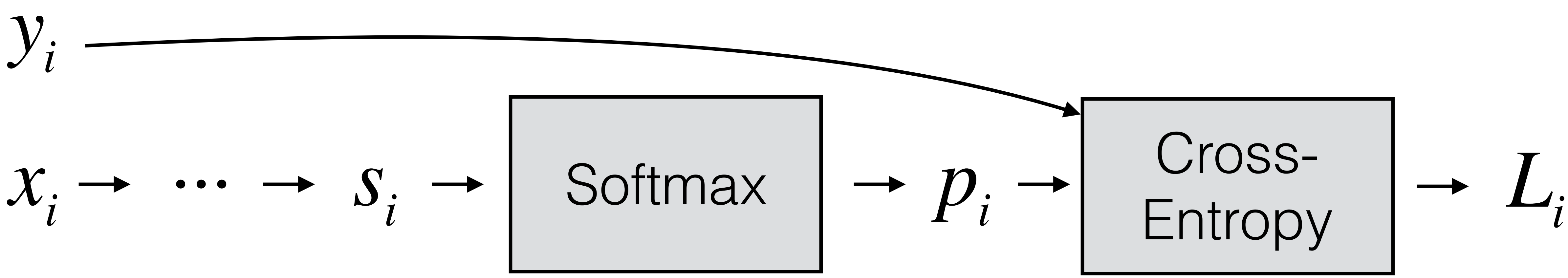

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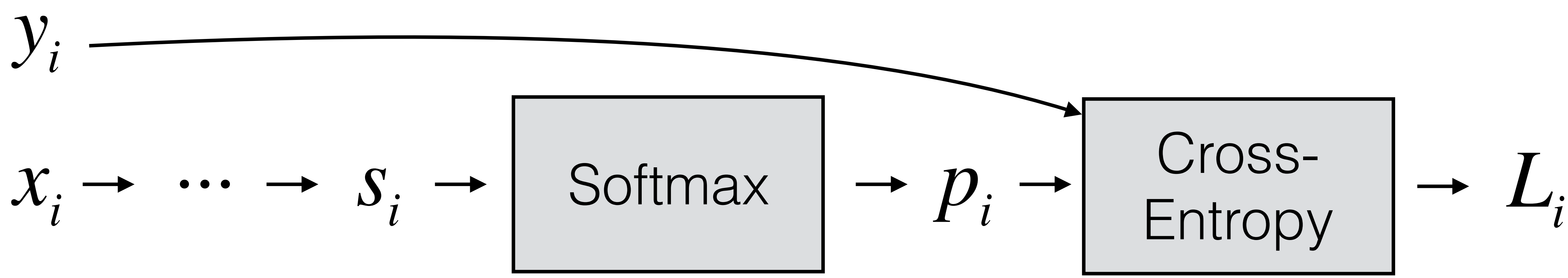

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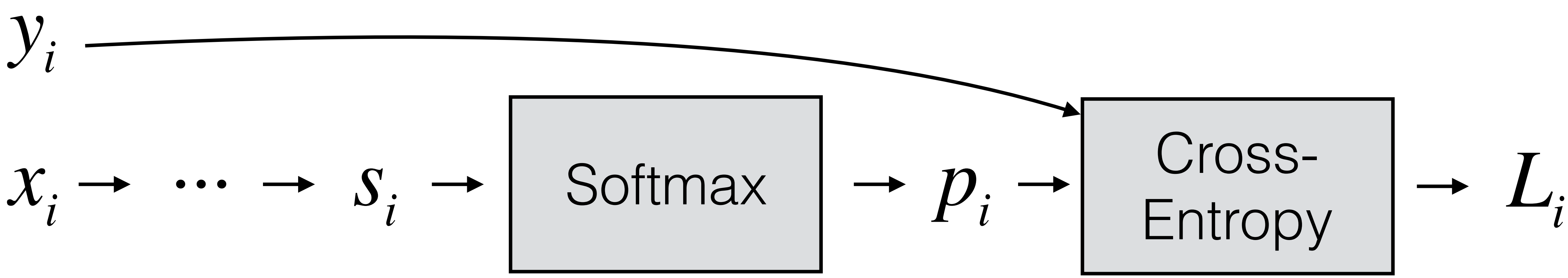

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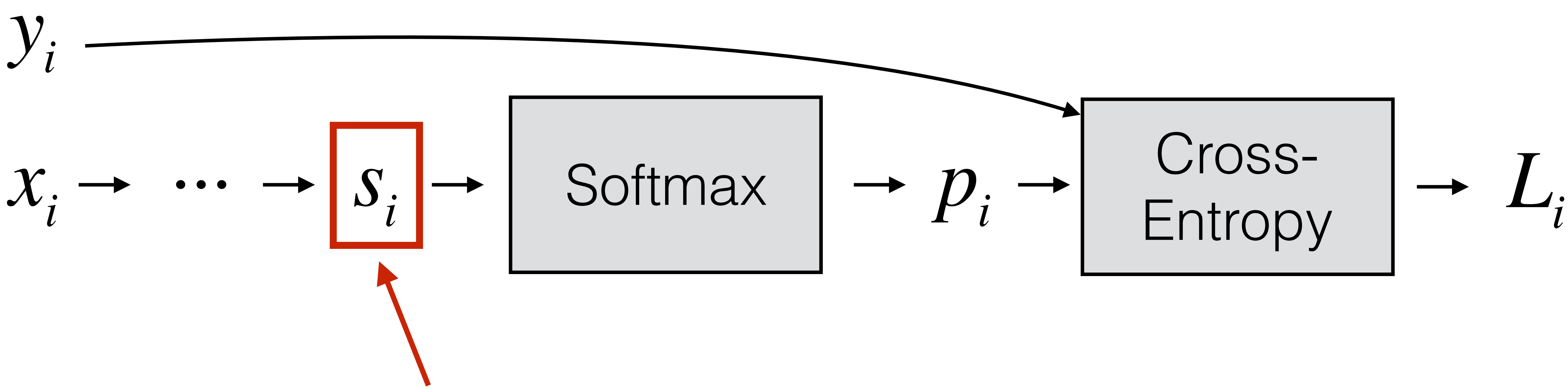

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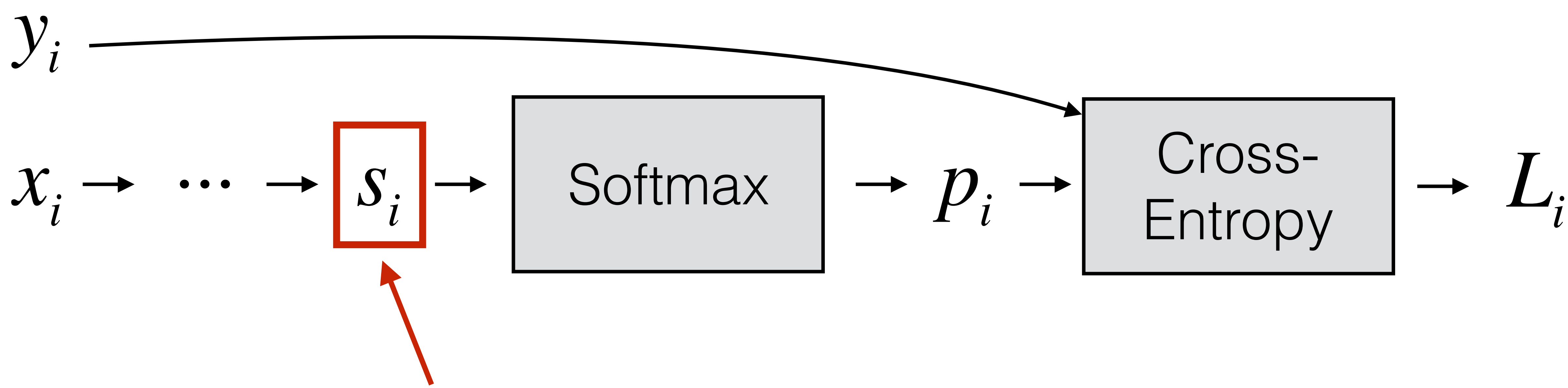

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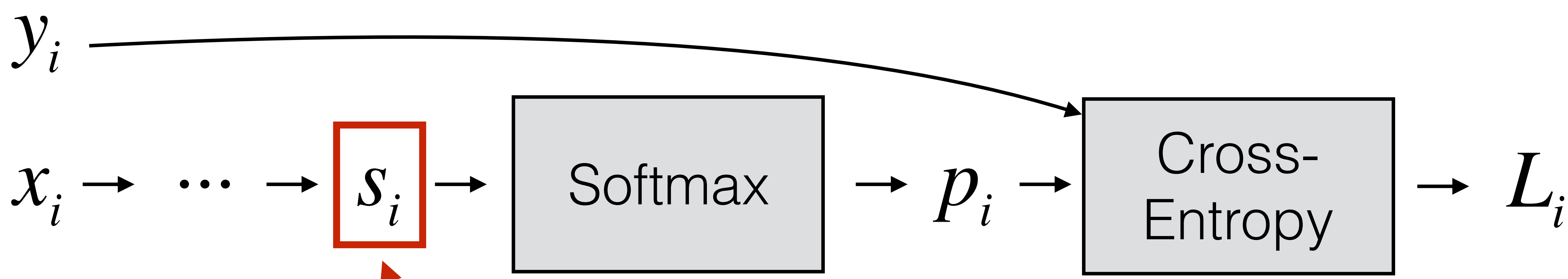
(You will derive this in PA5)

Example: Softmax (for N inputs)



Derivative: $$\frac{\partial L}{\partial s_{i,j}} = \frac{p_{i,j} - t_{i,j}}{N}$$ where $t_i = [0 \; \ldots \; 1 \ldots 0]$

(Entry y_i set to 1)

(You will derive this in PA5)

Now we can continue backpropagating to the layer before “f”

What about the weights?

To get the derivative of the weights, use the chain rule again!

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Example: 2D weights, 1D bias, 1D hidden activations:

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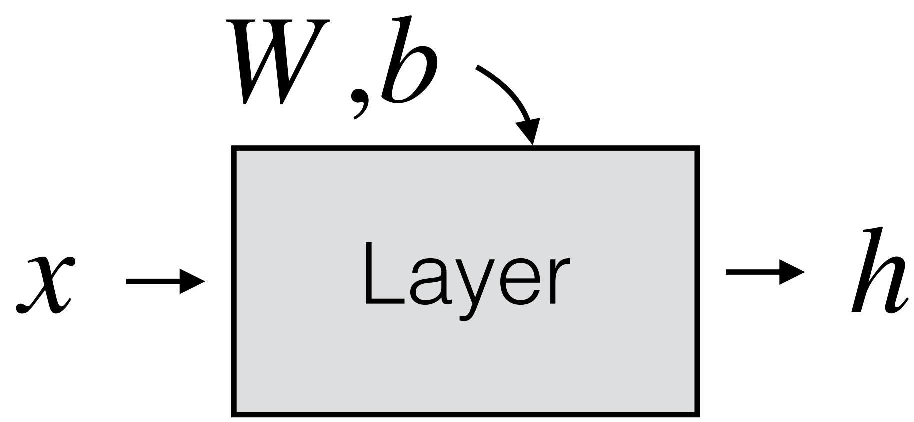

$$h = h(x; W)$$

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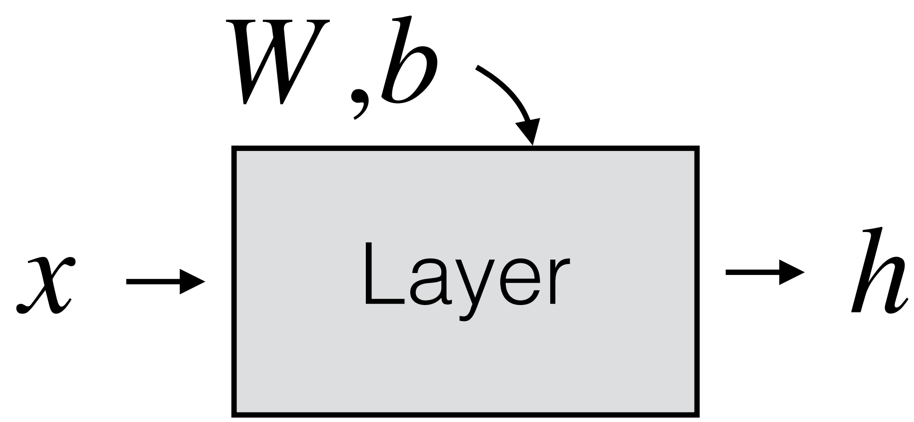
Example: 2D weights, 1D bias, 1D hidden activations:



$$h = h(x; W)$$

$$\frac{\partial L}{\partial W_{ij}} = \sum_k \frac{\partial L}{\partial h_k} \frac{\partial h_k}{\partial W_{ij}}$$

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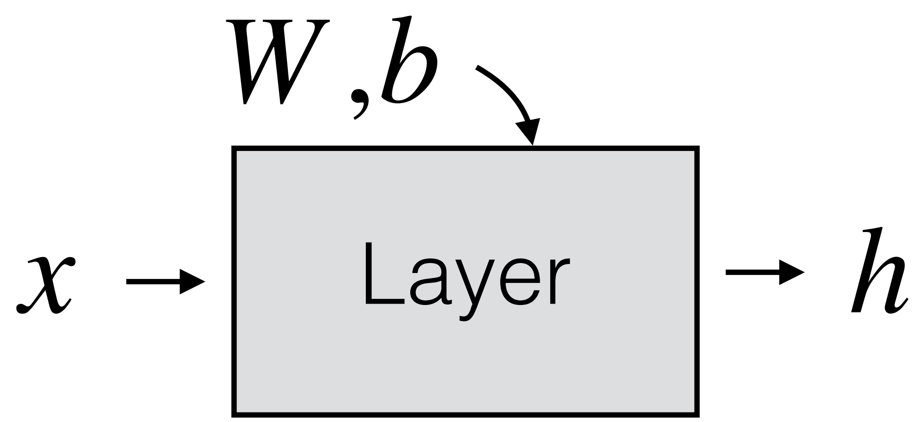

$$h = h(x; W)$$

$$\frac{\partial L}{\partial W_{ij}} = \sum_k \frac{\partial L}{\partial h_k} \frac{\partial h_k}{\partial W_{ij}}$$

$$\frac{\partial L}{\partial b_i} = \sum_k \frac{\partial L}{\partial h_k} \frac{\partial h_k}{\partial b_i}$$

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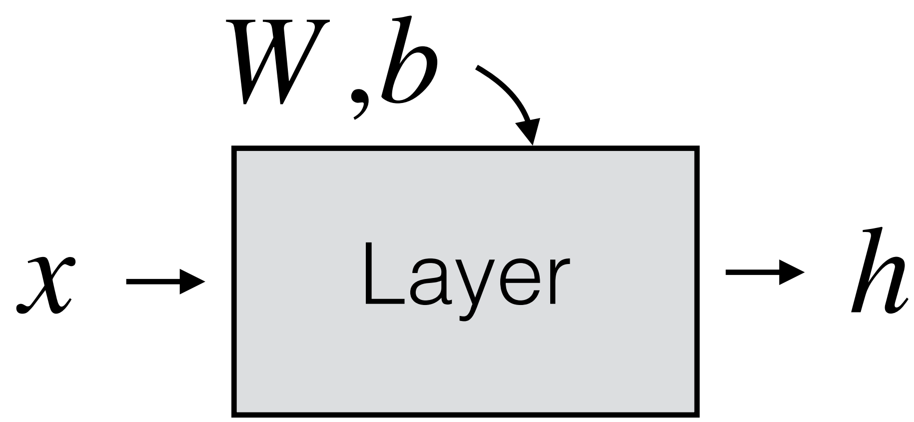
Example: 2D weights, 1D bias, 1D hidden activations:



$$h = h(x; W)$$

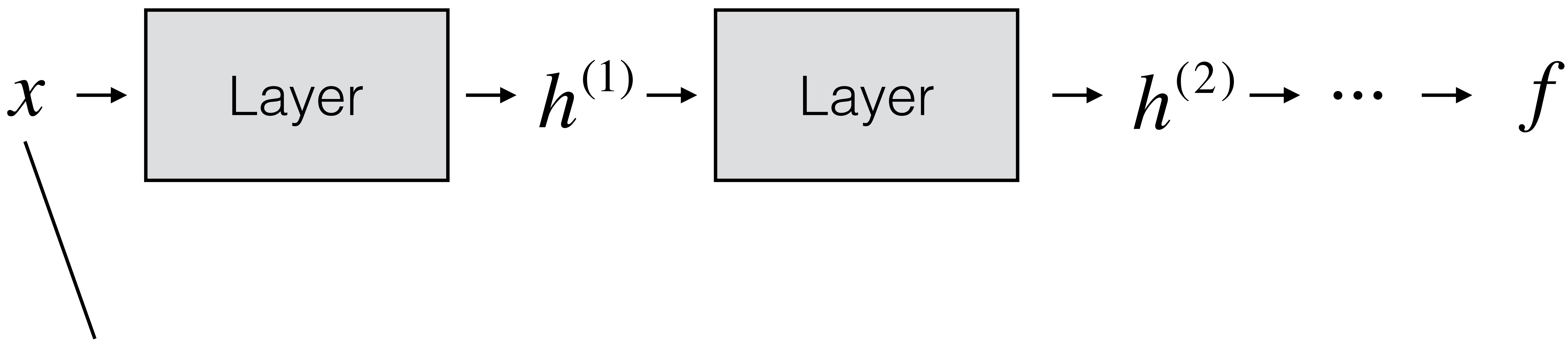
$$\frac{\partial L}{\partial W_{ij}} = \sum_k \frac{\partial L}{\partial h_k} \frac{\partial h_k}{\partial W_{ij}} \qquad \frac{\partial L}{\partial b_i} = \sum_k \frac{\partial L}{\partial h_k} \frac{\partial h_k}{\partial b_i}$$

(the number of subscripts and summations changes depending on your layer and parameter sizes)

ConvNets

They're just neural networks with
3D activations and weight sharing

What shape should the activations have?



- The input is an image, which is 3D (RGB channel, height, width)

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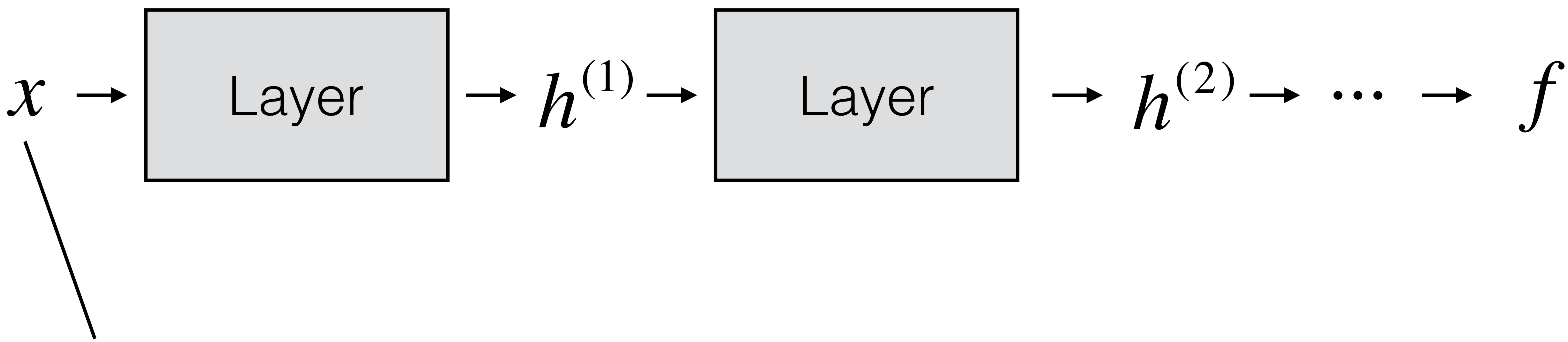
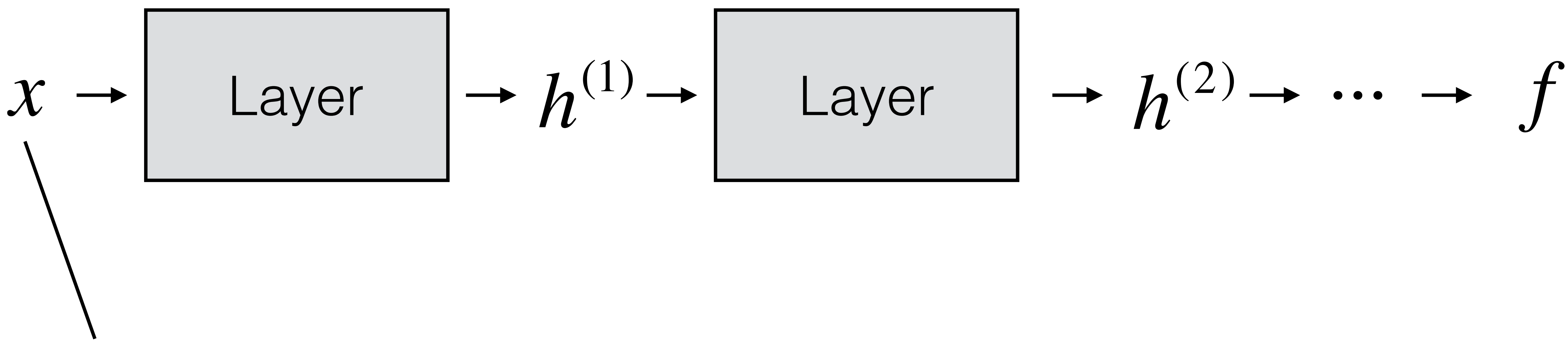


- The input is an image, which is 3D (RGB channel, height, width)

- We could flatten it to a 1D vector, but then we lose structure

What shape should the activations have?



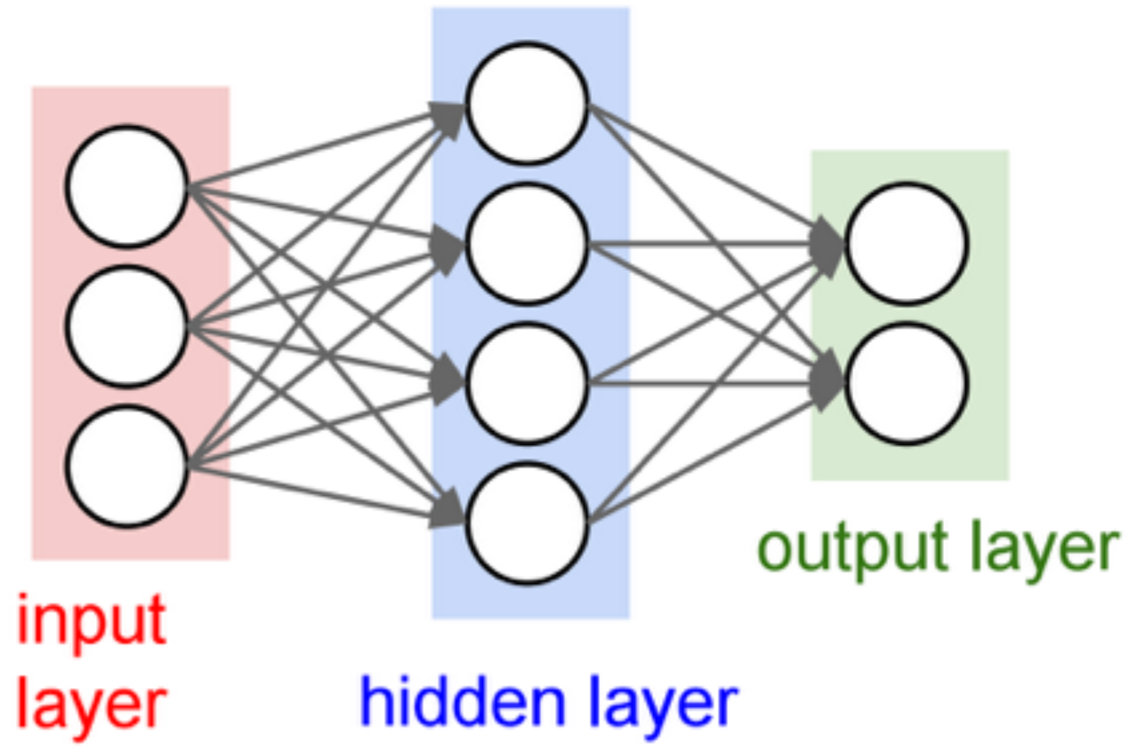
- The input is an image, which is 3D (RGB channel, height, width)

- We could flatten it to a 1D vector, but then we lose structure

- What about keeping everything in 3D?

3D Activations

before:

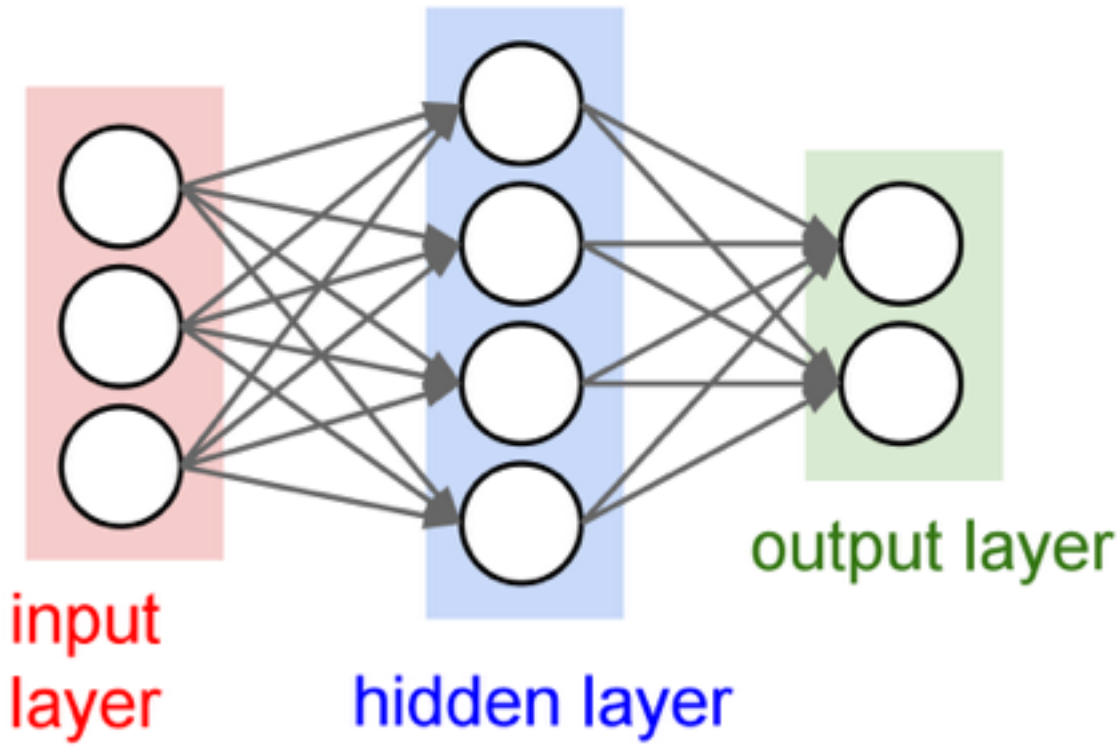


(1D vectors)

Figure: Andrej Karpathy

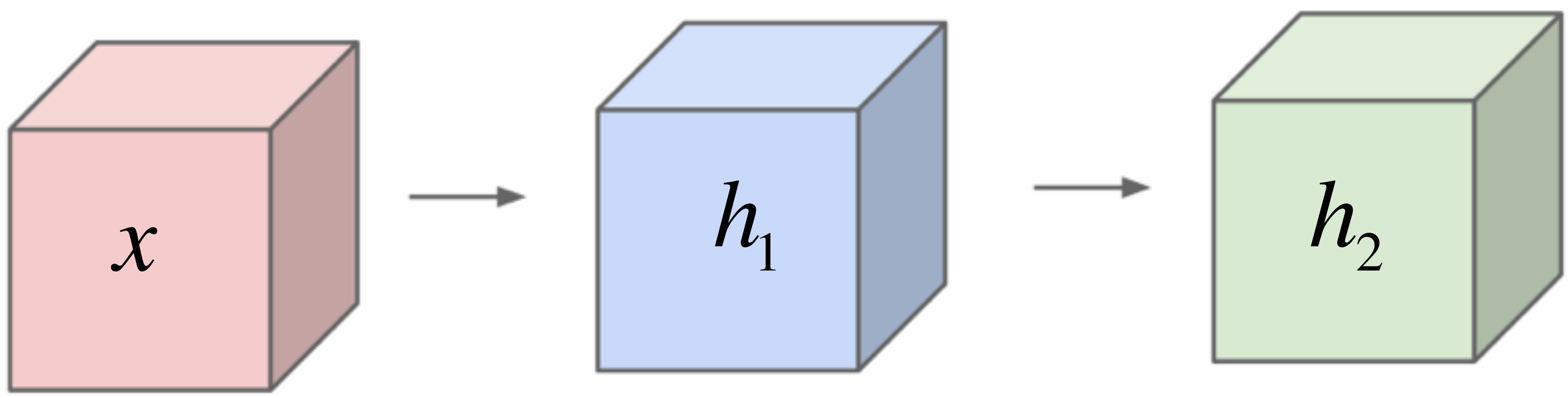
3D Activations

before:



(1D vectors)

now:



(3D arrays)

Figure: Andrej Karpathy

3D Activations

All Neural Net activations arranged in **3 dimensions:**

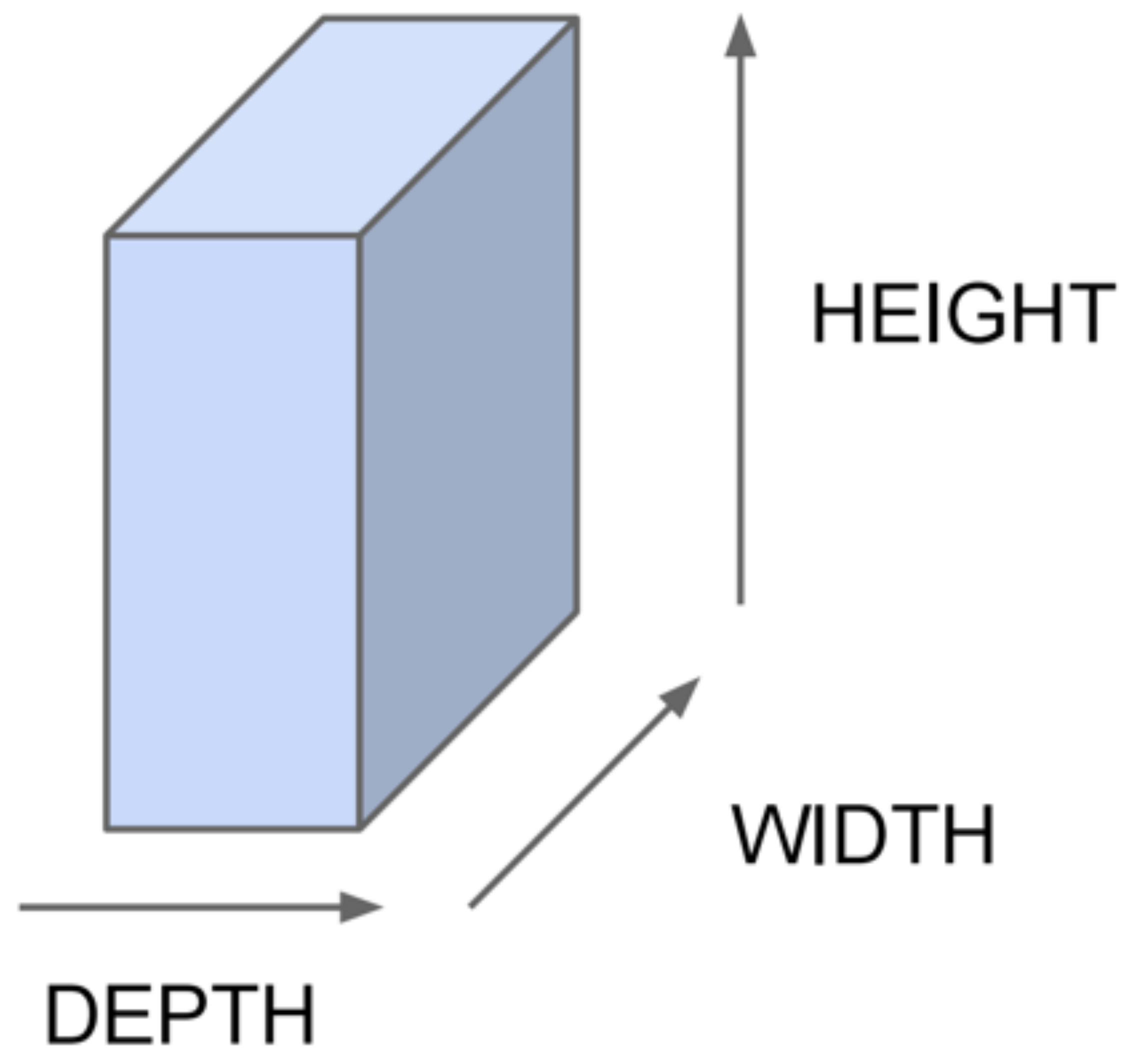


Figure: Andrej Karpathy

3D Activations

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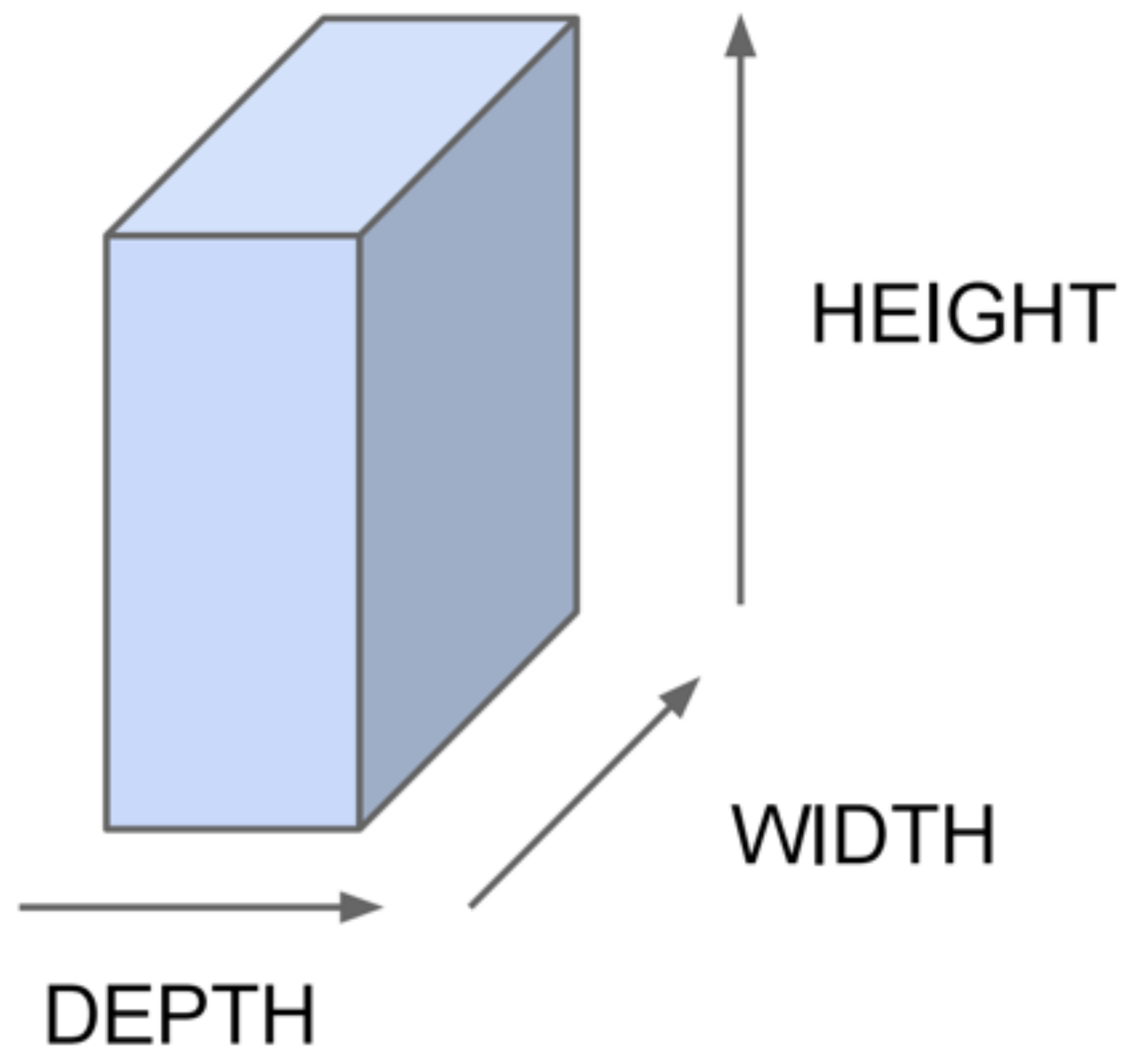


For example, a CIFAR-10 image is a 3x32x32 volume (3 depth — RGB channels, 32 height, 32 width)

Figure: Andrej Karpathy

3D Activations

1D Activations:

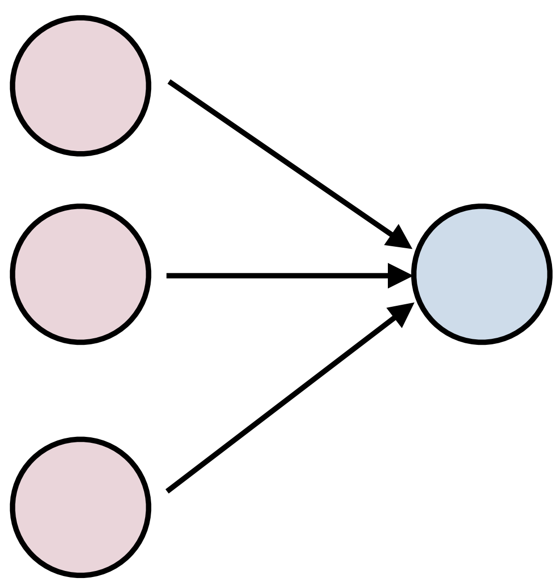
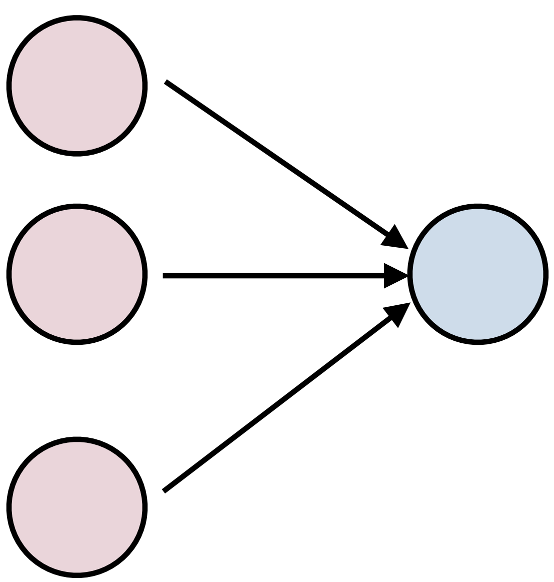


Figure: Andrej Karpathy

3D Activations

1D Activations:



3D Activations:

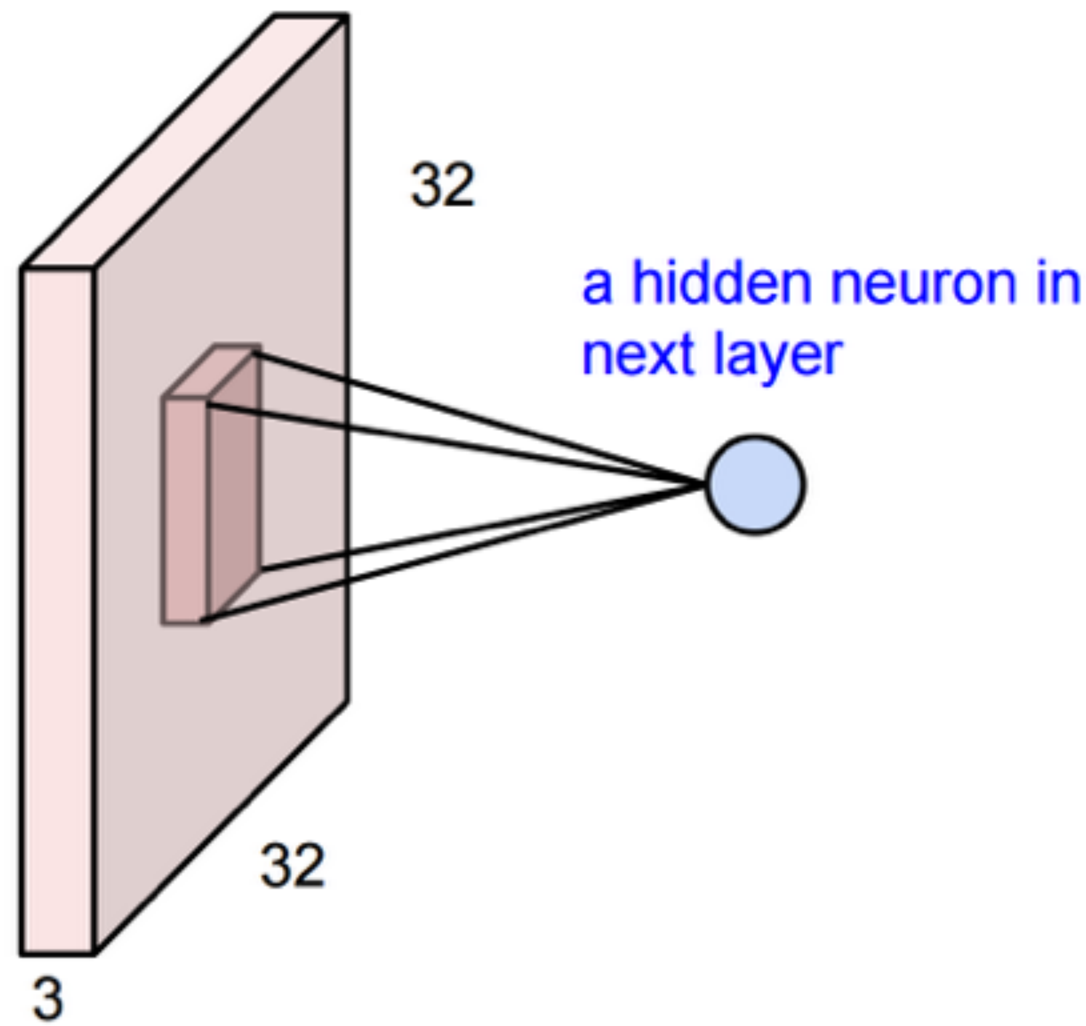
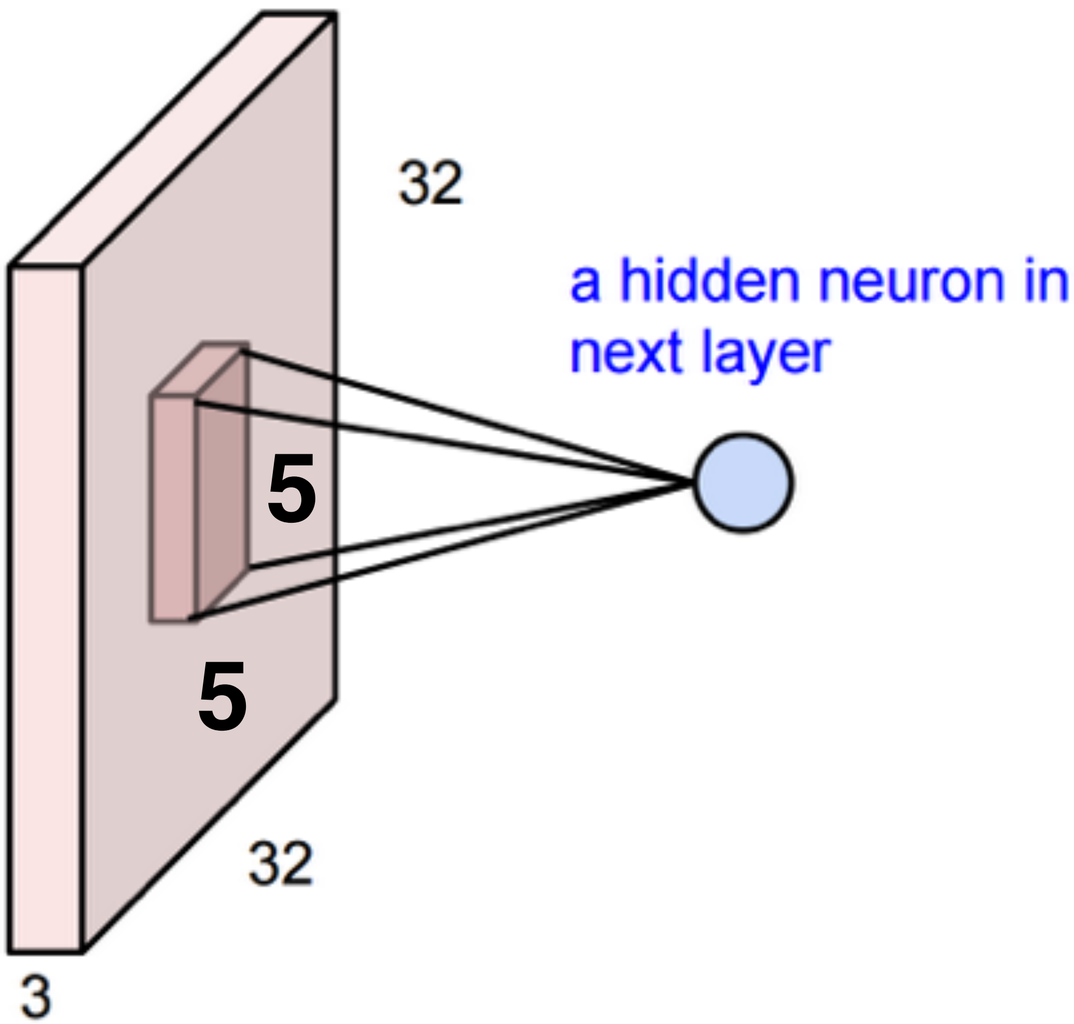


Figure: Andrej Karpathy

3D Activations



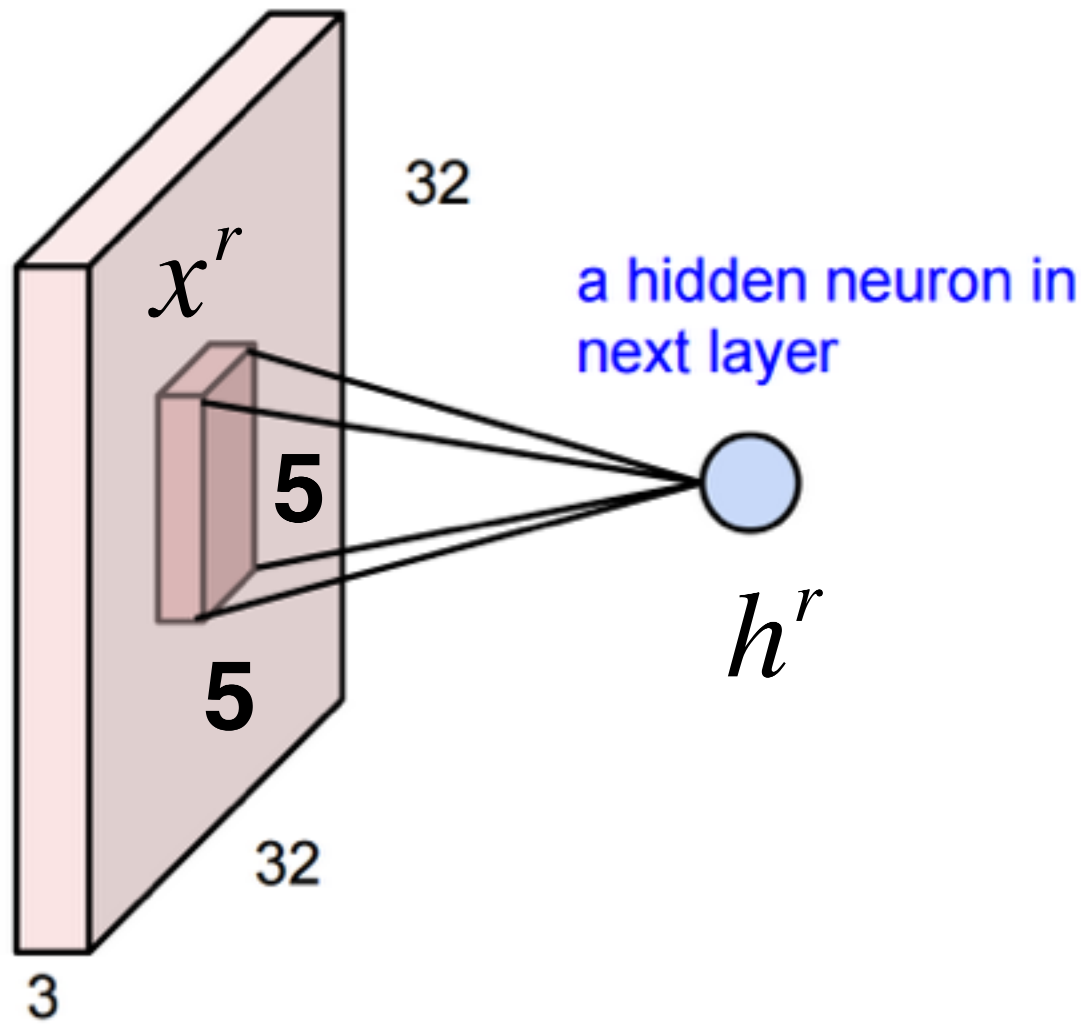
- The input is 3x32x32

- This neuron depends on a 3x5x5 chunk of the input

- The neuron also has a 3x5x5 set of weights and a bias (scalar)

Figure: Andrej Karpathy

3D Activations

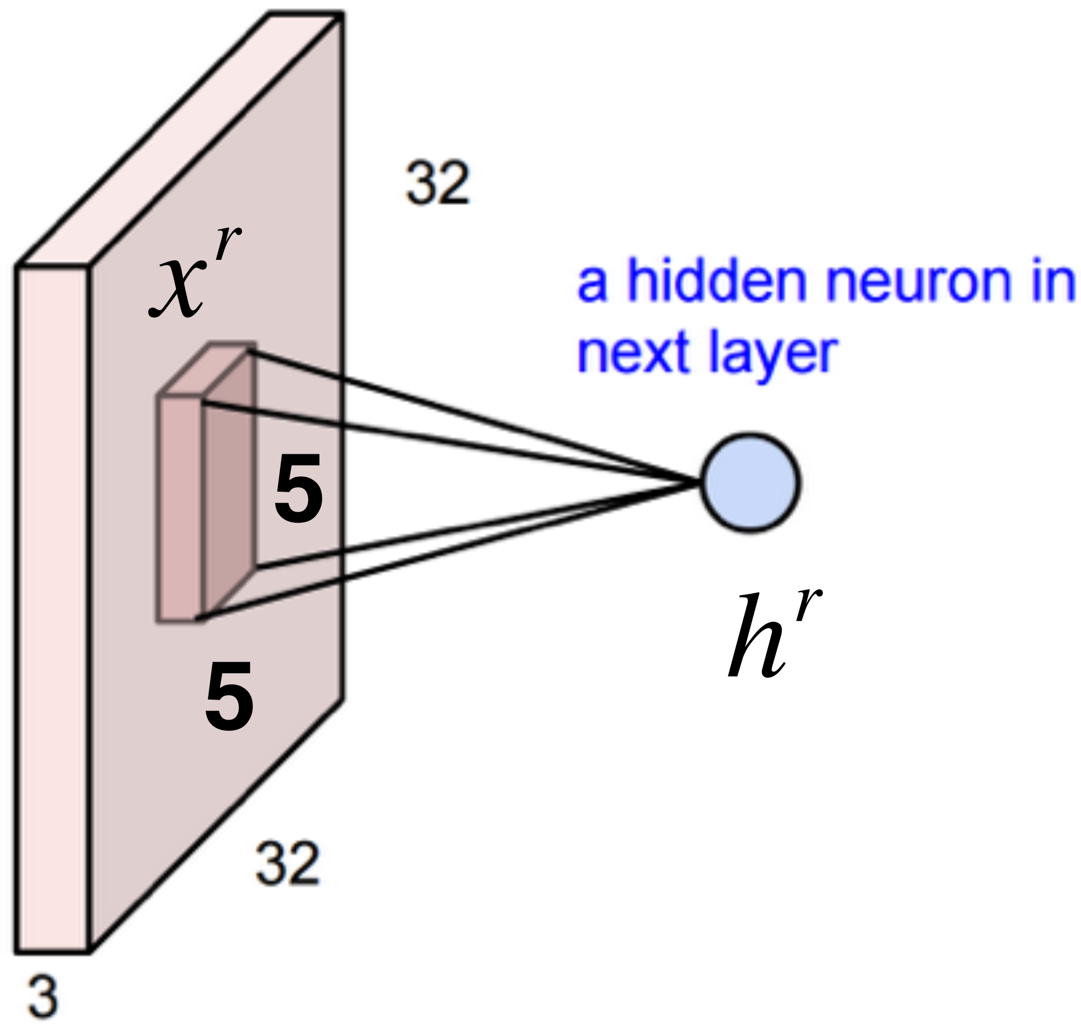


Example: consider the region of the input "x^r"

With output neuron h^r

Figure: Andrej Karpathy

3D Activations



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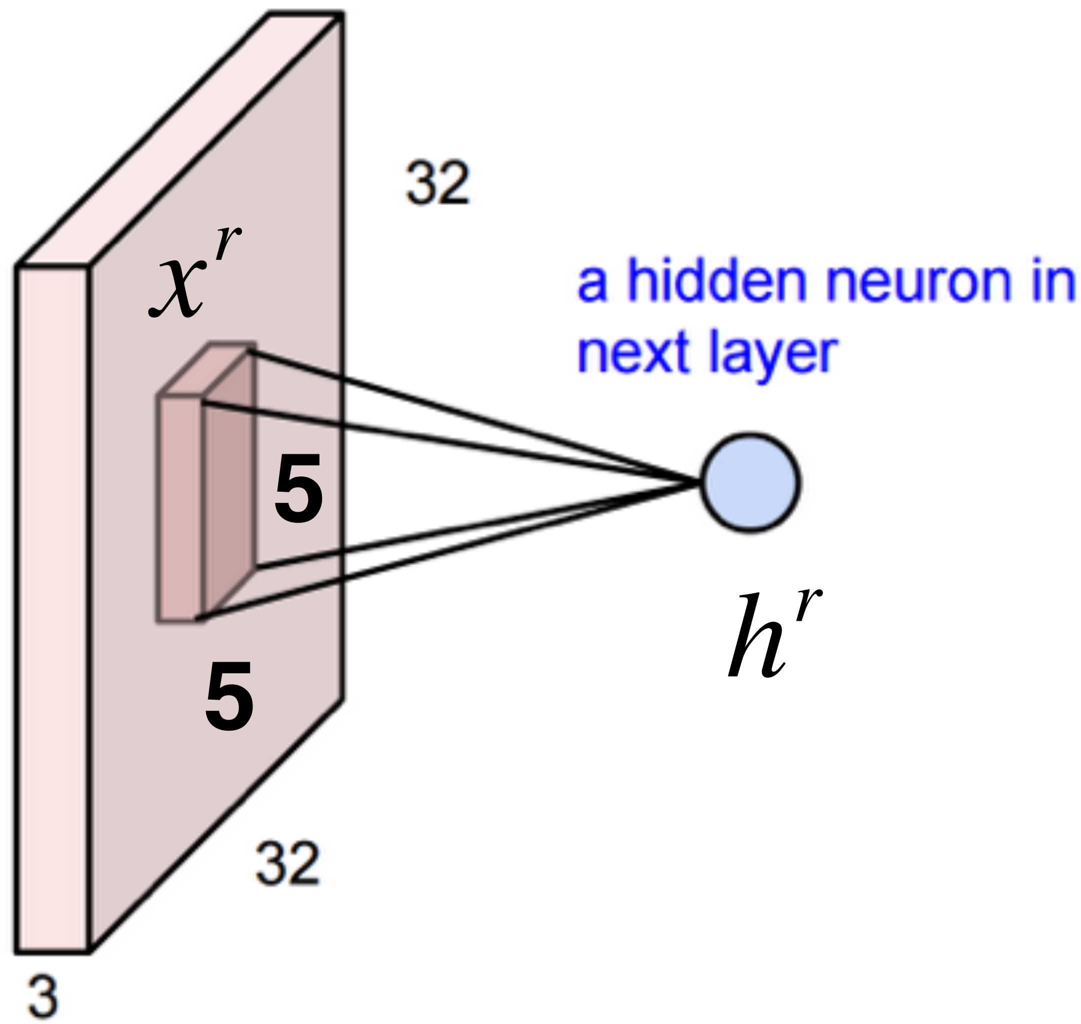
Then the output is:

$$h^r = \sum_{ijk} x^r{}_{ijk} W_{ijk} + b$$

Figure: Andrej Karpathy

3D Activations



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Sum over 3 axes

Figure: Andrej Karpathy

3D Activations

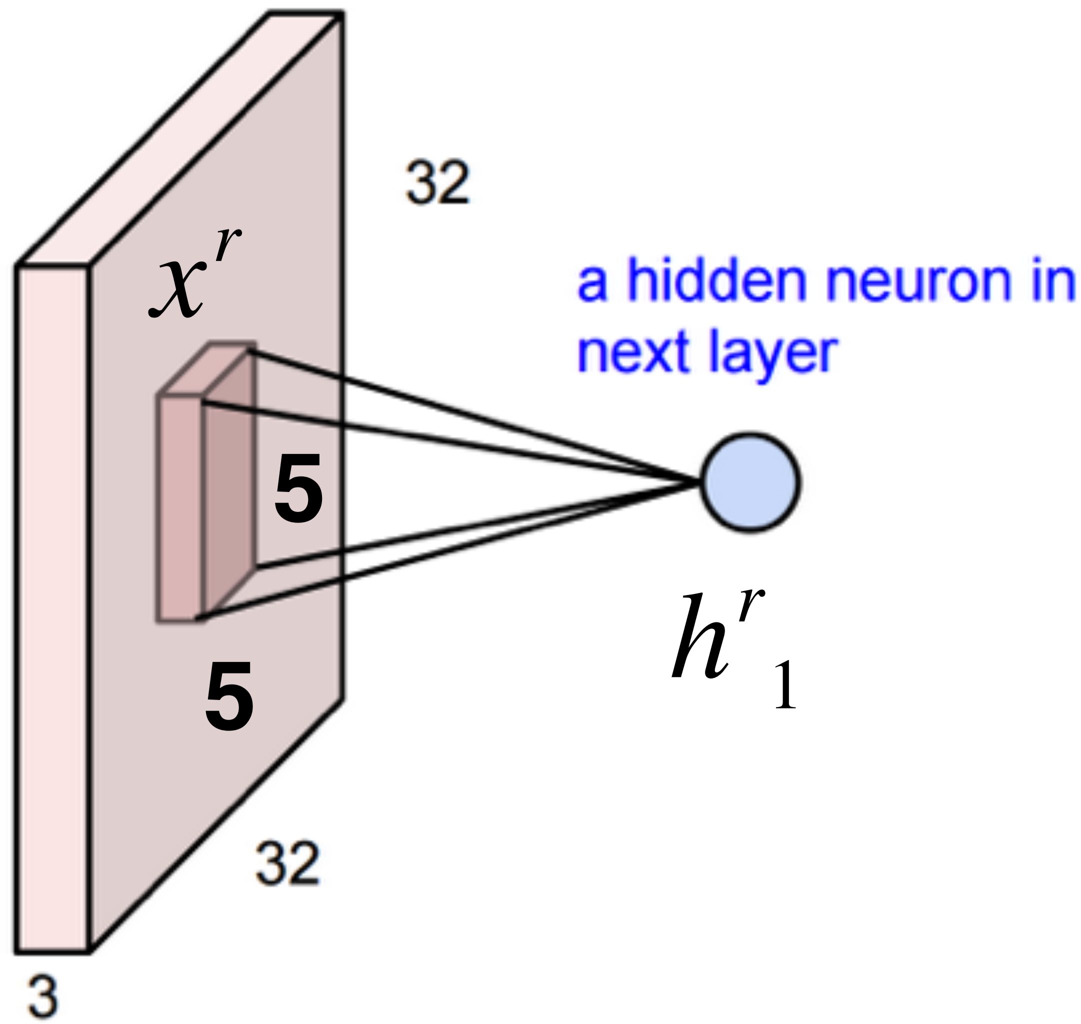


Figure: Andrej Karpathy

3D Activations

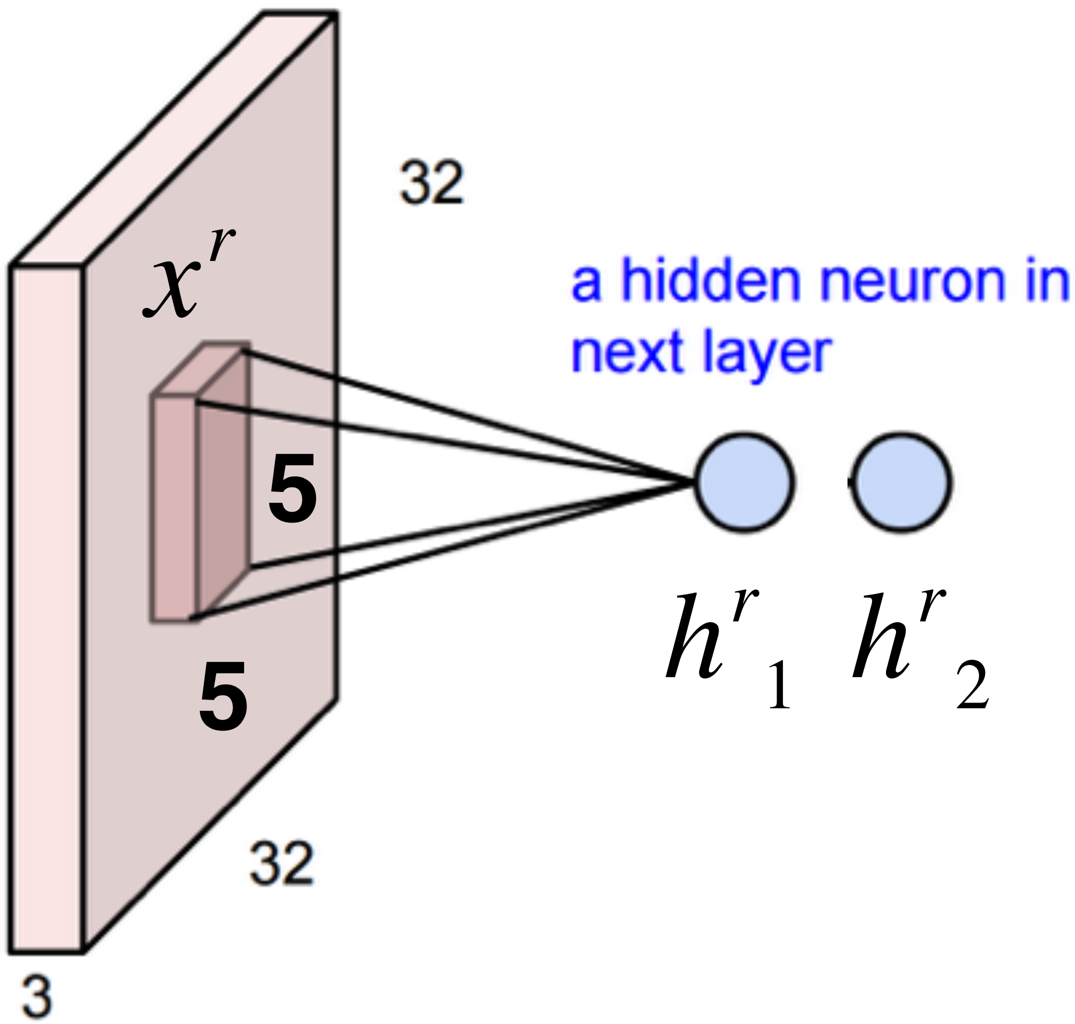
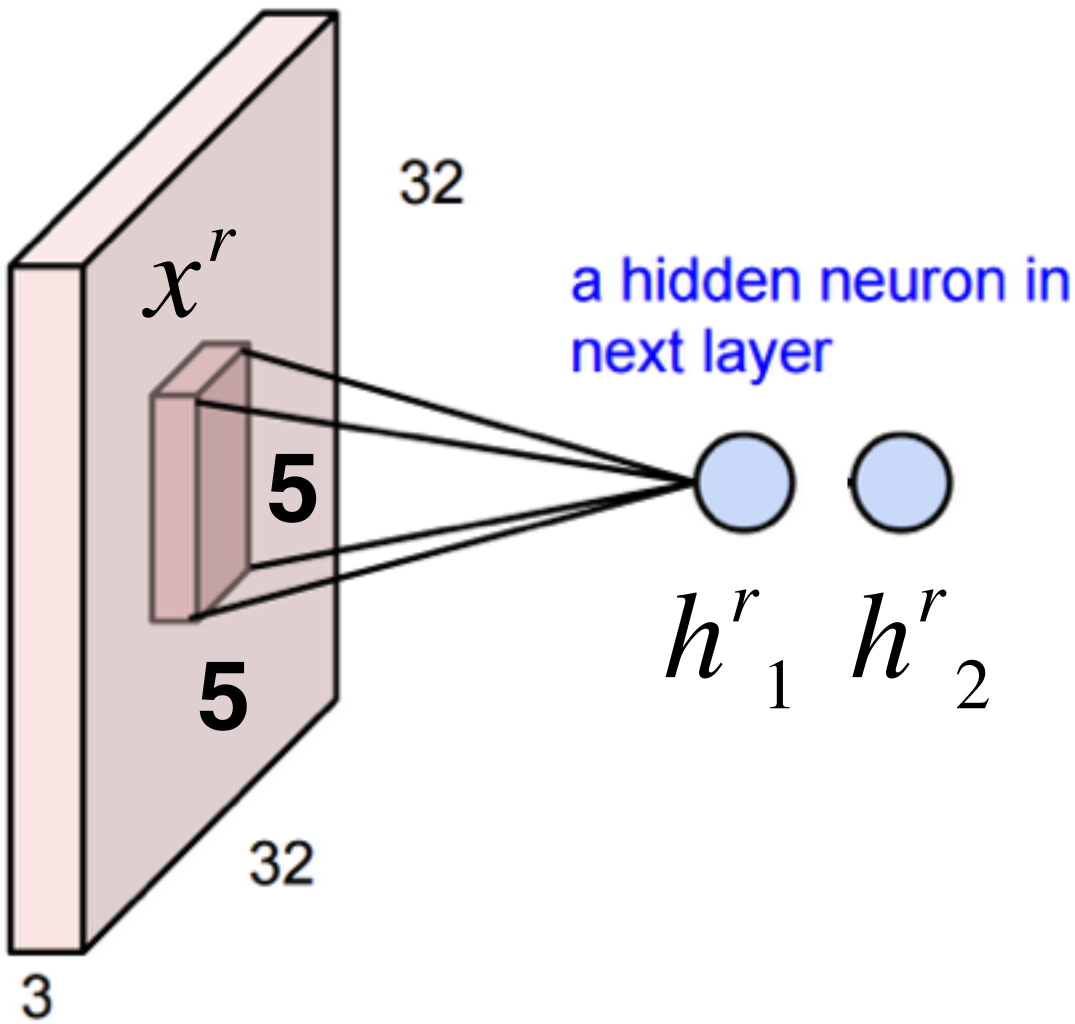


Figure: Andrej Karpathy

3D Activations



With **2** output neurons

$$h^r{}_1 = \sum_{ijk} x^r{}_{ijk} W_{1ijk} + b_1$$

$$h^r{}_2 = \sum_{ijk} x^r{}_{ijk} W_{2ijk} + b_2$$

Figure: Andrej Karpathy

3D Activations

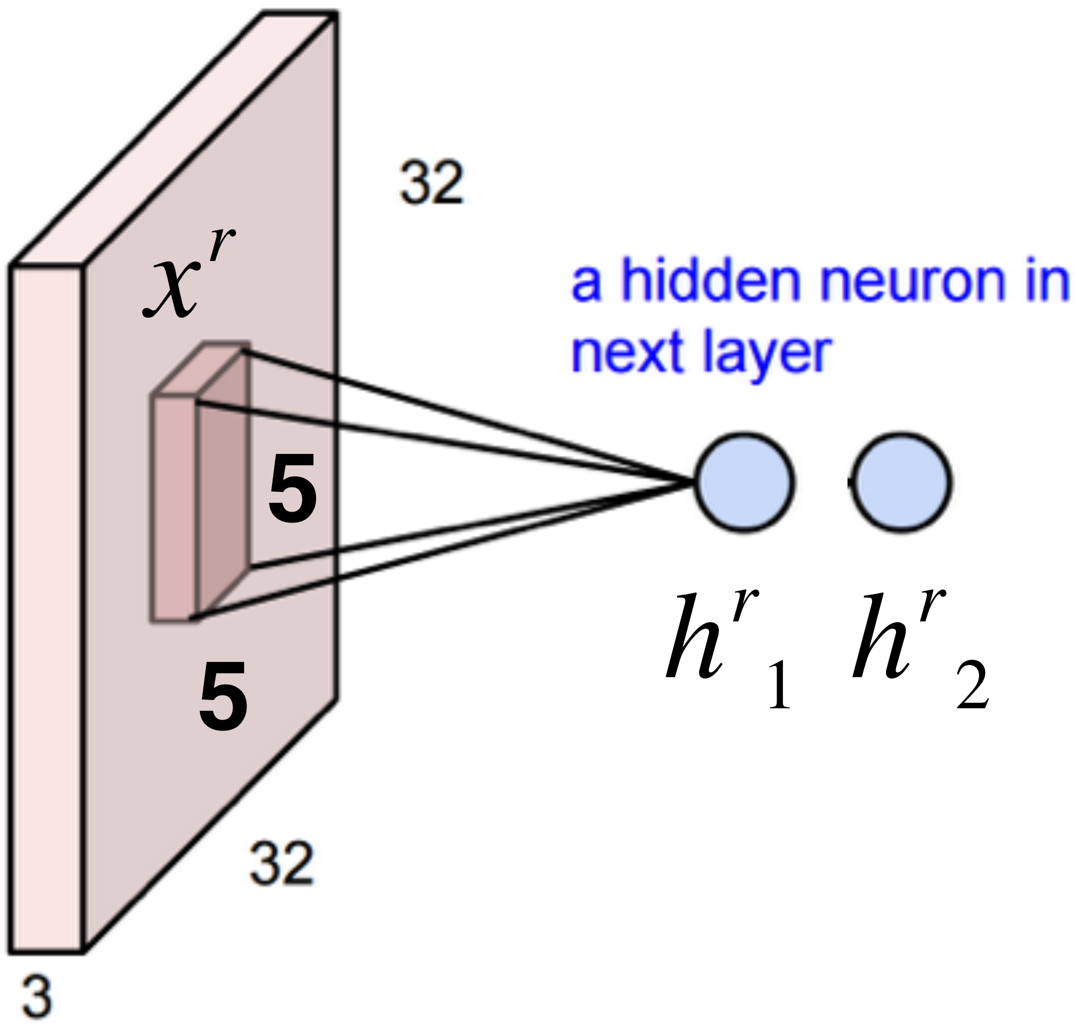


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Figure: Andrej Karpathy

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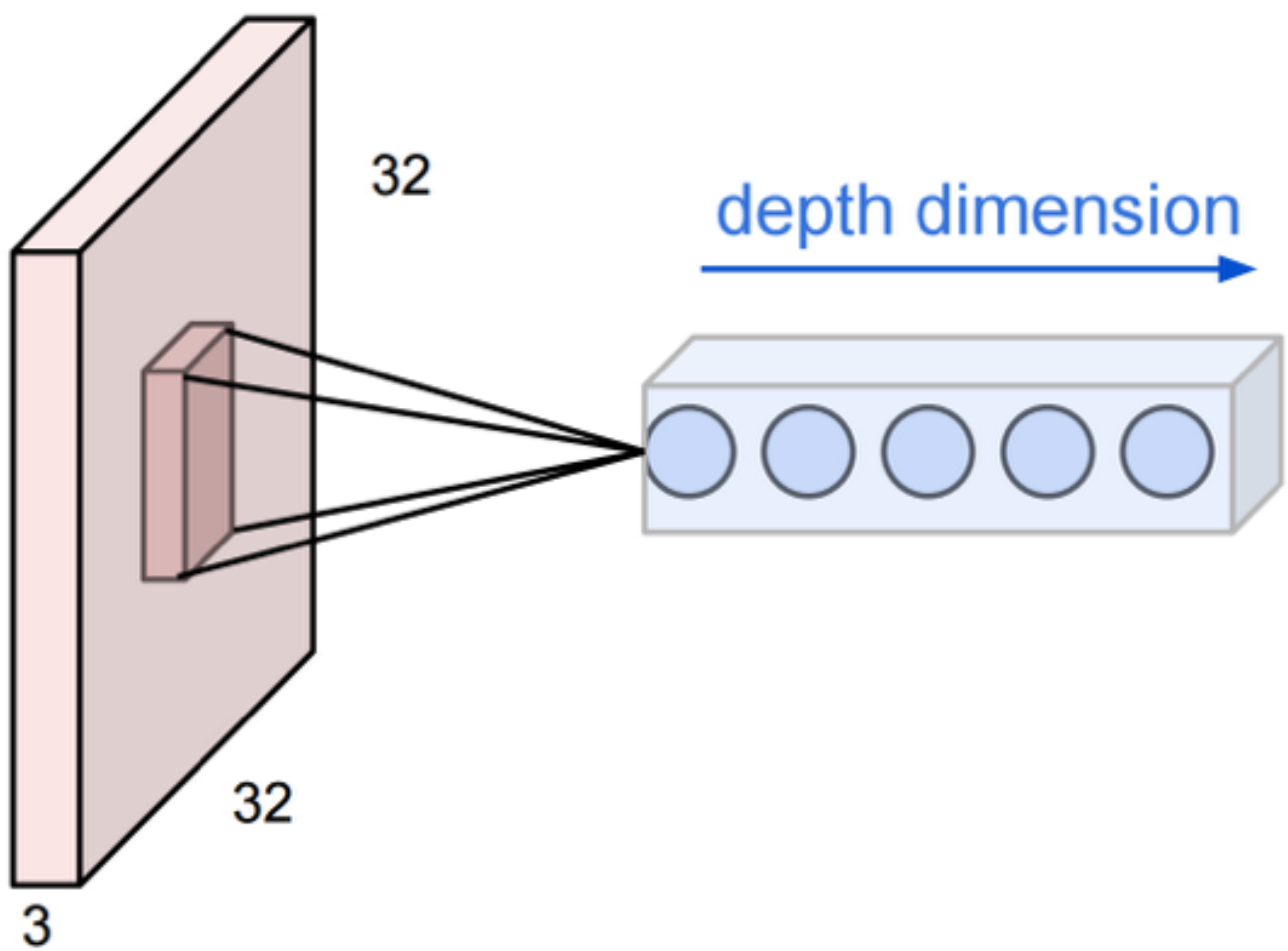
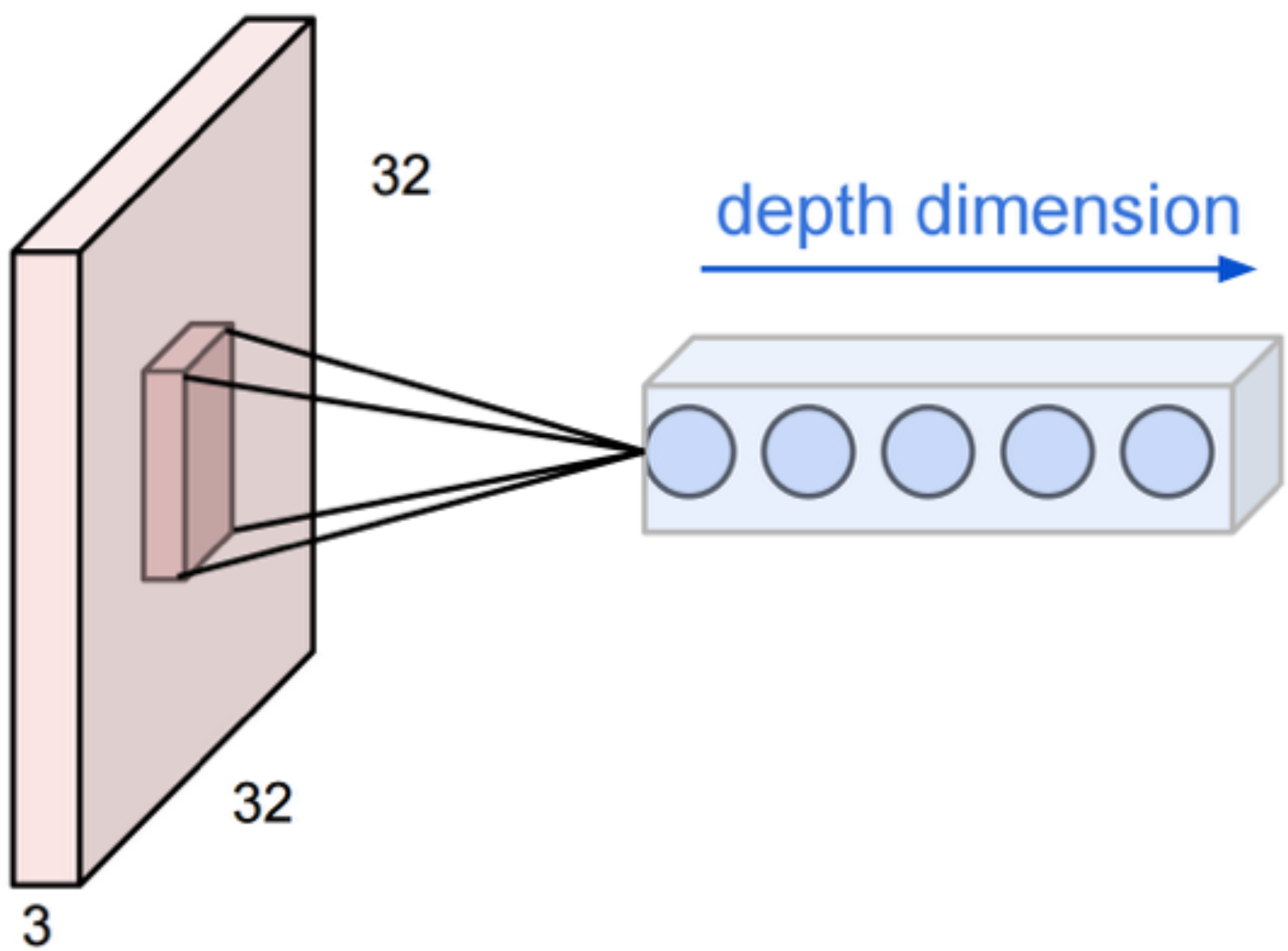


Figure: Andrej Karpathy

3D Activations

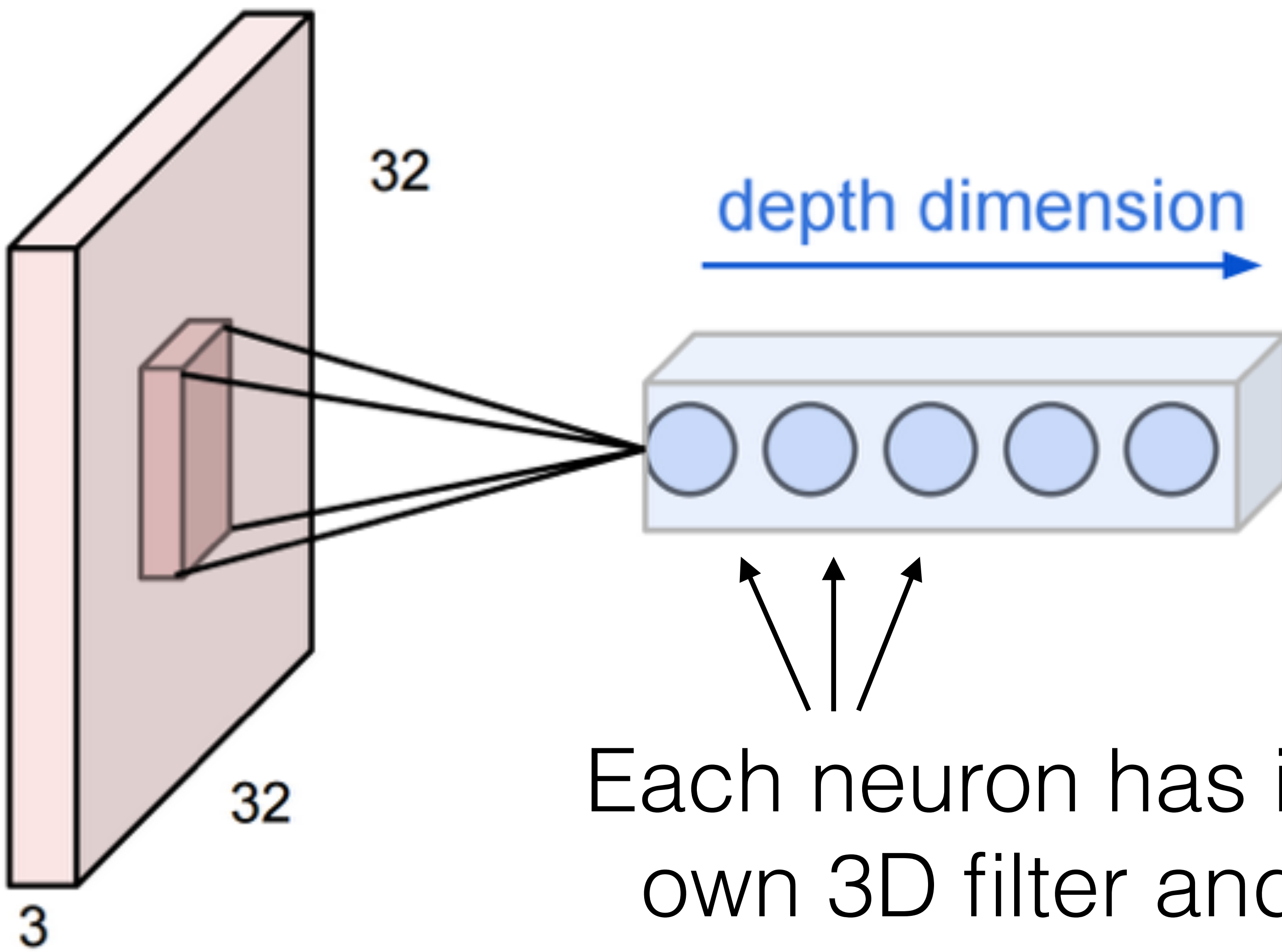


We can keep adding more outputs

These form a column in the output volume: [depth x 1 x 1]

Figure: Andrej Karpathy

3D Activations



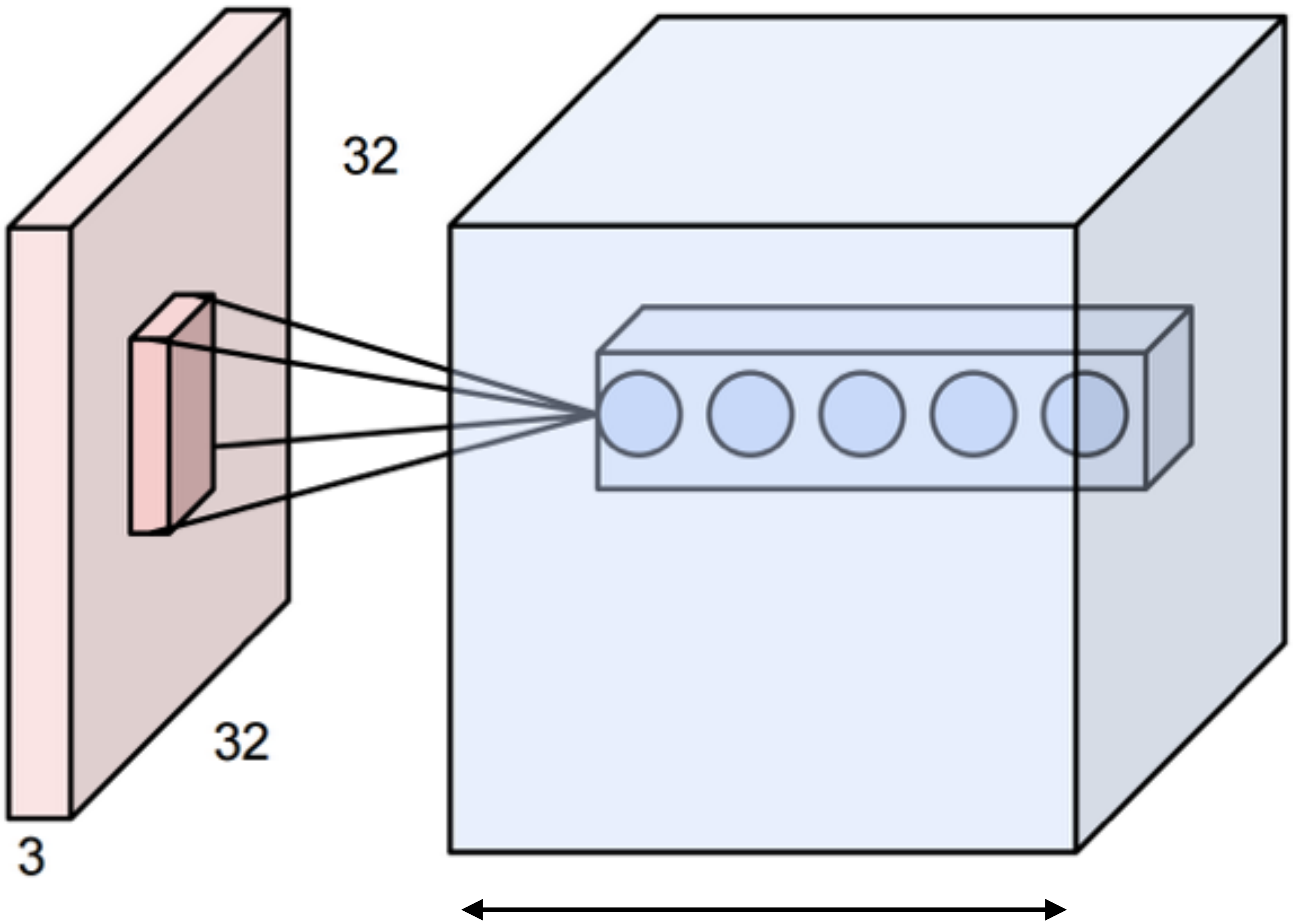
Each neuron has its own 3D filter and own (scalar) bias

We can keep adding more outputs

These form a column in the output volume: [depth x 1 x 1]

Figure: Andrej Karpathy

3D Activations

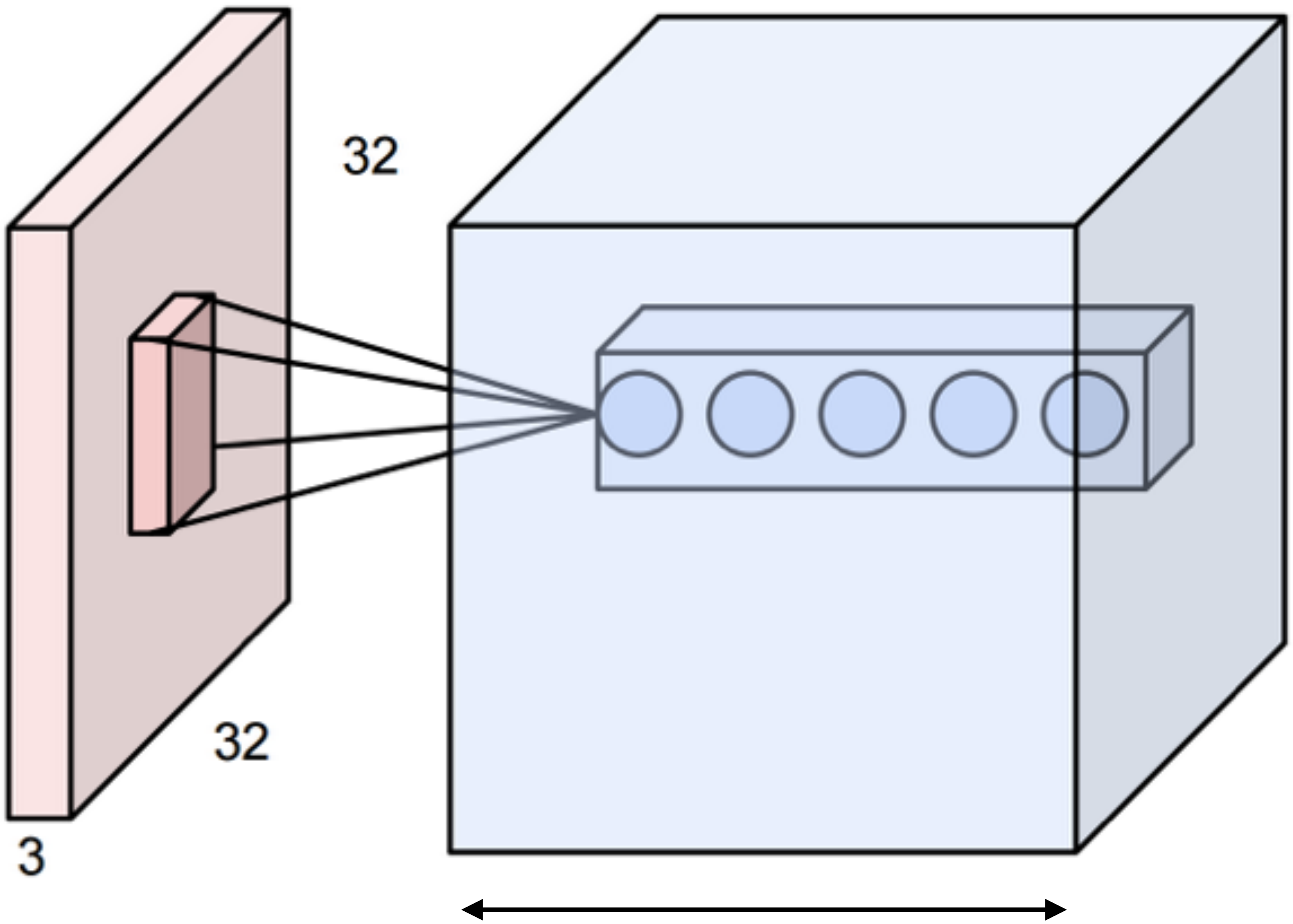


Now repeat this across the input

D sets of weights (also called filters)

Figure: Andrej Karpathy

3D Activations



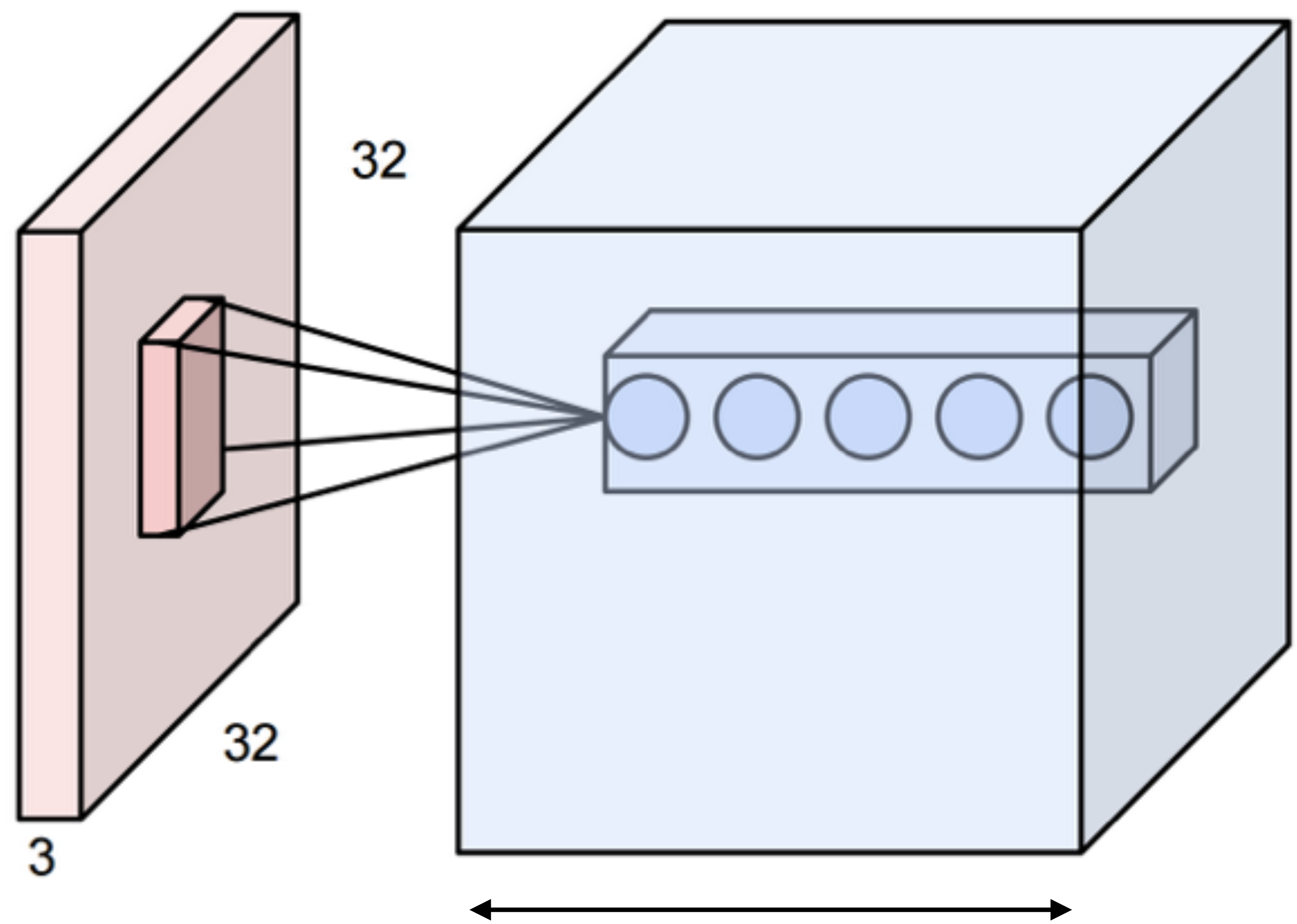
D sets of weights
(also called filters)

Now repeat this across the input

Weight sharing:
Each filter shares the same weights (but each depth index has its own set of weights)

Figure: Andrej Karpathy

3D Activations



D sets of weights
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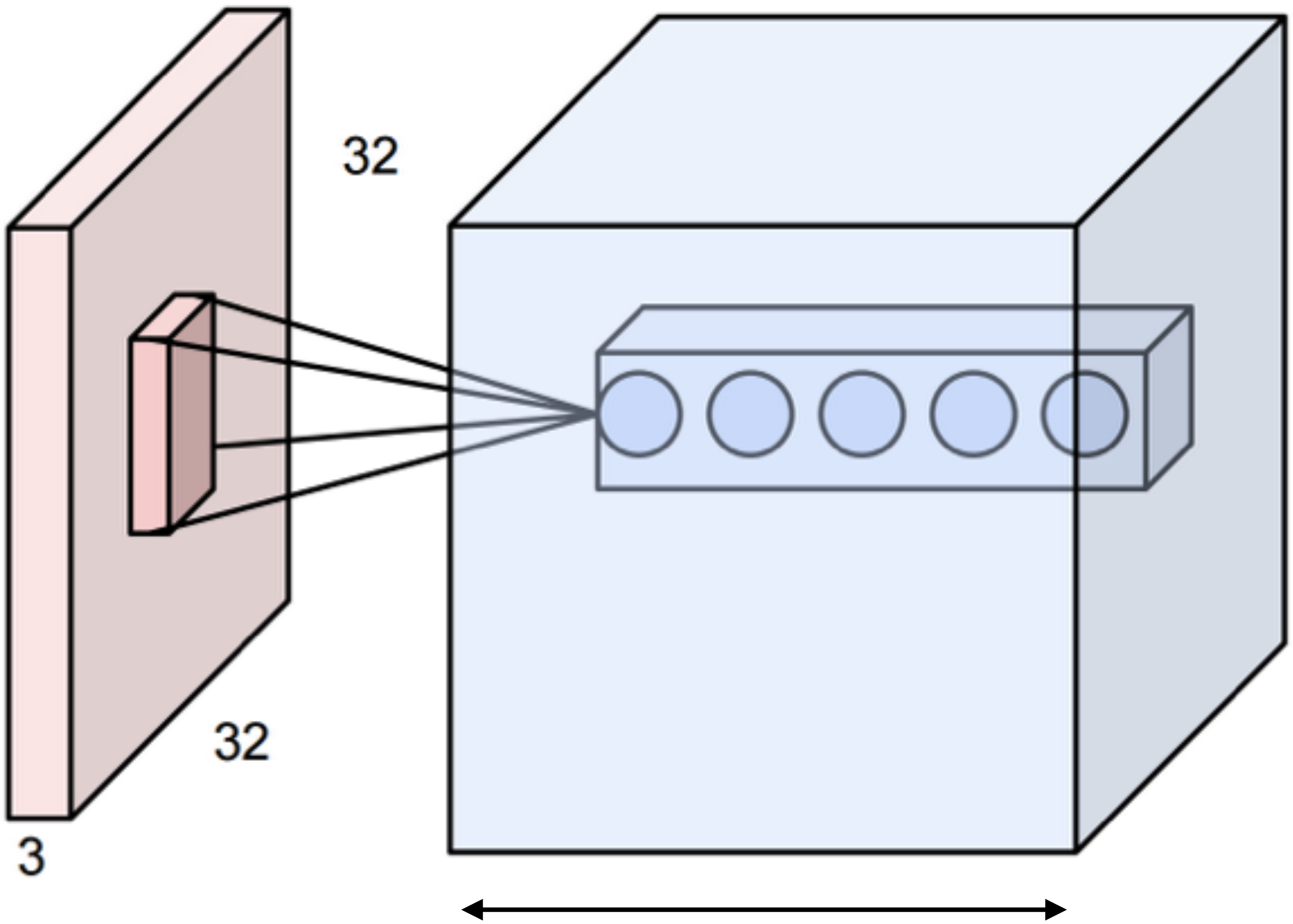
Figure: Andrej Karpathy

3D Activations

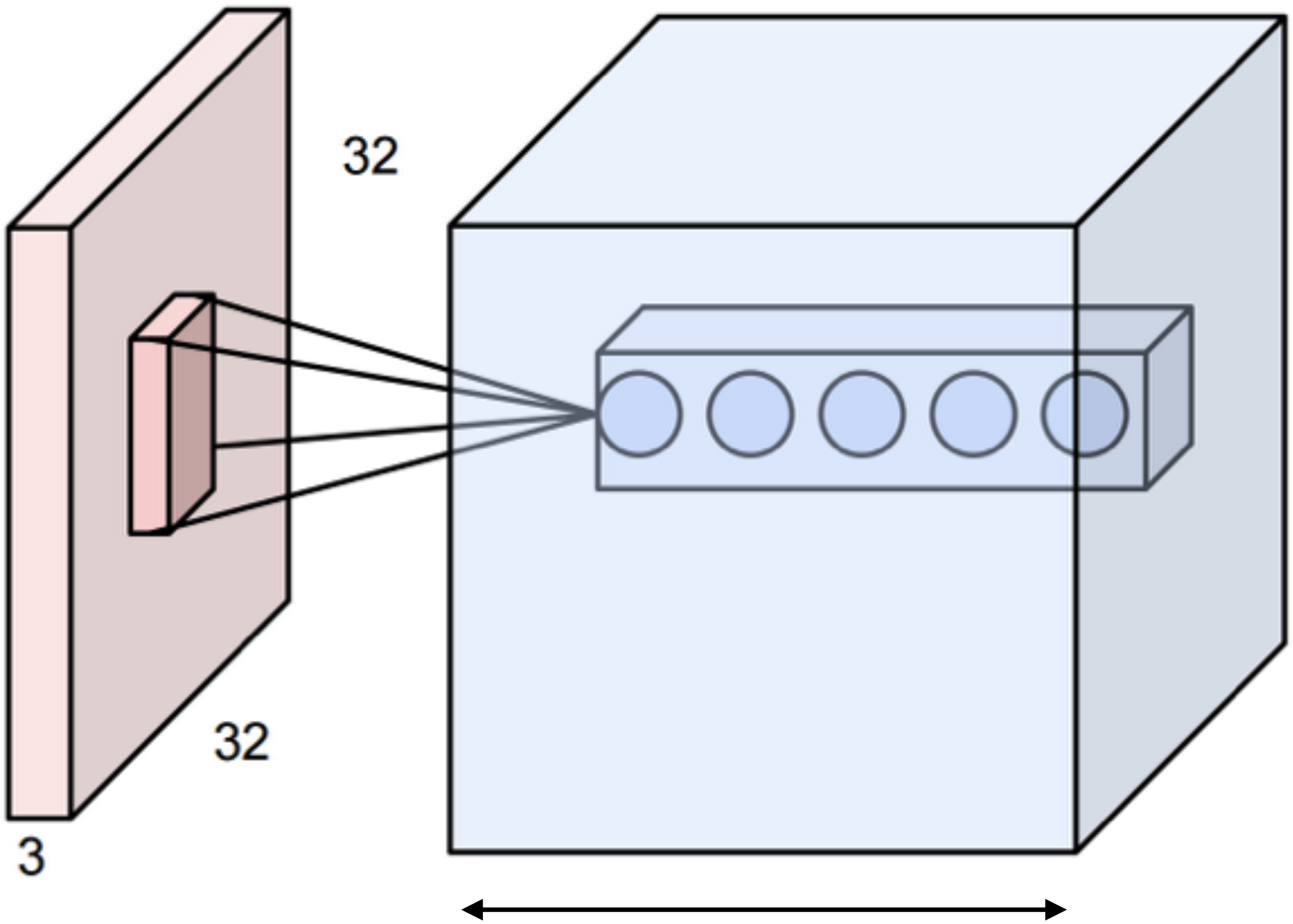


D sets of weights
(also called filters)

With weight sharing, this is called **convolution**

Figure: Andrej Karpathy

3D Activations



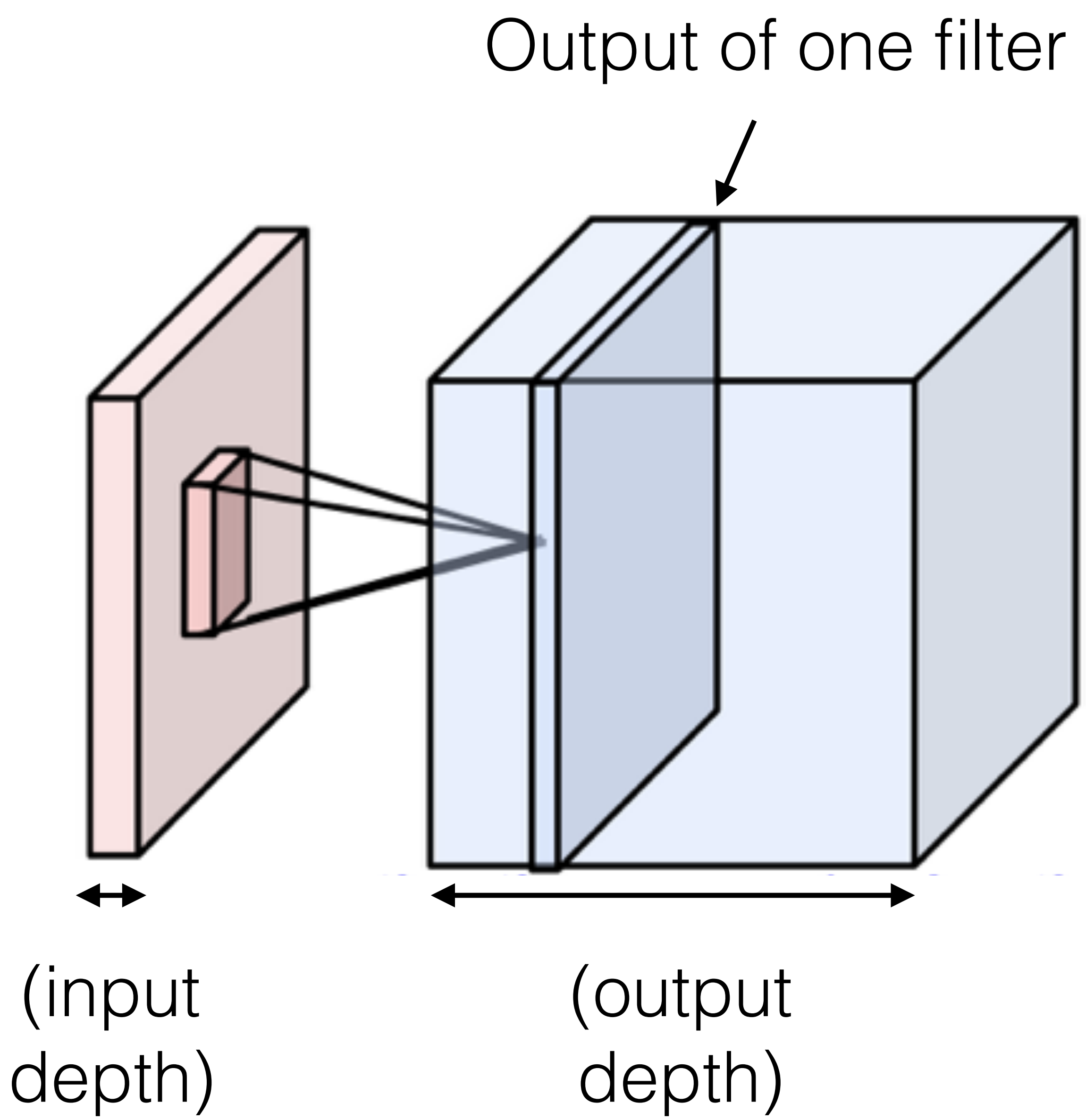
D sets of weights
(also called filters)

With weight sharing, this is called **convolution**

Without weight sharing, this is called a **locally connected layer**

Figure: Andrej Karpathy

3D Activations

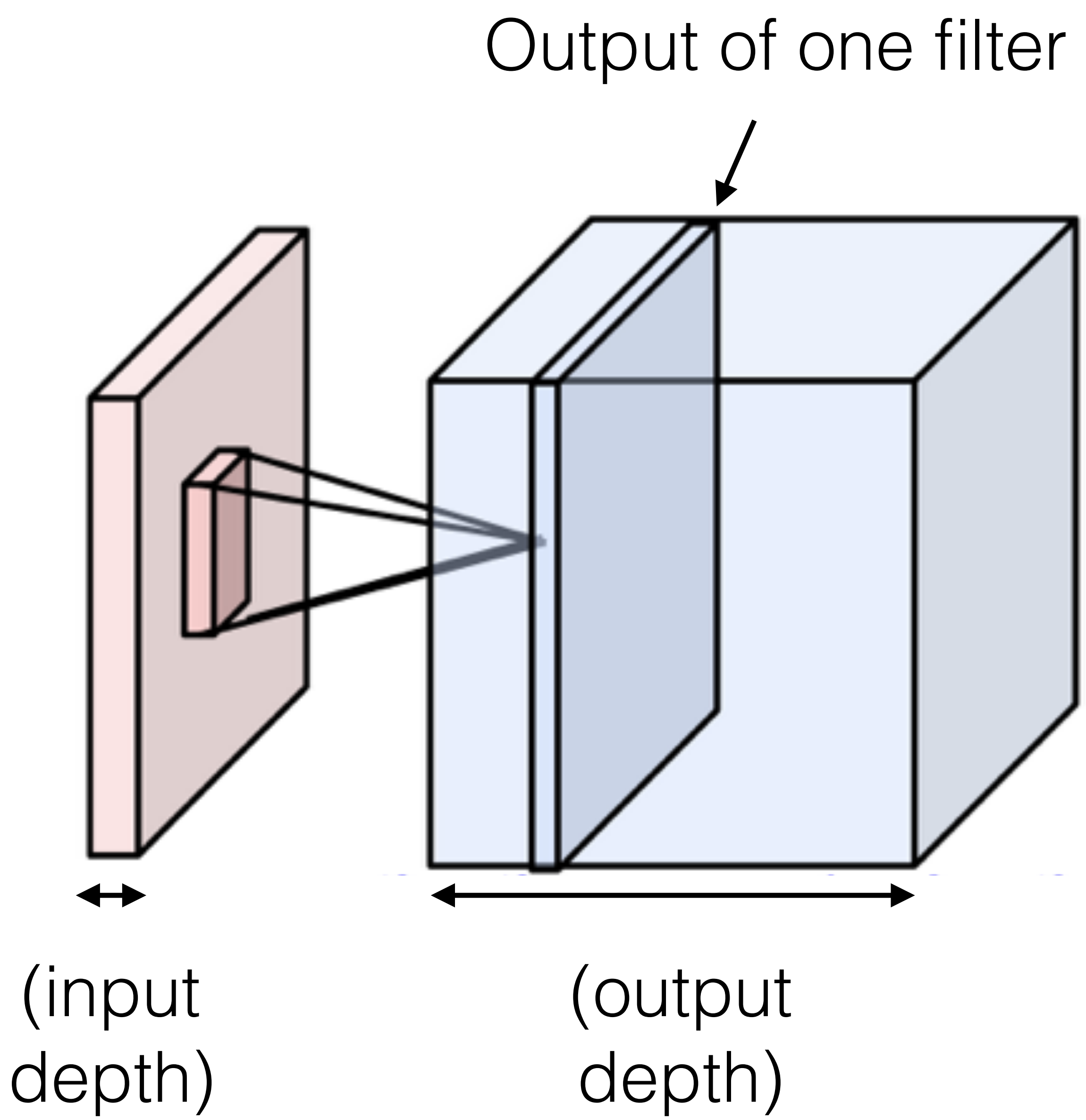


One set of weights gives one slice in the output

To get a 3D output of depth *D*, use *D* different filters

In practice, ConvNets use many filters (~64 to 1024)

3D Activations



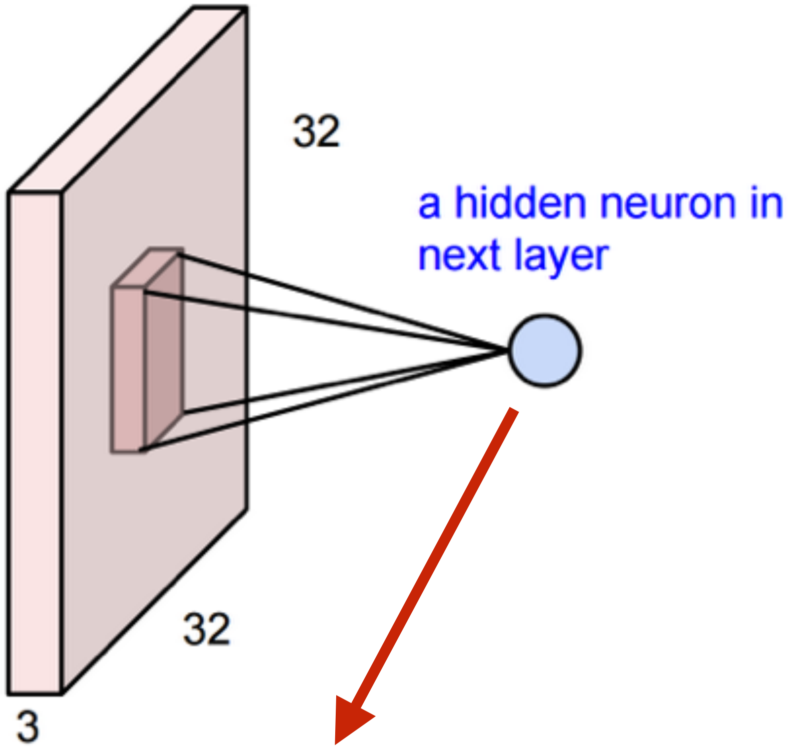
One set of weights gives one slice in the output

To get a 3D output of depth *D*, use *D* different filters

In practice, ConvNets use many filters (~64 to 1024)

All together, the weights are **4** dimensional:
(output depth, input depth, kernel height, kernel width)

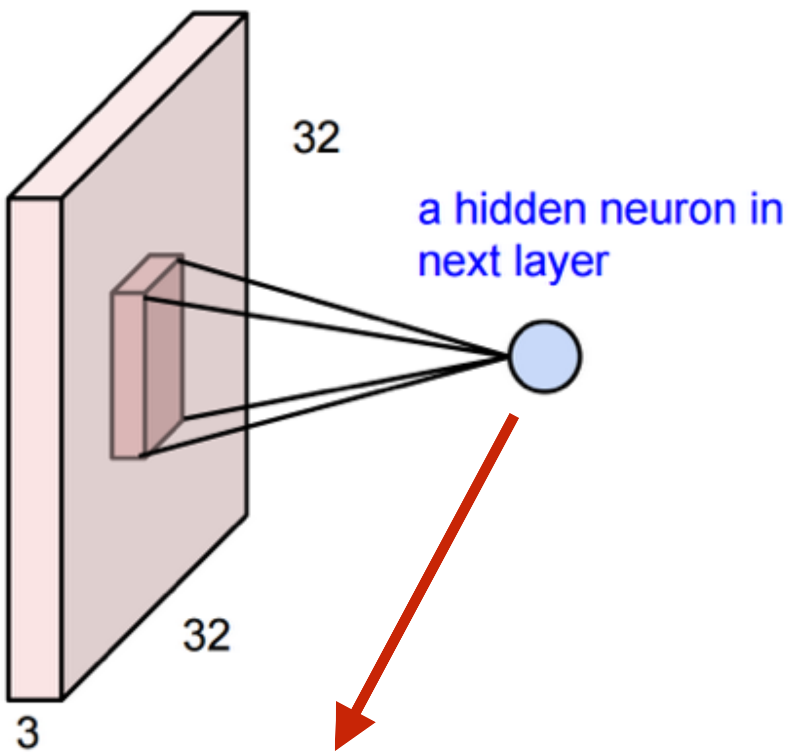
3D Activations




```
out[n, 0, r, c] =
```

Let’s code this up in NumPy

3D Activations

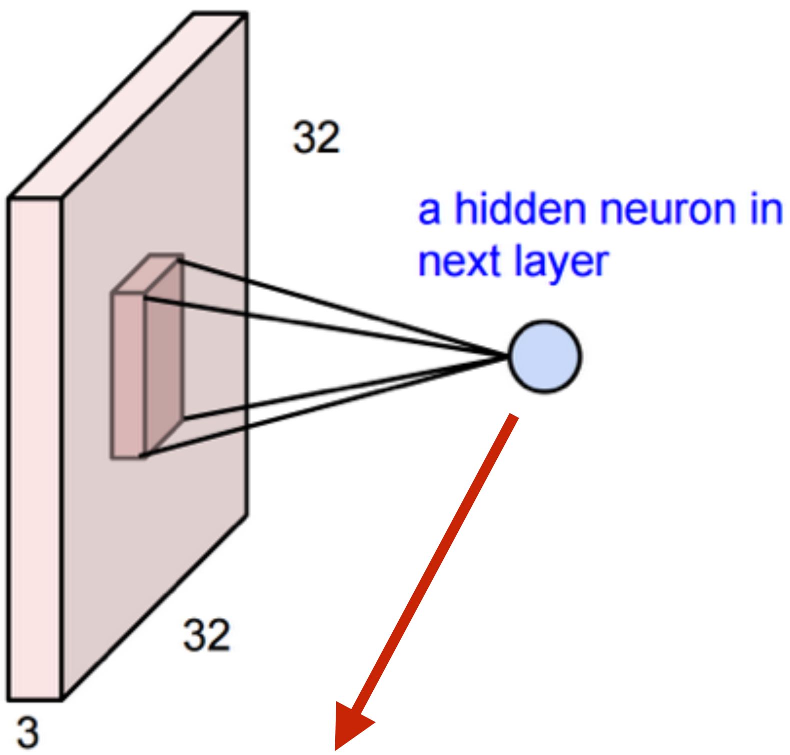


Let's code this up in NumPy

```
out[n, 0, r, c] =
```

n^{th} example

3D Activations



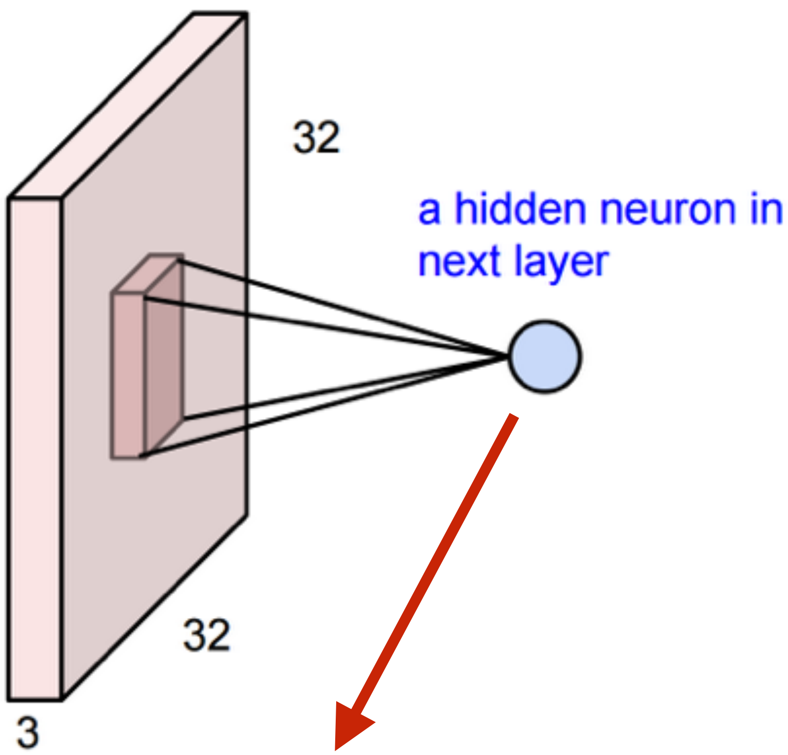

```
out[n, 0, r, c] =
```

first filter

n^{th} example

Let's code this up in NumPy

3D Activations




```
out[n, 0, r, c] =
```

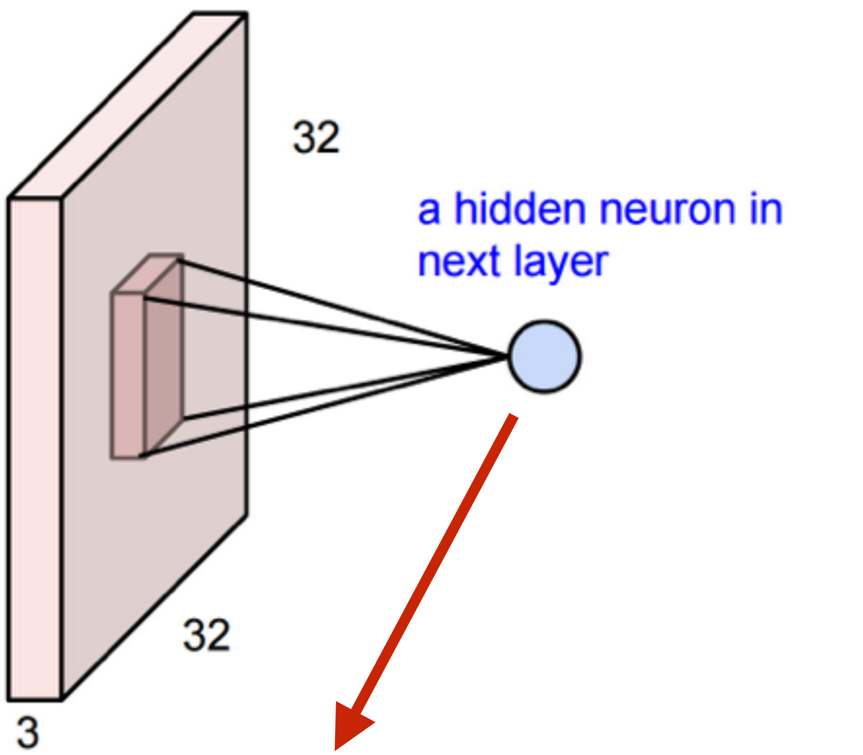
output position

first filter

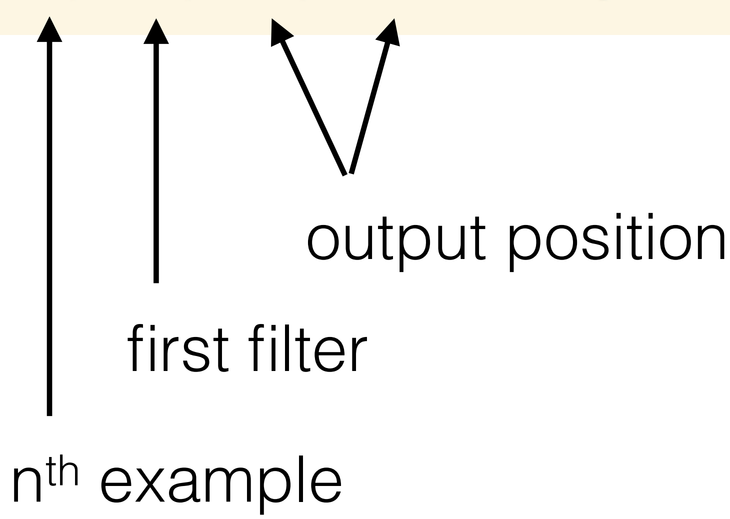
n^{th} example

Let’s code this up in NumPy

3D Activations




```
out[n, 0, r, c] = np.sum(
```



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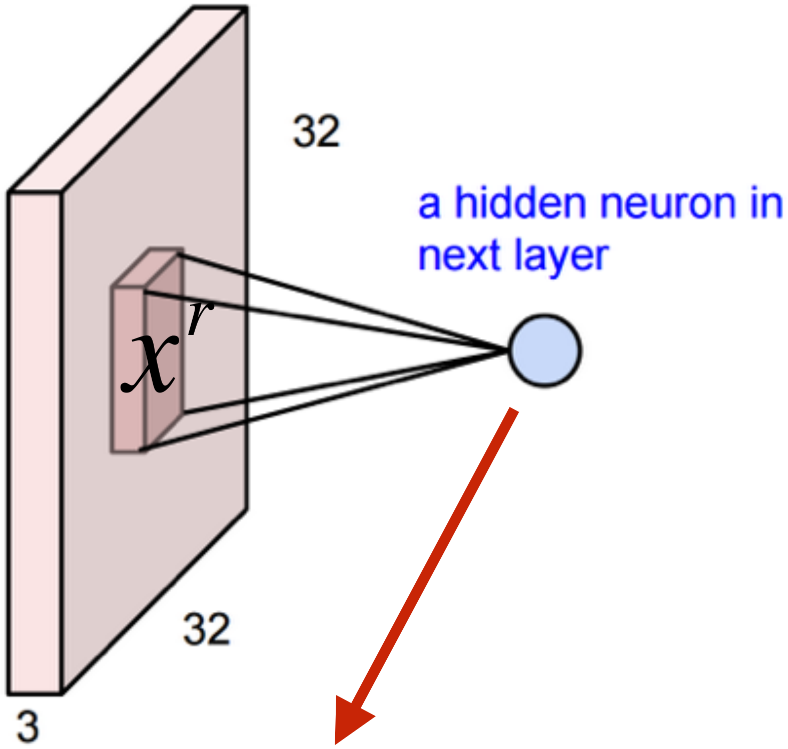
3D Activations



Let’s code this up in NumPy

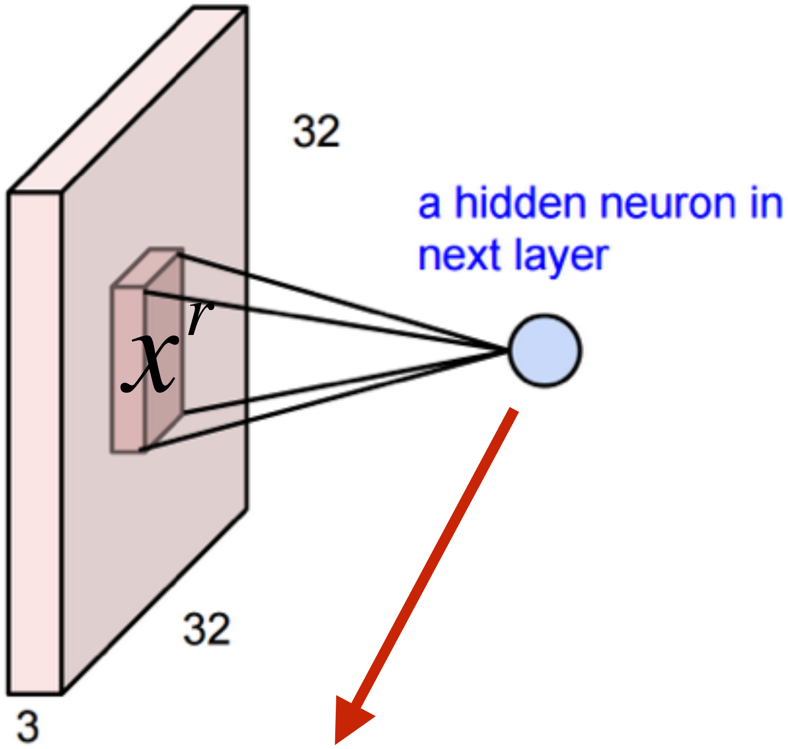
```
out[n, 0, r, c] = np.sum(X[n, :, r0:r1, c0:c1]
```

output position

first filter

n^{th} example

3D Activations



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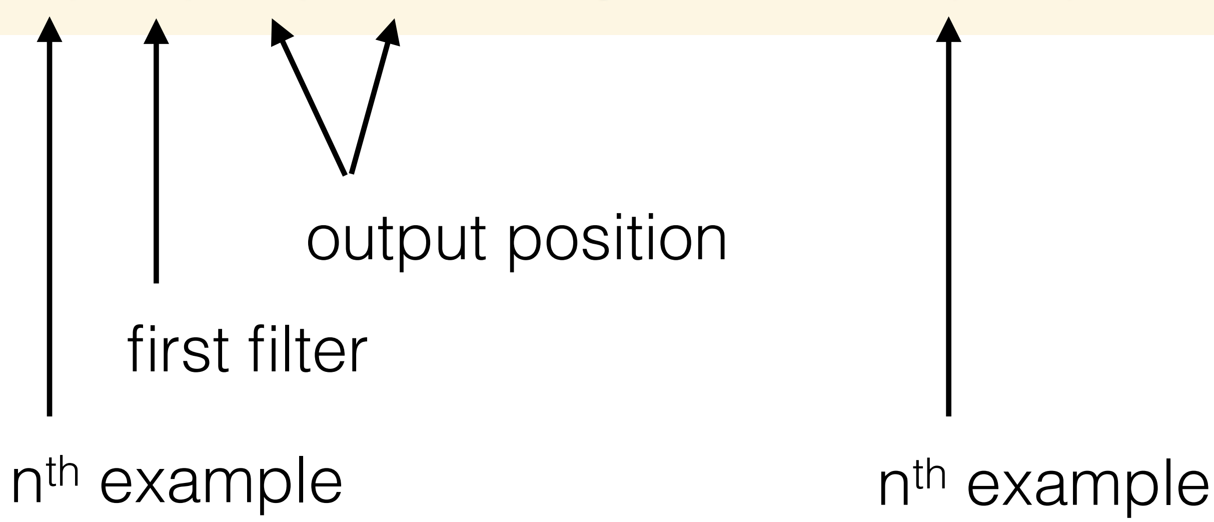
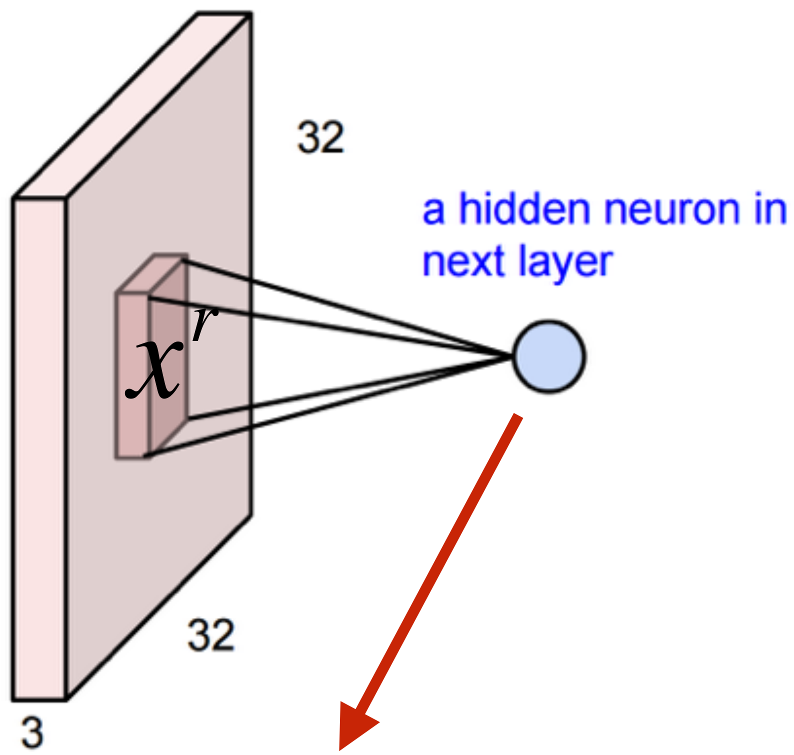


3D Activations

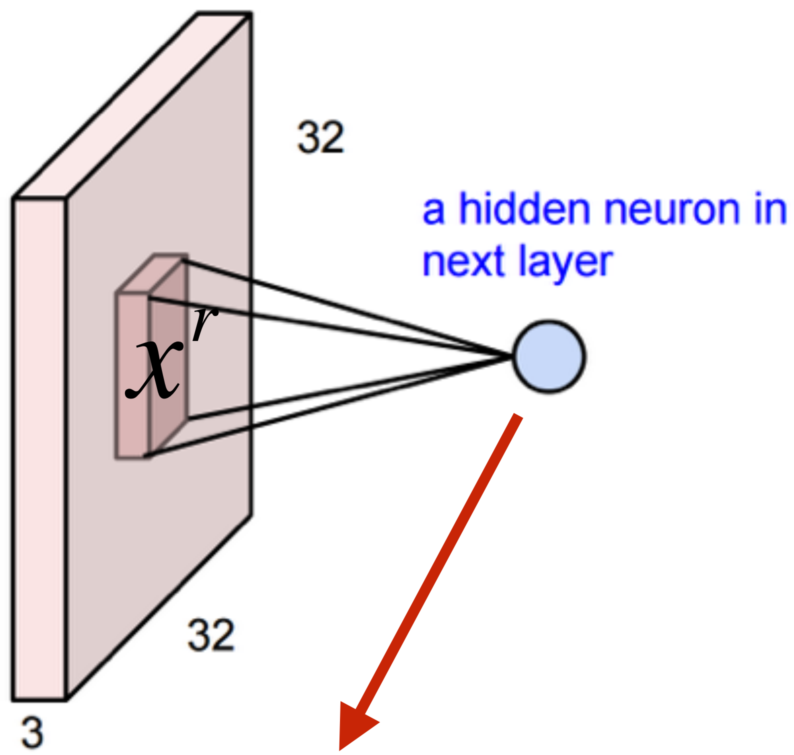


Let's code this up in NumPy

```
out[n, 0, r, c] = np.sum(X[n, :, r0:r1, c0:c1]
```

- out[n → nth example
- 0 → first filter
- r, c → output position
- X[n → nth example
- : → all input channels

3D Activations



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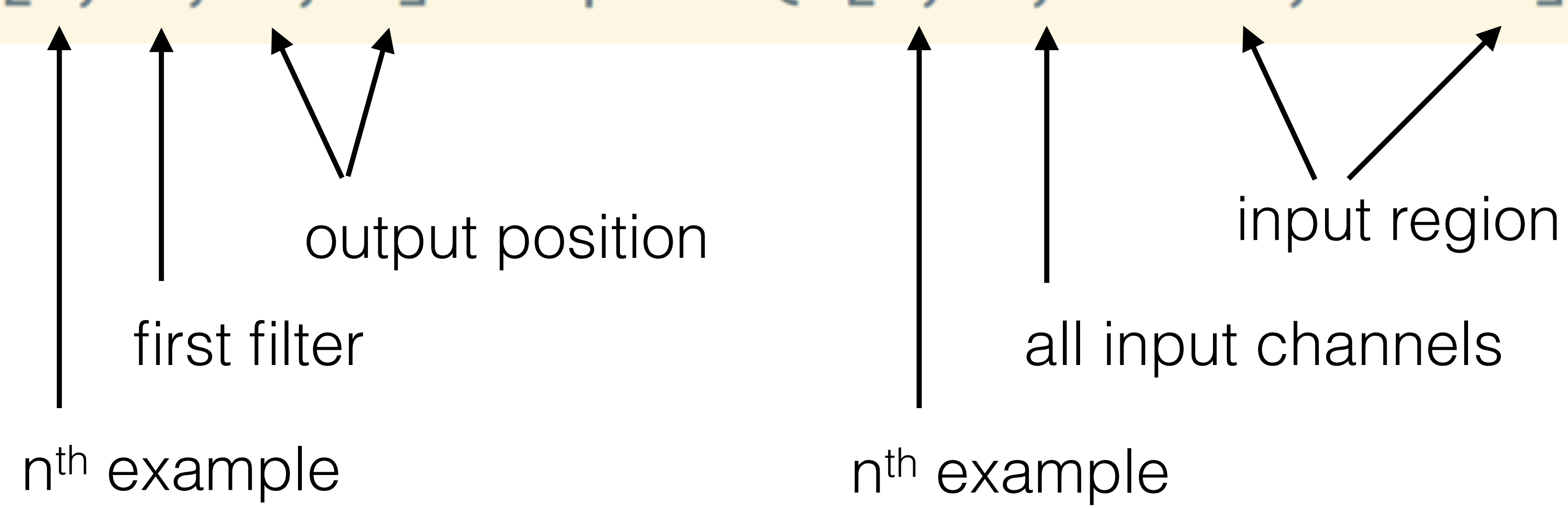
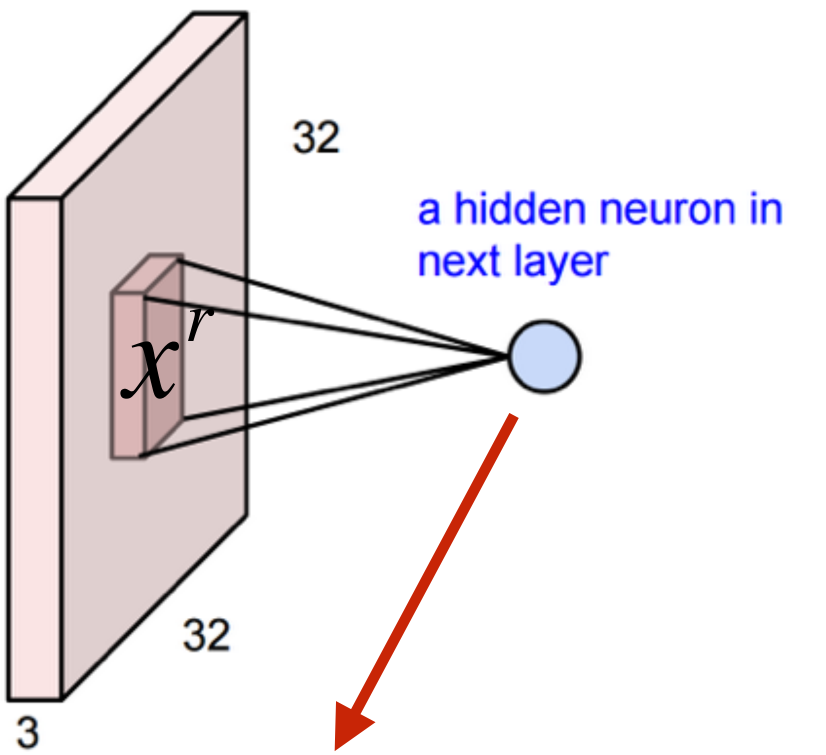
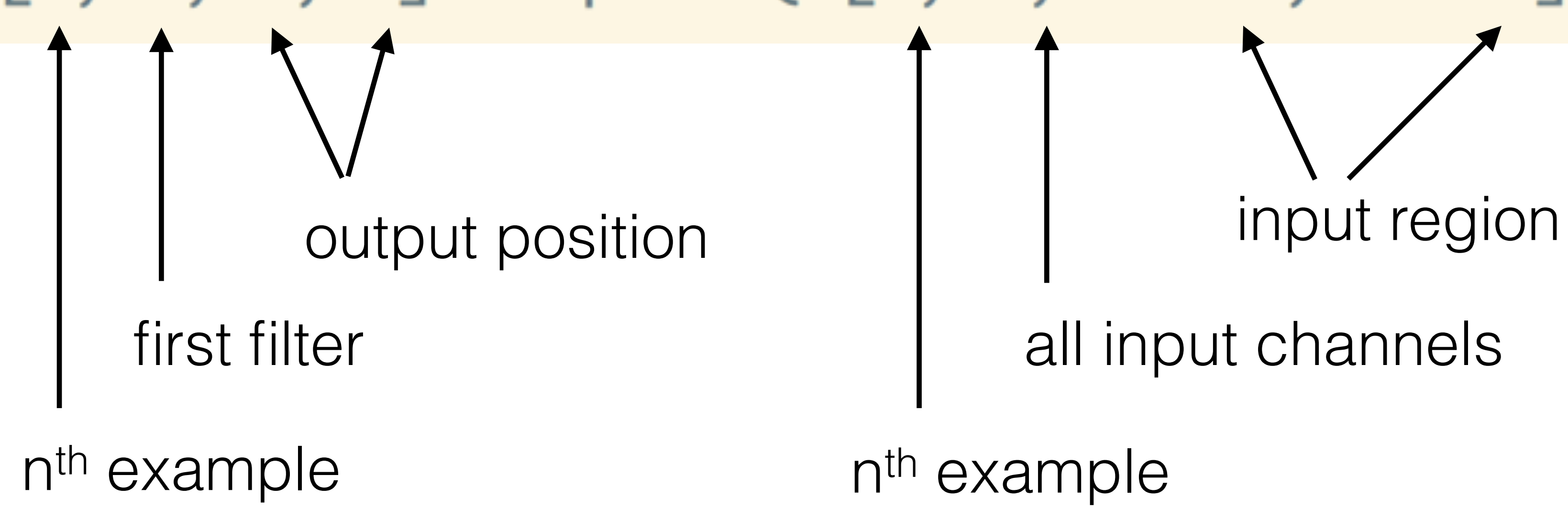


3D Activations

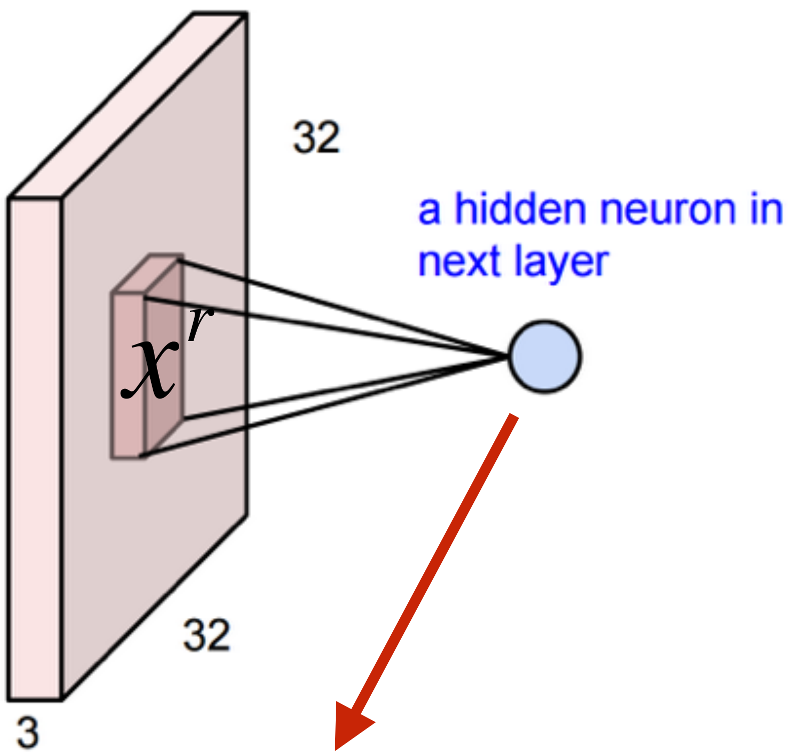


Let's code this up in NumPy

```
out[n, 0, r, c] = np.sum(X[n, :, r0:r1, c0:c1] * W[0, :, :, :]) + b[0]
```



3D Activations



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out[n, 0, r, c] = np.sum(X[n, :, r0:r1, c0:c1] * W[0, :, :, :]) + b[0]
```

output position

first filter

n^{th} example

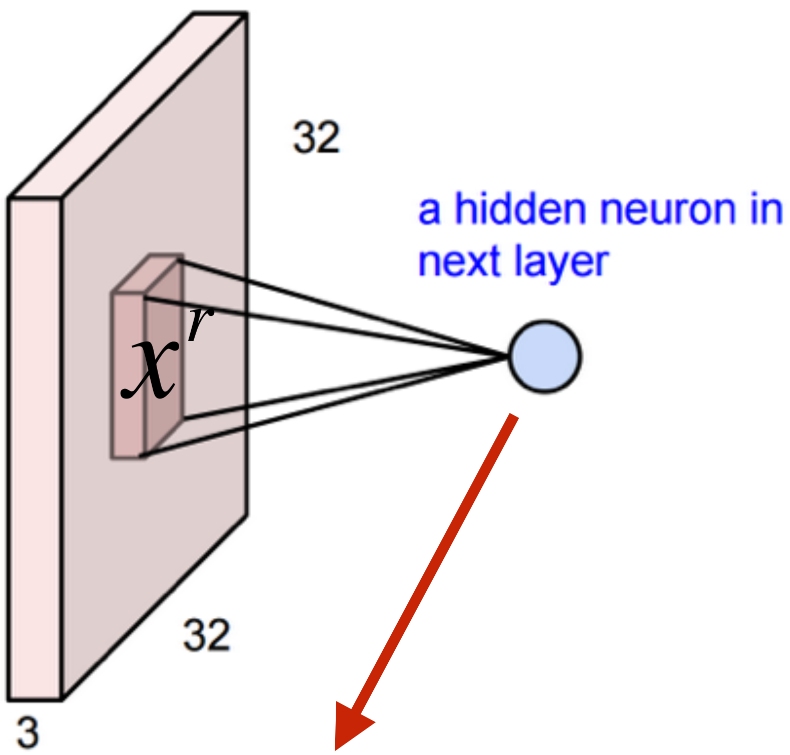
input region

all input channels

n^{th} example

first filter

3D Activations



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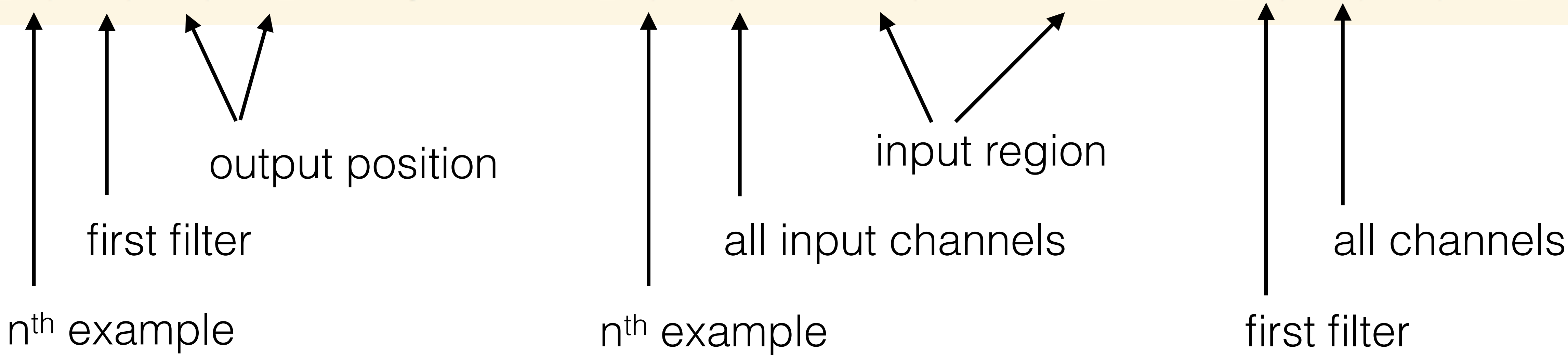
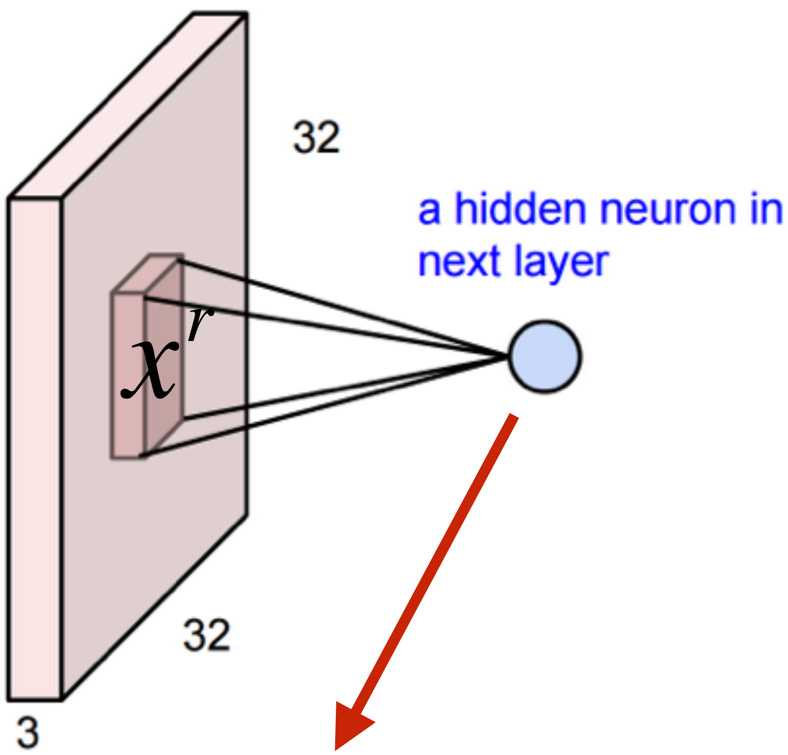


3D Activations



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output position

first filter

n^{th} example

input region

all input channels

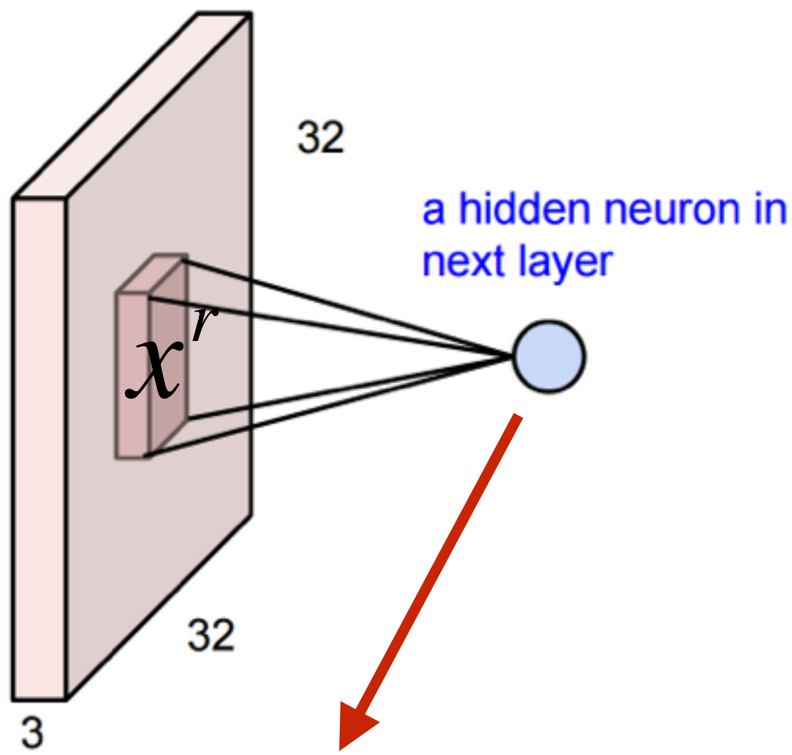
n^{th} example

all positions

all channels

first filter

3D Activations



Let's code this up in NumPy

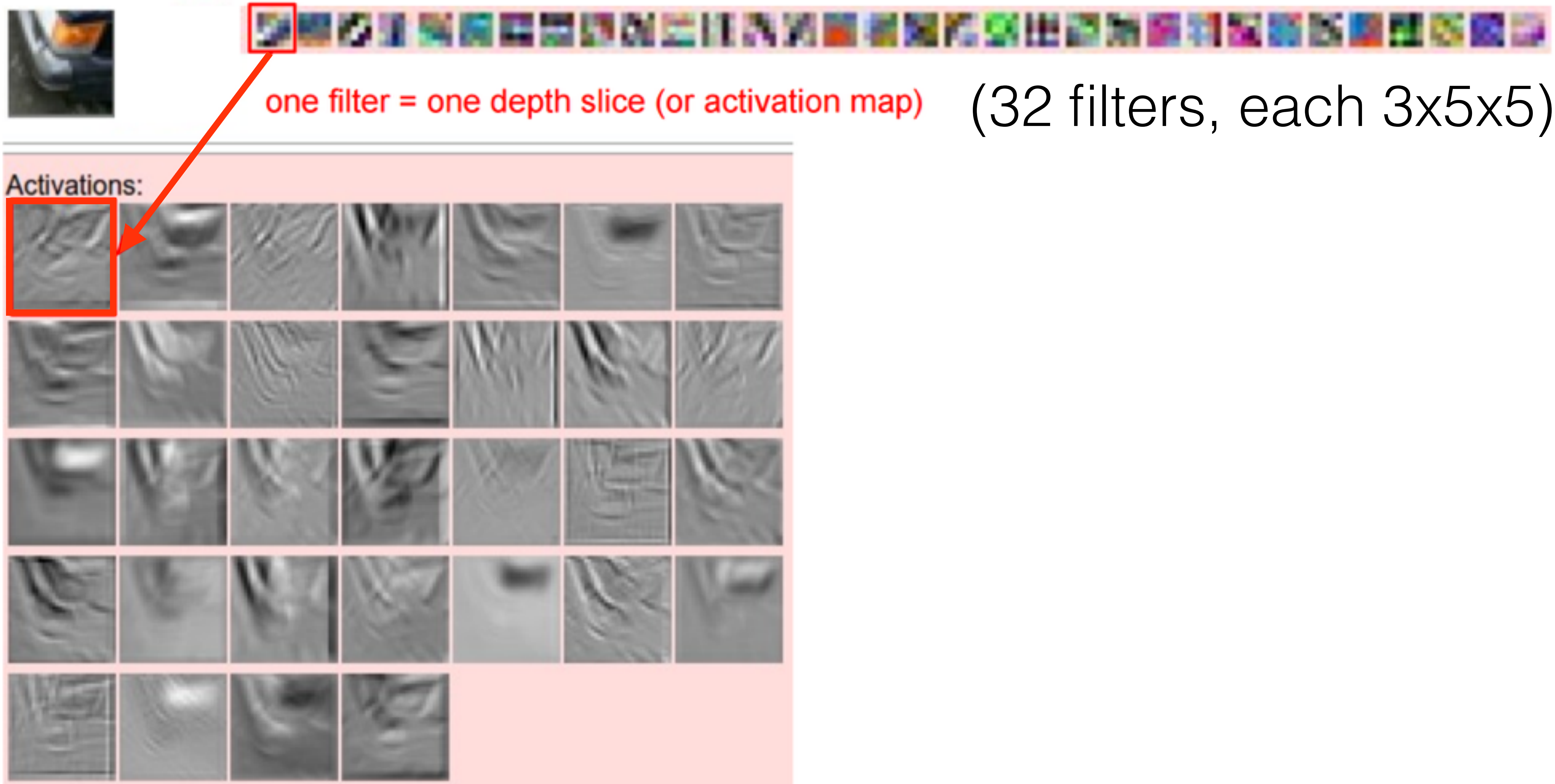
```
out[n, 0, r, c] = np.sum(X[n, :, r0:r1, c0:c1] * W[0, :, :, :]) + b[0]
```

- out[n → n^{th} example
- 0 → first filter
- r, c → output position
- X[n → n^{th} example
- : → all input channels
- r0:r1, c0:c1 → input region
- W[0 → first filter
- : → all channels
- :, : → all positions
- b[0] → bias

3D Activations

We can unravel the 3D cube and show each layer separately:

(Input)



(32 filters, each 3x5x5)

Figure: Andrej Karpathy

3D Activations

We can unravel the 3D cube and show each layer separately:

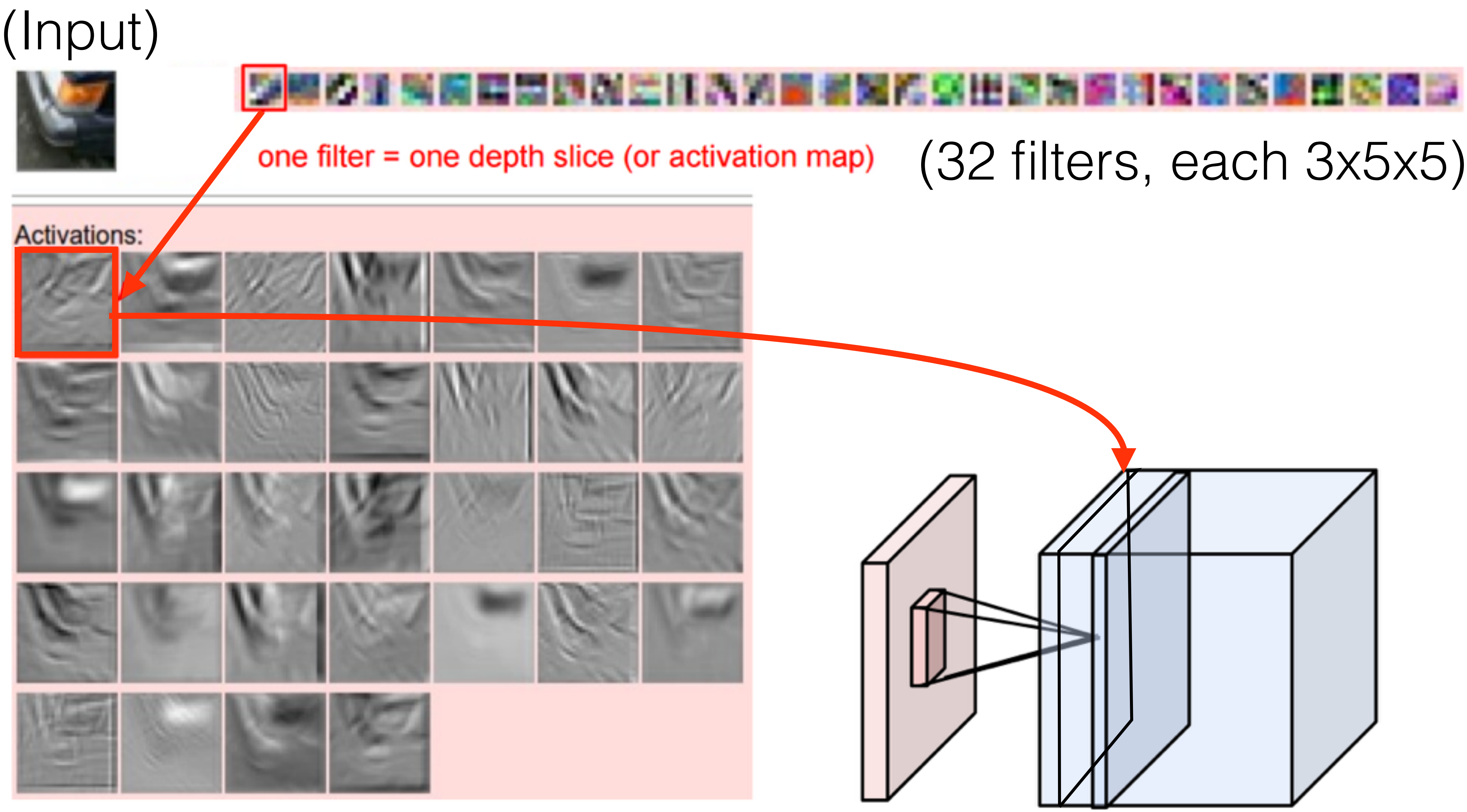


Figure: Andrej Karpathy

3D Activations

We can unravel the 3D cube and show each layer separately:

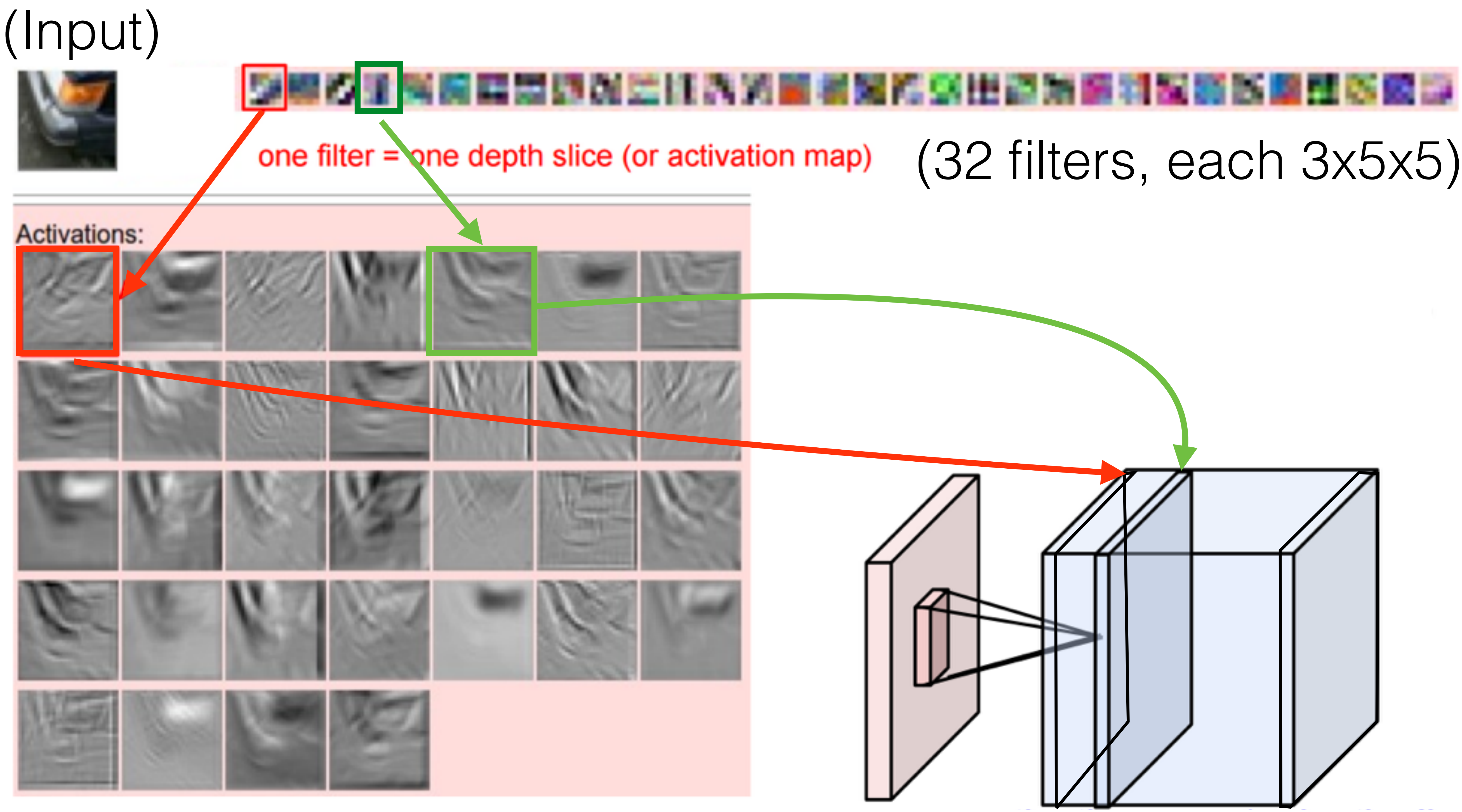


Figure: Andrej Karpathy

3D Activations

We can unravel the 3D cube and show each layer separately:

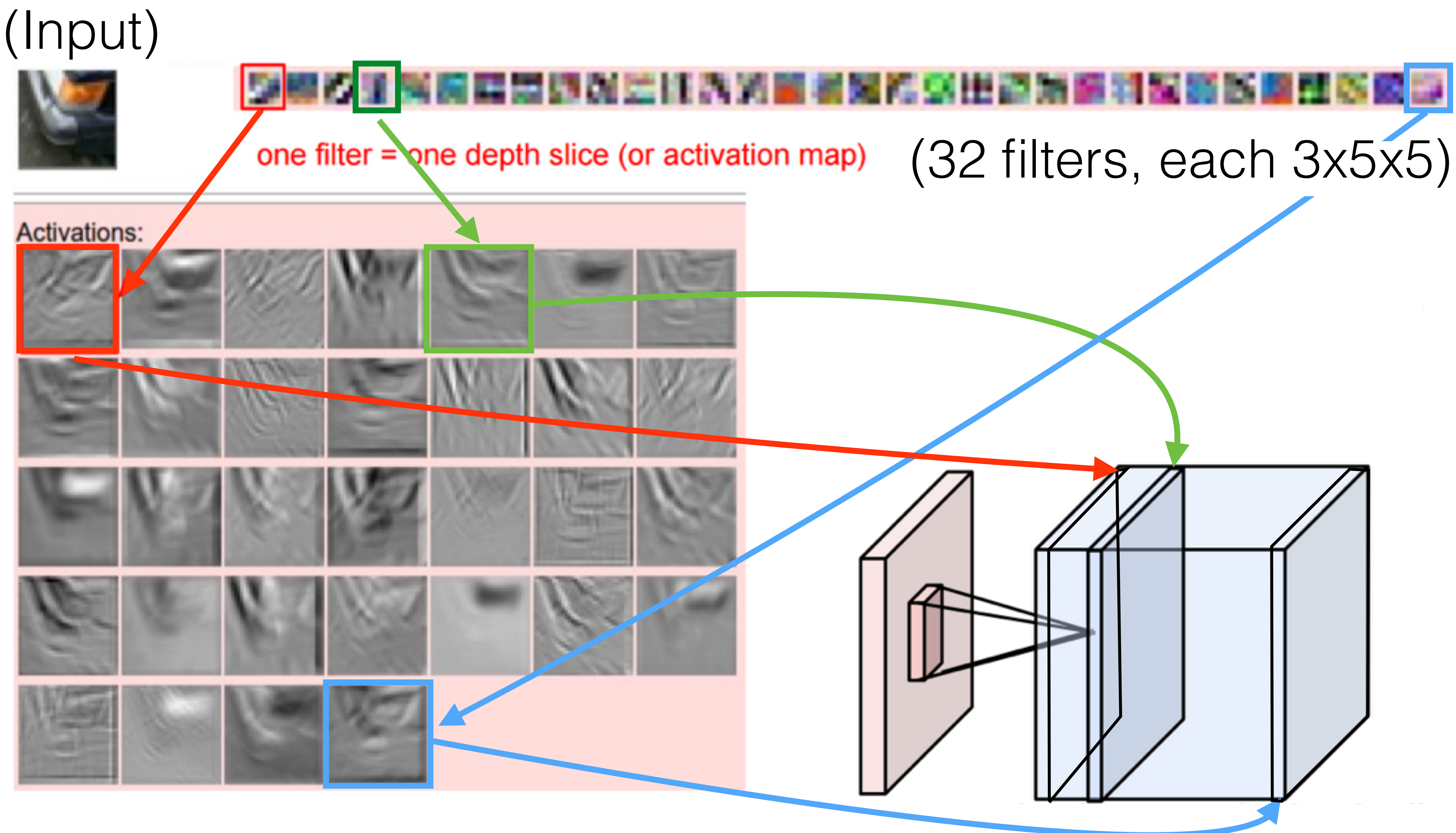


Figure: Andrej Karpathy

Questions?