CS4670 / 5670: Computer Vision

KavitaBala

Lecture 19: Cameras

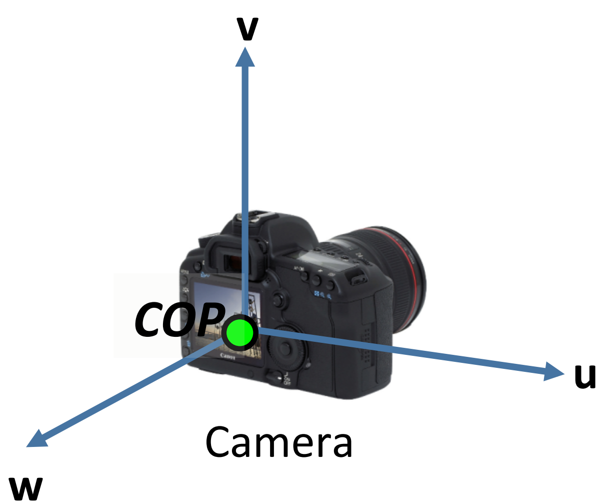
Source: S. Lazebnik

Announcements

- Prelim next Thu
  - Everything till Lecture 17 (Monday)

- My office hours
  - Wed 2:30-3:00->Tuesday 2:30-3:00

A Tale of Two Coordinate Systems



Camera

“The World”

Two important coordinate systems:

1. *World* coordinate system
2. *Camera* coordinate system

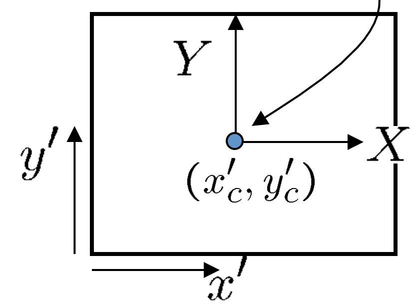
Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principal point (x'_c, y'_c), pixel size (s_x, s_y)
- blue parameters are called "extrinsics," red are "intrinsics"

Projection equation

$$\mathbf{X} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \Pi \mathbf{X}$$



- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

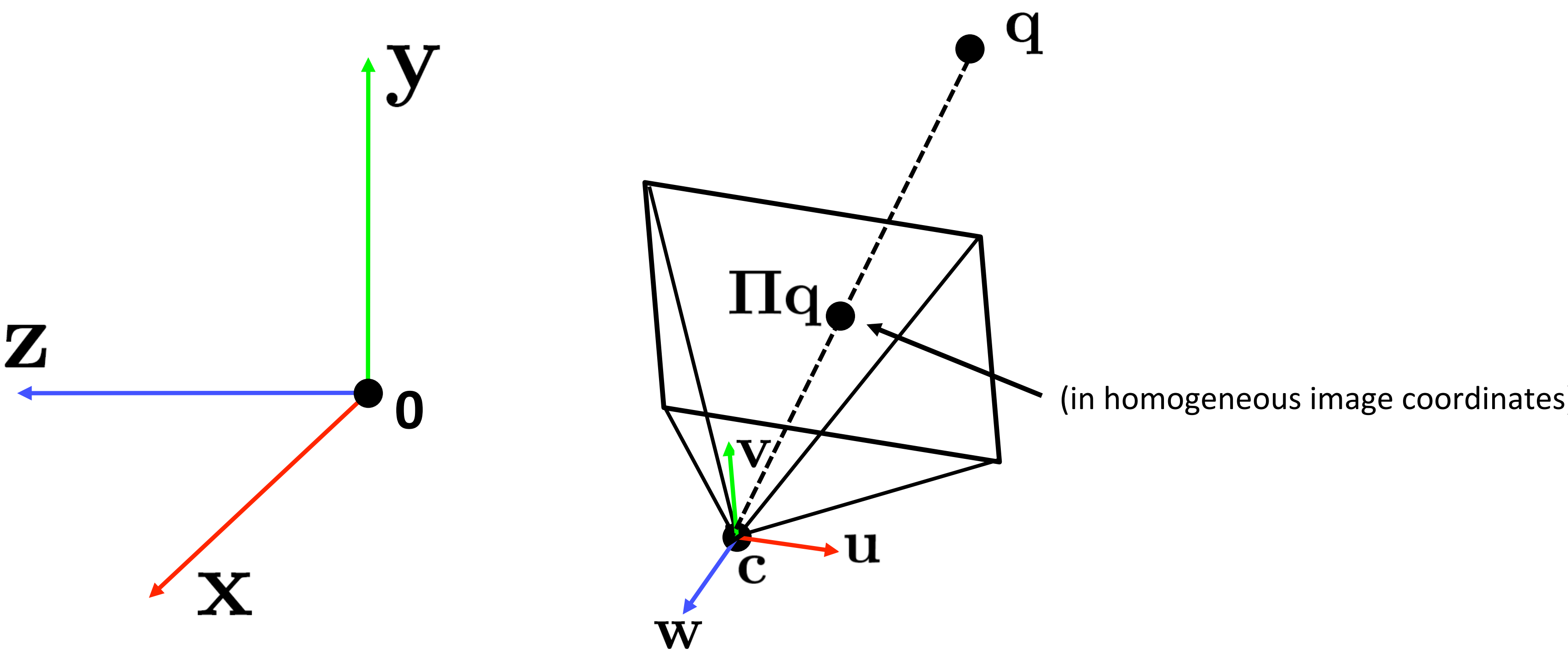
identity matrix

$$\mathbf{\Pi} = \begin{bmatrix} -fs_x & 0 & x'_c \\ 0 & -fs_y & y'_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}$$

intrinsics   projection   rotation   translation

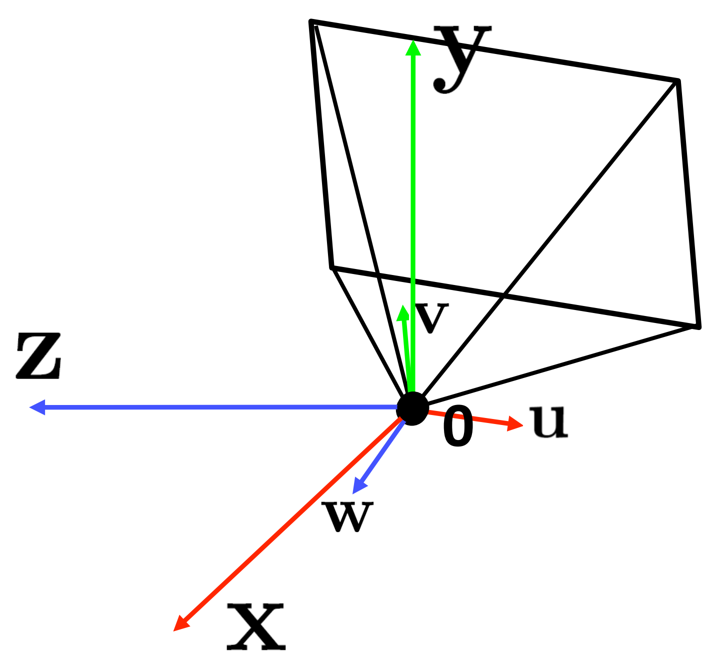
- The definitions of these parameters are **not** completely standardized
  - especially intrinsics—varies from one book to another

Projection matrix



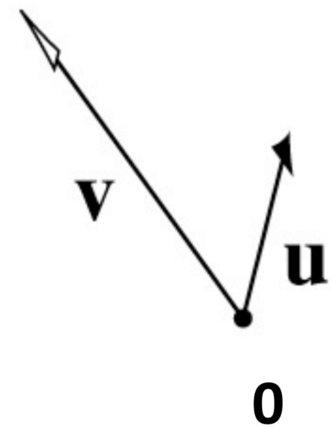
Extrinsics

- How do we get the camera to “canonical form”?
  - (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



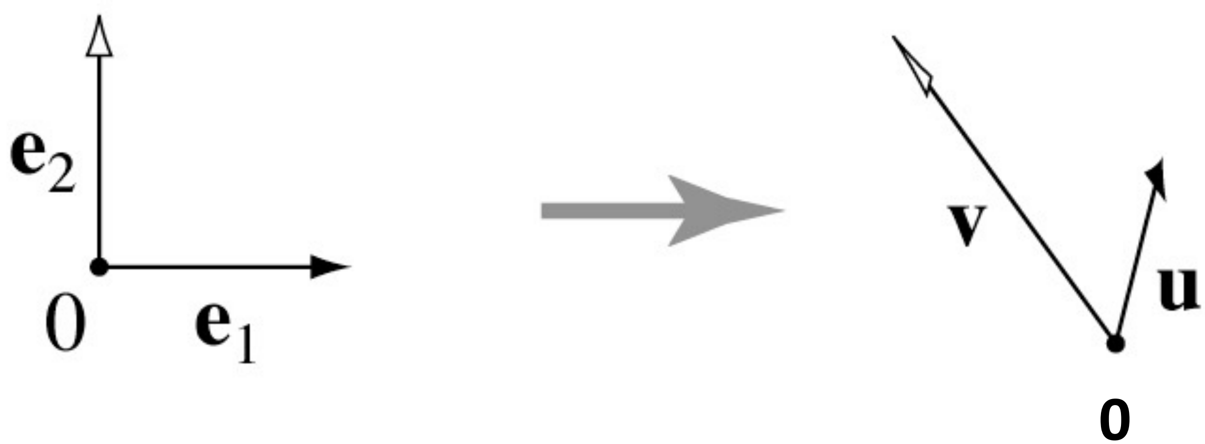
Affine change of coordinates

- Camera coordinate frame
  – point plus basis
- Need to change representation of point from one basis to another
- Canonical: origin (0,0) w/ axes e1, e2



Another way of thinking about this

- Change of coordinates



Recap from 4620

Coordinate frame summary

- Frame = point plus basis
- Frame matrix (uv-to-e1e2) is

$$F = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{o} \\ 0 & 0 & 1 \end{bmatrix}$$

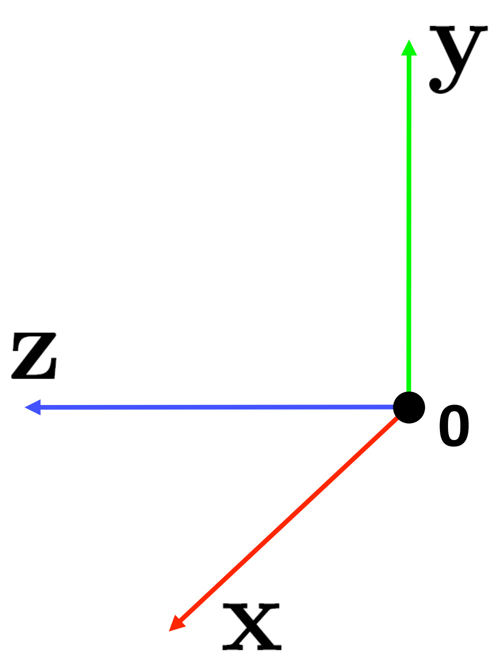
- Move points to and from frame by multiplying with *F*

$$p_e = F p_F \quad p_F = F^{-1} p_e$$

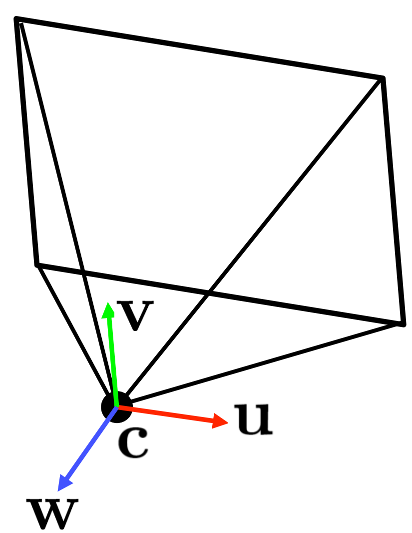
Recap from 4620

Extrinsics

- How to go between camera and world?
  - (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



Step 1: Translate by -**c**



Extrinsics

- How to go between camera and world?
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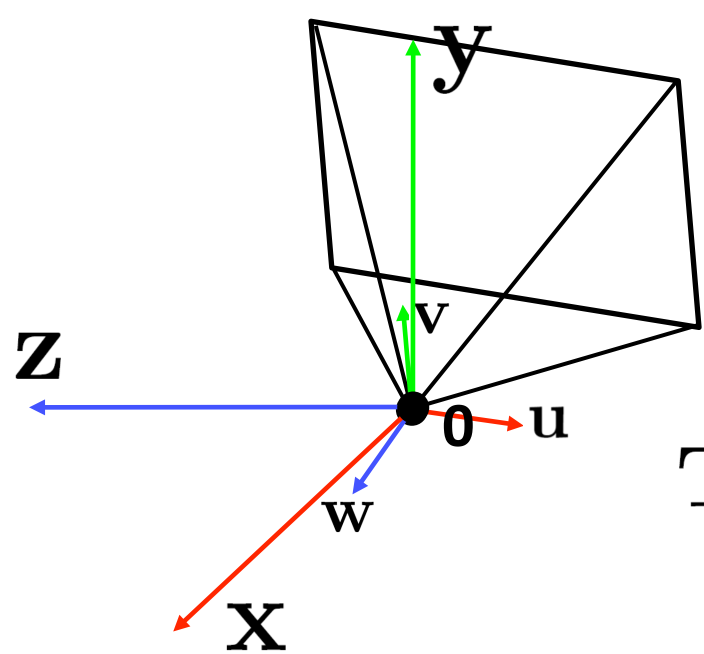
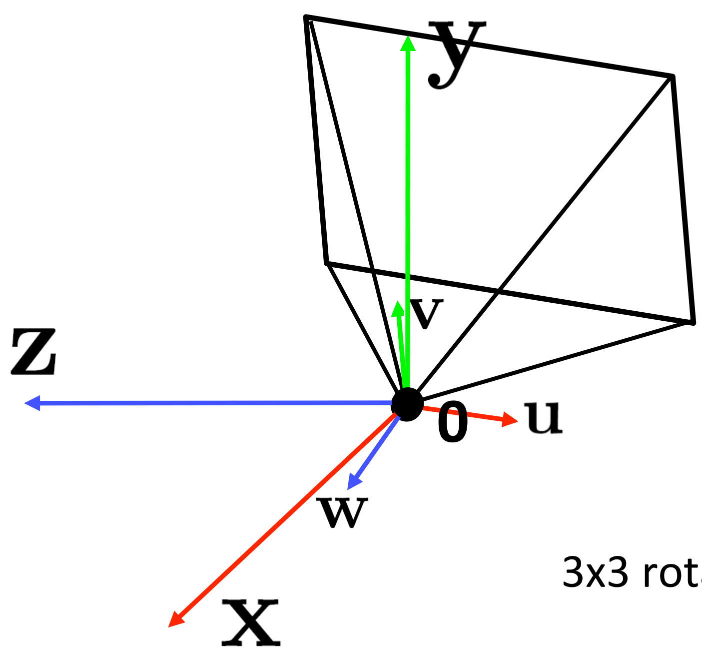


Step 1: Translate by -**c**

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{3\times3} & -\mathbf{c} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

Extrinsics

- How to go between camera and world?
  - (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)



Step 1: Translate by -**c**

Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

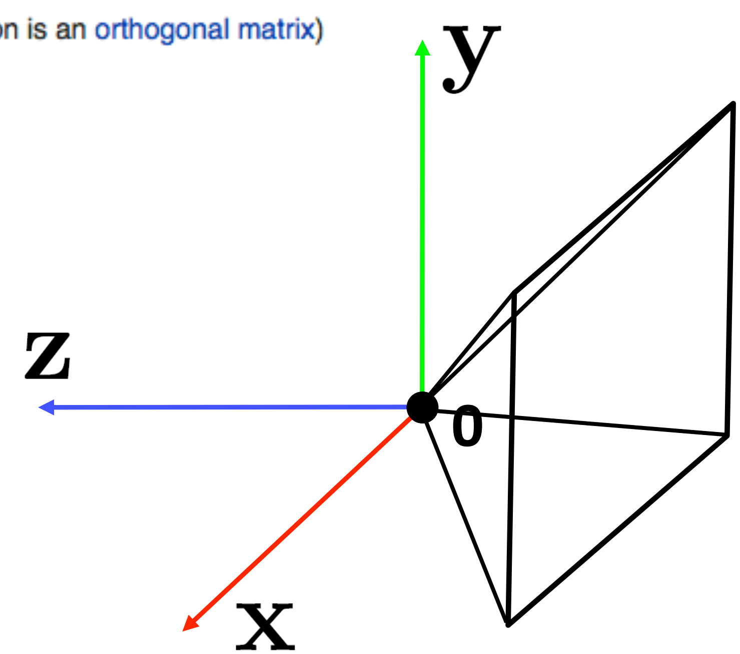
3x3 rotation matrix

Extrinsics

- How to go between camera and world?
  - (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)

For any rotation matrix R acting on $\mathbb{R}^n$,

- $R^T = R^{-1}$ (The rotation is an orthogonal matrix)



Step 1: Translate by -**c**
Step 2: Rotate by **R**

$$\mathbf{R} = \begin{bmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{bmatrix}$$

Perspective projection

$$\underbrace{\begin{bmatrix} -f & 0 & 0 \\ 0 & -f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(intrinsics)

(converts from 3D rays in camera coordinate system to pixel coordinates)

in general, $\mathbf{K} = \begin{bmatrix} -f & s & c_x \\ 0 & -\alpha f & c_y \\ 0 & 0 & 1 \end{bmatrix}$ (upper triangular matrix)

α : **aspect ratio** (1 unless pixels are not square)

s : **skew** (0 unless pixels are shaped like rhombi/parallelograms)

(c_x, c_y) : **principal point** ((0,0) unless optical axis doesn't intersect projection plane at origin)

Projection matrix

$$\Pi = \underbrace{\mathbf{K}}_{\text{intrinsics}} \underbrace{\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3\times 3} & -\mathbf{c} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}}_{\text{translation}}}_{\left[\ \mathbf{R}\ \middle|\ \underbrace{-\mathbf{Rc}}_{\text{(}\mathbf{t}\text{ in book's notation)}}\ \right]}$$

$$\Pi = \mathbf{K}\left[\ \mathbf{R}\ \middle|\ -\mathbf{Rc}\ \right]$$

Focal length

- Can think of as "zoom"

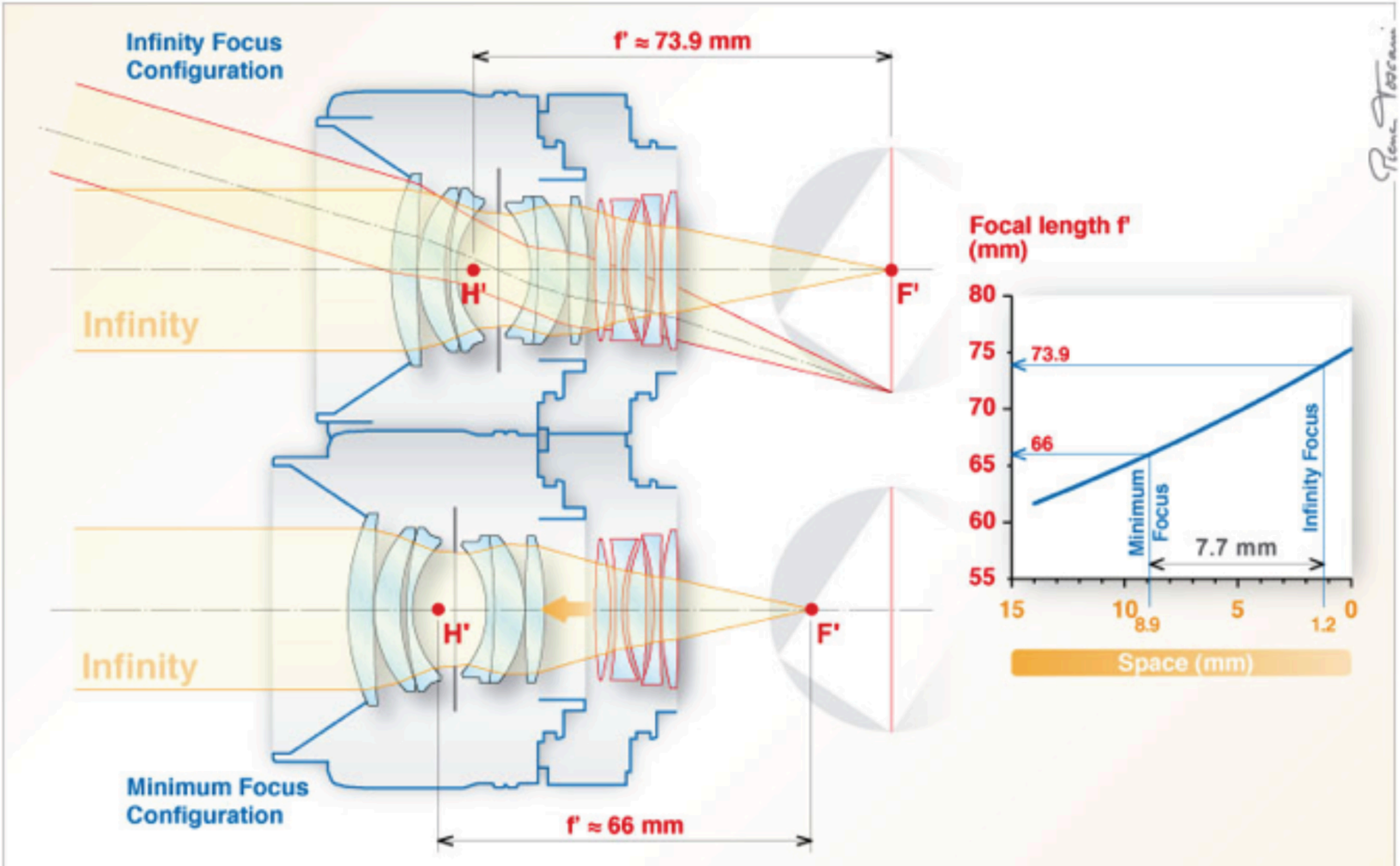
24mm

50mm

200mm

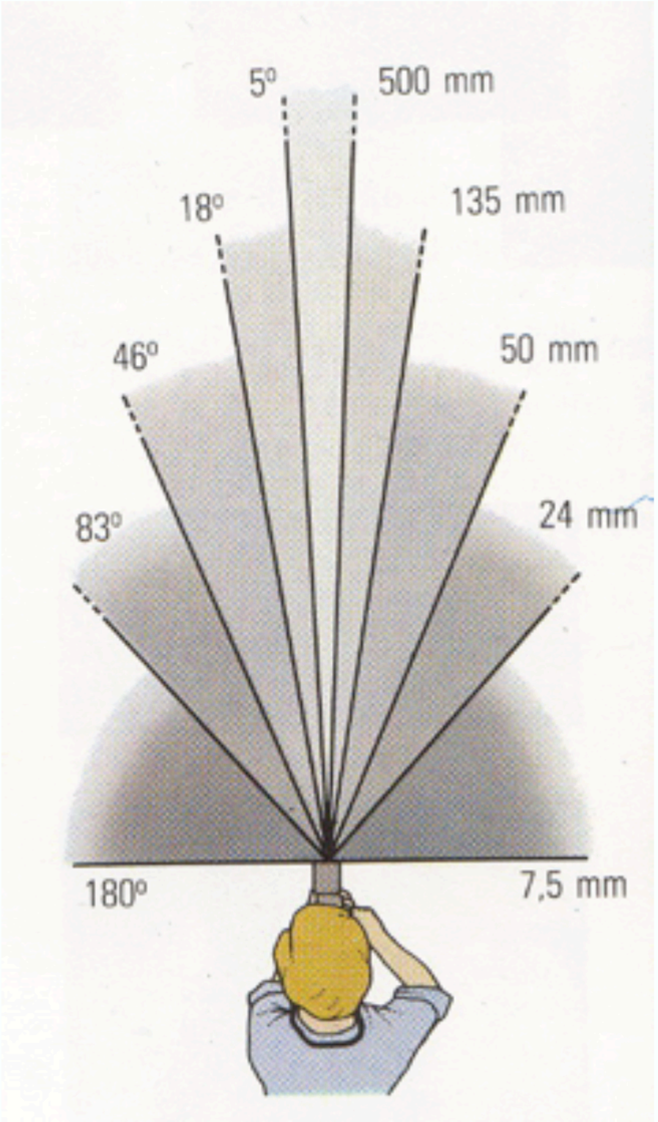
800mm

- Related to *field of view*



http://www.pierretoscani.com/echo_focal_length.html

Focal length in practice



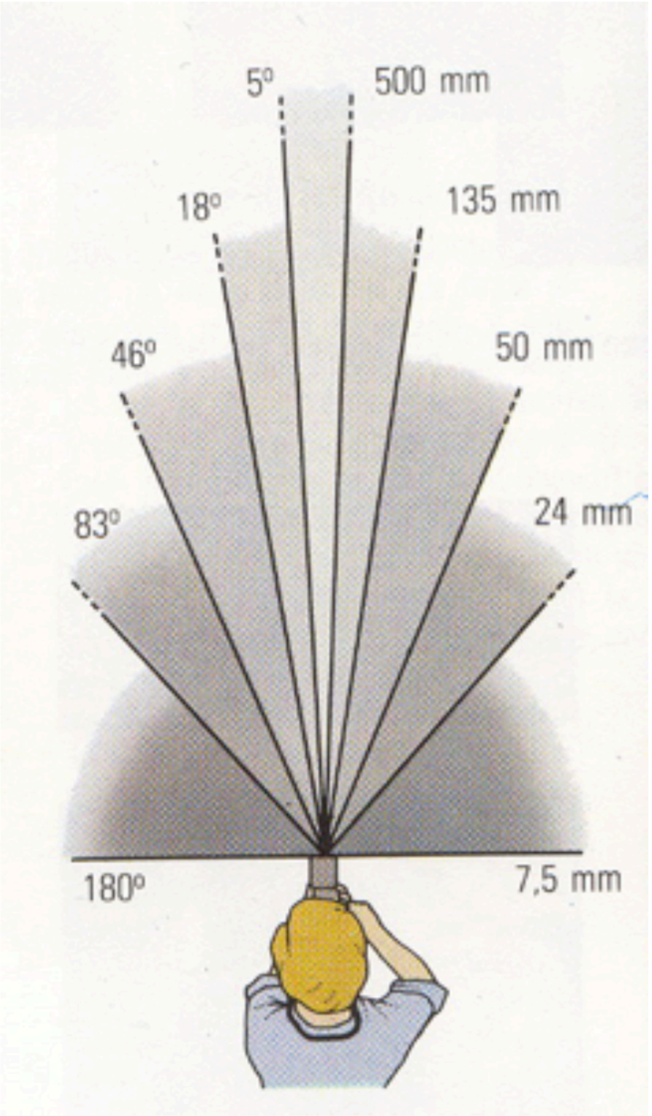
24mm

50mm

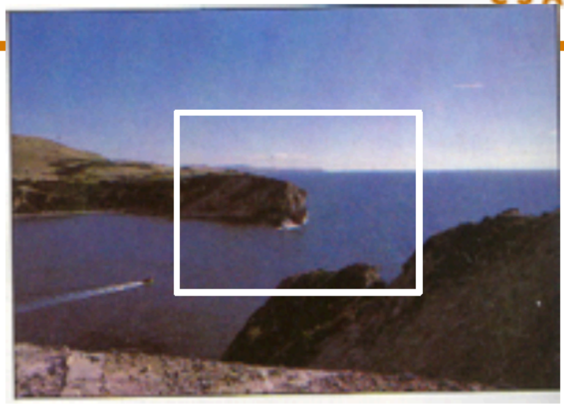
135mm

Fredo Durand

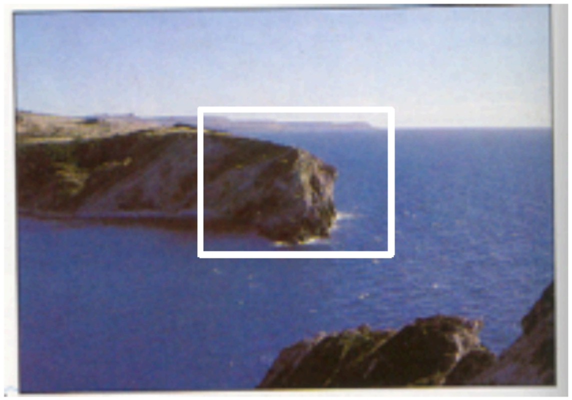
Focal length = cropping



24mm



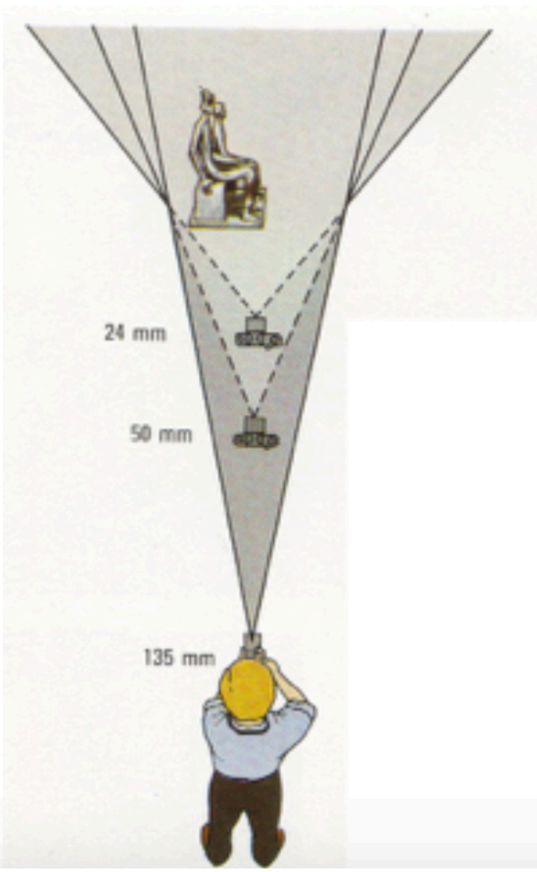
50mm



135mm

Focal length vs. viewpoint

- **Telephoto makes it easier to select background (a small change in viewpoint is a big change in background.**



Grand-angulaire 24 mm

Normal 50 mm

Longue focale 135 mm

Fredo Durand


Fredo Durand

- Hitchcock effect or Vertigo effect

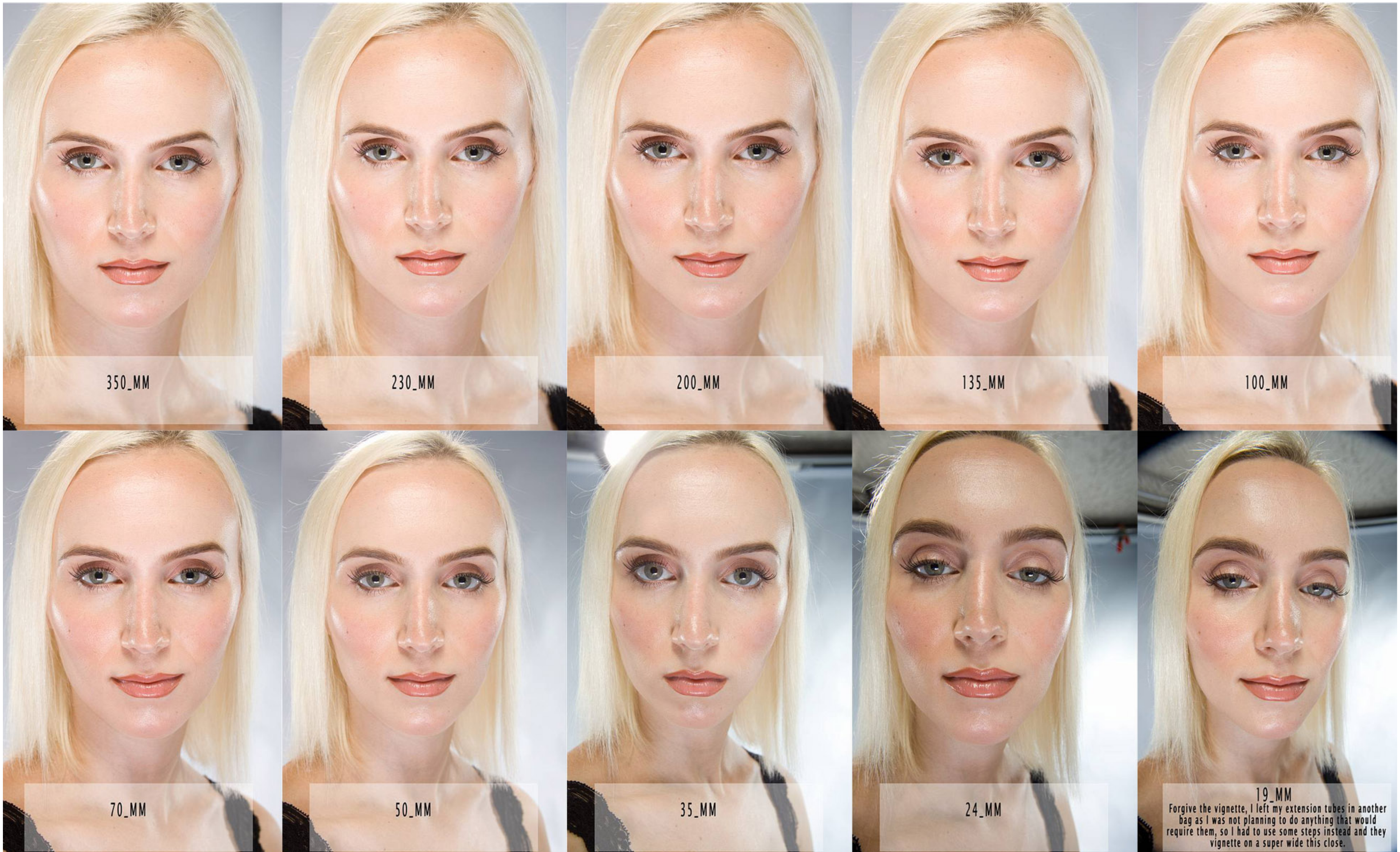
Wide angle

Standard

Telephoto

http://petapixel.com/2013/01/11/how-focal-length-affects-your-subjects-apparent-weight-as-seen-with-a-cat/

Fredo Durand



http://stepheneastwood.com/tutorials/lensdistortion/strippage.htm

Distortion

- 2 types
  - Perspective distortion
  - Lens distortion

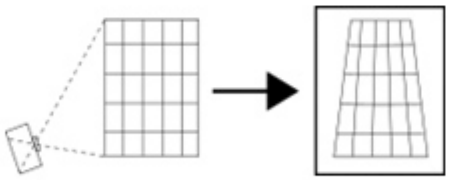
Perspective distortion

- Problem for architectural photography: converging verticals

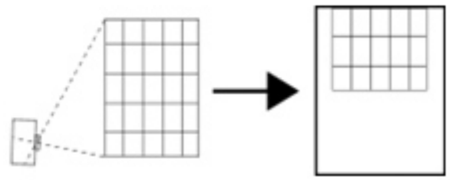
Source: F. Durand

Perspective distortion

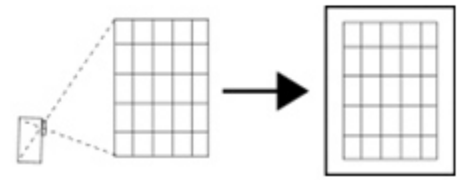
- Problem for architectural photography: converging verticals



Tilting the camera upwards results in converging verticals

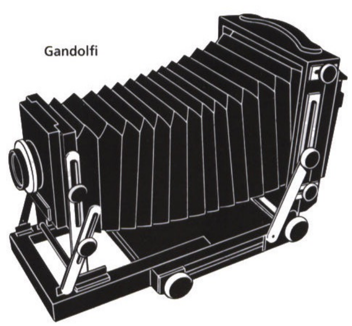


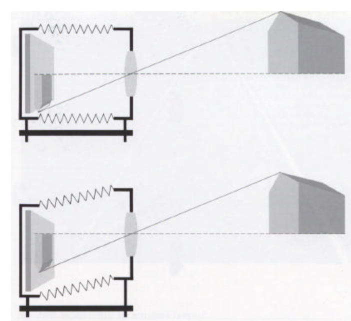
Keeping the camera level, with an ordinary lens, captures only the bottom portion of the building



Shifting the lens upwards results in a picture of the entire subject

- Solution: view camera (lens shifted w.r.t. film)





http://en.wikipedia.org/wiki/Perspective_correction_lens

Source: F. Durand

Perspective distortion

- Problem for architectural photography: converging verticals
- Result:

Source: F. Durand

Perspective distortion

- What does a sphere project to?

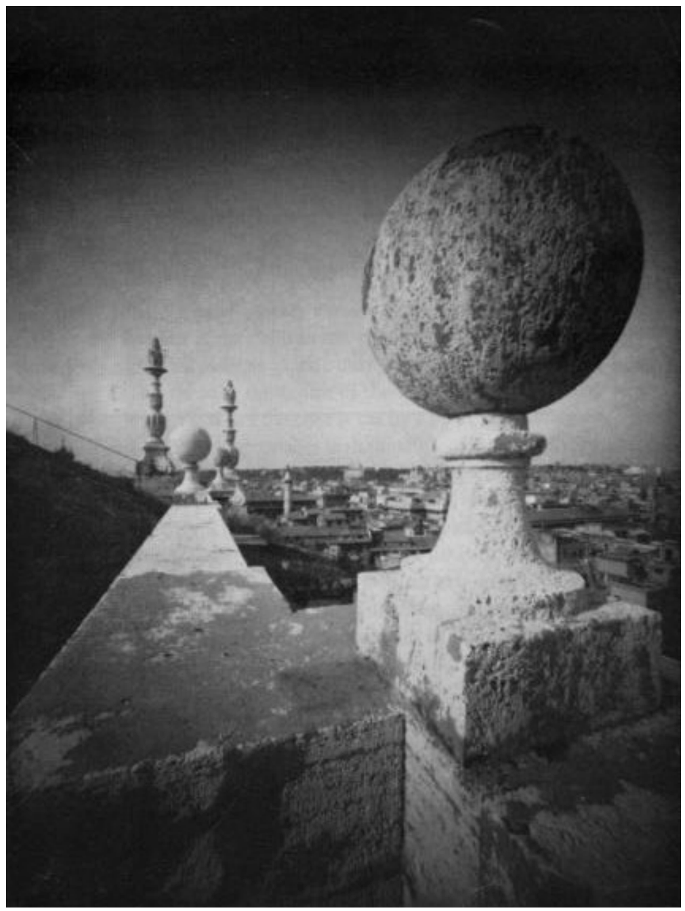
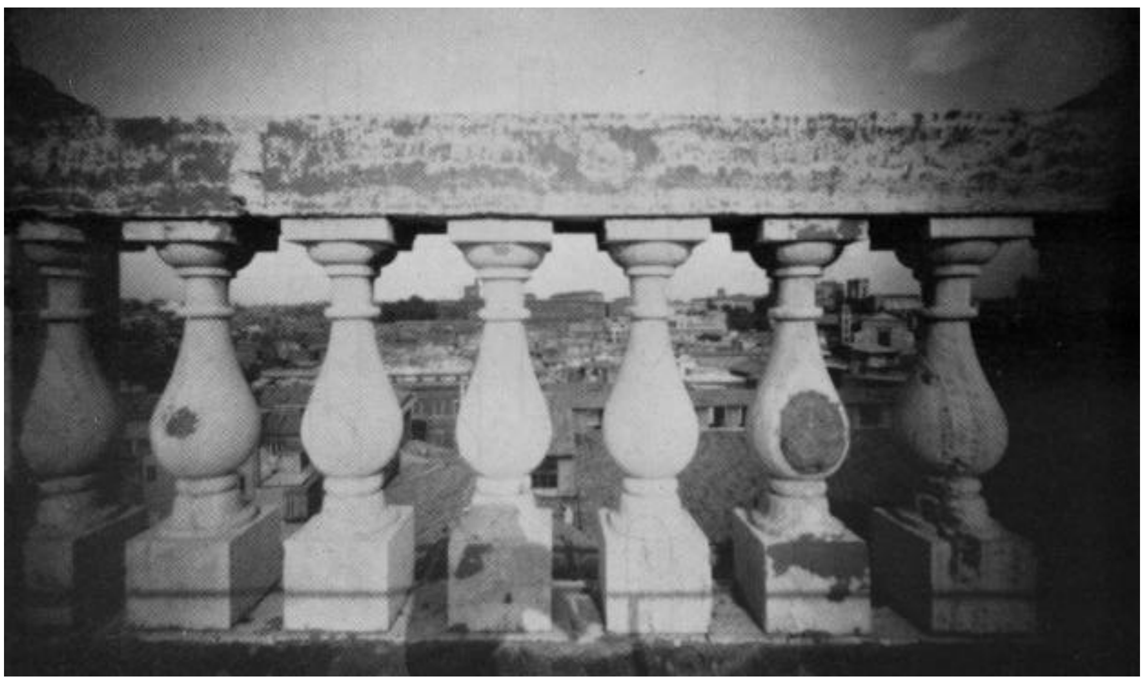
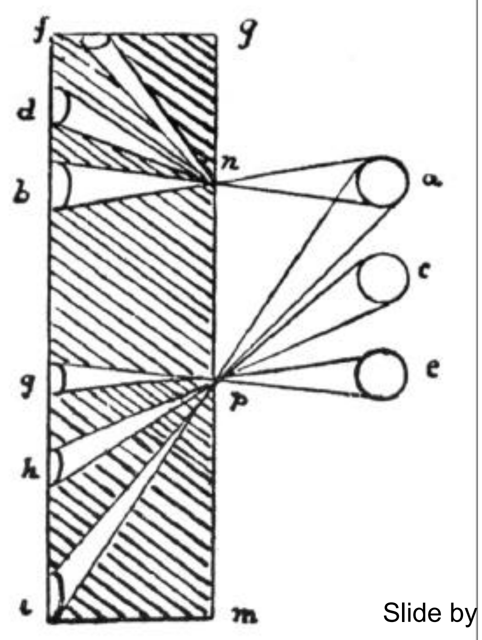


Image source: F. Durand

Perspective distortion

- The exterior columns appear bigger
- The distortion is not due to lens flaws
- Problem pointed out by Da Vinci

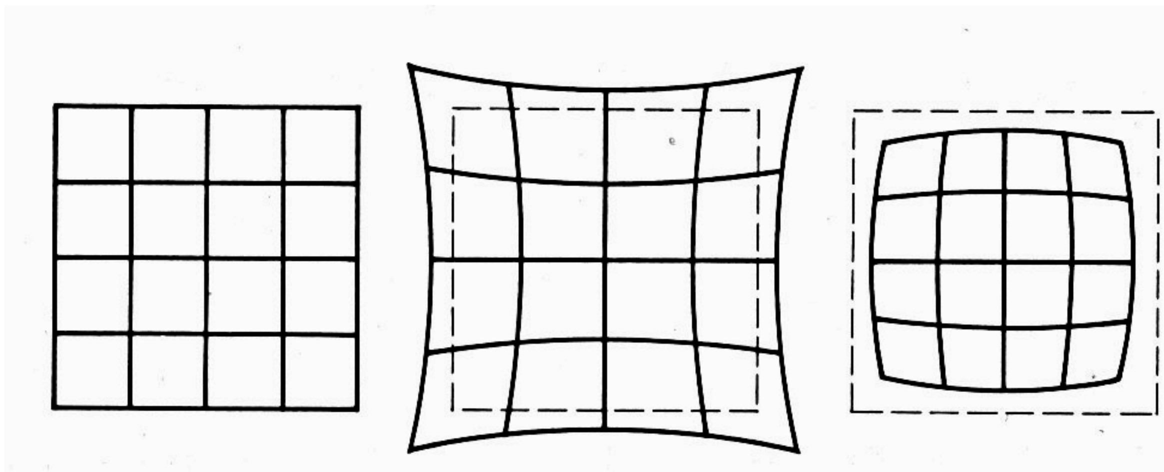




Slide by F. Durand

Perspective distortion: People

Distortion due to lens


No distortion   Pin cushion   Barrel

- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens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Modeling distortion

- Radial distortion model

- Apply after projection, but before camera intrinsic: f and (xc, yc) translation

Project $(\widehat{x}, \widehat{y}, \widehat{z})$ to "normalized" image coordinates

$$x'_n = \widehat{x}/\widehat{z}$$
$$y'_n = \widehat{y}/\widehat{z}$$

Modeling distortion

Apply radial distortion

$$\begin{aligned} r^2 &= {x'_n}^2 + {y'_n}^2 \\ x'_d &= x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4) \\ y'_d &= y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4) \end{aligned}$$

Apply focal length
translate image center

$$\begin{aligned} x' &= f x'_d + x_c \\ y' &= f y'_d + y_c \end{aligned}$$

- To model lens distortion
  - Use above projection operation instead of standard projection matrix multiplication

Correcting radial distortion

from [Helmut Dersch]

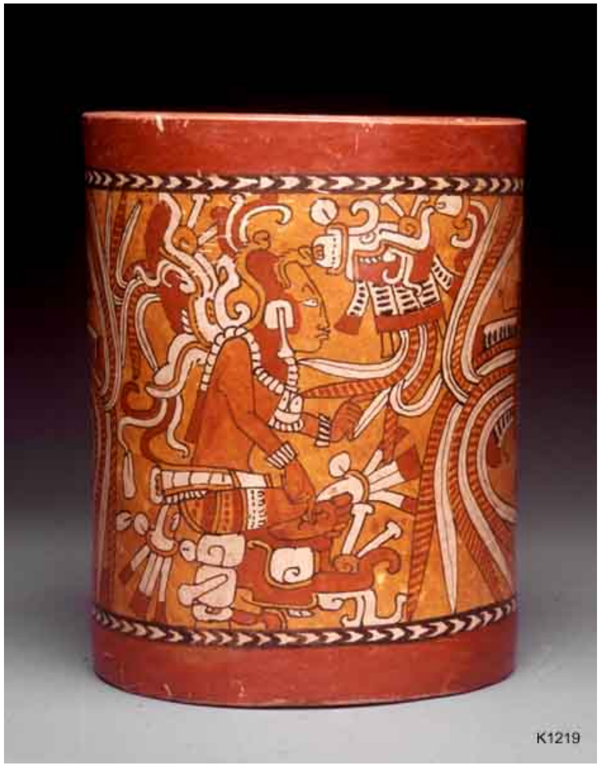
Other types of projection

- Lots of intriguing variants…
- (I'll just mention a few fun ones)

360 degree field of view…

- Basic approach
  - Take a photo of a parabolic mirror with an orthographic lens (Nayar)
  - Or buy one a lens from a variety of omnicam manufacturers…
    - See http://www.cis.upenn.edu/~kostas/omni.html

Rotating sensor (or object)



http://research.famsi.org/kerrmaya.html

Also known as “cyclographs”, “peripheral images”