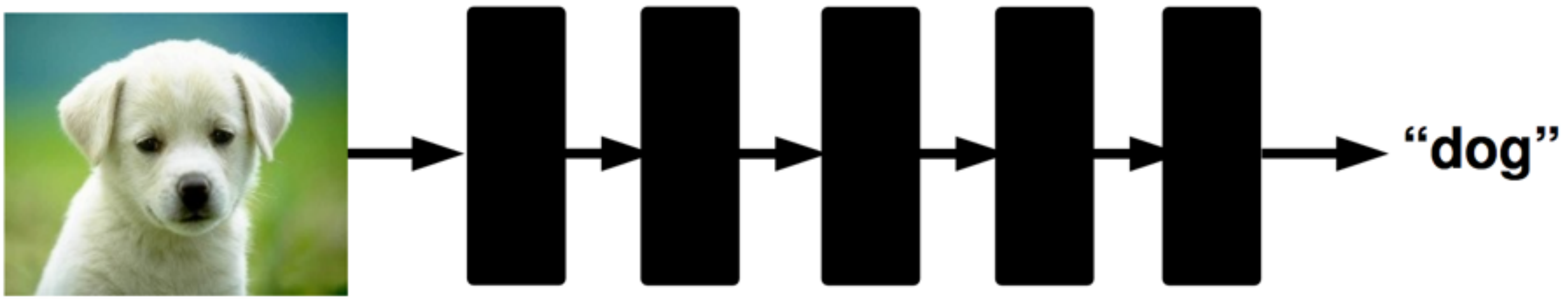
CS4670/5670: Computer Vision

Kavita Bala and Sean Bell

Lecture 30: Neural Nets and CNNs



Today

- PA 4 due this week
- PA 5 out this week. Due on Monday May 4

- HW 2 due next week

- Class on Monday (Charter Day)

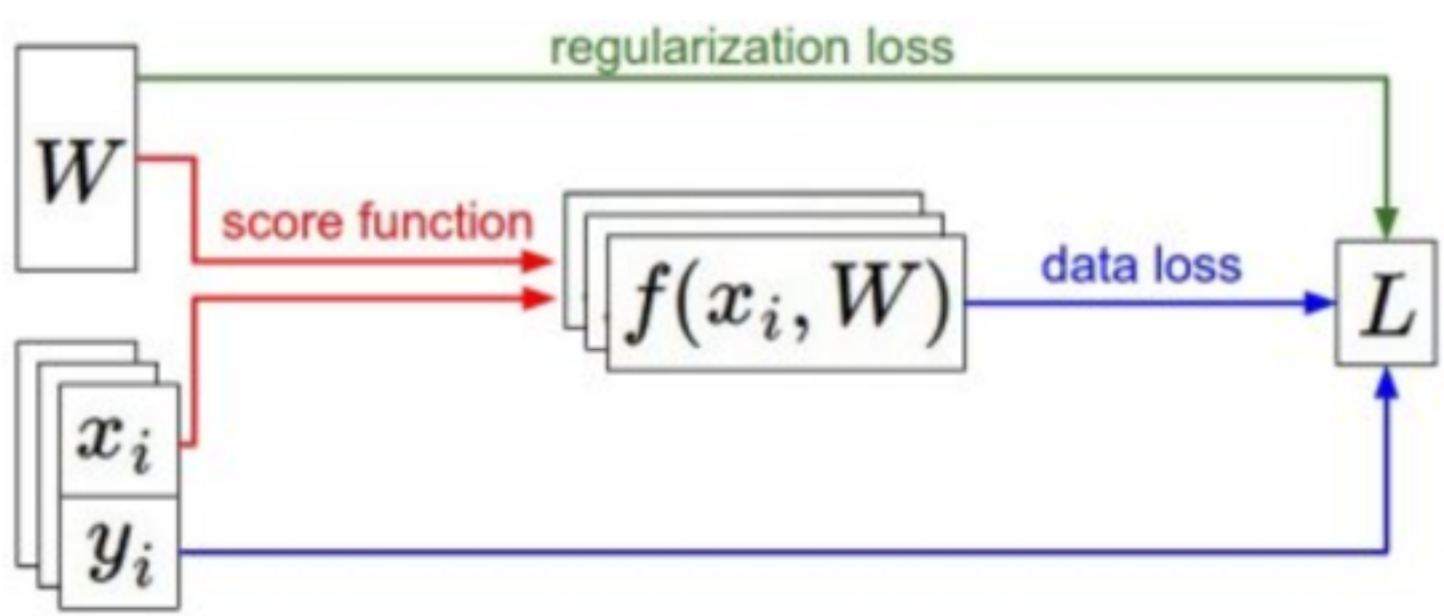
Where are we?

1. Score function

$$f(x_i, W, b) = Wx_i + b$$

2. Loss function

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} \left[\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) \right] + \lambda R(W)$$



Where are we?

- Have score function and loss function
  - Will generalize the score function
- Find W and b to minimize loss

Where are we?

- Gradient descent to optimize loss functions
  - Batch gradient descent, stochastic gradient descent
  - Momentum
- Where do we get gradients of loss?
  - Compute them
  - Backpropagation and chain rule (more today)

- classification

Top 5:
pencil sharpener
pool table
hand blower
oil filter
packet

Groundtruth:
pencil sharpener

- localization

Groundtruth:
white wolf
white wolf (2)
white wolf (3)
white wolf (4)
white wolf (5)

- detection

Groundtruth:
tv or monitor
tv or monitor (2)
tv or monitor (3)
person
remote control
remote control (2)

- segmentation

difficulty

Image classification: training

Training
Labels

Training
Images

Image classification: training

Training Labels

Training Images

Image classification: training

Training Labels

Training Images

Classifier Training

Image classification: training

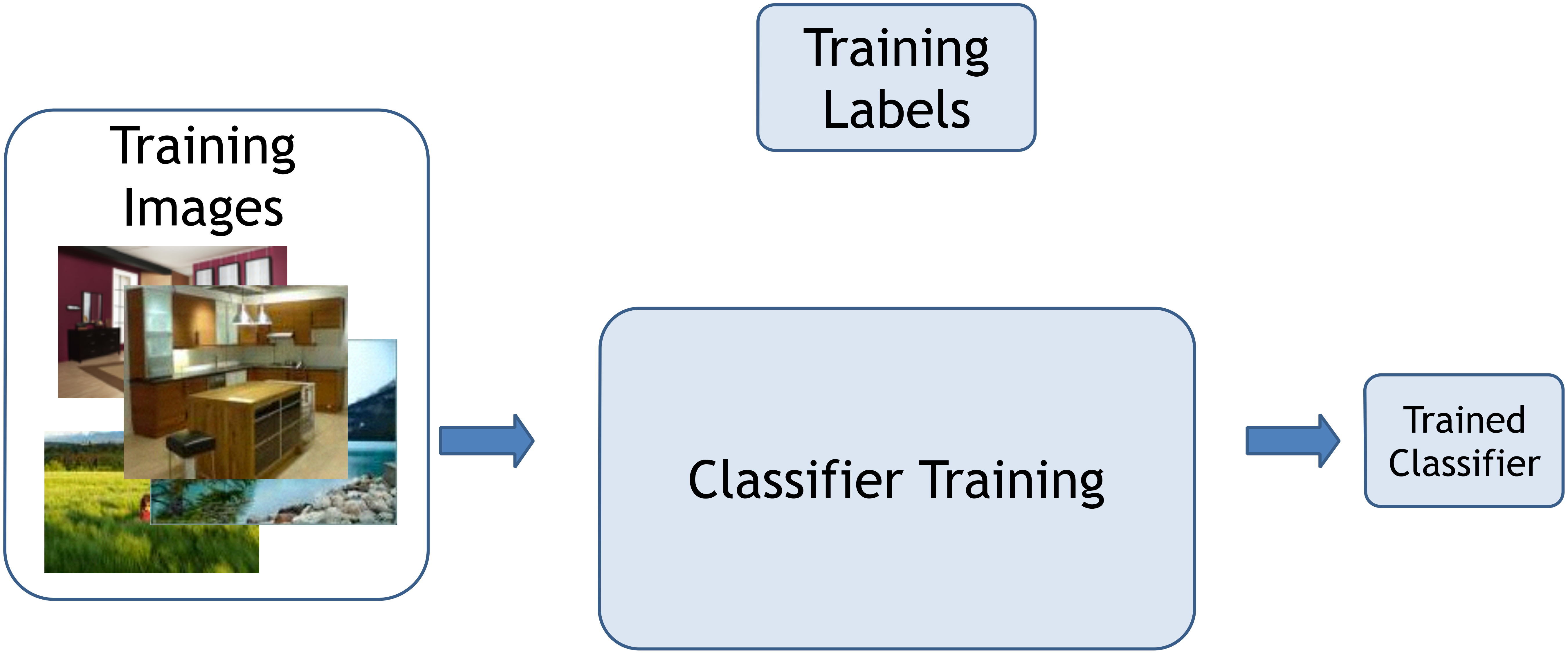


Image classification: training

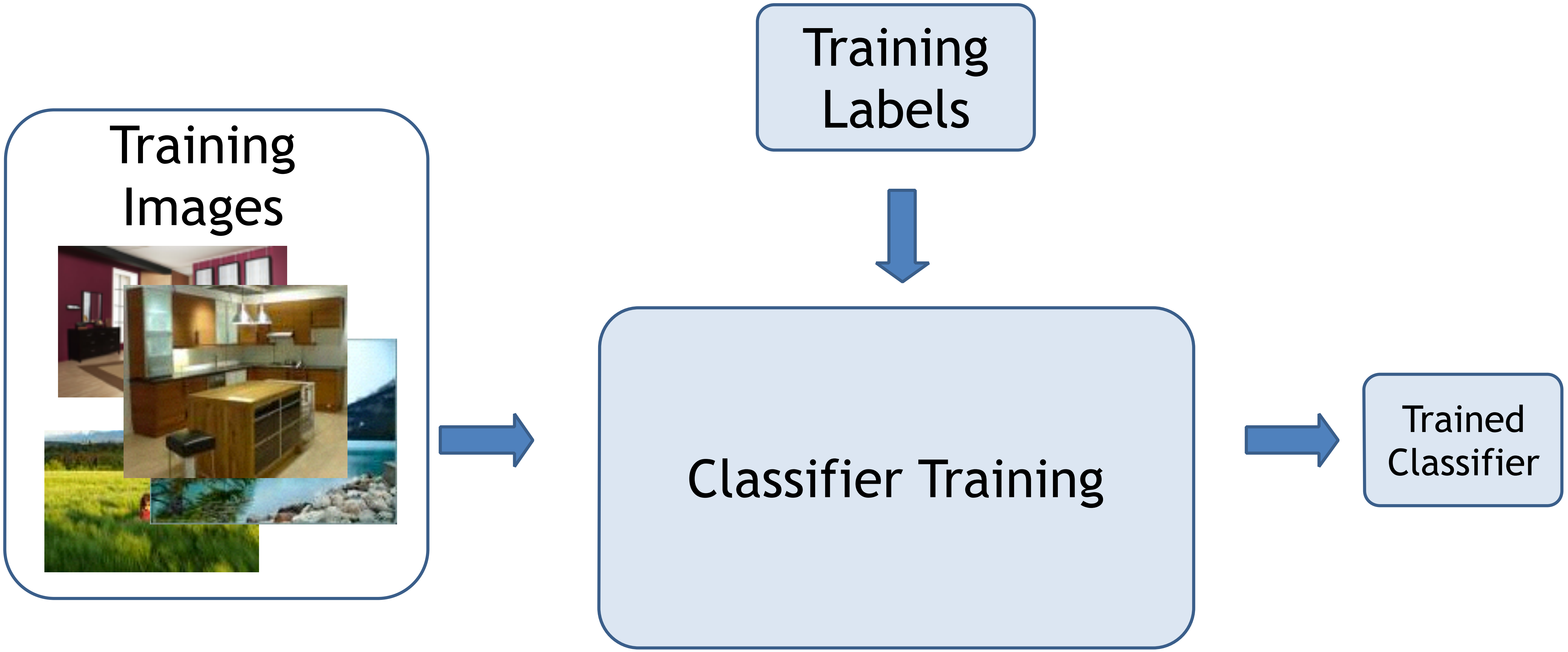


Image classification: testing

Test Image

Image classification: testing

Test Image

Image classification: testing

Test Image

Trained Classifier

Image classification: testing

Test Image

Trained Classifier

Prediction

Outdoor

Traditional Recognition Approach

Input data (pixels) → feature representation (hand-crafted) → Learning Algorithm (e.g., SVM)

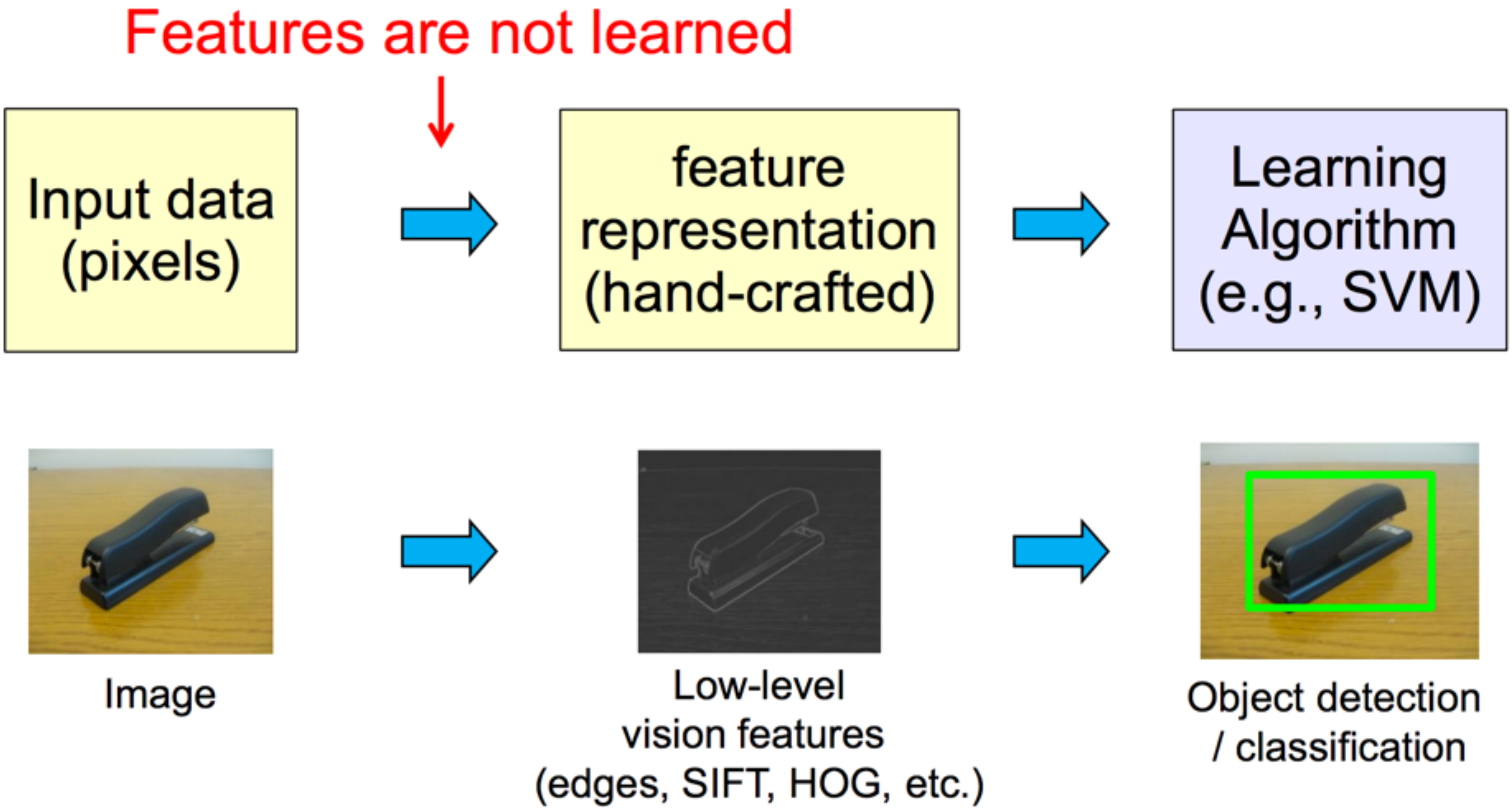
Image

Low-level vision features (edges, SIFT, HOG, etc.)

Object detection / classification

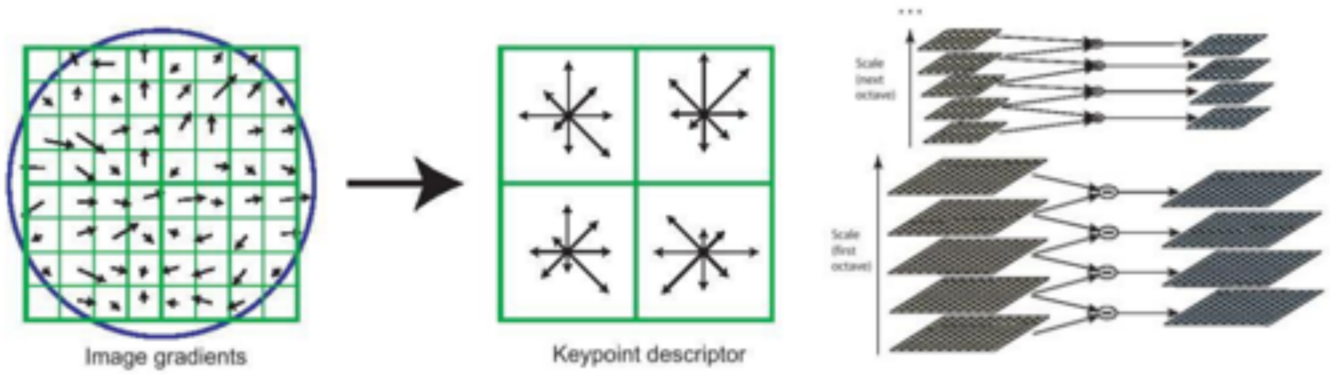
Slide: R. Fergus

Traditional Recognition Approach

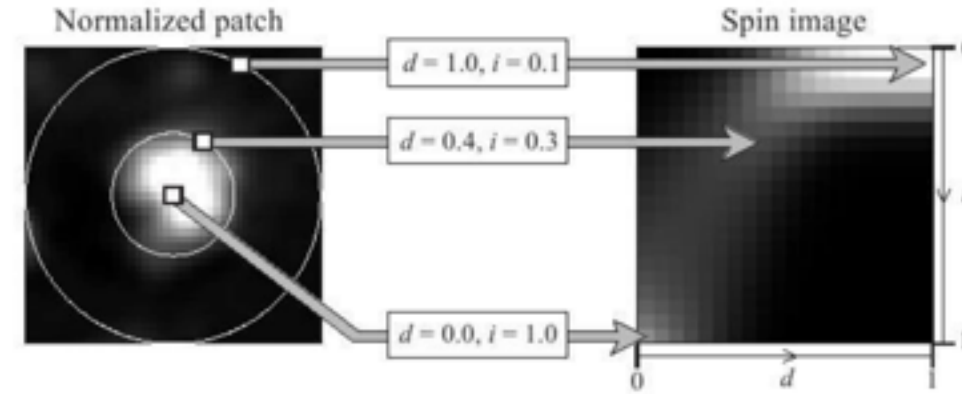


Slide: R. Fergus

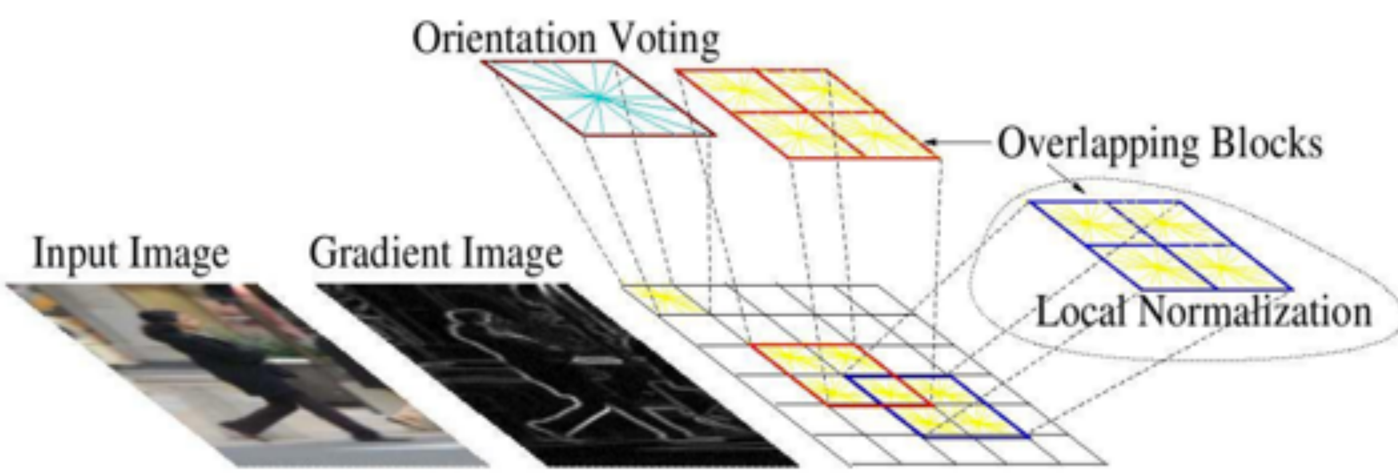
Computer vision features



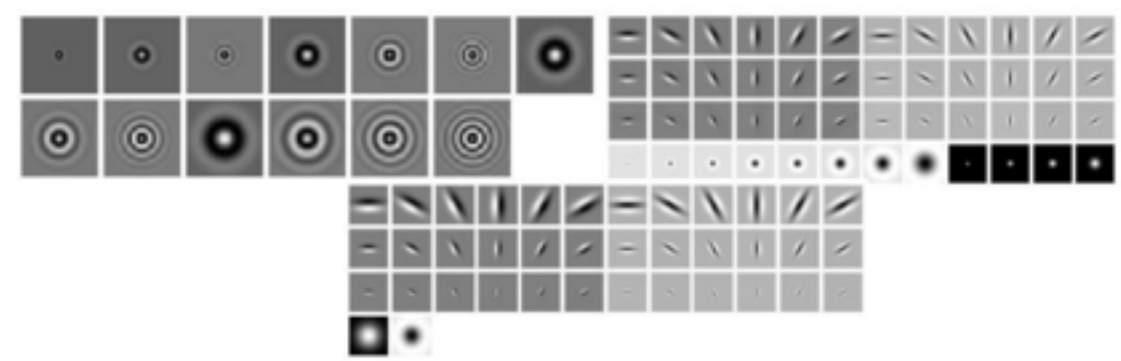
SIFT



Spin image



HoG



Textons

and many others:

SURF, MSER, LBP, Color-SIFT, Color histogram, GLOH, ...

Slide: R. Fergus

Mid-Level Representations

- Mid-level cues

"Tokens" from Vision by D.Marr:

- Object parts:

- Difficult to hand-engineer → What about learning them?

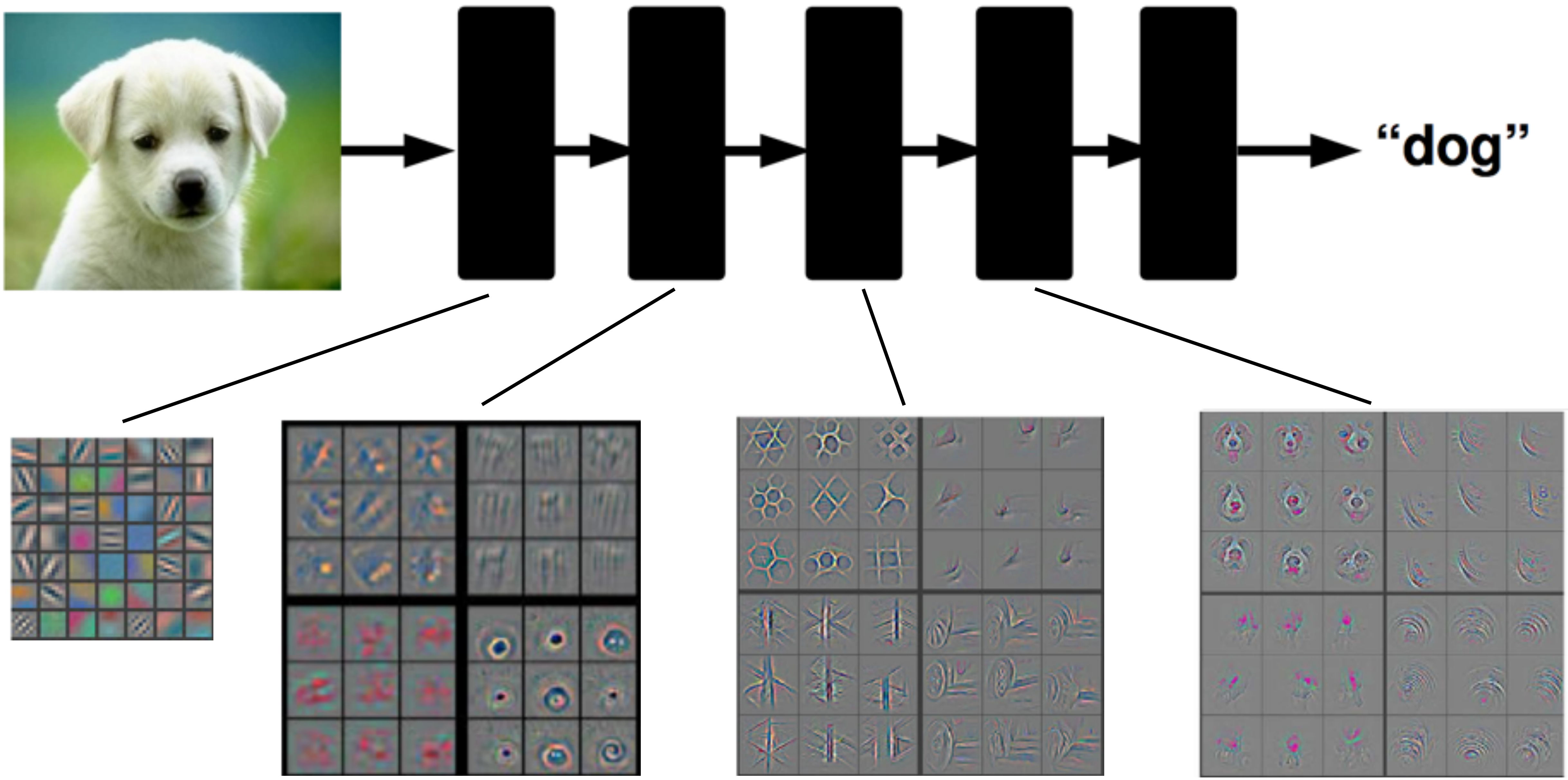
Slide: R. Fergus

Learning Feature Hierarchy

- Learn hierarchy
- All the way from pixels → classifier
- One layer extracts features from output of previous layer

Slide: R. Fergus

Feature hierarchy with CNNs
End-to-end models



[Zeiler and Fergus, "Visualizing and Understanding Convolutional Networks", 2013]

IMAGENET Large Scale Visual Recognition Challenge

Year 2010

NEC-UIUC

Dense grid descriptor: HOG, LBP

↓

Coding: local coordinate, super-vector

↓

Pooling, SPM

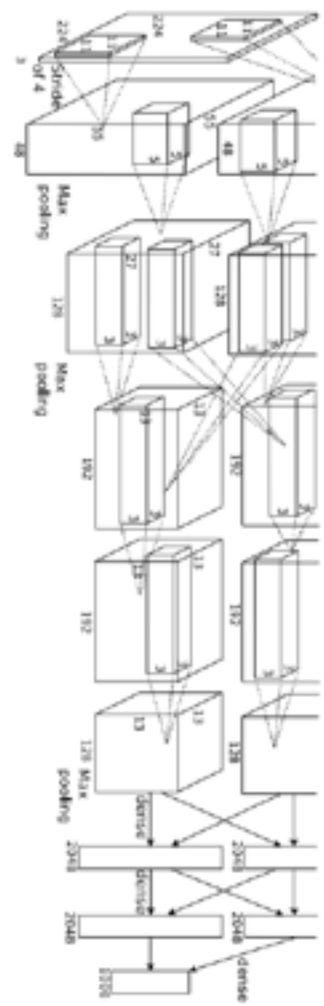
↓

Linear SVM

[Lin CVPR 2011]

Year 2012

SuperVision



[Krizhevsky NIPS 2012]

Year 2014

GoogLeNet

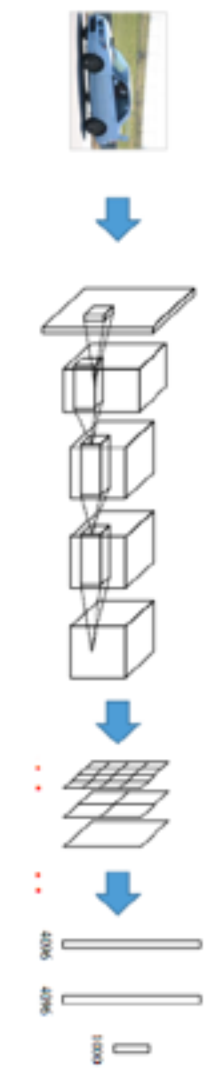
[Szegedy arxiv 2014]

VGG

image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
FC-1000
softmax

[Simonyan arxiv 2014]

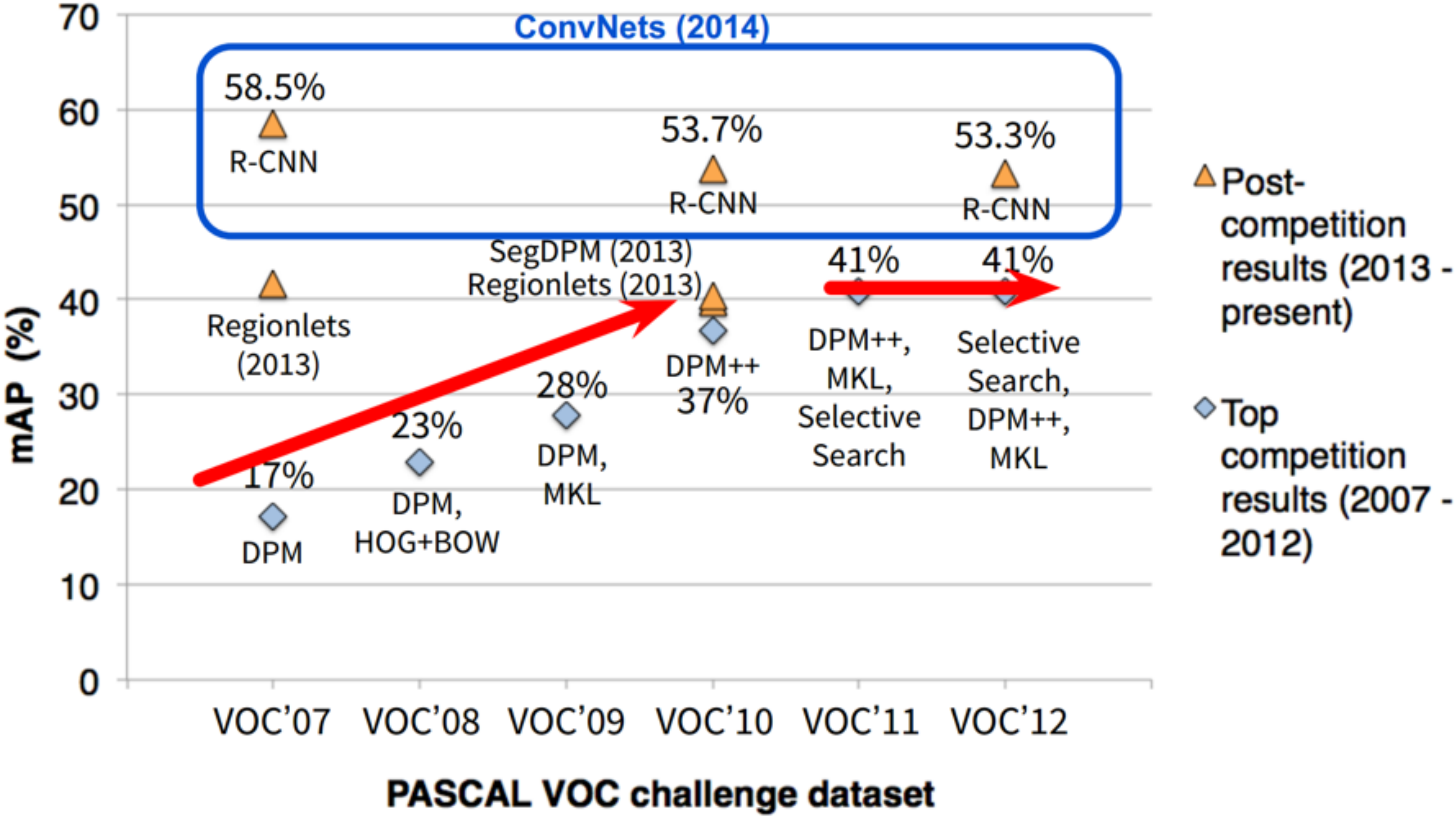
MSRA



[He arxiv 2014]

Recent history of object detection

- **Large improvements** using Deep Learning [Girshick'13/14]



Rich feature hierarchies for accurate object detection and semantic segmentation. Girshick, Ross, Jeff Donahue, Trevor Darrell, and Jitendra Malik. *arXiv preprint arXiv:1311.2524* (2013).

The PASCAL Visual Object Classes Challenge - a Retrospective, Everingham, M., Eslami, S. M. A., Van Gool, L., Williams, C. K. I., Winn, J. and Zisserman, A. Accepted for International Journal of Computer Vision, 2014

Convolutional Neural Networks

CS 4670
Sean Bell

(Sep 2014)

code.flickr.com

Posted on October 20, 2014 by Rob Hess, Clayton Mellina, and Friends

← Previous

Introducing: Flickr PARK or BIRD

Zion National Park Utah by Les Haines

OR

Secretary Bird by Bill Gracey

Slide: Flickr

code.flickr.com

flickr.com Flickr jobs!

To play, drag an image from the examples or from your desktop.

PARK or BIRD

Want to know if your photo is from a U.S. national park? Want to know if it contains a bird? Just drag it into the box to the left, and we'll tell you. We'll use the GPS embedded in your photo (if it's there) to see whether it's from a park, and we'll use our super-cool computer vision skills to try to see whether it's a bird (which is a hard problem, but we do a pretty good job at it).

To try it out, just drag any photo from your desktop into the upload box, or try dragging any of our example images. We'll give you your answers below!

Want to know more about PARK or BIRD, including why the heck we did this? Just click here for more info →

PARK? BIRD?

EXAMPLE PHOTOS

Photo credits

Slide: Flickr

code.flickr.com

flickr.com Flickr jobs!

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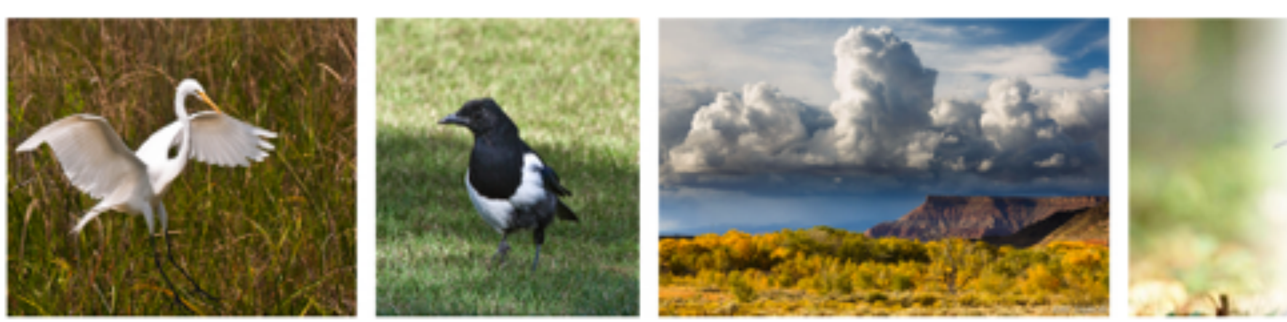

Photo credits

PARK?

YES

Ah yes, Bryce Canyon is truly beautiful.

BIRD?

NO

Beautiful clouds, but I don't see any birds flying up there.

Slide: Flickr

code.flickr.com

flickr.com Flickr jobs!

PARK or BIRD

Want to know if your photo is from a U.S. national park? Want to know if it contains a bird? Just drag it into the box to the left, and we'll tell you. We'll use the GPS embedded in your photo (if it's there) to see whether it's from a park, and we'll use our super-cool computer vision skills to try to see whether it's a bird (which is a hard problem, but we do a pretty good job at it).

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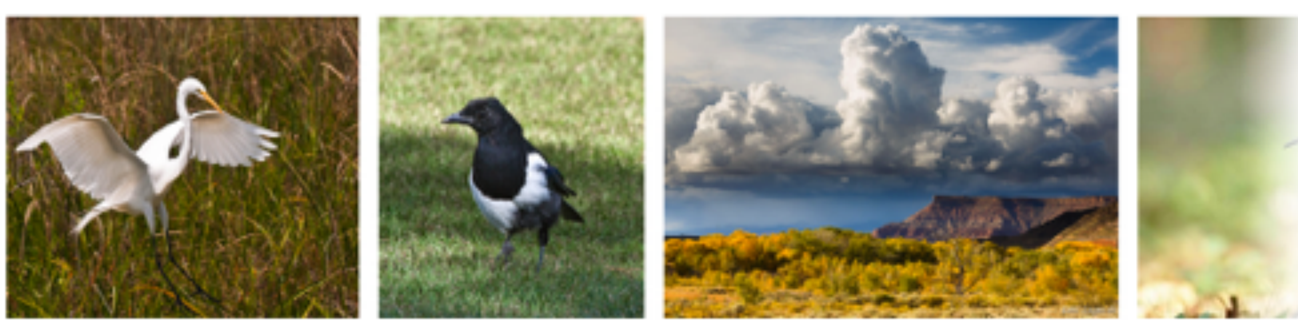


Photo credits

PARK?

YES

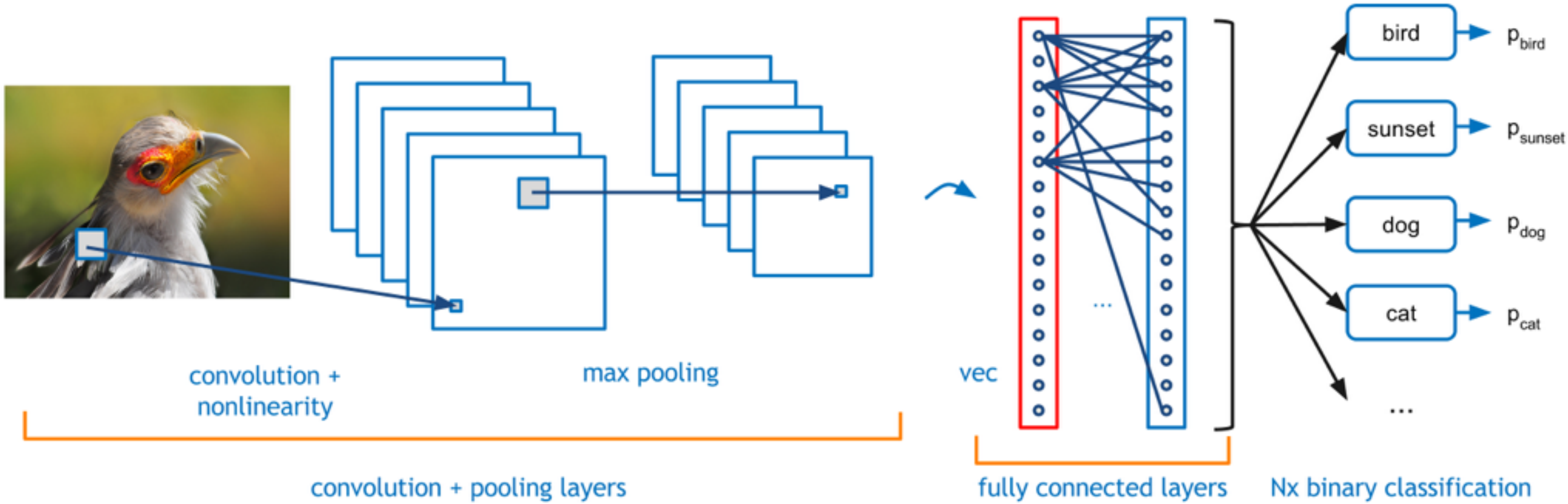
Hey, yeah! I went to Everglades once!

BIRD?

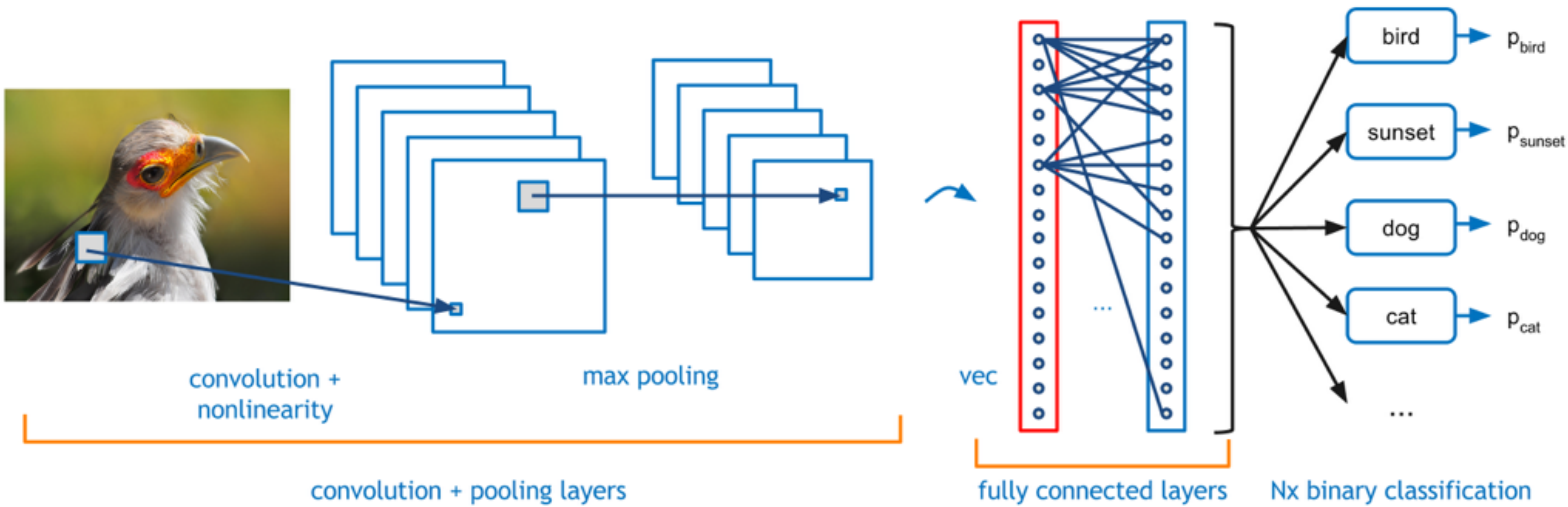
YES

Hey! Nice bird shot!

Slide: Flickr



Slide: Flickr



This week, we'll learn what this is, how to compute it, and how to learn it

Slide: Flickr

What is a Convolutional Neural Network (CNN)?

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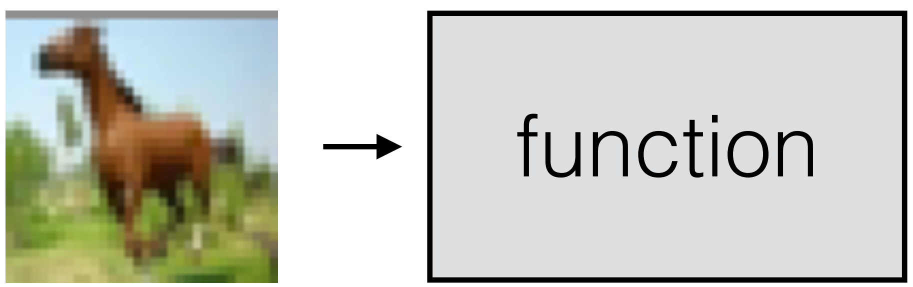

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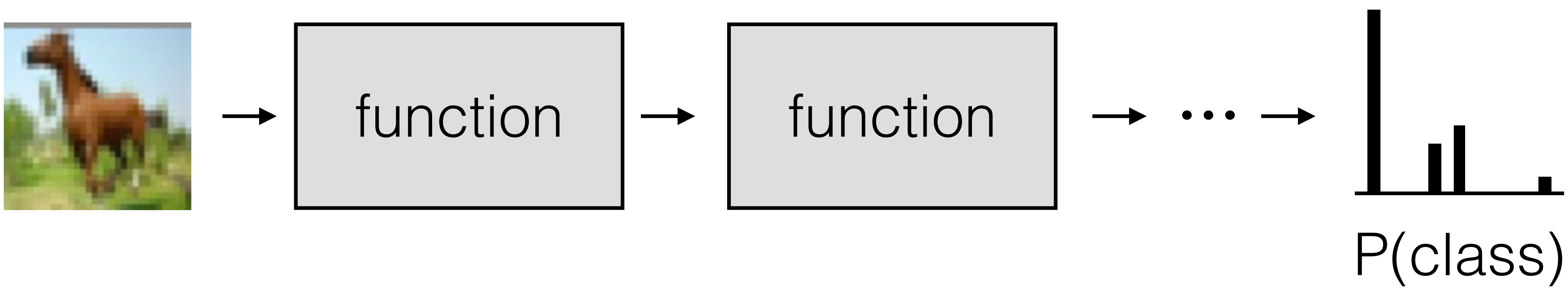

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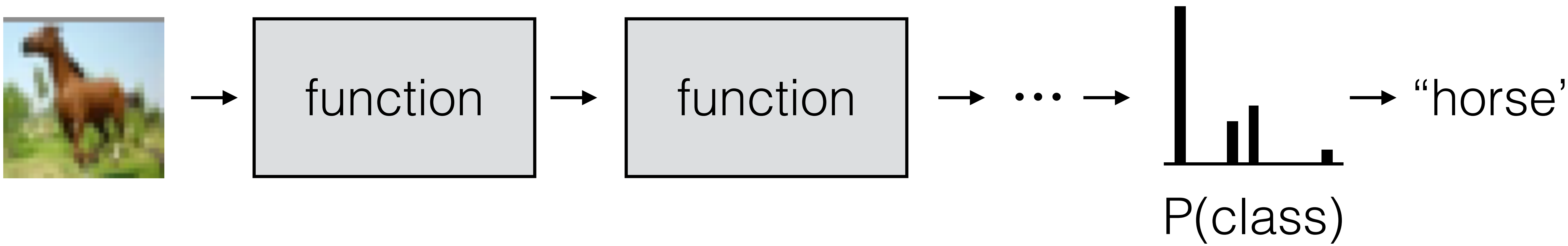

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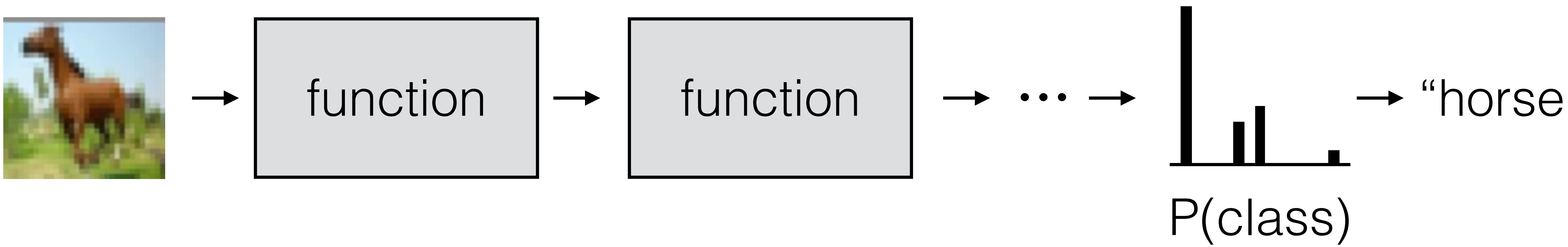
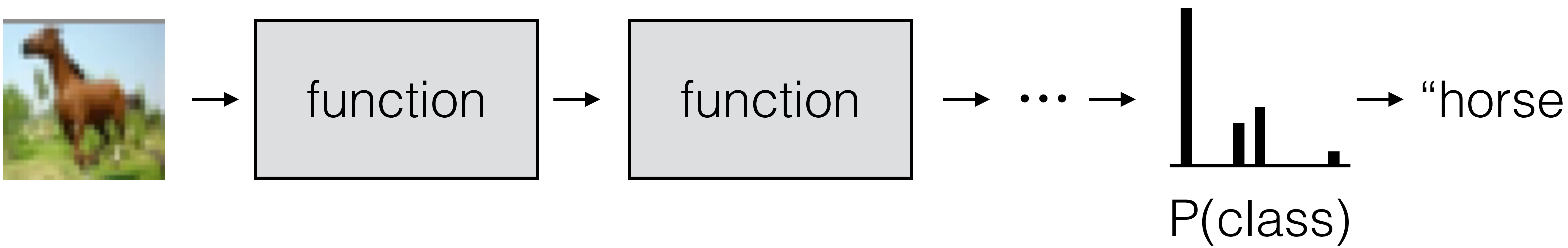


Key questions:

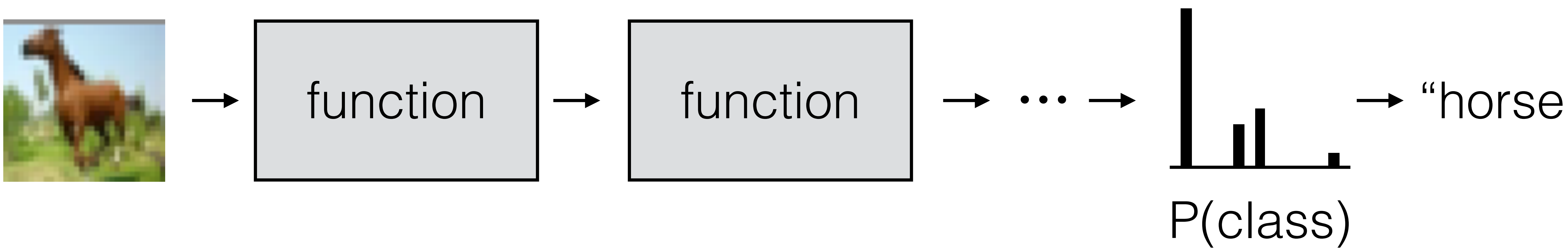
What is a Convolutional Neural Network (CNN)?



Key questions:

- What kinds of of functions should we use?

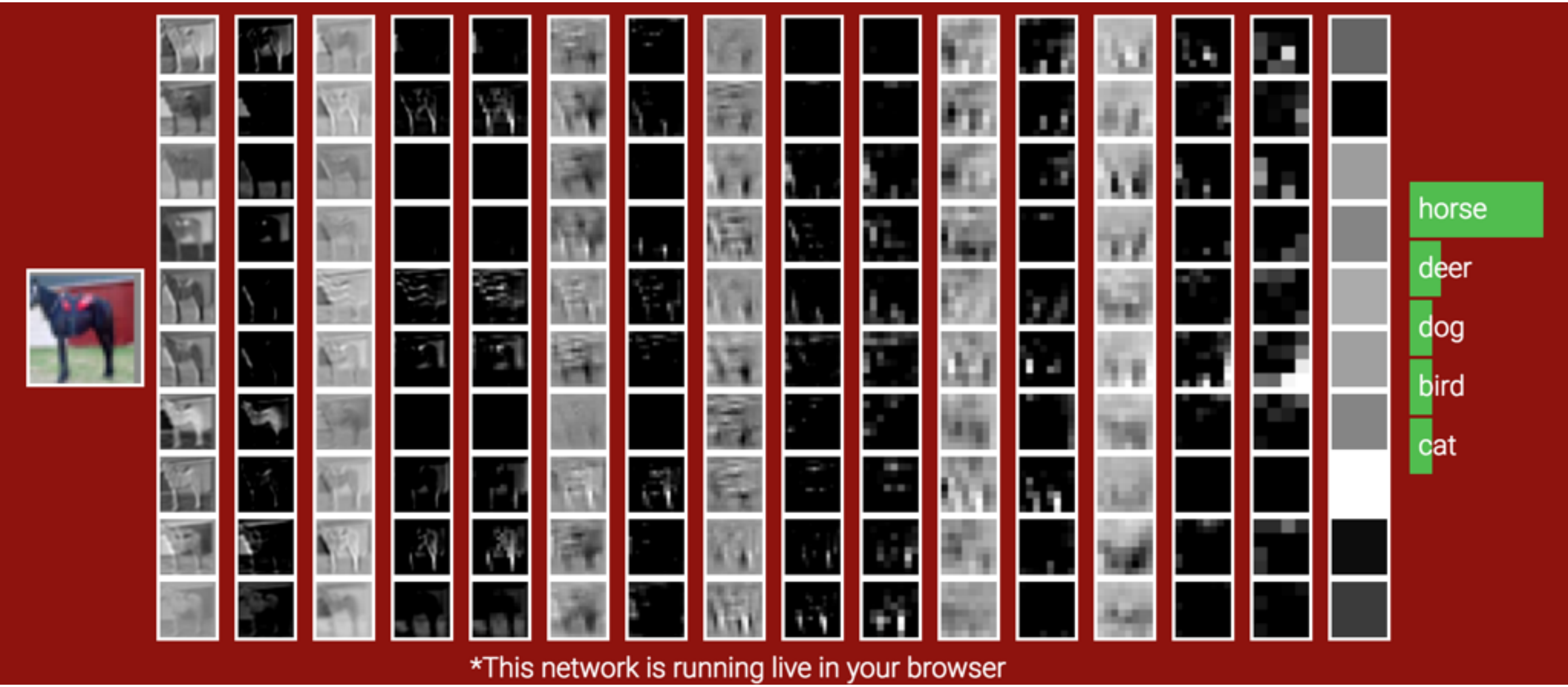
What is a Convolutional Neural Network (CNN)?



Key questions:

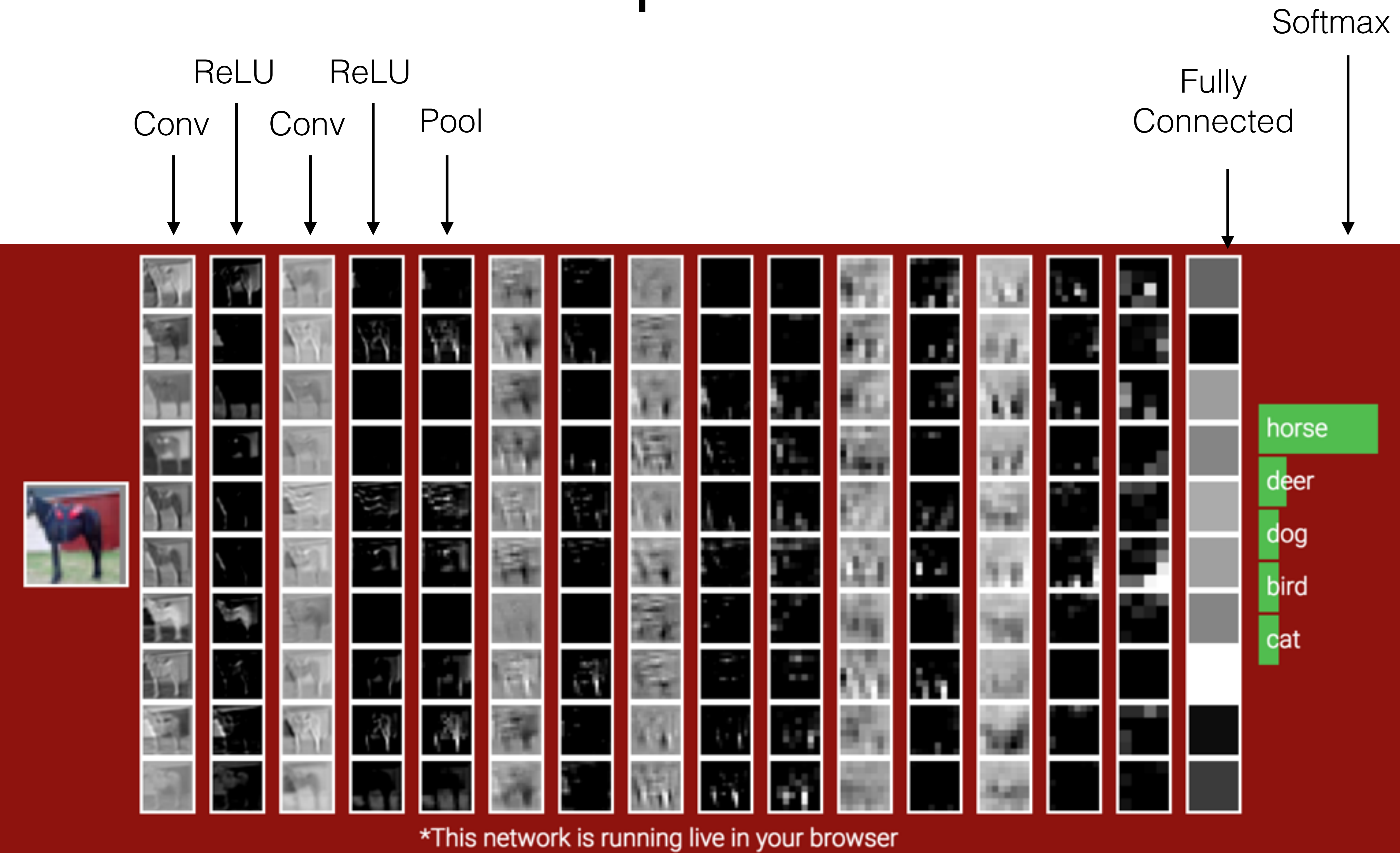
- What kinds of of functions should we use?
- How do we learn the parameters for those functions?

Example CNN



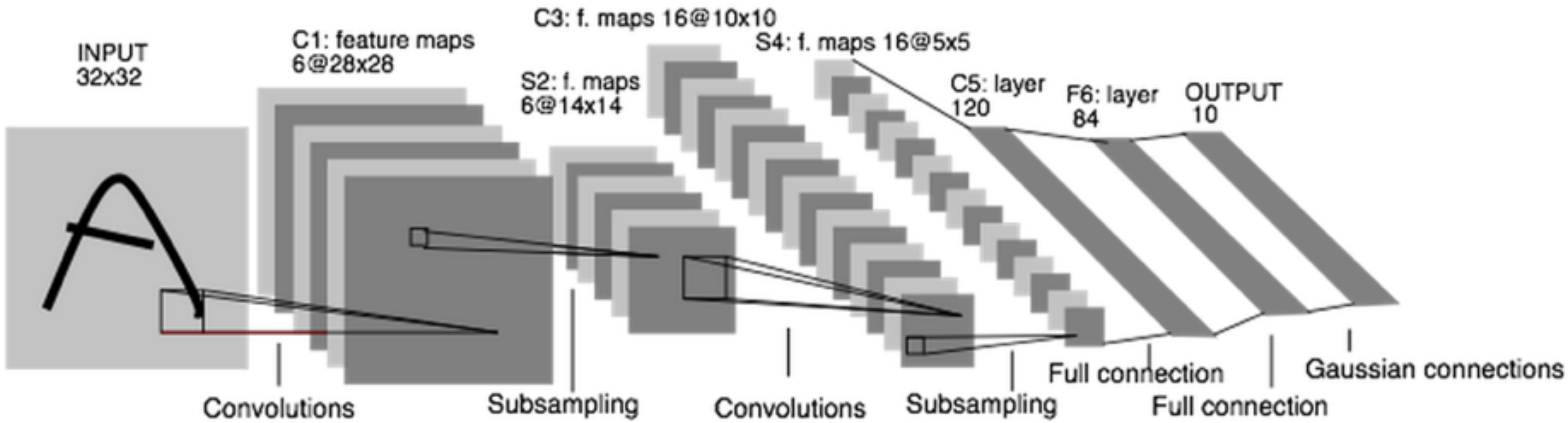
[Andrej Karpathy]

Example CNN



[Andrej Karpathy]

CNNs in 1989: "LeNet"



LeNet: a classifier for handwritten digits. [LeCun 1989]

CNNs in 2012: “SuperVision” (aka “AlexNet”)

“AlexNet” — Won the ILSVRC2012 Challenge

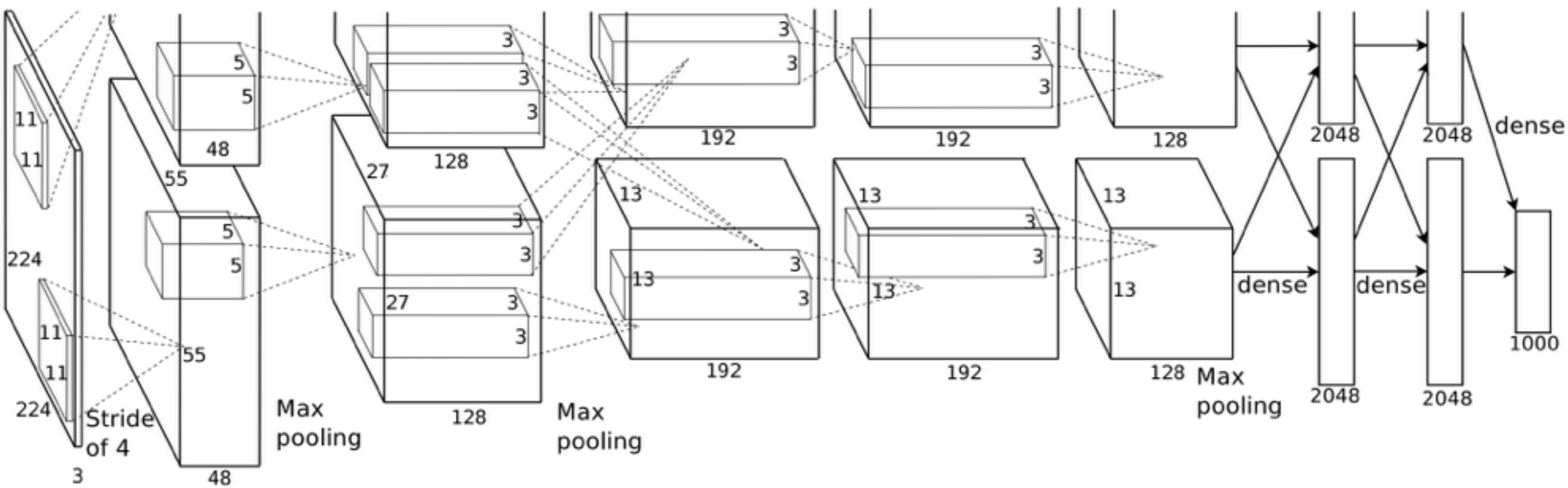


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network’s input is 150,528-dimensional, and the number of neurons in the network’s remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

[Krizhevsky, Sutskever, Hinton. NIPS 2012]

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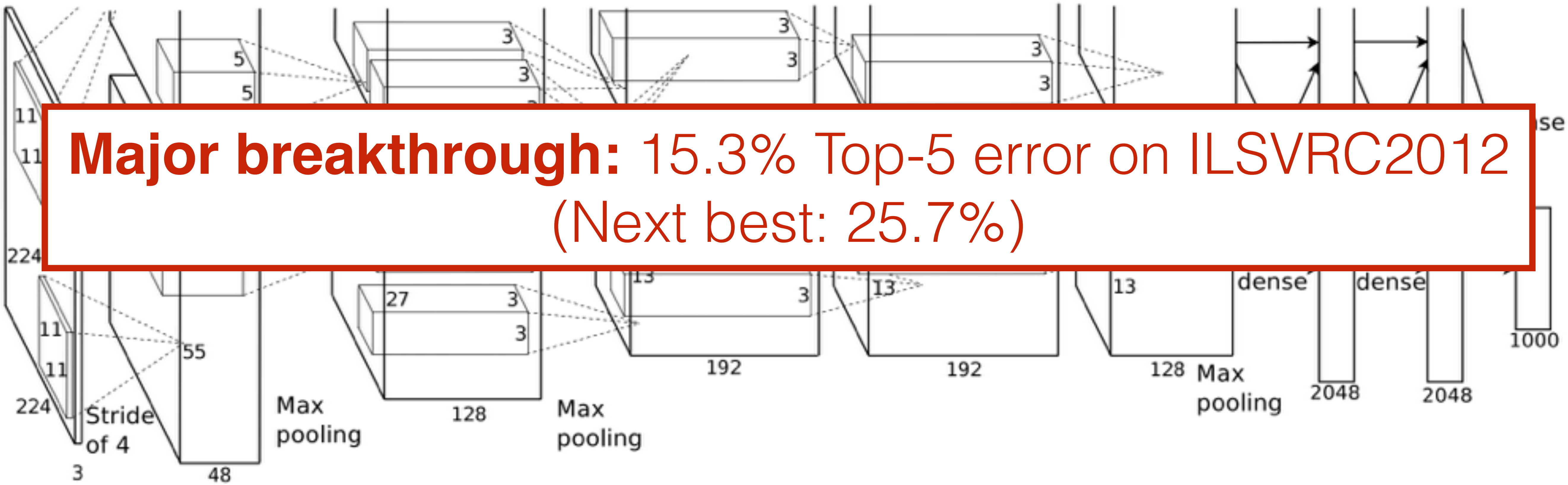

Major breakthrough: 15.3% Top-5 error on ILSVRC2012 (Next best: 25.7%)

Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network’s input is 150,528-dimensional, and the number of neurons in the network’s remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

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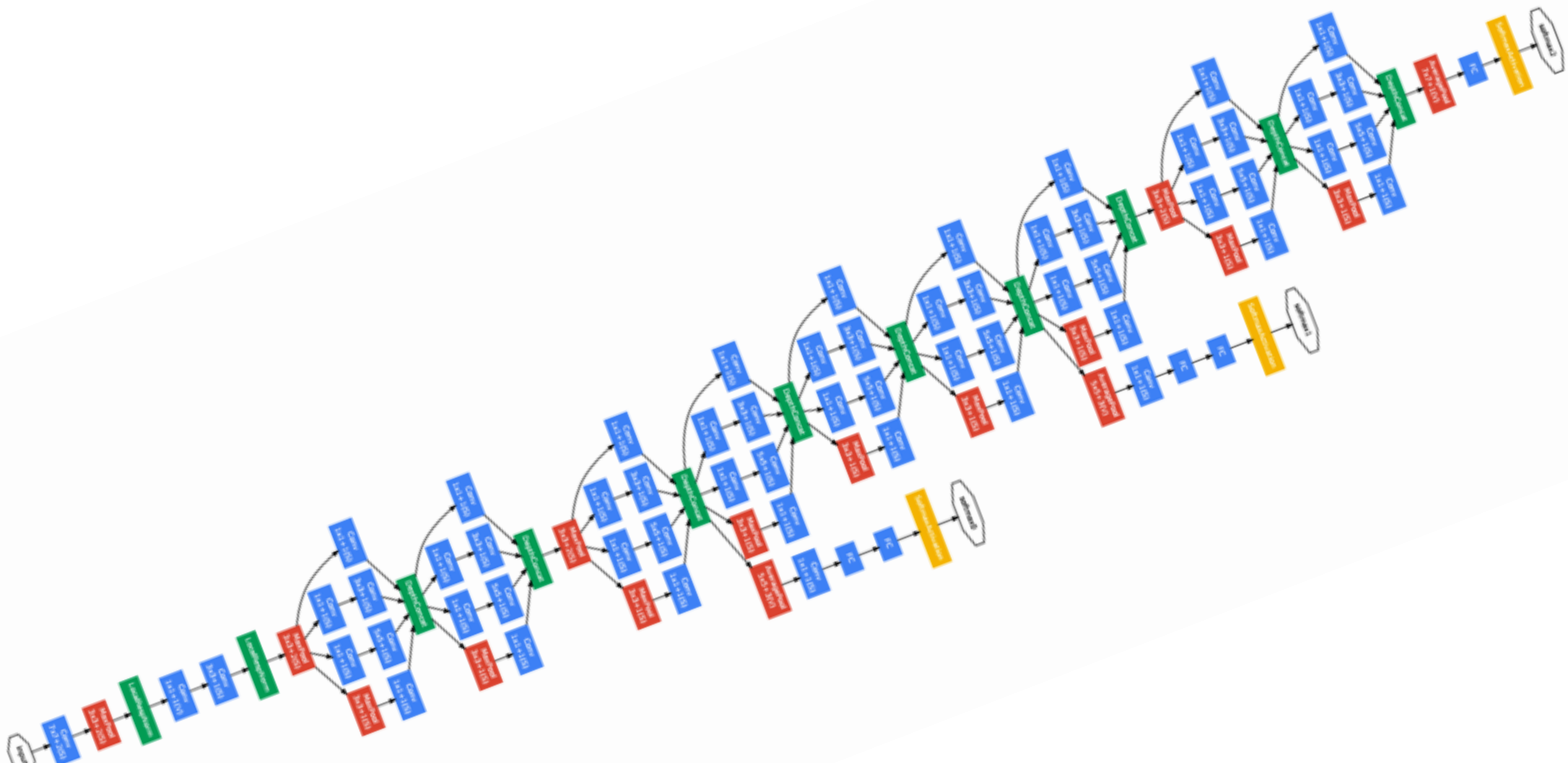
CNNs in 2014: “GoogLeNet”

“GoogLeNet” — Won the ILSVRC2014 Challenge



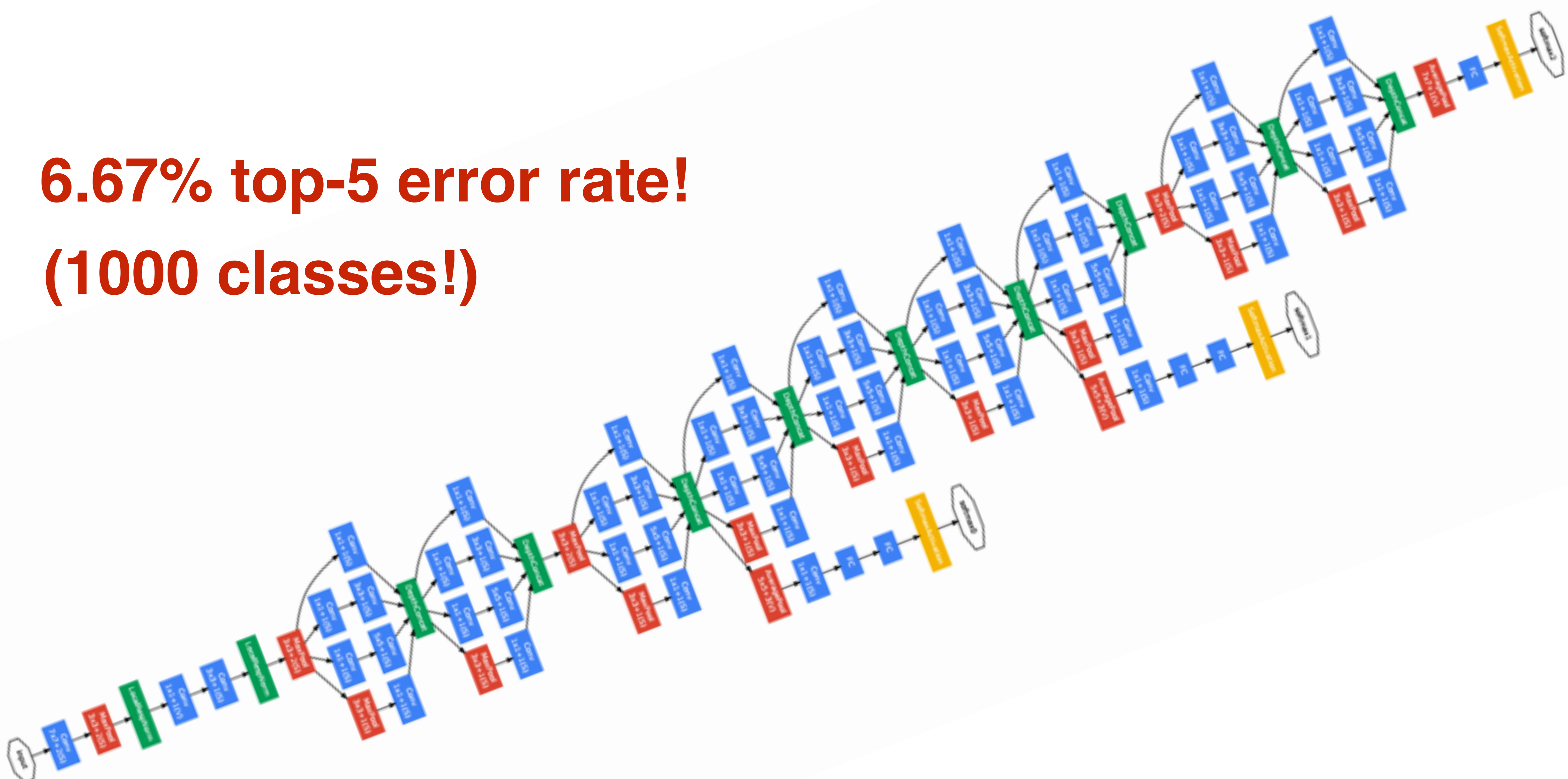
[Szegedy et al, arXiv 2014]

CNNs in 2014: “GoogLeNet”

“GoogLeNet” — Won the ILSVRC2014 Challenge

6.67% top-5 error rate!

(1000 classes!)



[Szegedy et al, arXiv 2014]

CNNs in 2014: “VGGNet”

“VGGNet” — Second Place in the ILSVRC2014 Challenge

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

No fancy picture, sorry

[Simonyan et al, arXiv 2014]

CNNs in 2014: “VGGNet”

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maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

No fancy picture, sorry

7.3% top-5 error rate

[Simonyan et al, arXiv 2014]

CNNs in 2014: “VGGNet”

“VGGNet” — Second Place in the ILSVRC2014 Challenge

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conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

No fancy picture, sorry

7.3% top-5 error rate

(and 1st place in the detection challenge)

[Simonyan et al, arXiv 2014]

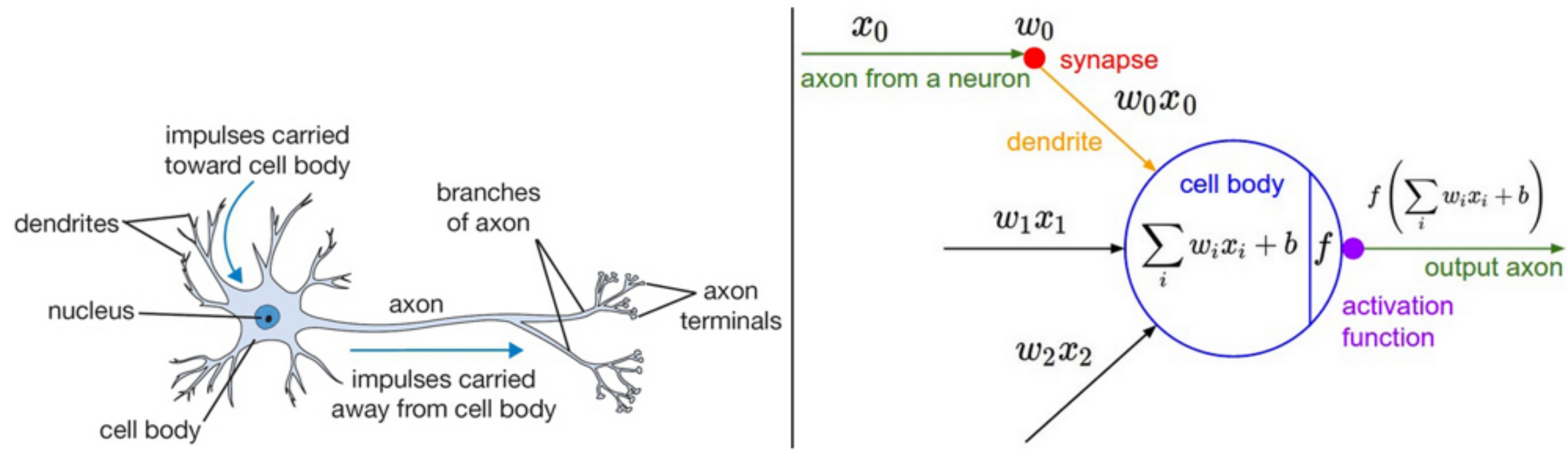
Neural Networks

(First we'll cover Neural Nets, then build up to Convolutional Neural Nets)

Inspiration from Biology

Figure: Andrej Karpathy

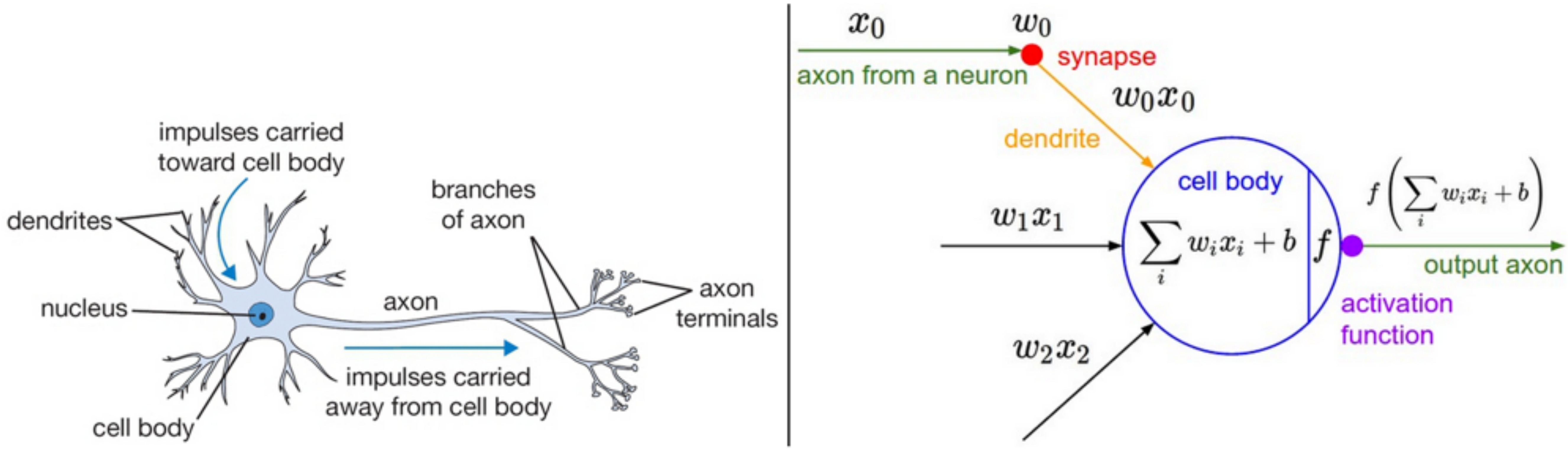
Inspiration from Biology



A cartoon drawing of a biological neuron (left) and its mathematical model (right).

Figure: Andrej Karpathy

Inspiration from Biology

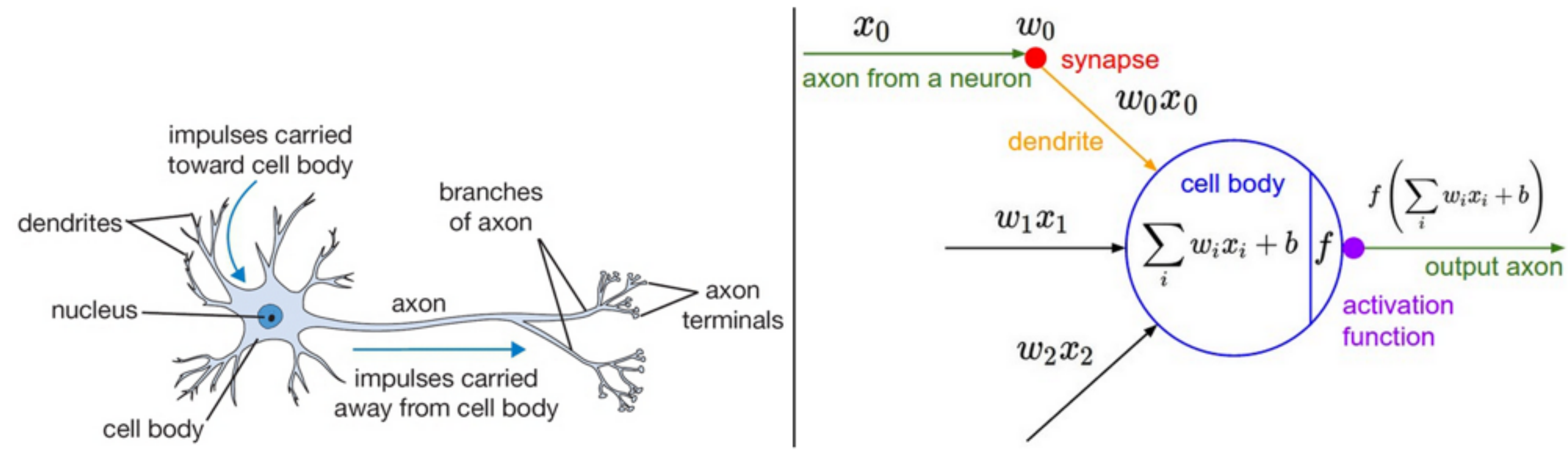


A cartoon drawing of a biological neuron (left) and its mathematical model (right).

Neural nets are **loosely inspired** by biology

Figure: Andrej Karpathy

Inspiration from Biology



A cartoon drawing of a biological neuron (left) and its mathematical model (right).

Neural nets are **loosely inspired** by biology

But they certainly are **not** a model of how the brain works, or even how neurons work

Figure: Andrej Karpathy

Simple Neural Net: 1 Layer

Let's consider a simple 1-layer network:

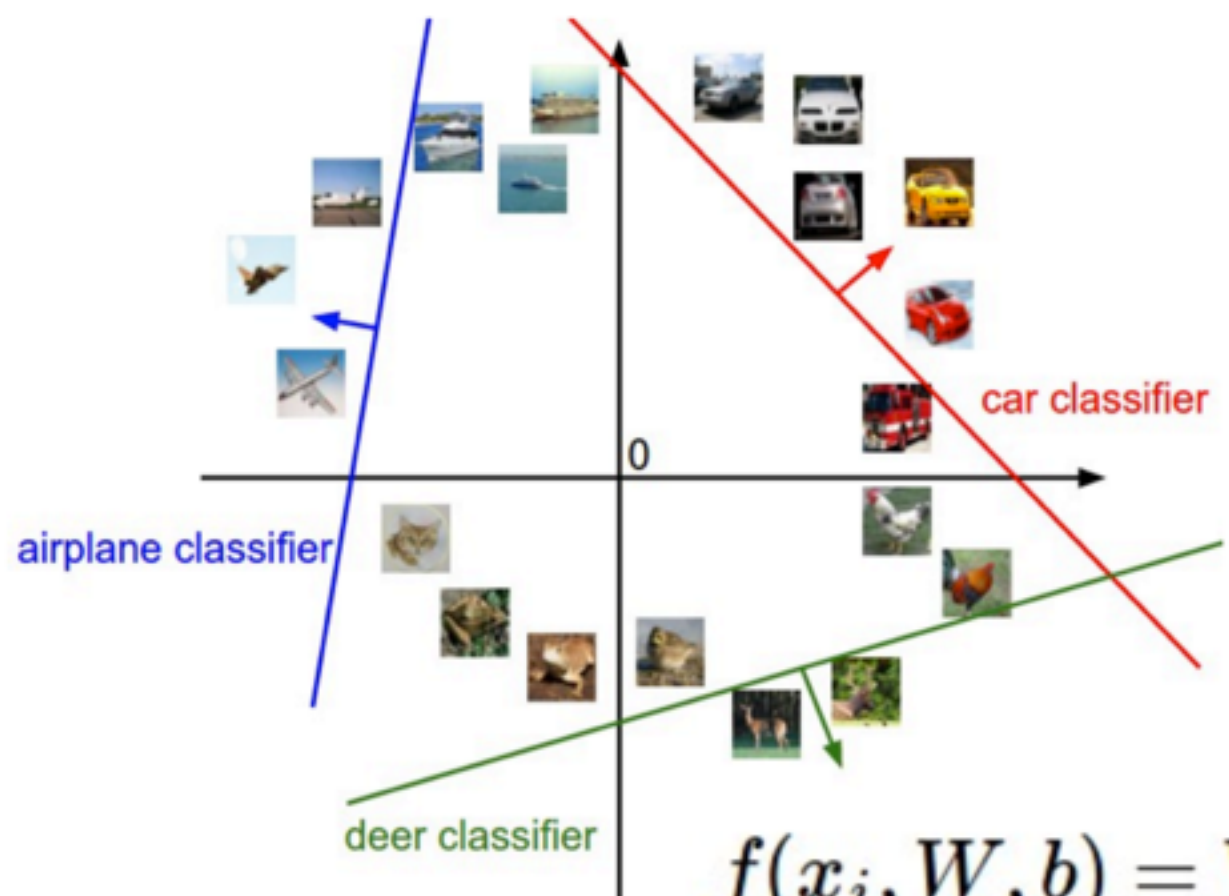
$$x \rightarrow \boxed{Wx+b} \rightarrow f$$

Simple Neural Net: 1 Layer

Let’s consider a simple 1-layer network:

$$x \rightarrow \boxed{Wx + b} \rightarrow f$$

This is the same as what you saw last class:



$$f(x_i, W, b) = Wx_i + b$$

1 Layer Neural Net

Block Diagram:

$$x \rightarrow \boxed{Wx + b} \rightarrow f$$

(Input)

(class scores)

1 Layer Neural Net

Block Diagram:

$$x \rightarrow \boxed{Wx + b} \rightarrow f$$

(Input) (class scores)

Expanded Block Diagram:

$$\underbrace{W}_{M \times D} \; \underbrace{x}_{D \times 1} + \underbrace{b}_{M \times 1} = \underbrace{f}_{M \times 1}$$

M, D, 1, D, M, 1, M, 1

M classes
D features
1 example

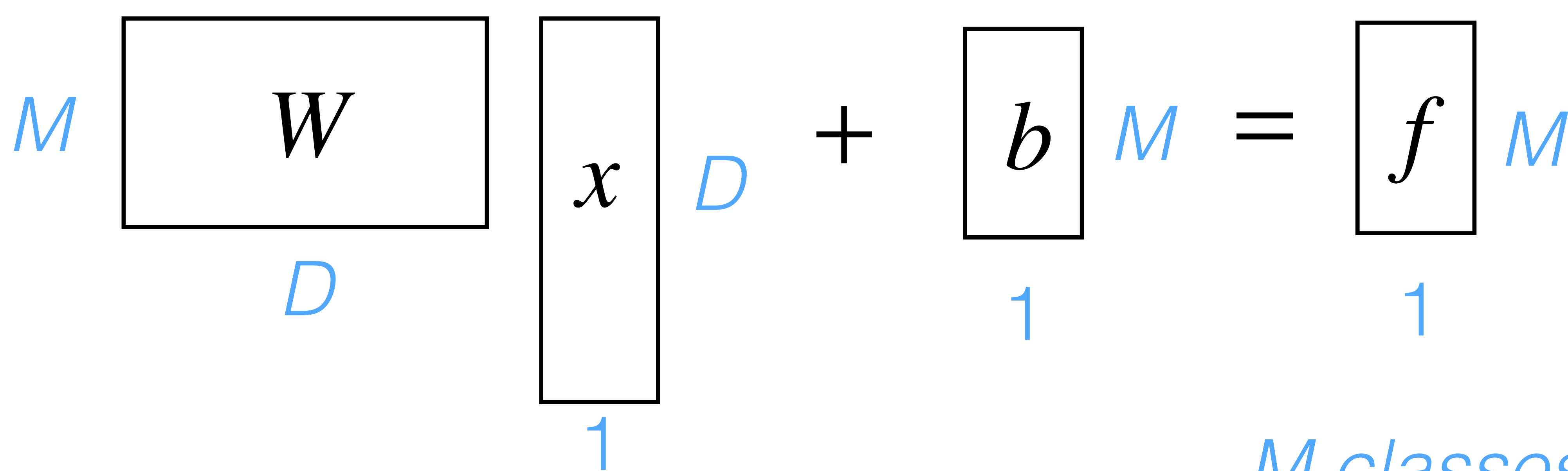
1 Layer Neural Net

Block Diagram:

$$x \rightarrow \boxed{Wx + b} \rightarrow f$$

(Input) (class scores)

Expanded Block Diagram:



M classes
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1 example

NumPy:

```
f = np.dot(W, x) + b
```

1 Layer Neural Net

1 Layer Neural Net

- How do we process N inputs at once?

1 Layer Neural Net

- How do we process *N* inputs at once?
- It's most convenient to have the first dimension (row) represent which example we are looking at, so we need to transpose everything

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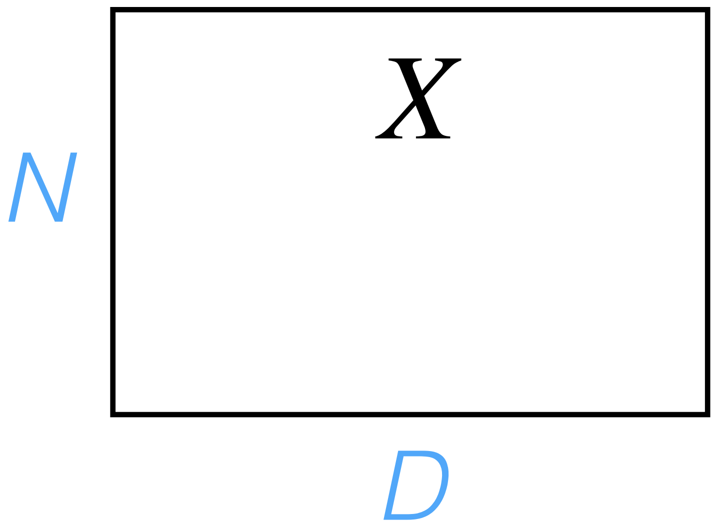

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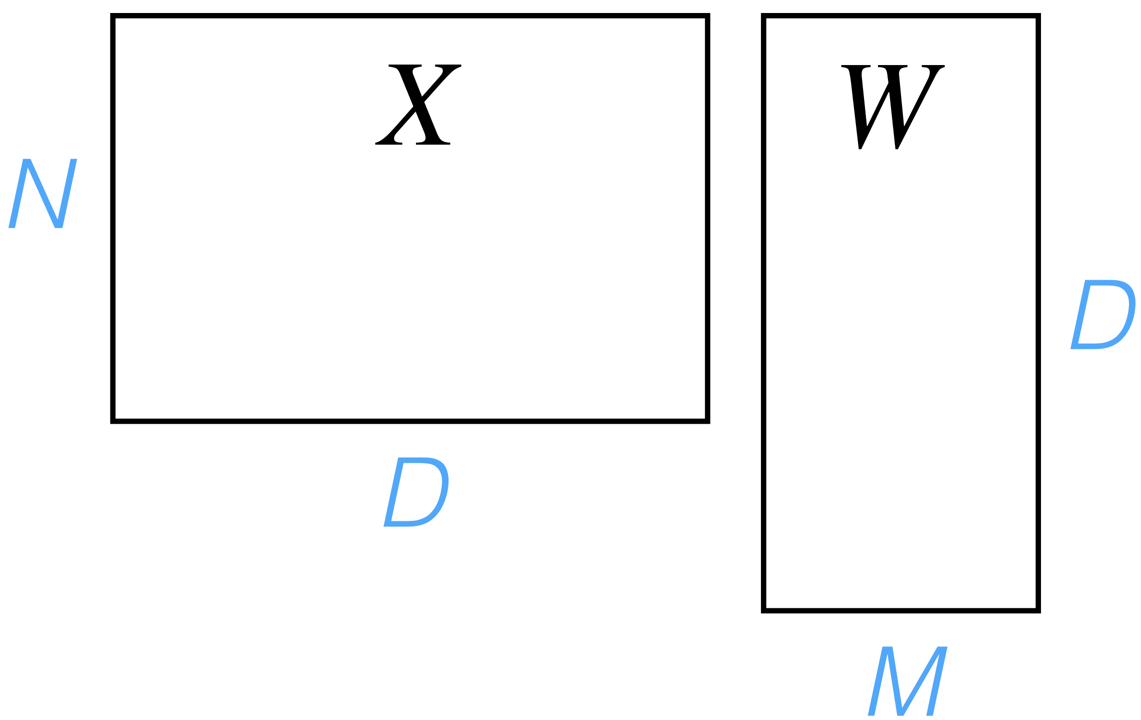

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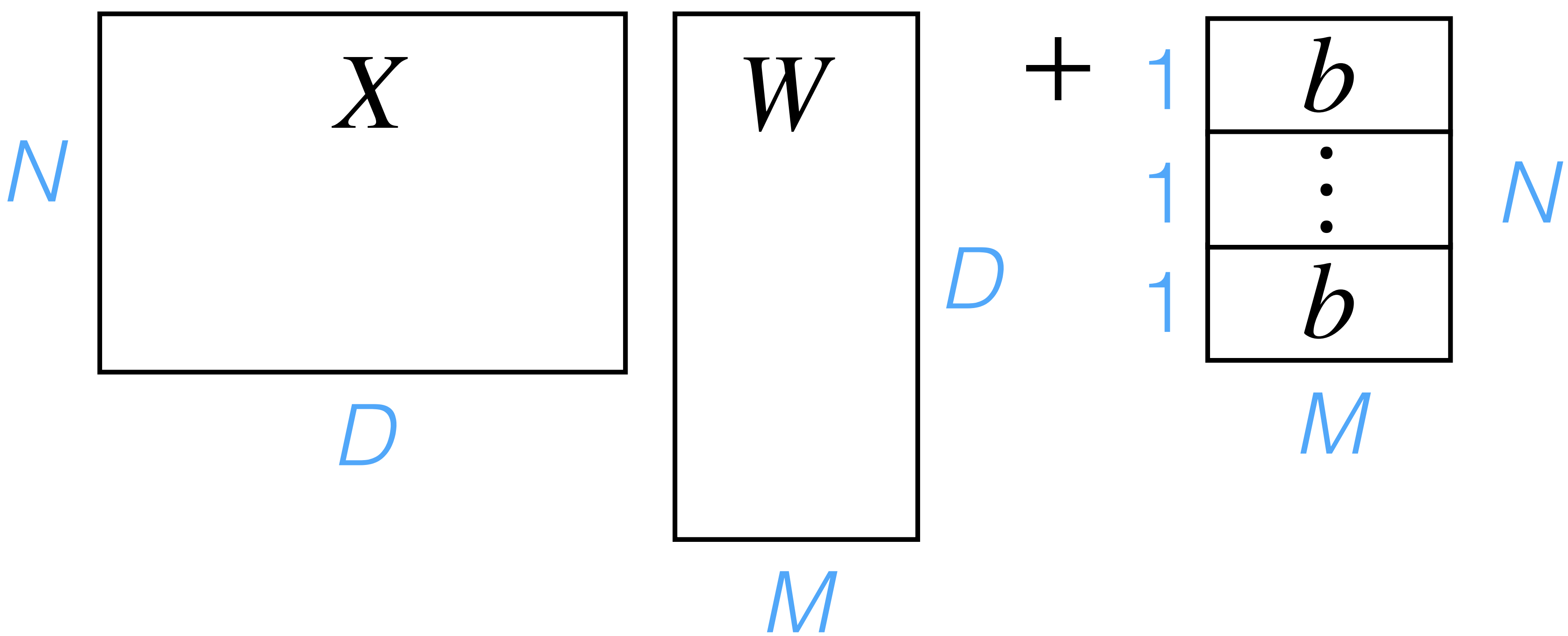

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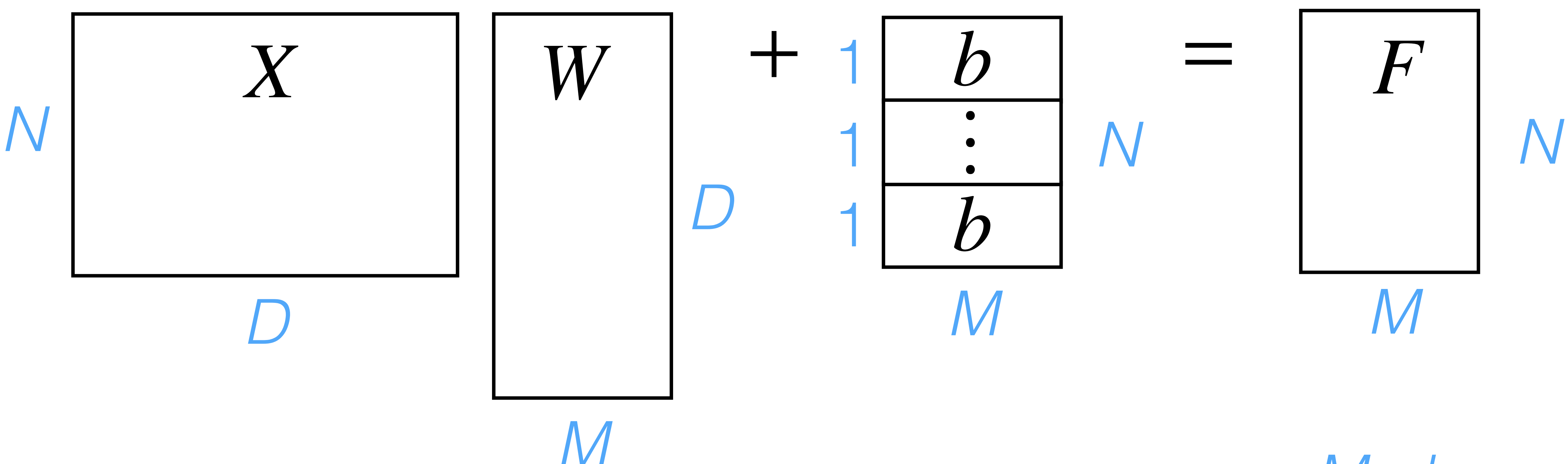

M classes
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N examples

1 Layer Neural Net

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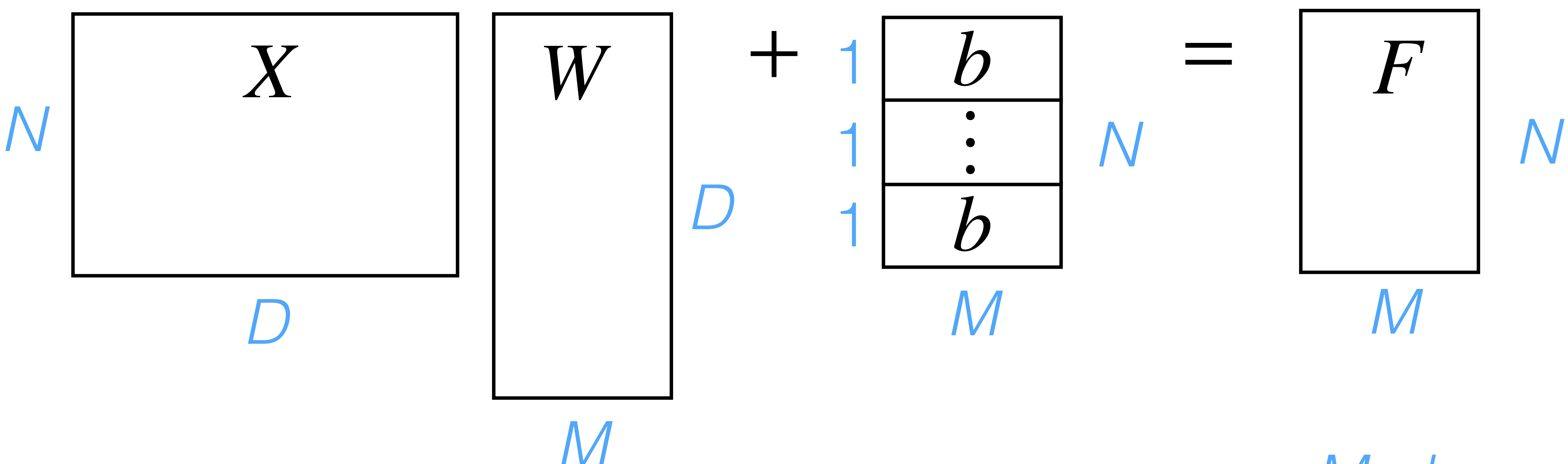

Note: Often, if the weights are transposed, they are still called "W"

M classes
D features
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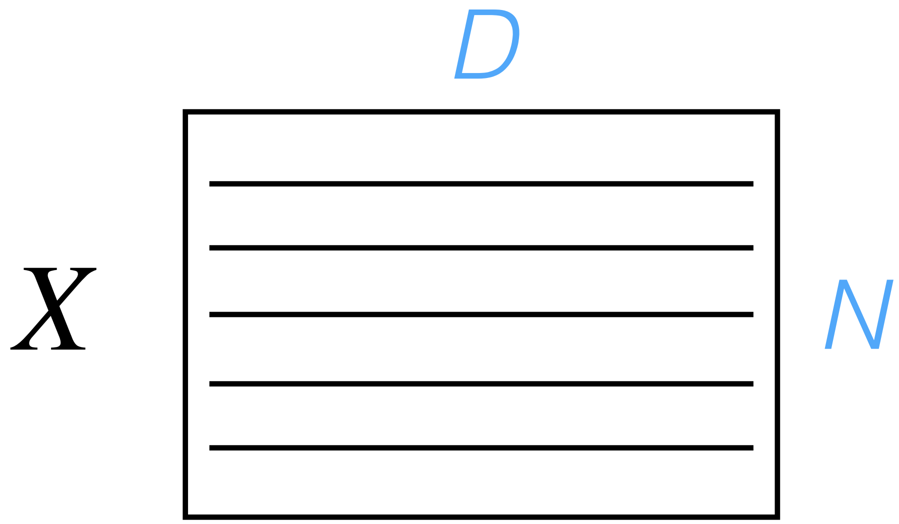
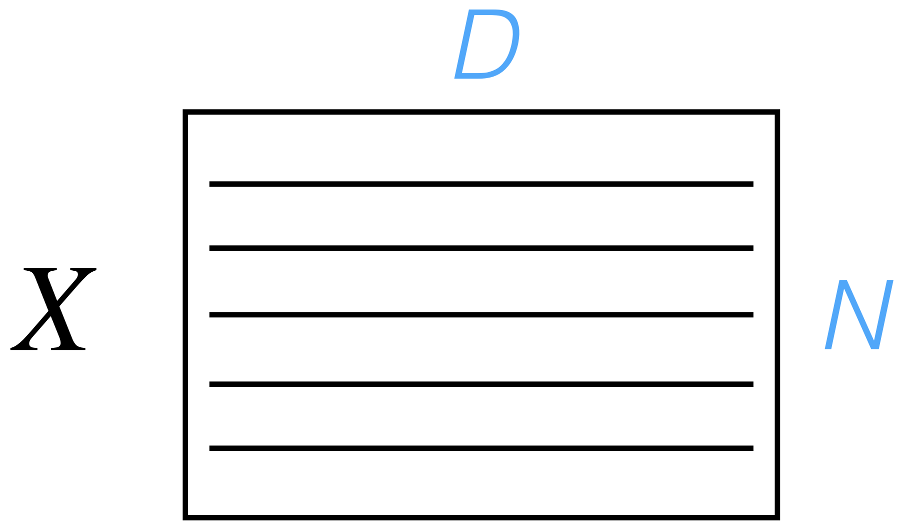
1 Layer Neural Net

1 Layer Neural Net



Each **row** is one input example

1 Layer Neural Net



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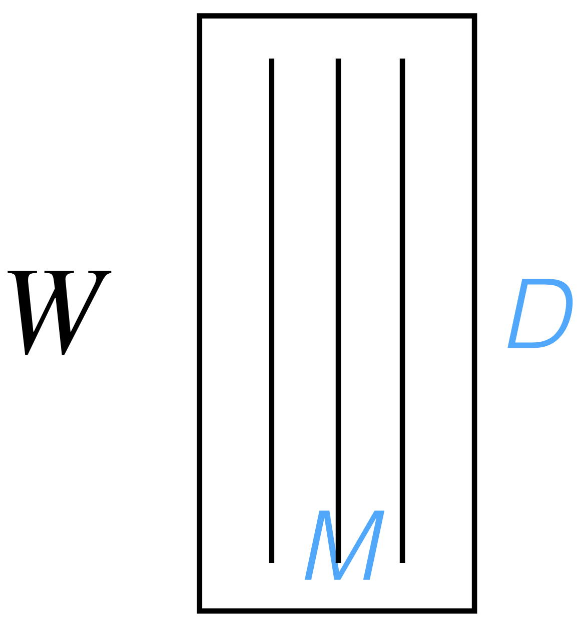
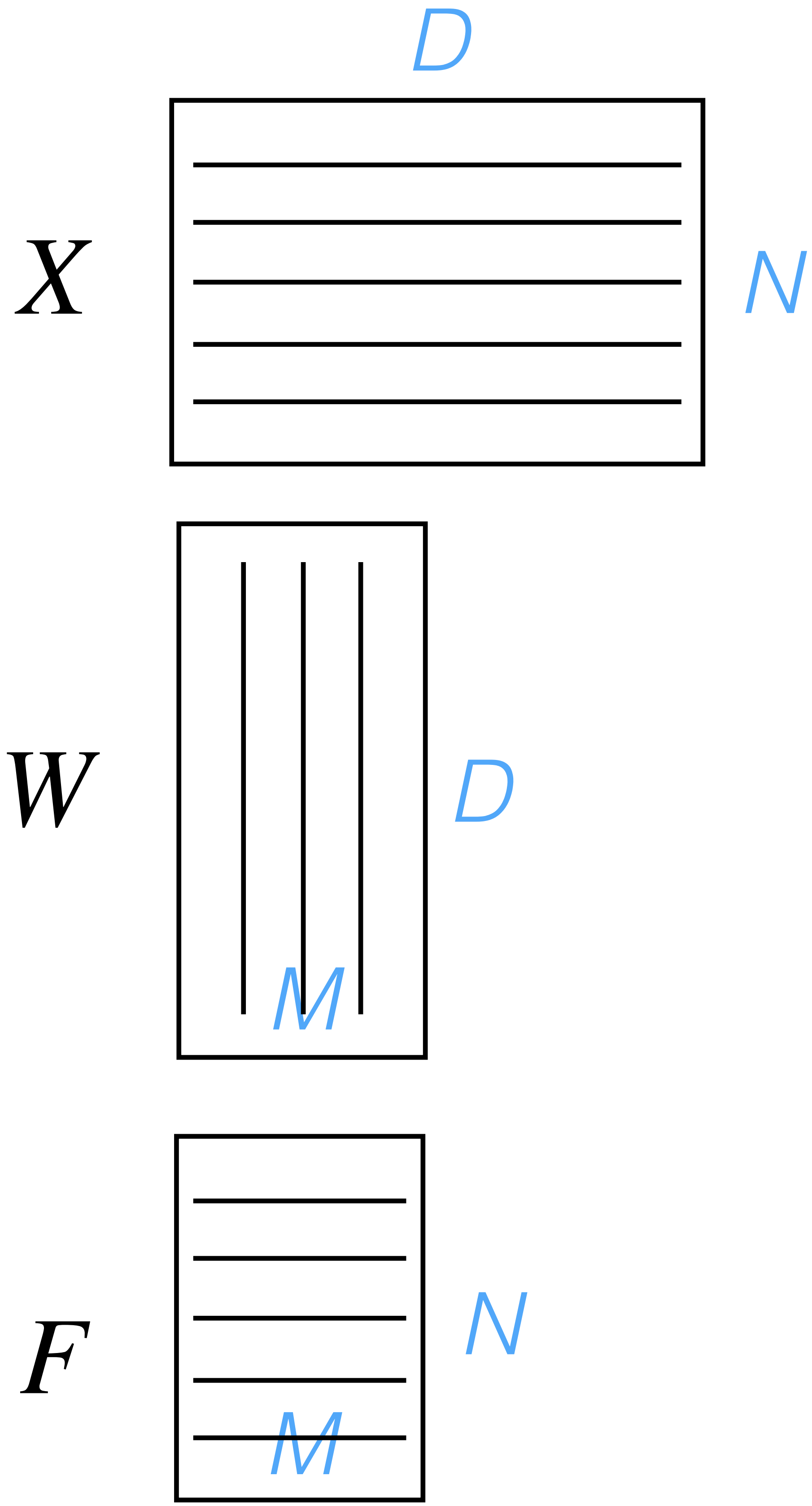


Each **column** is are the weights for one class

1 Layer Neural Net



Each **row** is one input example

Each **column** is are the weights for one class

Each **row** is are the predicted scores for one example

1 Layer Neural Net

Implementing this with NumPy:

First attempt — let's try this:

```
F = np.dot(X, W) + b
```

1 Layer Neural Net

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1 Layer Neural Net

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N x D

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- NumPy needs to know how to expand "b" from 1D to 2D

1 Layer Neural Net

Implementing this with NumPy:

First attempt — let's try this:

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F = np.dot(X, W) + b
```

D x M

Doesn't work — why?

- NumPy needs to know how to expand "b" from 1D to 2D
- This is called "broadcasting"

1 Layer Neural Net

Implementing this with NumPy:

```
F = np.dot(X, W) + b[np.newaxis, :]
```

What does “np.newaxis” do?

1 Layer Neural Net

Implementing this with NumPy:

```
F = np.dot(X, W) + b[np.newaxis, :]
```

What does “np.newaxis” do?

```
In [3]: b
Out[3]: array([0, 1, 2])
```

b = [0, 1, 2]

1 Layer Neural Net

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F = np.dot(X, W) + b[np.newaxis, :]
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What does “np.newaxis” do?

```
In [3]: b
Out[3]: array([0, 1, 2])

In [4]: b[np.newaxis, :]
Out[4]: array([[0, 1, 2]])
```

b = [0, 1, 2]

Make “b” a row vector

1 Layer Neural Net

Implementing this with NumPy:

```
F = np.dot(X, W) + b[np.newaxis, :]
```

What does "np.newaxis" do?

```
In [3]: b
Out[3]: array([0, 1, 2])

In [4]: b[np.newaxis, :]
Out[4]: array([[0, 1, 2]])

In [5]: b[:, np.newaxis]
Out[5]:
array([[0],
       [1],
       [2]])
```

b = [0, 1, 2]

Make "b" a row vector

Make "b" a column vector

1 Layer Neural Net

Implementing this with NumPy:

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F = np.dot(X, W) + b[np.newaxis, :]
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What does “np.newaxis” do?

1 Layer Neural Net

Implementing this with NumPy:

```python
F = np.dot(X, W) + b[np.newaxis, :]
```

What does “np.newaxis” do?

```
In [12]: b[np.newaxis, :] + np.zeros((3, 3))
Out[12]:
array([[ 0.,  1.,  2.],
       [ 0.,  1.,  2.],
       [ 0.,  1.,  2.]])
```

Row vector
(repeat along
rows)

1 Layer Neural Net

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In [13]: b[:, np.newaxis] + np.zeros((3, 3))
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array([[ 0.,  0.,  0.],
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```

Row vector (repeat along rows)

Column vector (repeat along columns)

2 Layer Neural Net

What if we just added another layer?

$$x \rightarrow \boxed{W^{(1)}x + b^{(1)}} \rightarrow h$$

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Let’s expand out the equation:

2 Layer Neural Net

What if we just added another layer?

$$x \rightarrow \boxed{W^{(1)}x + b^{(1)}} \rightarrow h \rightarrow \boxed{W^{(2)}h + b^{(2)}} \rightarrow f$$

Let's expand out the equation:

$$f = W^{(2)}(W^{(1)}x + b^{(1)}) + b^{(2)}$$

$$= (W^{(2)}W^{(1)})x + (W^{(2)}b^{(1)} + b^{(2)})$$

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But this is just the same as a 1 layer net with:

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$$W = W^{(2)}W^{(1)} \qquad b = W^{(2)}b^{(1)}+b^{(2)}$$

2 Layer Neural Net

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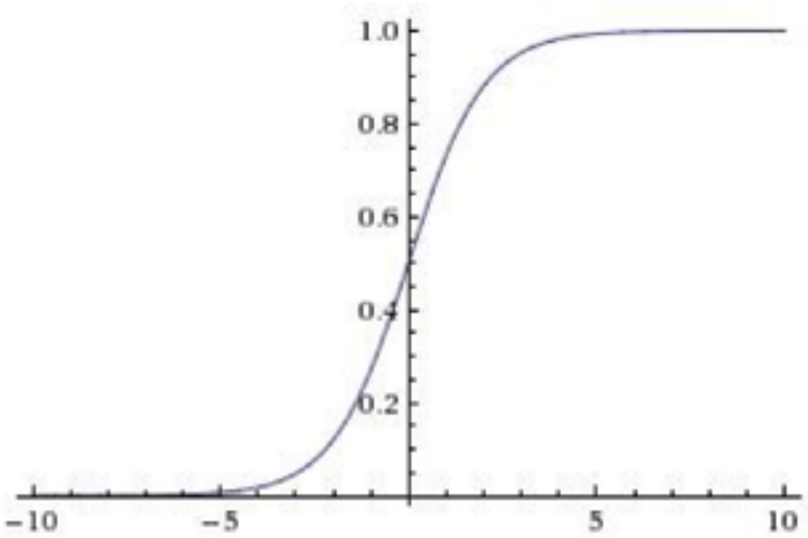
But this is just the same as a 1 layer net with:

$$W = W^{(2)}W^{(1)} \qquad b = W^{(2)}b^{(1)}+b^{(2)}$$

We need a **non-linear** operation between the layers

Nonlinearities

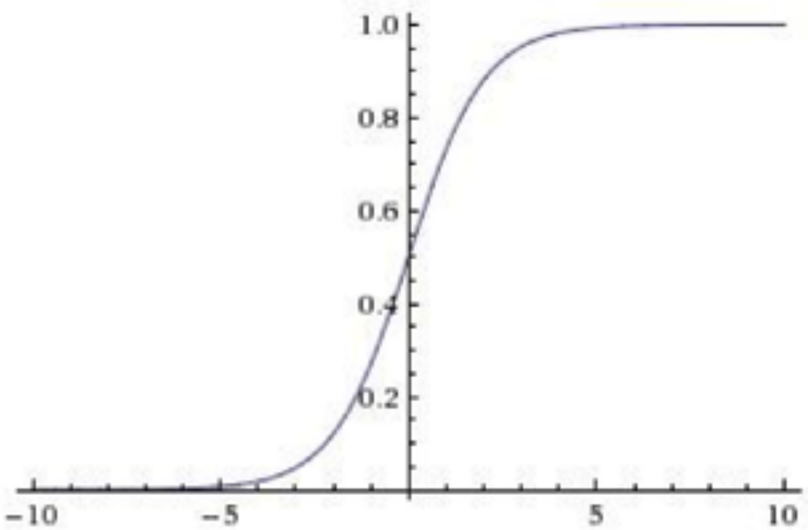
Nonlinearities



Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

Nonlinearities



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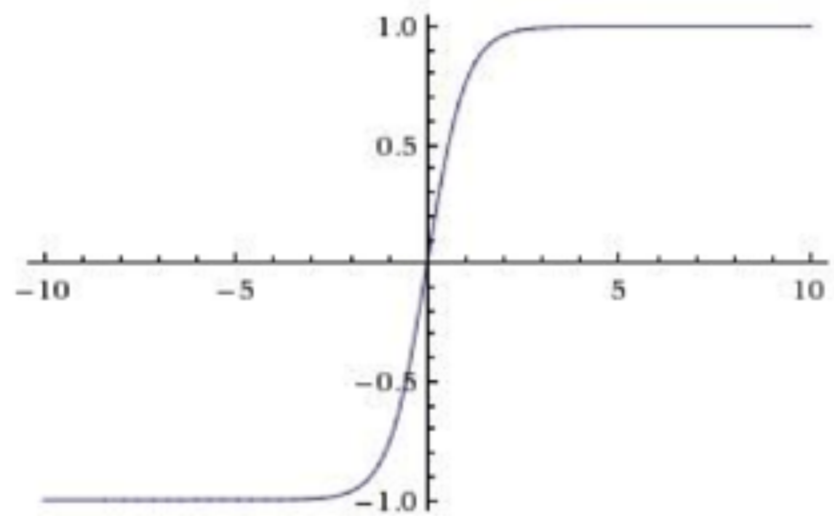
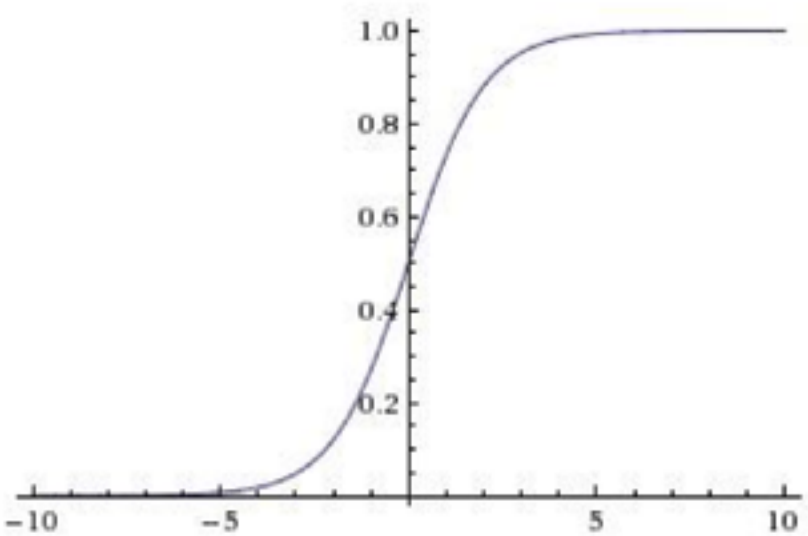


Tanh

Nonlinearities



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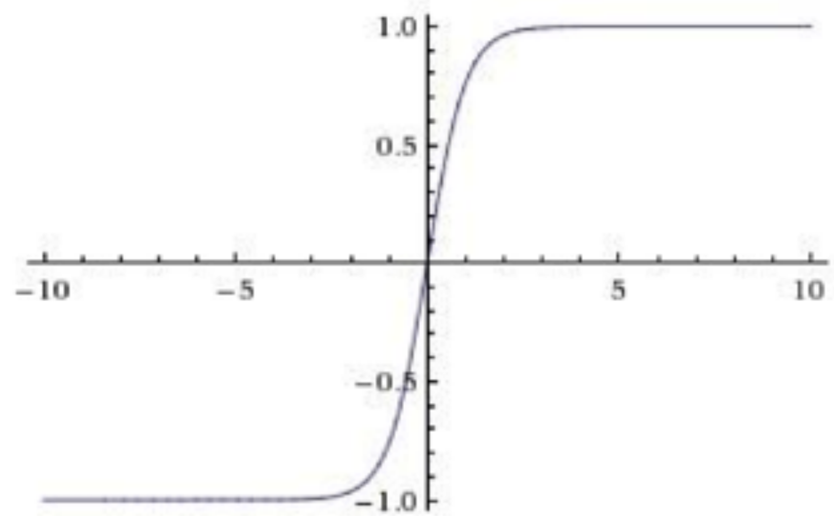

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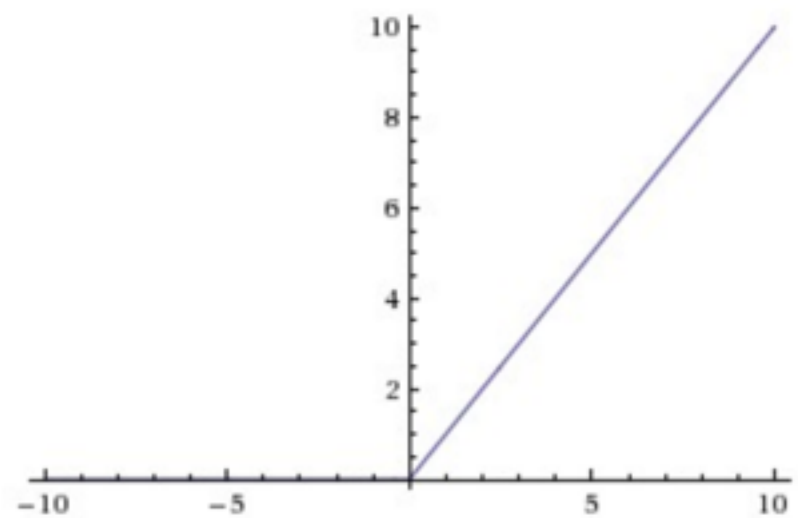
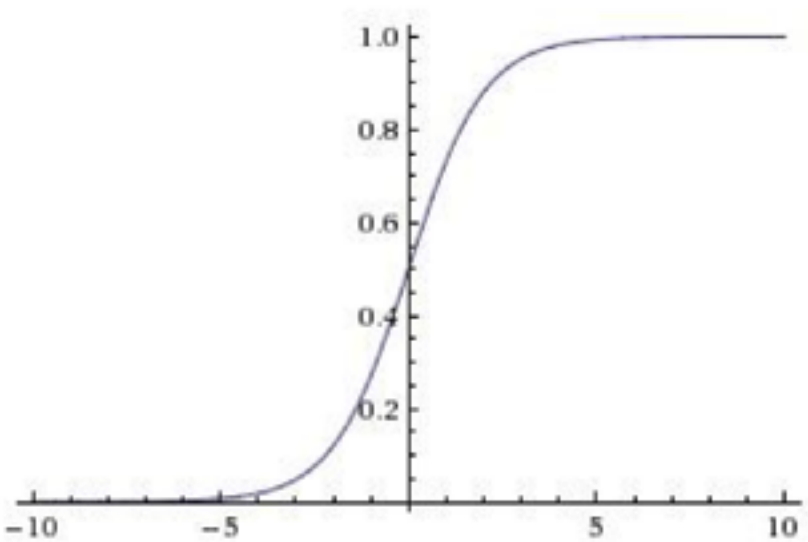


ReLU

Nonlinearities



Sigmoid

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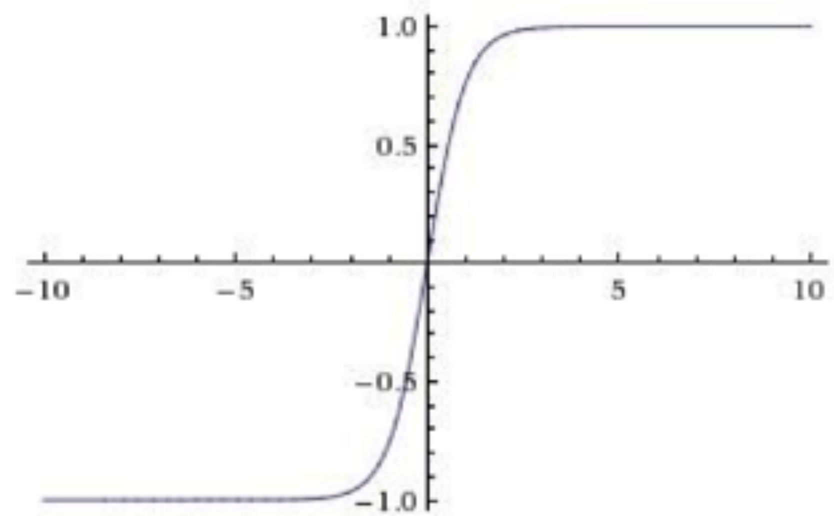
Historically popular

2 Big problems:
- Not zero centered
- They saturate



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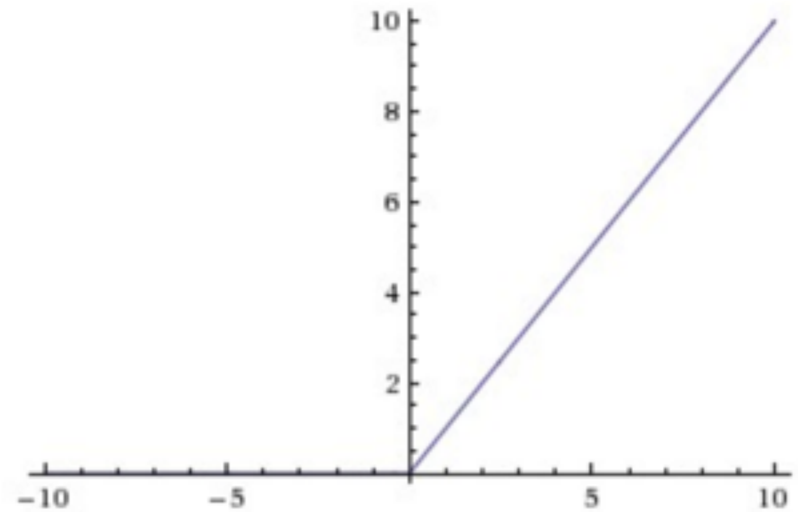

ReLU

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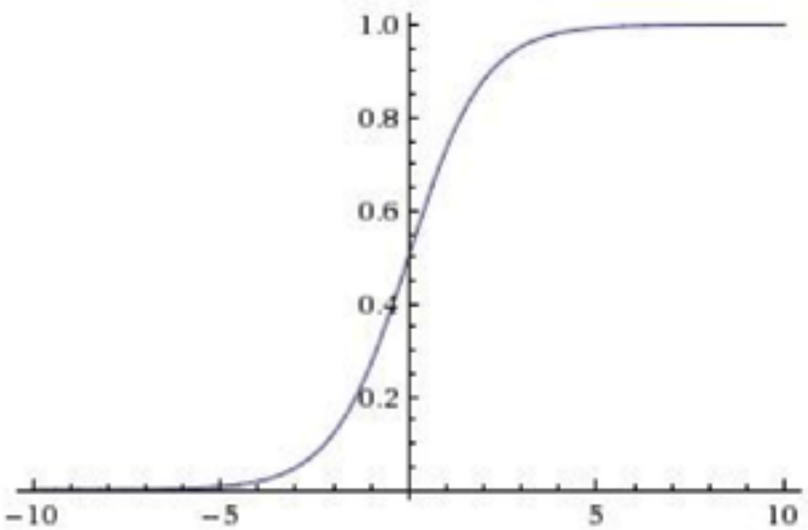

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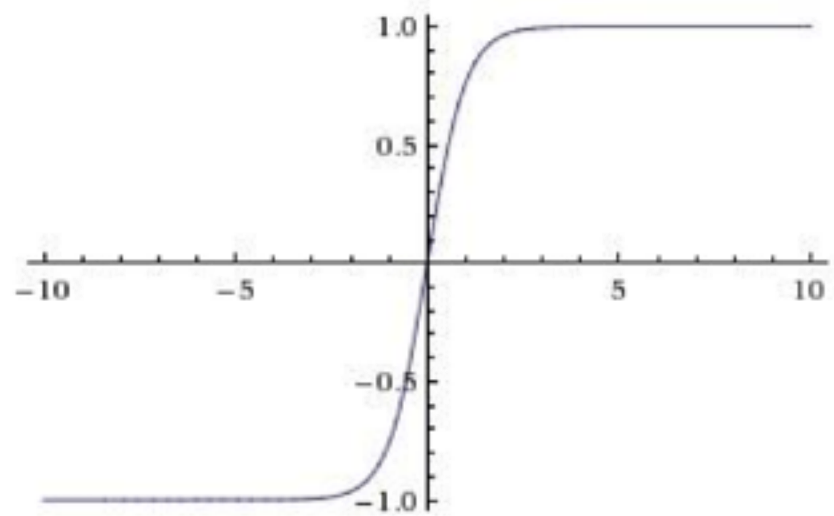
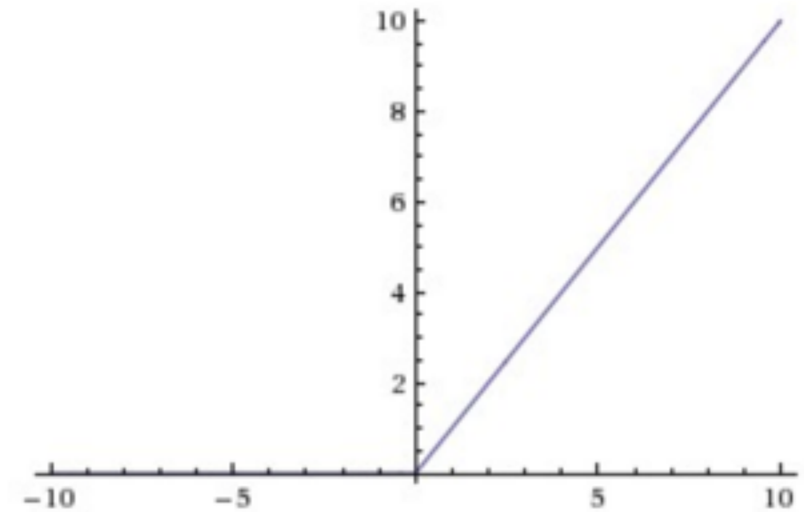


Tanh

- Zero-centered,
- But also saturates



ReLU

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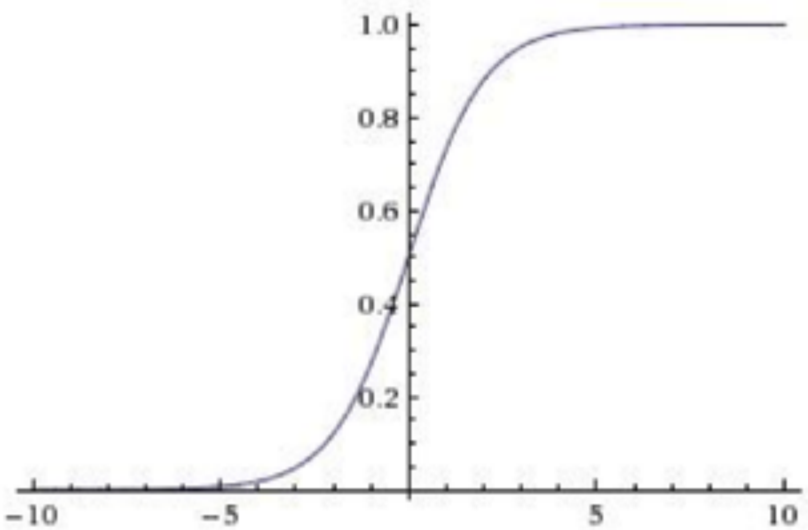

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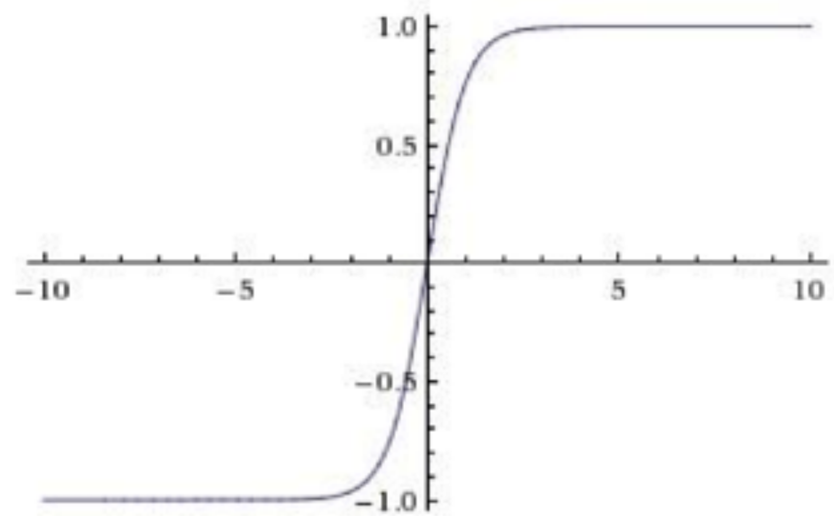

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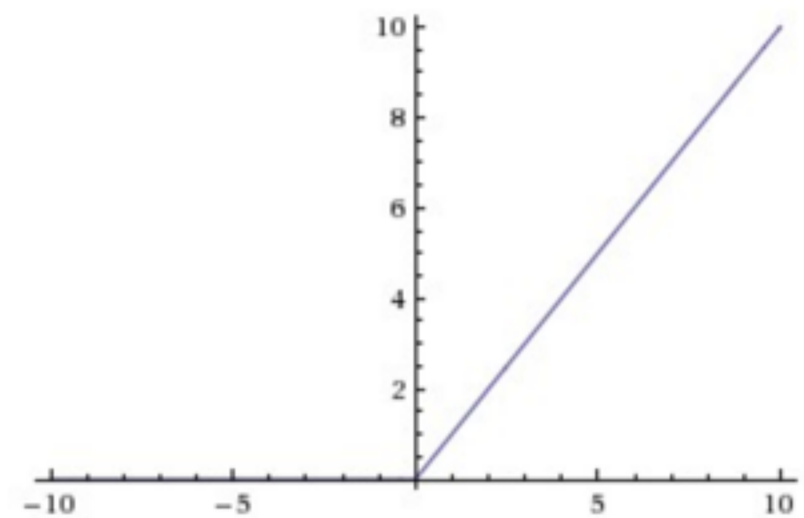

ReLU

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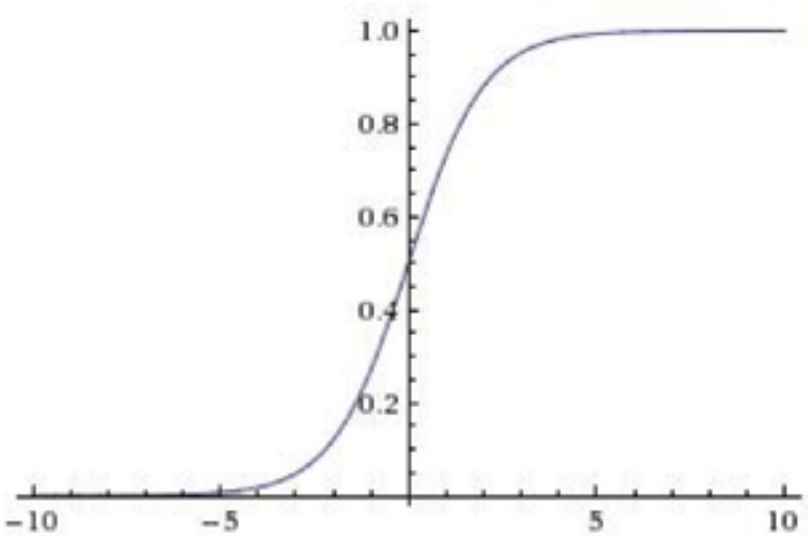
Nonlinearities



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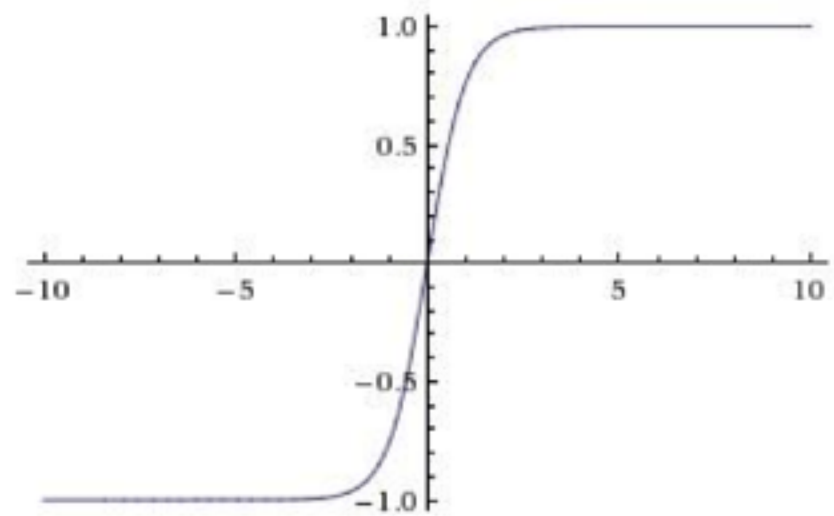

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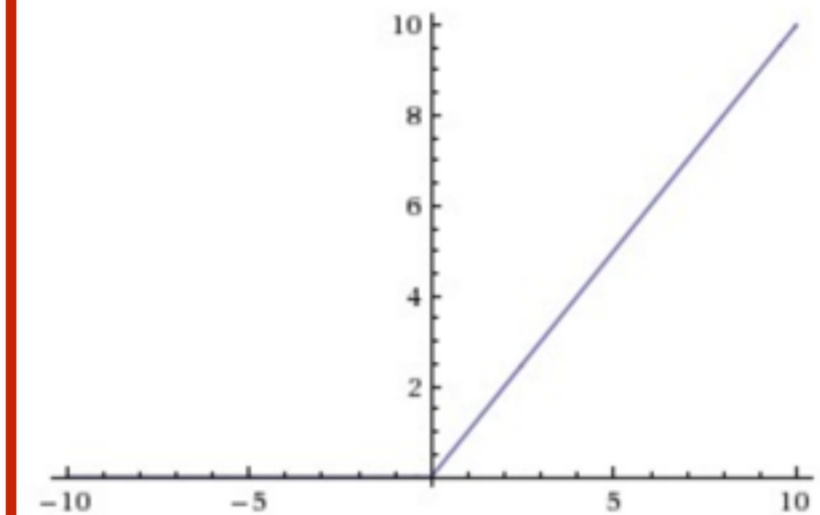
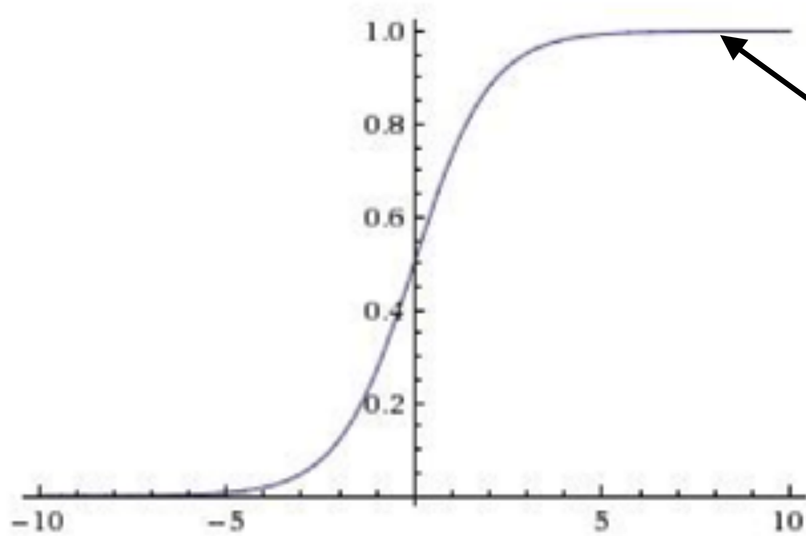


ReLU

Best in practice for classification

- No saturation
- Very efficient

Nonlinearities — Saturation



What happens if we reach this part?

Sigmoid

```
In [22]: sigmoid = lambda x: 1 / (1 + np.exp(-x))
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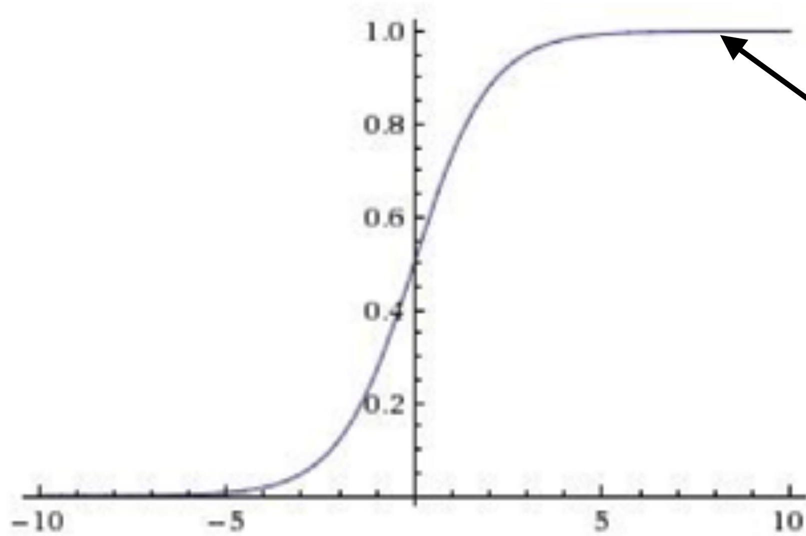
Nonlinearities — Saturation



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In [24]: sigmoid(np.array([-1, 2, 5]))
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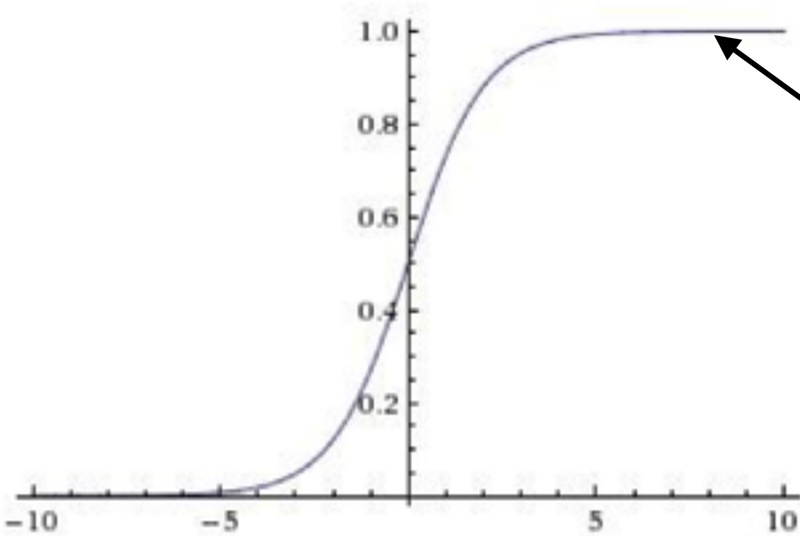

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In [25]: sigmoid(np.array([-1, 2, 50]))
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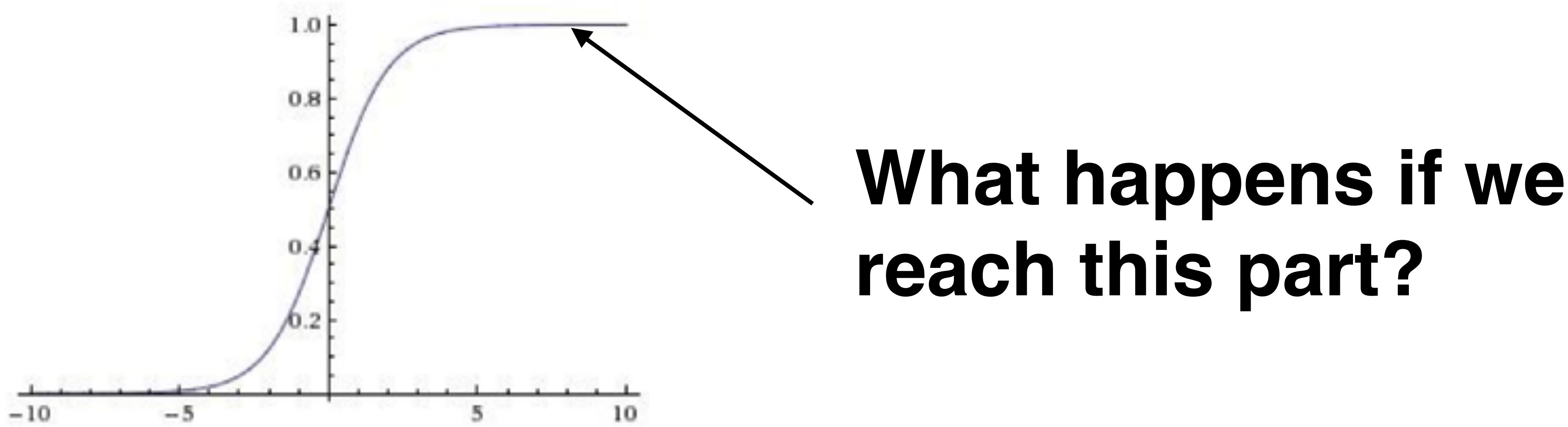
Nonlinearities — Saturation



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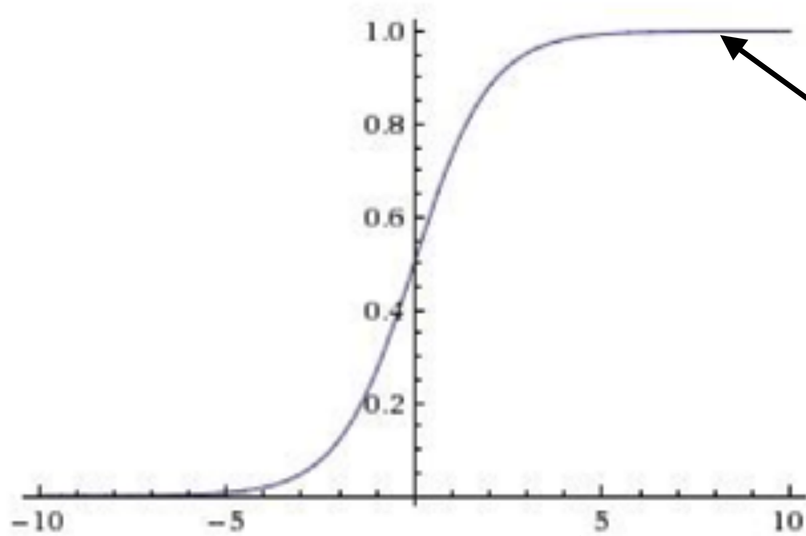
In [26]: sigmoid(np.array([100, 200, 50]))
Out[26]: array([ 1.,  1.,  1.])
```

Nonlinearities — Saturation



$$\sigma(x) = \frac{1}{1+e^{-x}}$$

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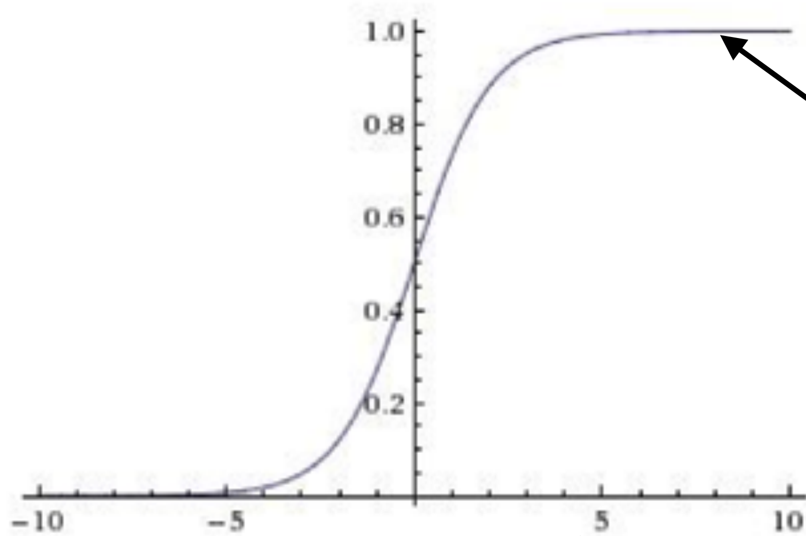
Nonlinearities — Saturation



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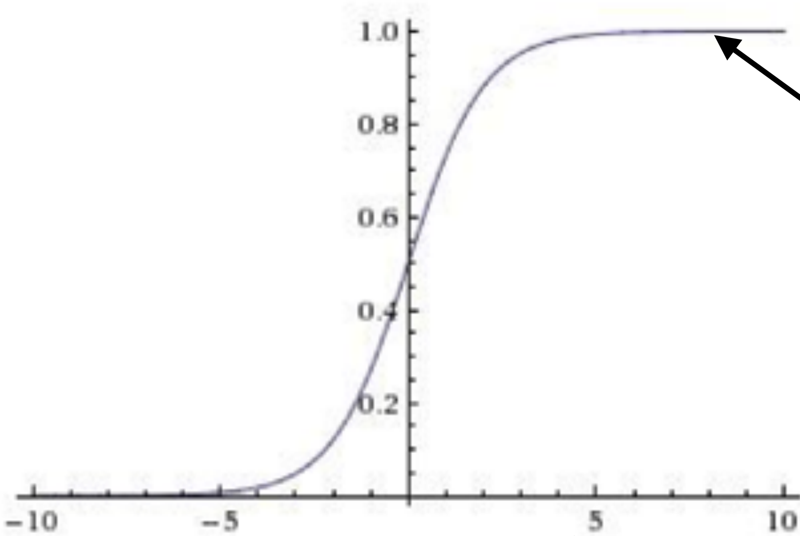
Nonlinearities — Saturation



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```
dsigmoid = lambda x: sigmoid(x) * (1 - sigmoid(x))
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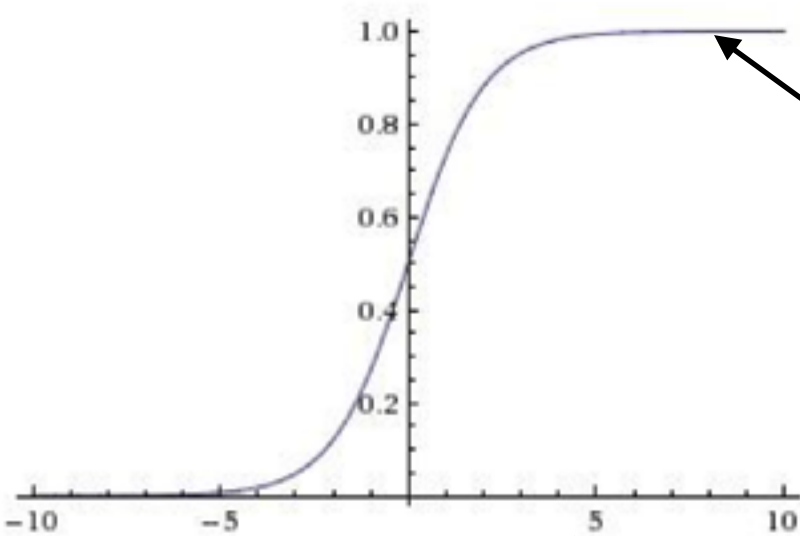

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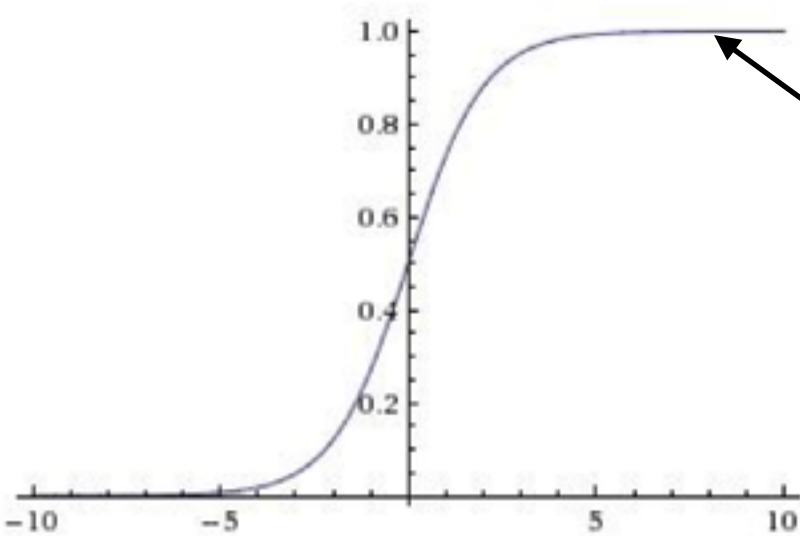

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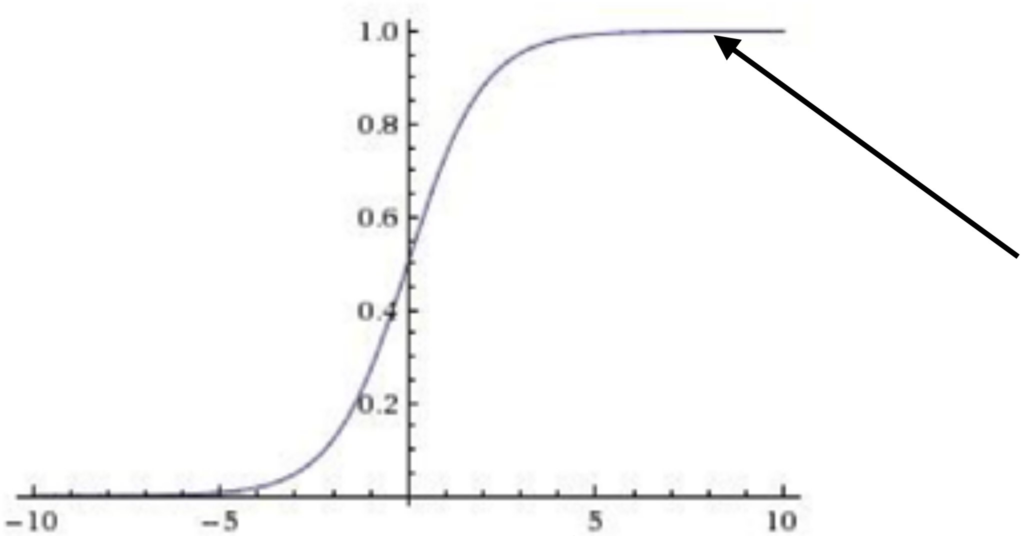
Nonlinearities — Saturation



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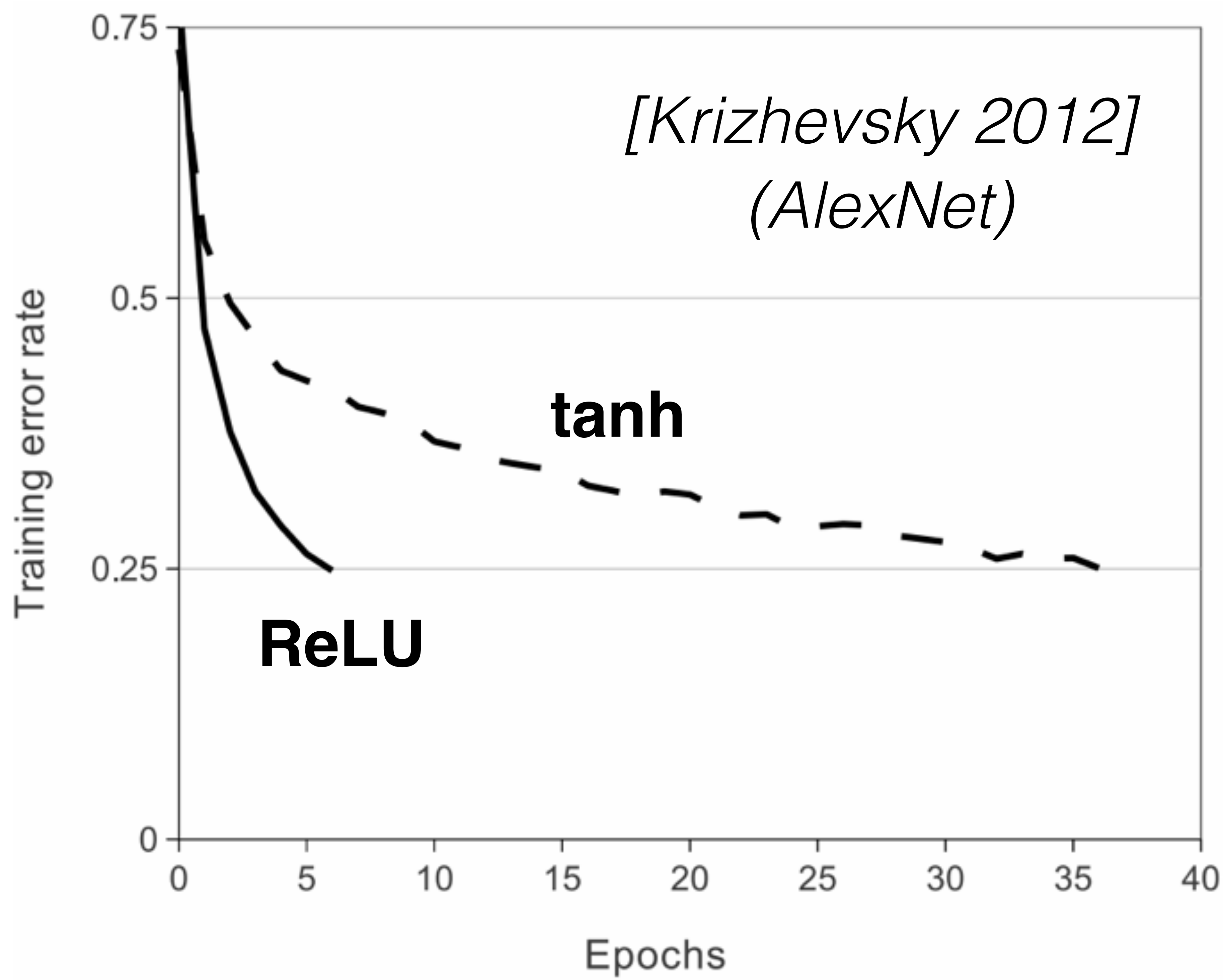
In [29]: dsigmoid(np.array([-1, 2, 5]))
Out[29]: array([ 0.19661193,  0.10499359,  0.00664806])

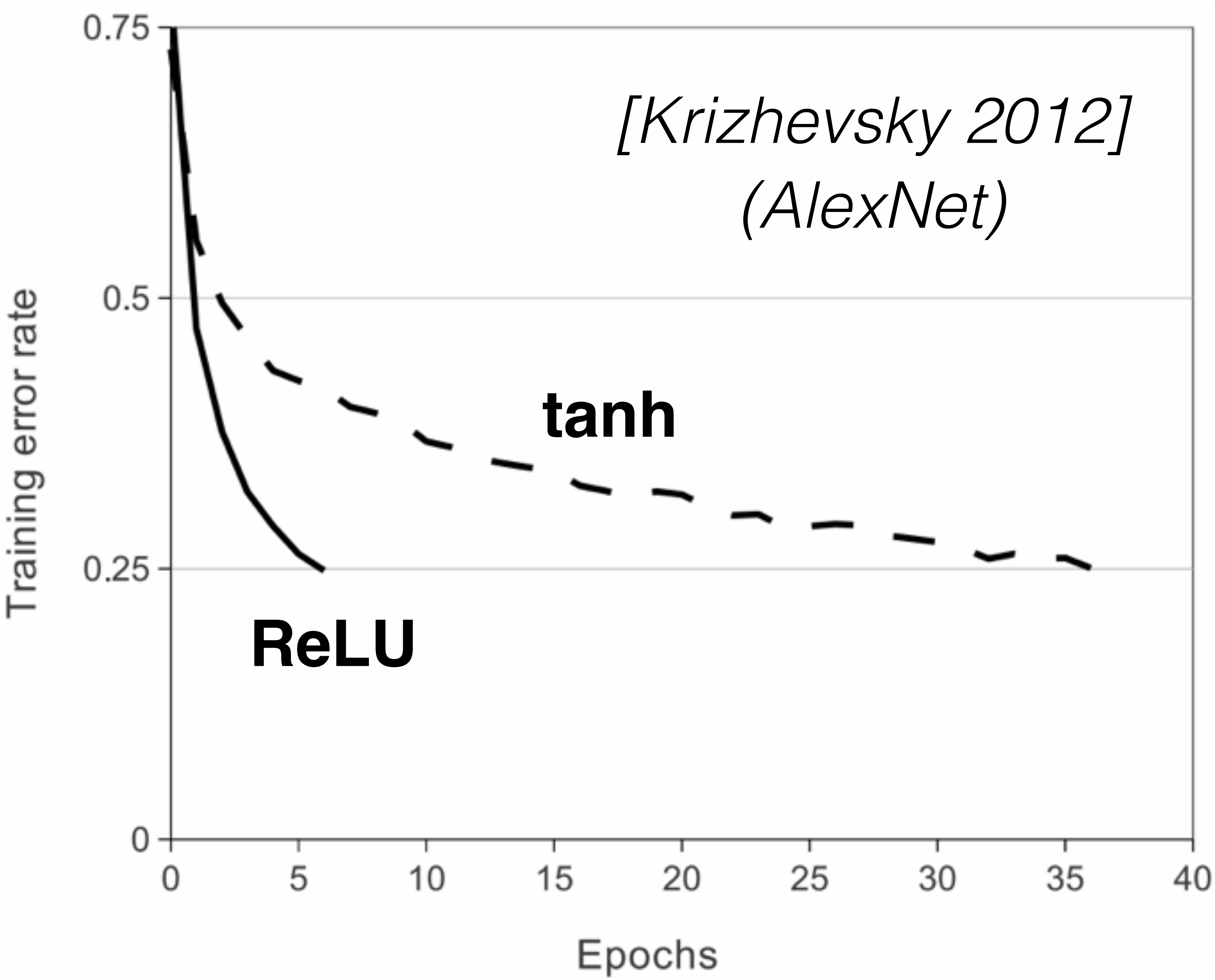
In [30]: dsigmoid(np.array([100, 200, 50]))
Out[30]: array([ 0.,  0.,  0.])
```

Saturation: the gradient is zero!

Nonlinearities



Nonlinearities



In practice, ReLU converges ~6x faster than Tanh for classification problems

ReLU in NumPy

Many ways to write ReLU — these are all equivalent:

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```
h1relu = h1.copy()
h1relu[h1 < 0] = 0
```

(c) Make a boolean mask where positive values are True, and then do an elementwise multiplication (since *int(True) = 1*):

```
h1relu = h1 * (h1 >= 0)
```

2 Layer Neural Net

2 Layer Neural Net

(Layer 1)

$$x \rightarrow \boxed{W^{(1)}x + b^{(1)}} \rightarrow$$

2 Layer Neural Net

(Layer 1) (Nonlinearity)

$$x \rightarrow \boxed{W^{(1)}x + b^{(1)}} \rightarrow \boxed{\max(0,\cdot)} \rightarrow h^{(1)}$$

(“hidden activations”)

2 Layer Neural Net

(Layer 1) (Nonlinearity) (Layer 2)

$$x \rightarrow \boxed{W^{(1)}x + b^{(1)}} \rightarrow \boxed{\max(0,\cdot)} \rightarrow h^{(1)} \rightarrow \boxed{W^{(2)}x + b^{(2)}} \rightarrow f$$

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Let's expand out the equation:

2 Layer Neural Net

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$$f = W^{(2)} \max(0, W^{(1)}x + b^{(1)}) + b^{(2)}$$

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Note: *any* nonlinear function will prevent this collapse, but not all nonlinear functions actually work in practice

2 Layer Neural Net

(Layer 1) (Nonlinearity) (Layer 2)

$$x \rightarrow \boxed{W^{(1)}x+b^{(1)}} \rightarrow \boxed{\max(0,\cdot)} \rightarrow h^{(1)} \rightarrow \boxed{W^{(2)}x+b^{(2)}} \rightarrow f$$

(“hidden activations”)

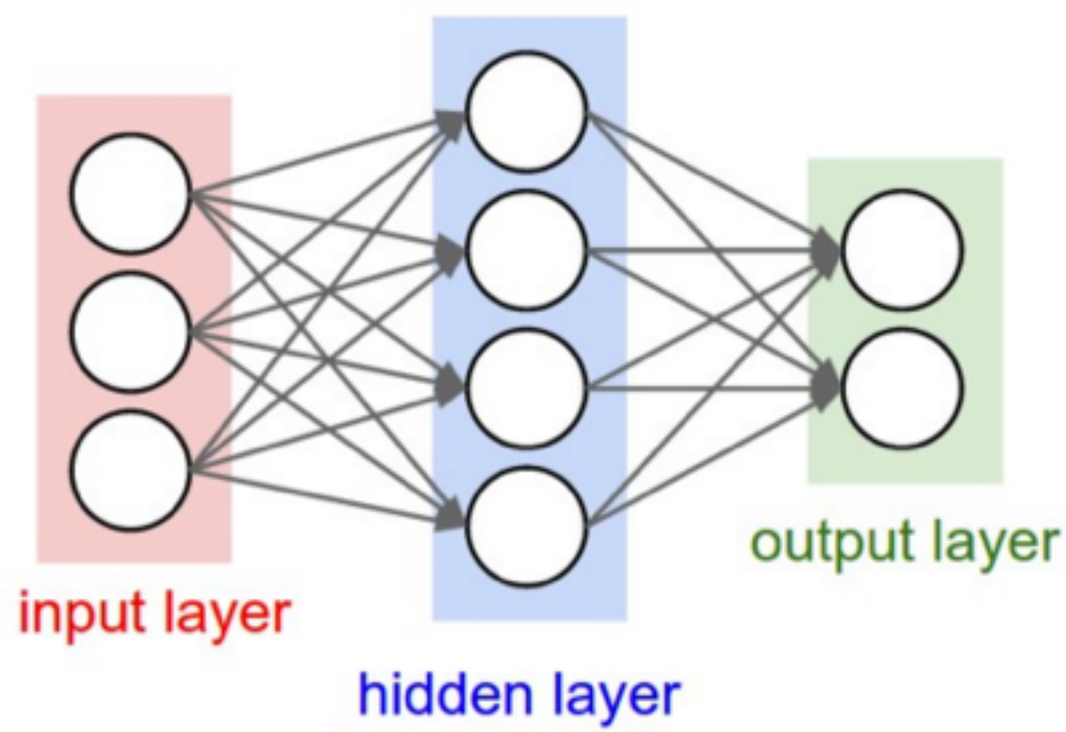
Note: Traditionally, the nonlinearity was considered part of the layer and is called an “activation function”

In this class, we will consider them separate layers, but be aware that many others consider them part of the layer

Neural Net:
Graphical Representation

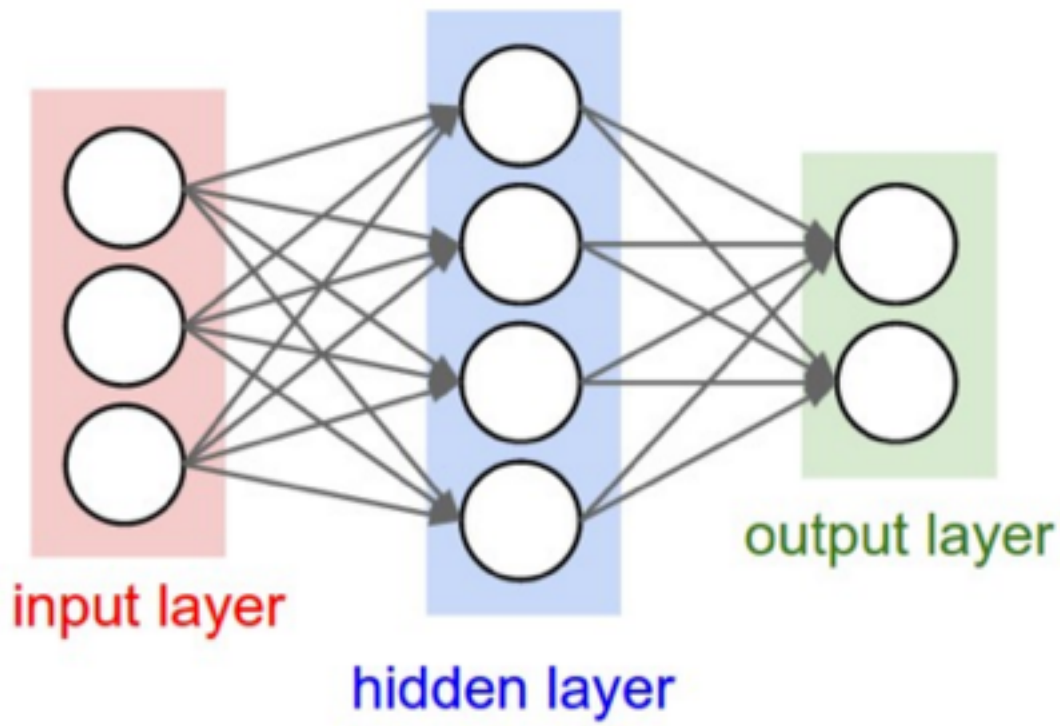
Neural Net: Graphical Representation

2 layers



Neural Net: Graphical Representation

2 layers

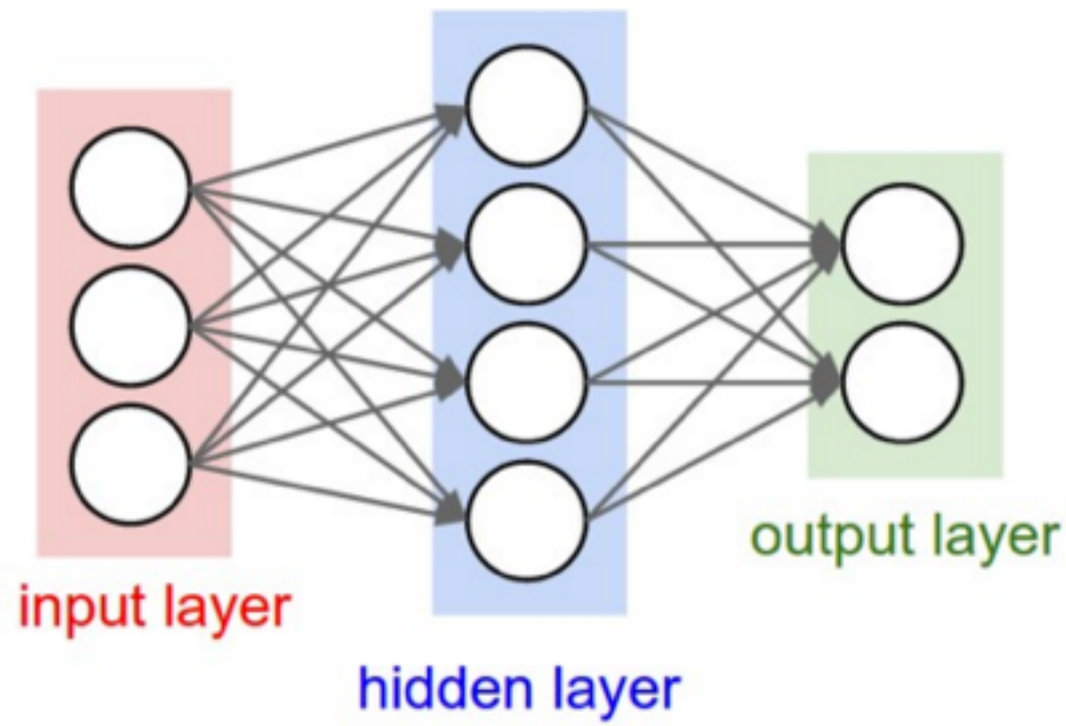


- Called **“fully connected”** because every output depends on every input.

- Also called **“affine”** or **“inner product”**

Neural Net: Graphical Representation

2 layers



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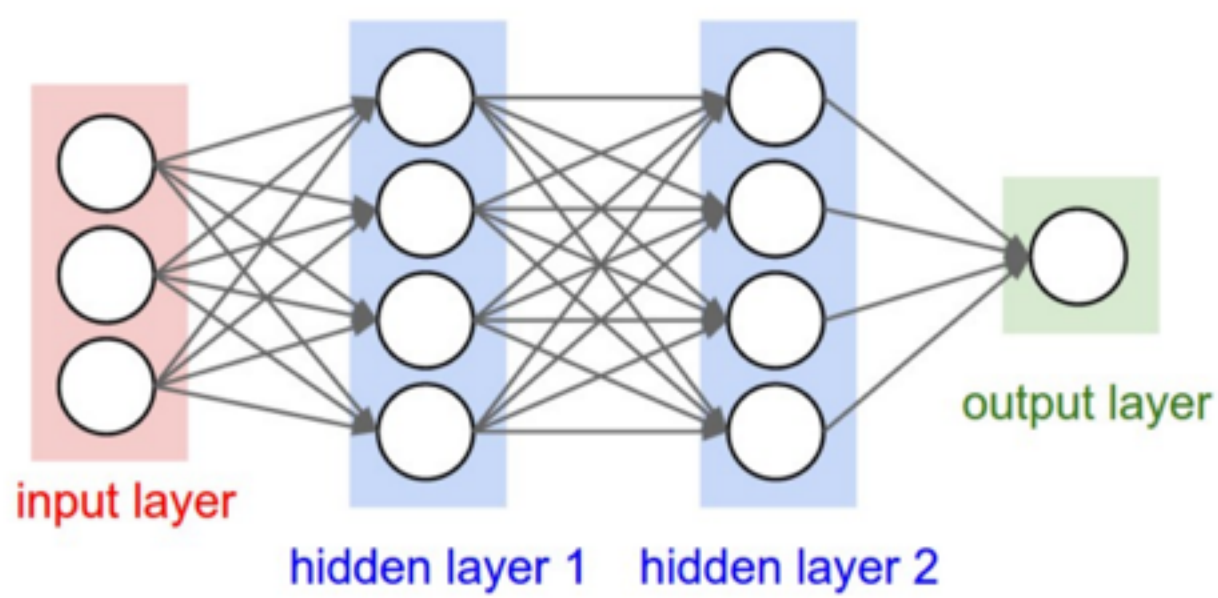
3 layers



Questions?

Neural Networks, More generally

$$x \rightarrow \boxed{\text{Function}} \rightarrow h^{(1)} \rightarrow \boxed{\text{Function}} \rightarrow h^{(2)} \rightarrow \cdots \rightarrow f$$

Neural Networks, More generally

$$x \to \boxed{\text{Function}} \to h^{(1)} \to \boxed{\text{Function}} \to h^{(2)} \to \cdots \to f$$

This can be:

- Fully connected layer
- Nonlinearity (ReLU, Tanh, Sigmoid)
- Convolution
- Pooling (Max, Avg)
- Vector normalization (L1, L2)
- *Invent new ones*

Neural Networks, More generally

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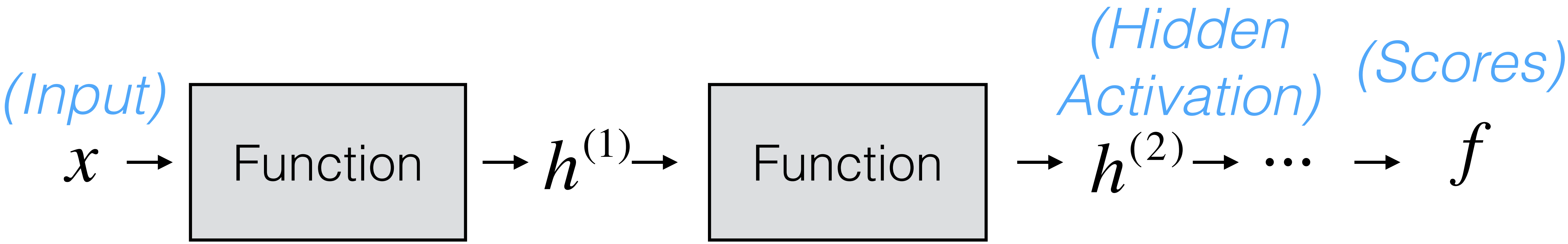
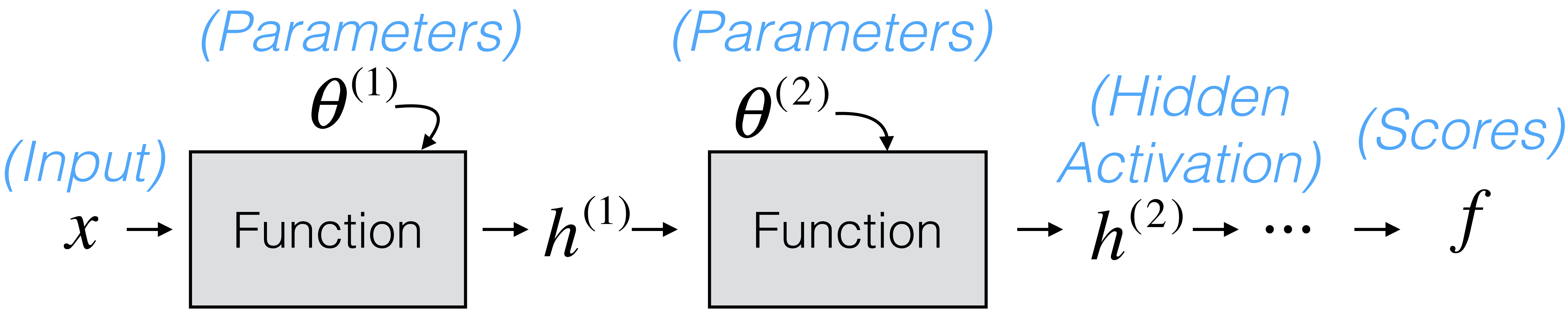
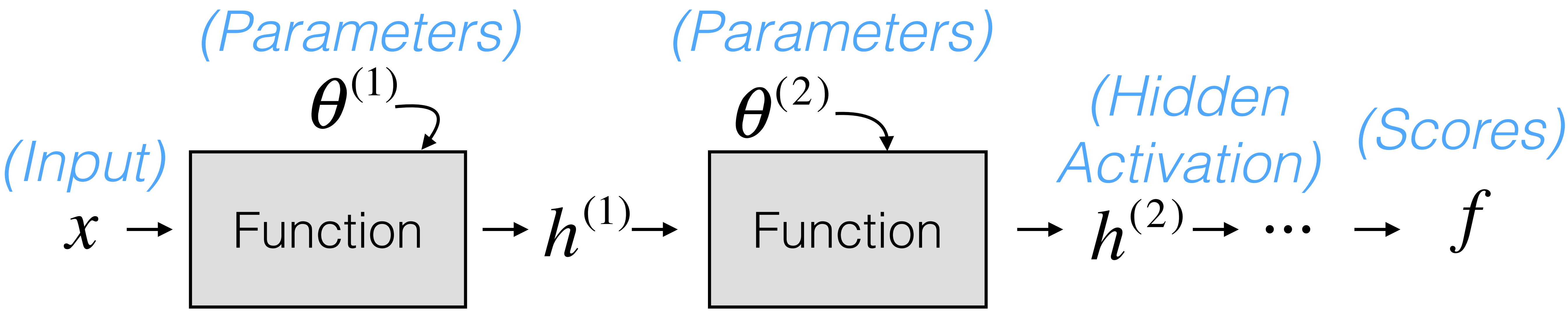
Neural Networks, More generally



Neural Networks, More generally

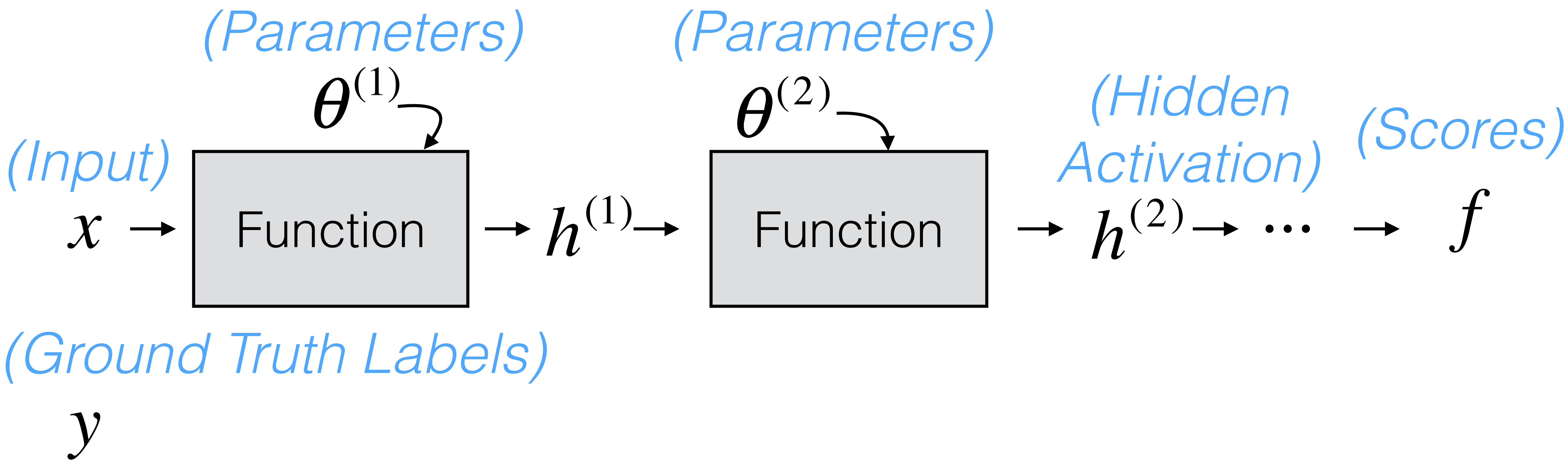


Neural Networks, More generally



Here, θ represents whatever parameters that layer is using (e.g. for a fully connected layer $\theta^{(1)} = \{ W^{(1)}, b^{(1)} \}$).

Neural Networks, More generally



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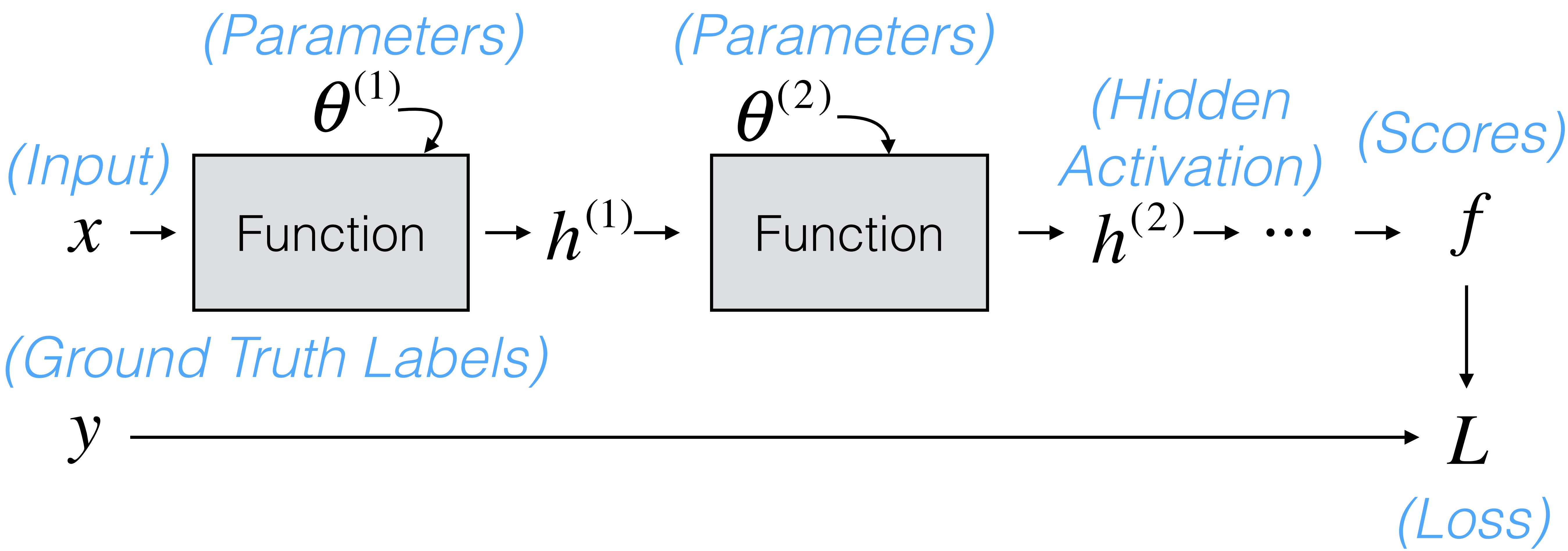

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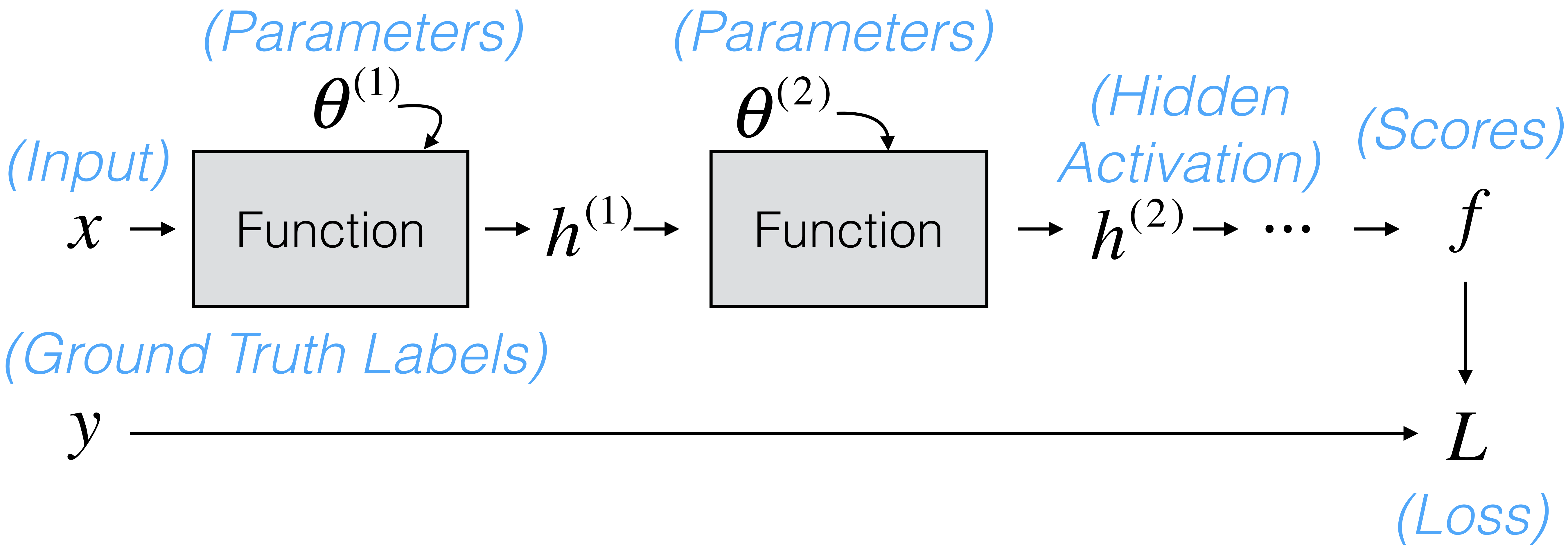
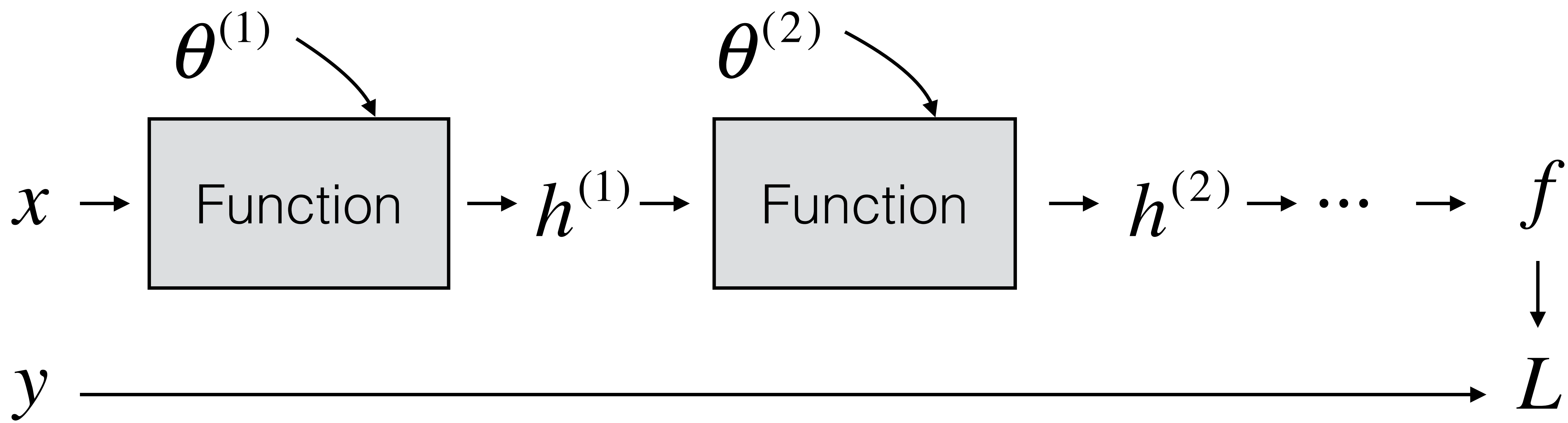
Neural Networks, More generally



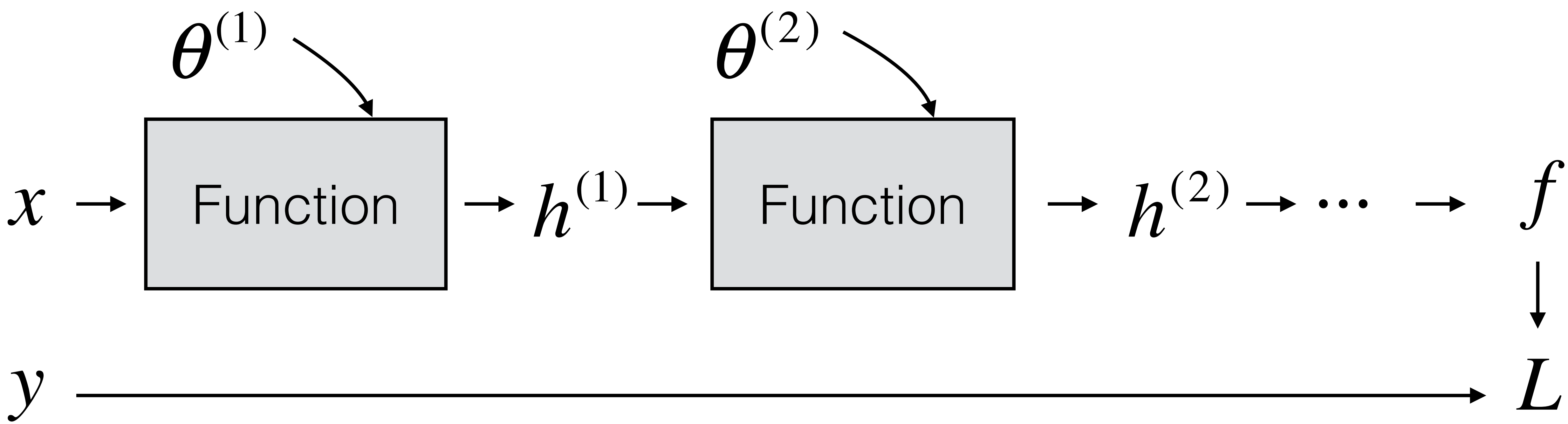
Here, θ represents whatever parameters that layer is using (e.g. for a fully connected layer $\theta^{(1)} = \{ W^{(1)}, b^{(1)} \}$).

Recall: the loss "L" measures how far the predictions "f" are from the labels "y". The most common loss is Softmax.

Neural Networks,
More generally

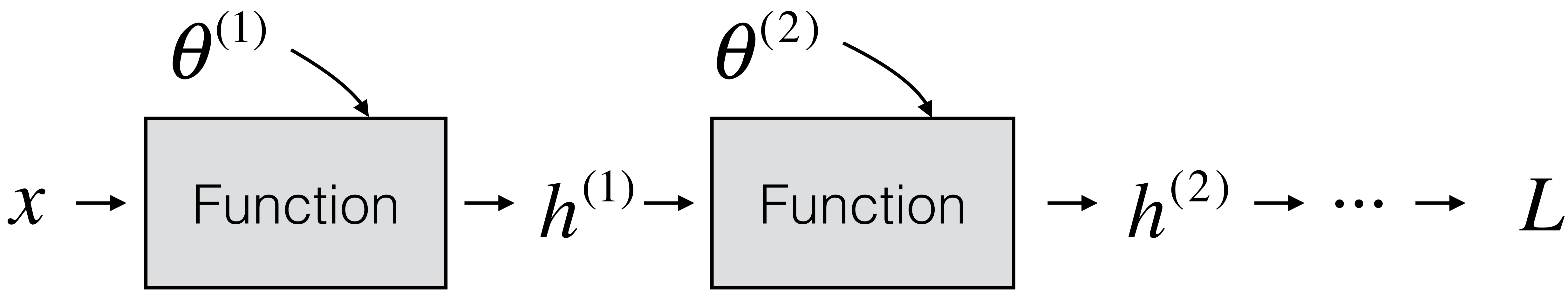


Neural Networks, More generally



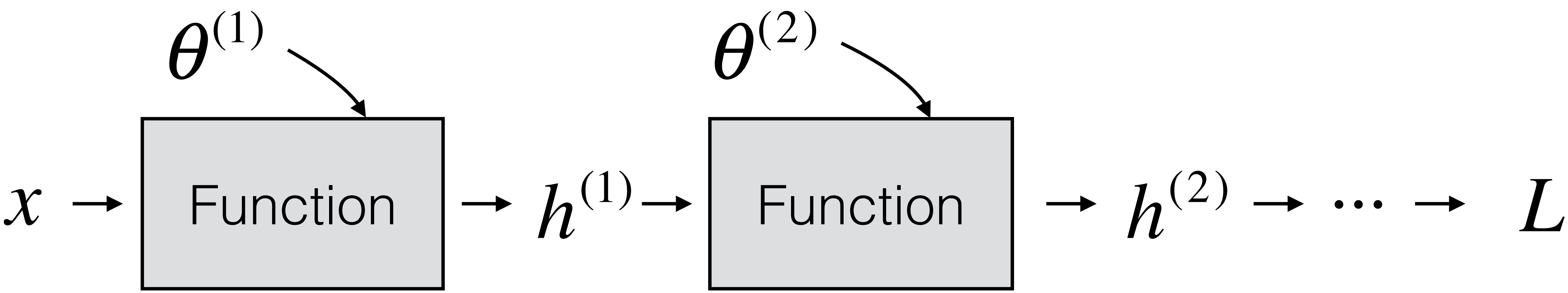
Goal: Find a value for parameters ($\theta^{(1)}$, $\theta^{(2)}$, …), so that the loss (L) is small

Neural Networks, More generally



- To keep things clean, we will sometimes hide “y”, but remember that it is always there

Neural Networks, More generally



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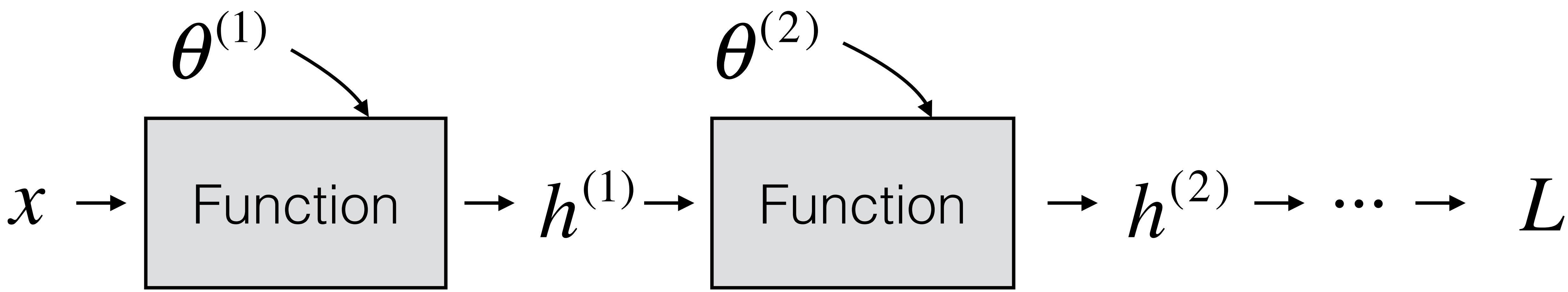
Neural Networks, More generally



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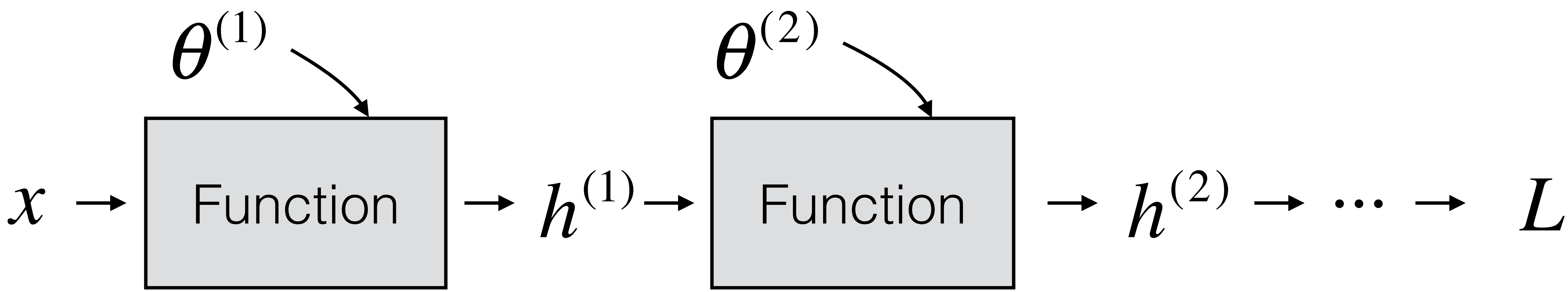
$$\theta \leftarrow \theta - \alpha \frac{\partial L}{\partial \theta}$$

Neural Networks, More generally



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- From last class, we learned that to improve the weights, we can take a step in the negative gradient direction:

$$\theta \leftarrow \theta - \alpha \frac{\partial L}{\partial \theta}$$

- How do we compute this?

Backpropagation

[Rumelhart, Hinton, Williams. Nature 1986]

Learning representations by back-propagating errors

David E. Rumelhart*, Geoffrey E. Hinton†
& Ronald J. Williams*

* Institute for Cognitive Science, C-015, University of California, San Diego, La Jolla, California 92093, USA
† Department of Computer Science, Carnegie-Mellon University, Pittsburgh, Philadelphia 15213, USA

We describe a new learning procedure, back-propagation, for networks of neurone-like units. The procedure repeatedly adjusts the weights of the connections in the network so as to minimize a measure of the difference between the actual output vector of the net and the desired output vector. As a result of the weight adjustments, internal 'hidden' units which are not part of the input or output come to represent important features of the task domain, and the regularities in the task are captured by the interactions of these units. The ability to create useful new features distinguishes back-propagation from earlier, simpler methods such as the perceptron-convergence procedure[1].

There have been many attempts to design self-organizing neural networks. The aim is to find a powerful synaptic modification rule that will allow an arbitrarily connected neural network to develop an internal structure that is appropriate for more difficult when we introduce hidden units whose actual or desired states are not specified by the task. (In perceptrons, there are 'feature analysers' between the input and output that are not true hidden units because their input connections are fixed by hand, so their states are completely determined by the input vector: they do not learn representations.) The learning procedure must decide under what circumstances the hidden units should be active in order to help achieve the desired input-output behaviour. This amounts to deciding what these units should represent. We demonstrate that a general purpose and relatively simple procedure is powerful enough to construct appropriate internal representations.

The simplest form of the learning procedure is for layered networks which have a layer of input units at the bottom; any number of intermediate layers; and a layer of output units at the top. Connections within a layer or from higher to lower layers are forbidden, but connections can skip intermediate layers. An input vector is presented to the network by setting the states of the input units. Then the states of the units in each layer are determined by applying equations (1) and (2) to the connections coming from lower layers. All units within a layer have their states set in parallel, but different layers have their states set sequentially, starting at the bottom and working upwards until the states of the output units are determined.

The total input, x_j, to unit j is a linear function of the outputs, y_i, of the units that are connected to j and of the weights, w_{ji}, on these connections

$$x_j = \sum_i y_i w_{ji} \qquad (1)$$

Backpropagation

[Rumelhart, Hinton, Williams. Nature 1986]

Learning representations by back-propagating errors

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Learning representations by back-propagating errors.

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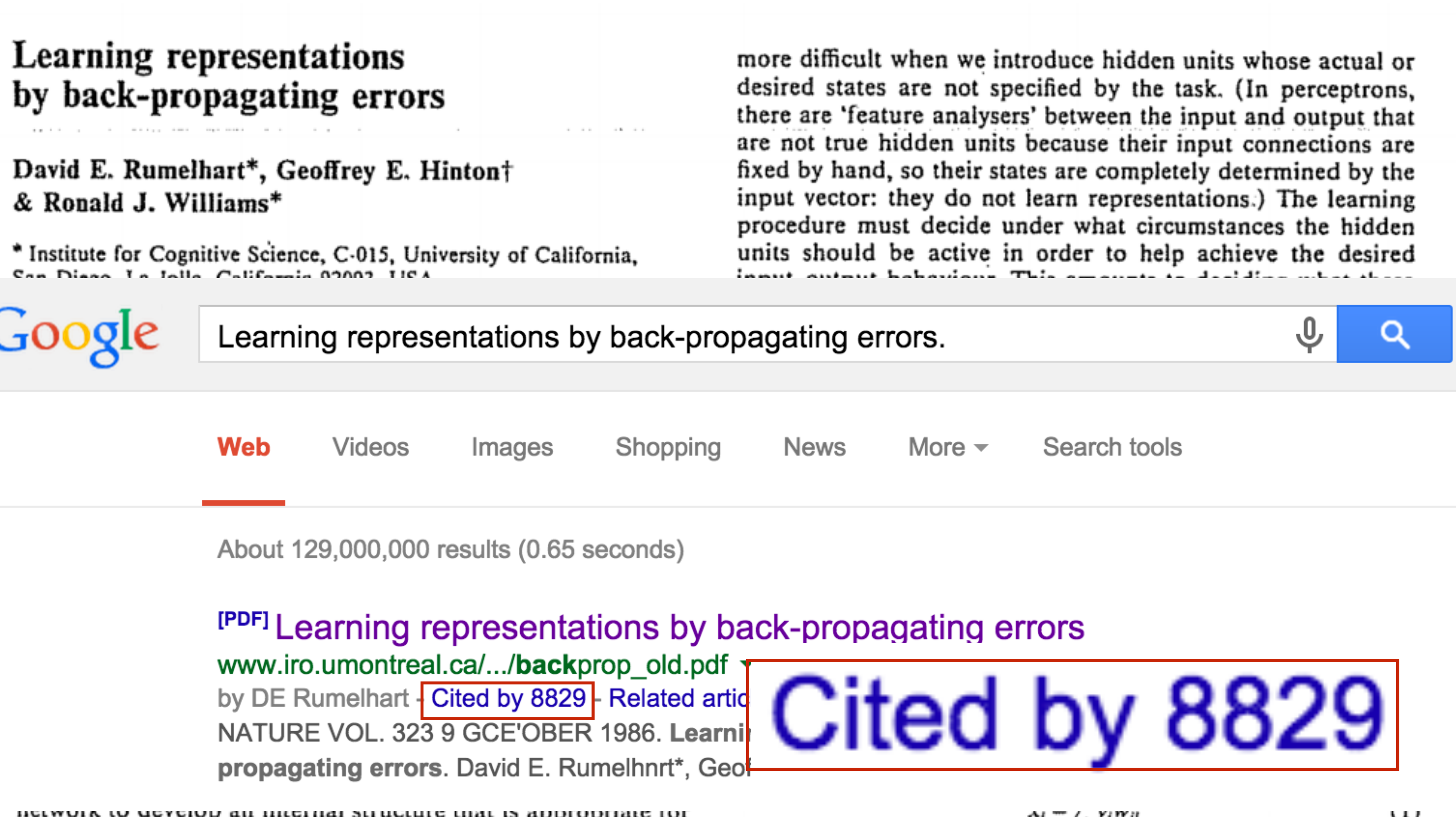
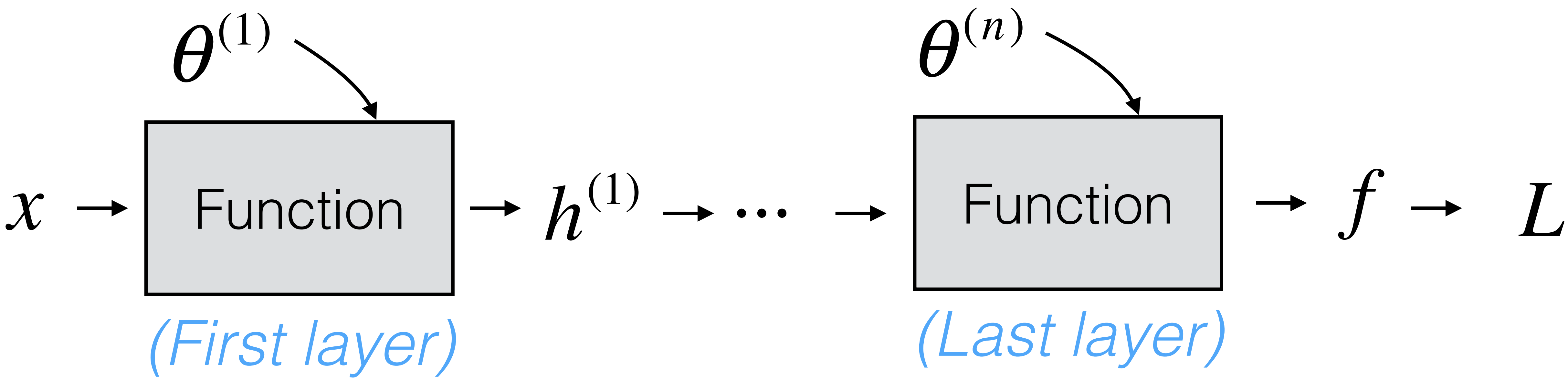
Backpropagation

[Rumelhart, Hinton, Williams. Nature 1986]



Backpropagation

Forward Propagation:



Backpropagation

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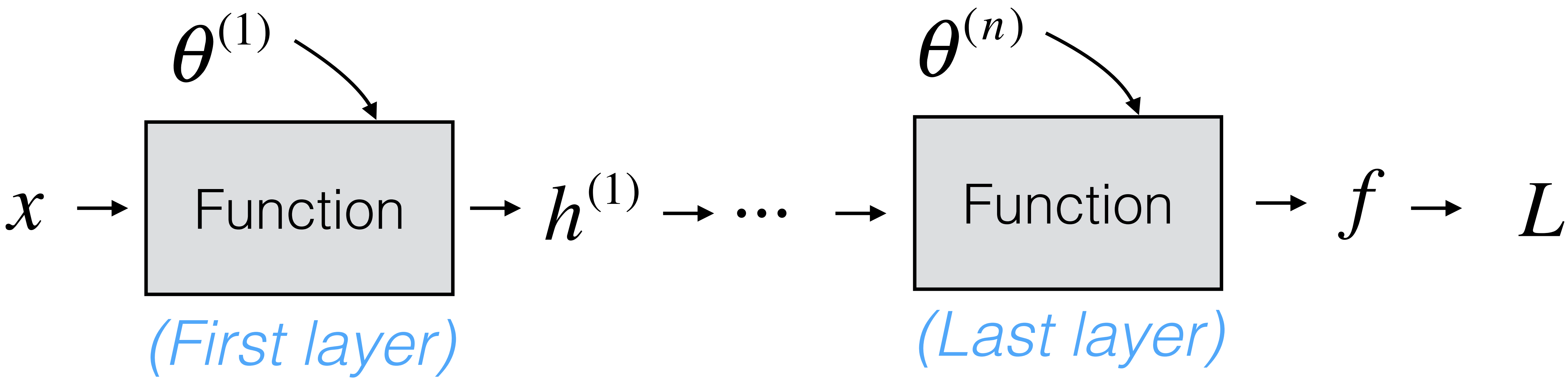
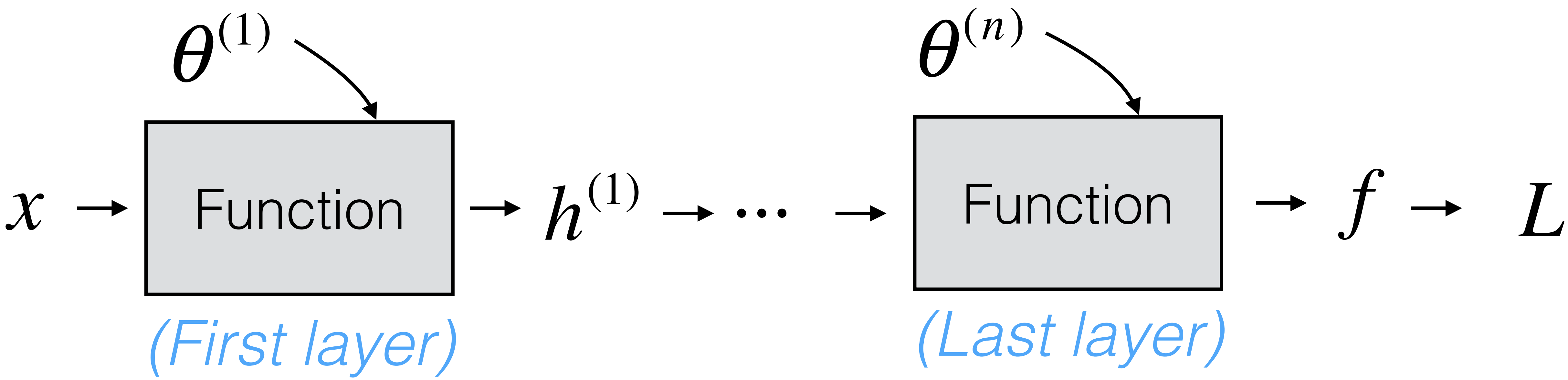


Backward Propagation:

Backpropagation

Forward Propagation:

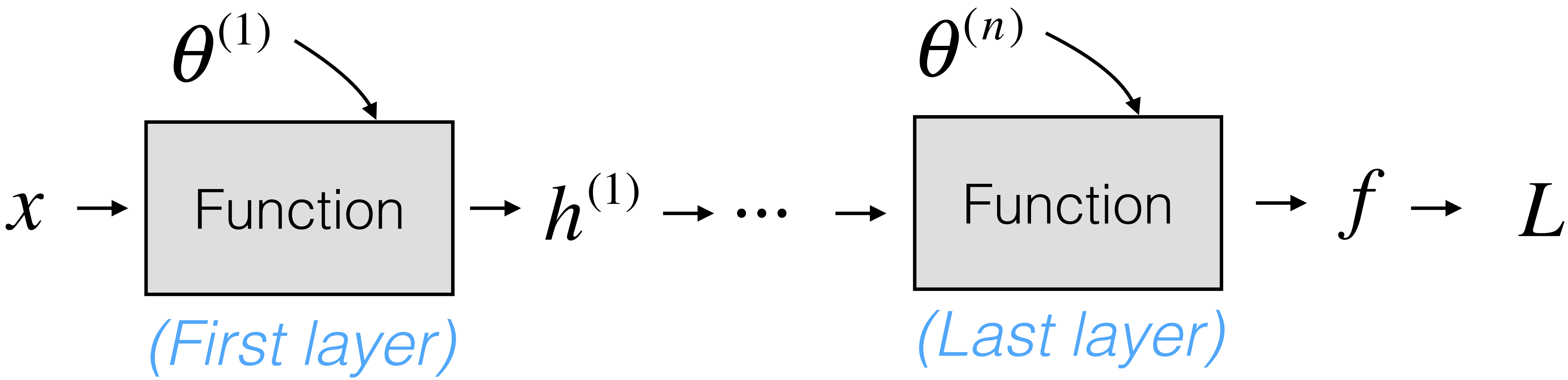


Backward Propagation:

L

Backpropagation

Forward Propagation:

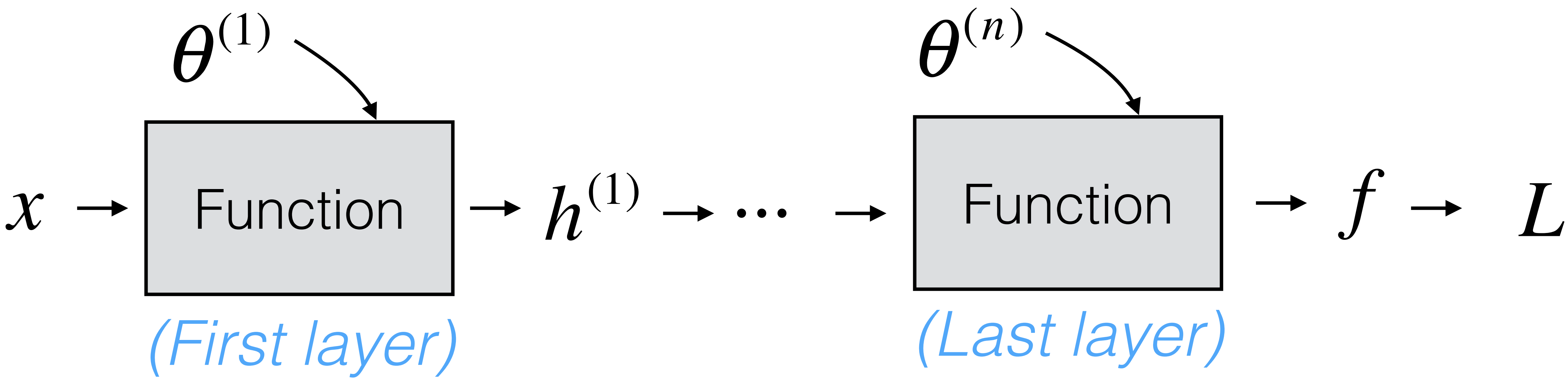


Backward Propagation:

$$\frac{\partial L}{\partial f} \leftarrow L$$

Backpropagation

Forward Propagation:



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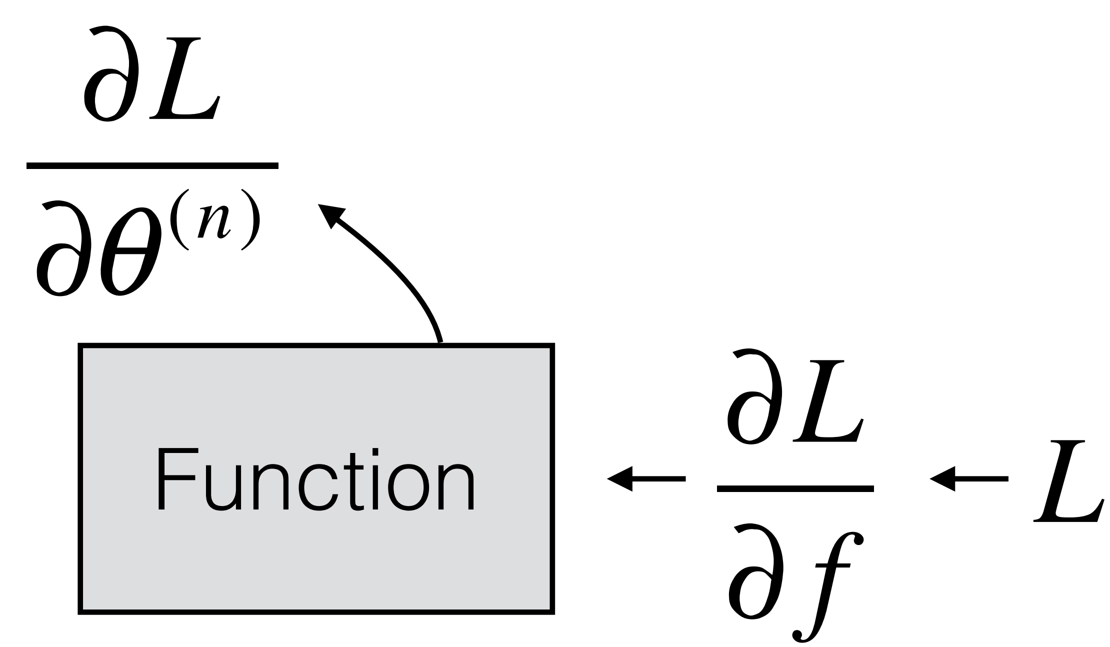

Backpropagation

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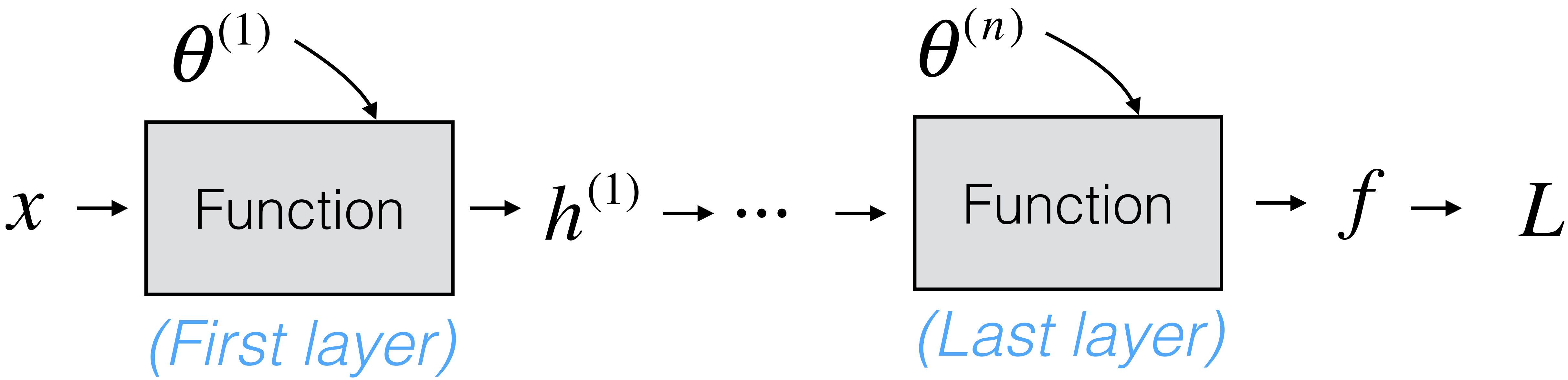

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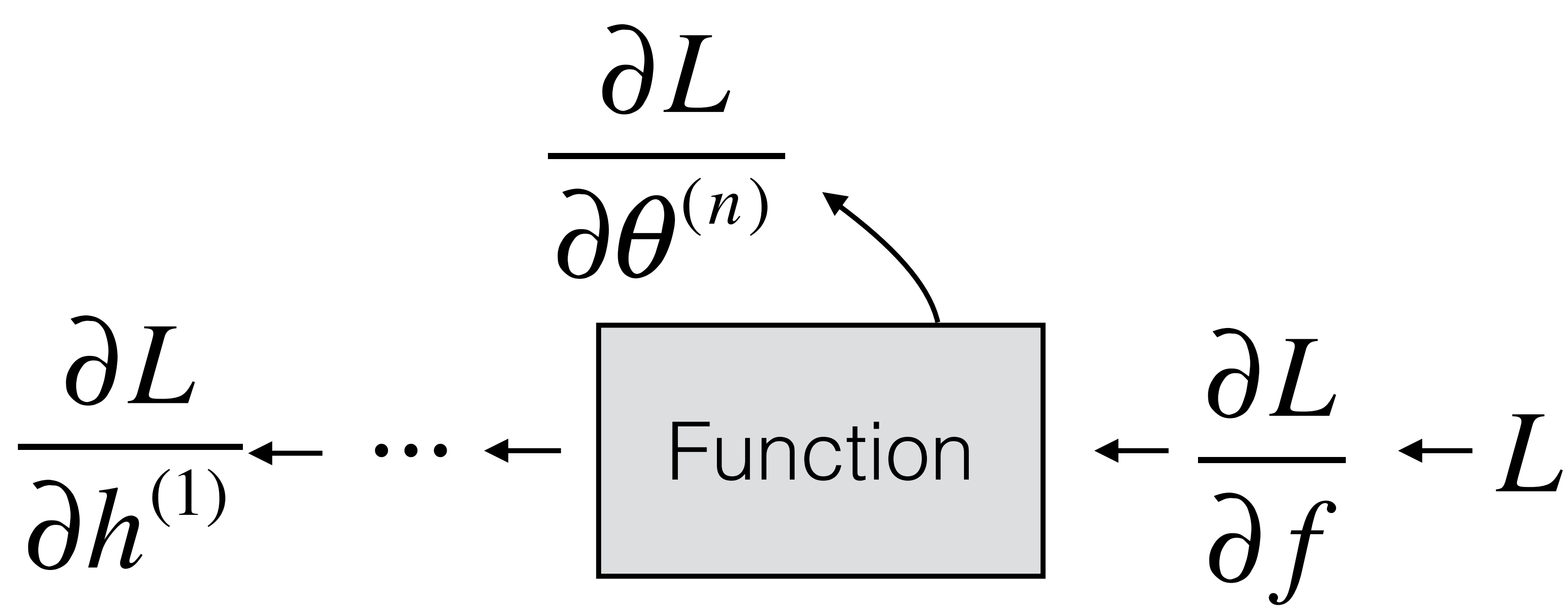

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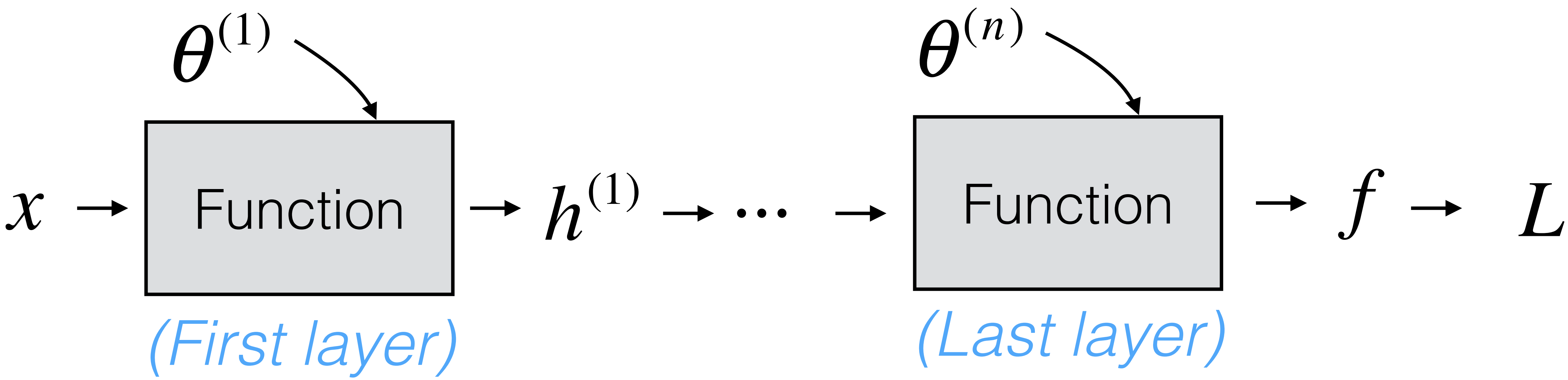

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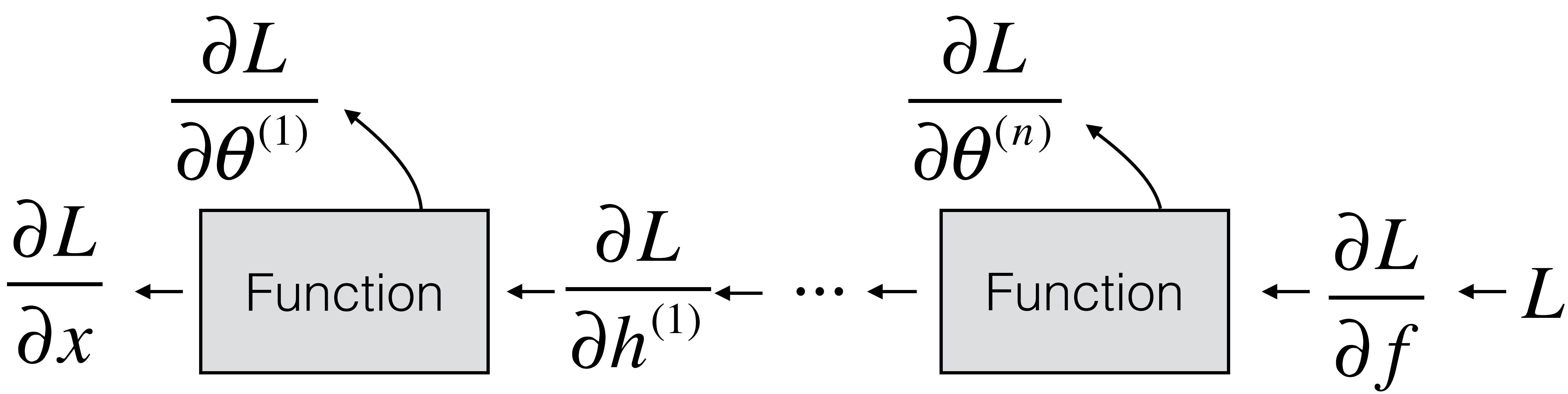

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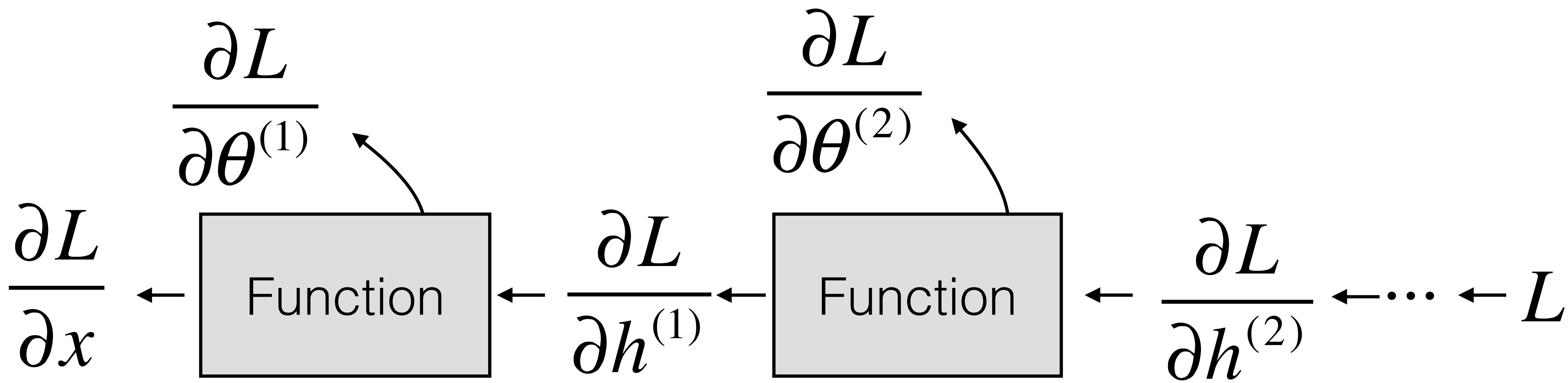

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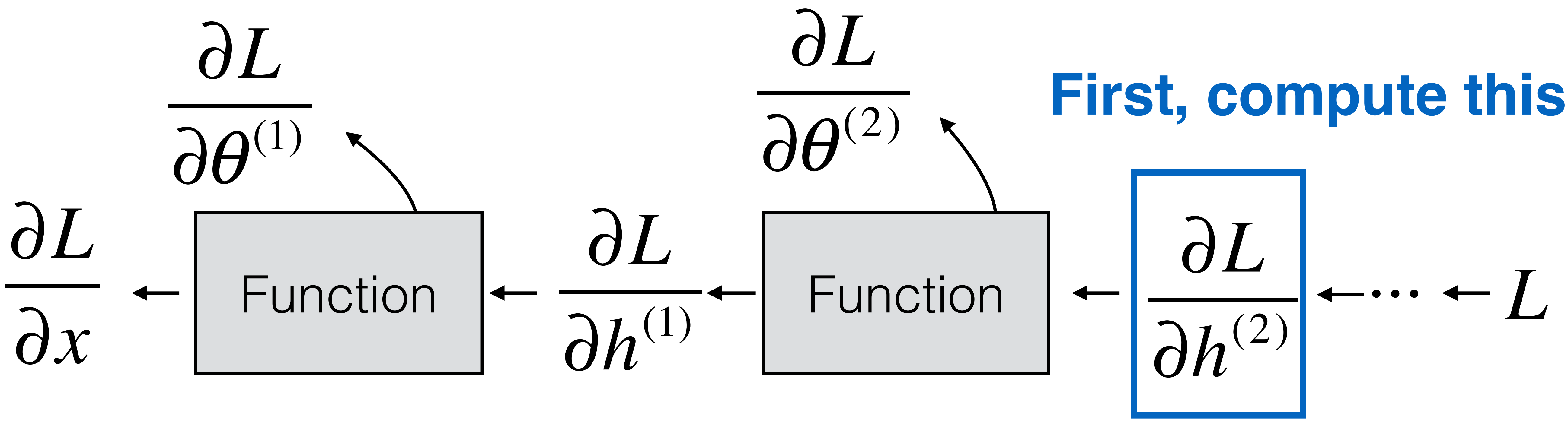
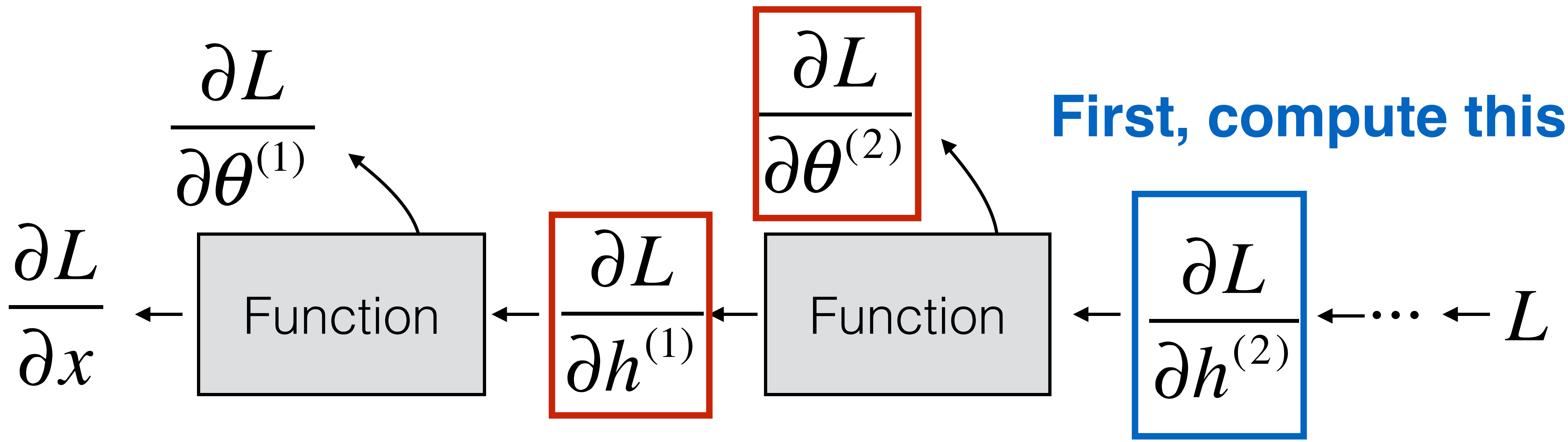
Backward Propagation:



Backpropagation

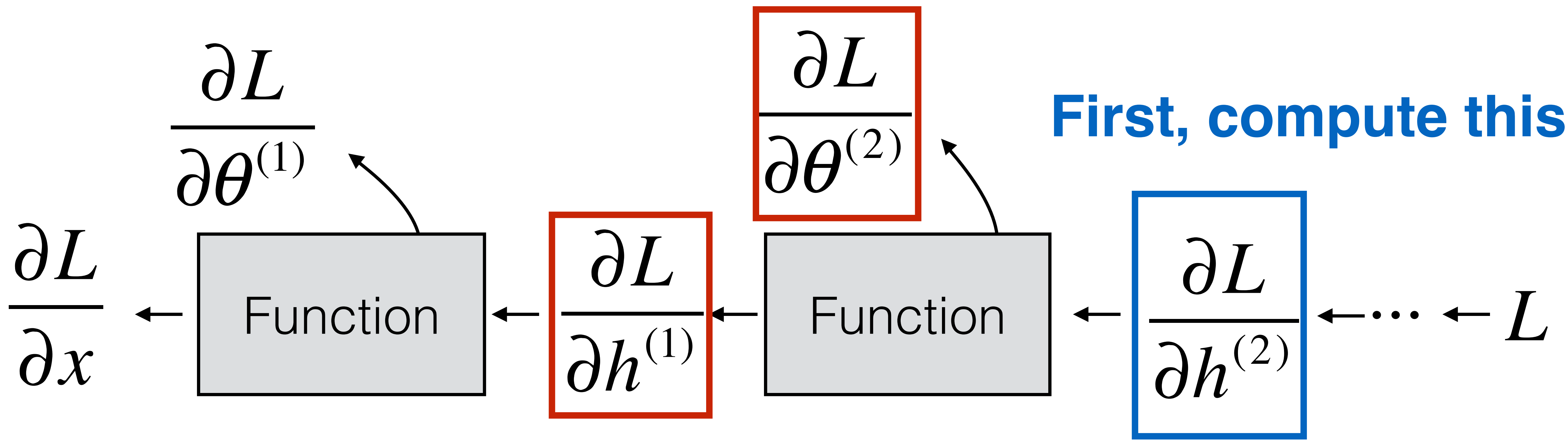
Backward Propagation:

Then compute this



Backpropagation

Backward Propagation:



Key observation: You can compute $\boxed{\frac{\partial L}{\partial \theta^{(2)}}}$ and $\boxed{\frac{\partial L}{\partial h^{(1)}}}$ given only $\boxed{\frac{\partial L}{\partial h^{(2)}}}$, and you can do it <u>one layer at a time.</u>

Chain rule recap

I hope everyone remembers the chain rule:

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial x}$$

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Chain rule recap

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Backward propagation: $\frac{\partial L}{\partial x} \longleftarrow \frac{\partial L}{\partial h} \longleftarrow \cdots$

… but what if x and y are multi-dimensional?