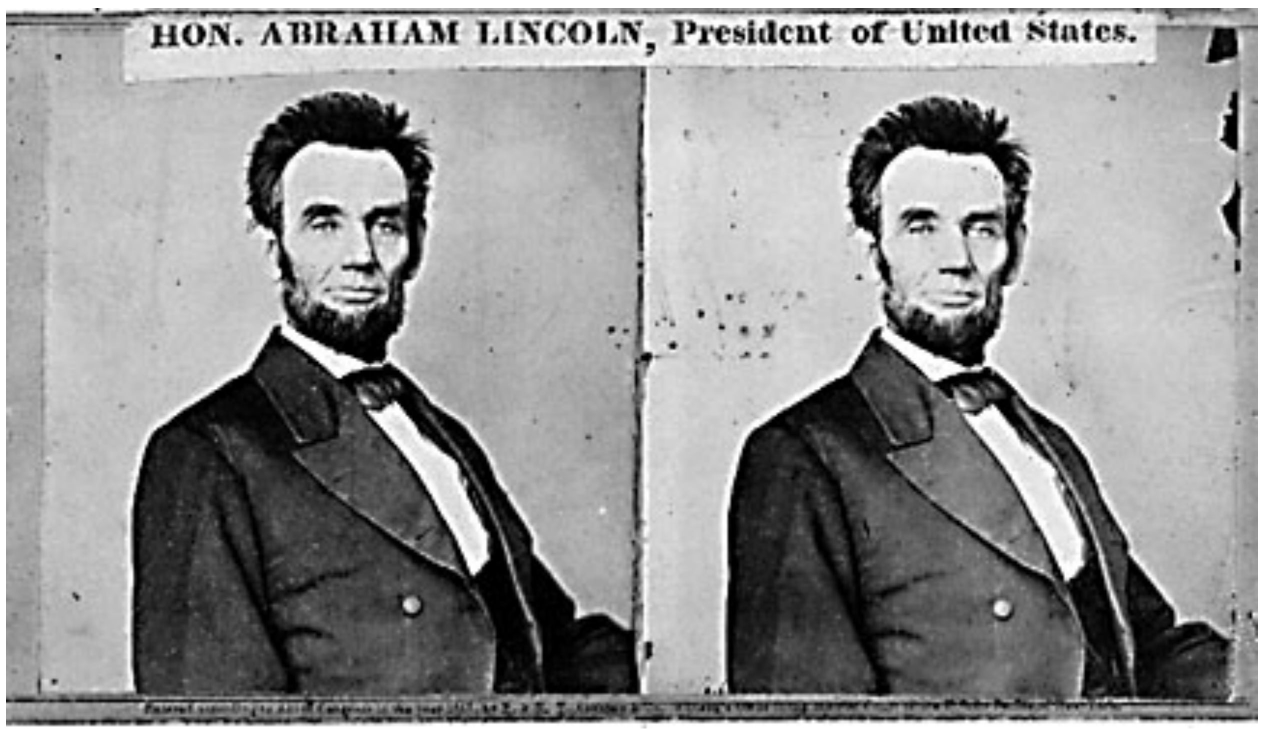
CS4670 / 5670: Computer Vision

Kavita Bala

Lec 22: Stereo



Road map

- What we've seen so far:
  - Low-level image processing: filtering, edge detecting, feature detection
  - Geometry: image transformations, panoramas, single-view modeling Fundamental matrices
- What's next:
  - Finishing up geometry: multi view stereo, structure from motion
  - Recognition
  - Image formation

Announcements

- Wed: photometric stereo
- No class Dragon Day

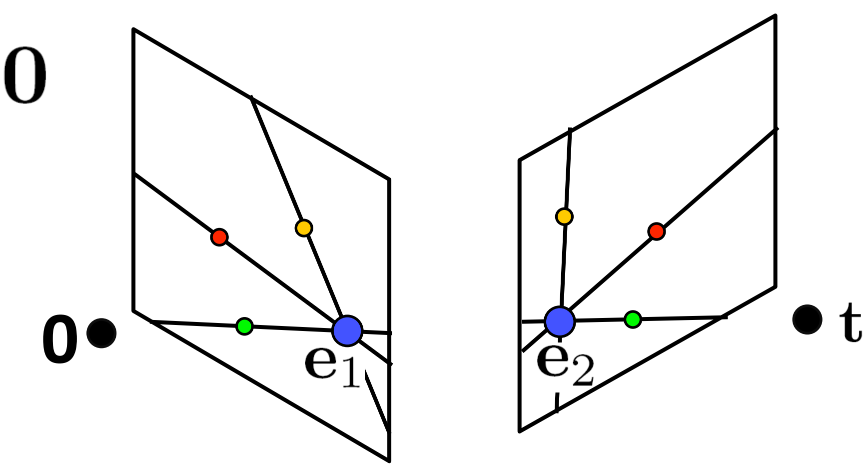
Fundamental matrix result

$$\mathbf{q}^T\mathbf{F}\mathbf{p} = 0$$

(Longuet-Higgins, 1981)

Properties of the Fundamental Matrix

- $\mathbf{F}\mathbf{p}$ is the epipolar line associated with $\mathbf{p}$
- $\mathbf{F}^T\mathbf{q}$ is the epipolar line associated with $\mathbf{q}$
- $\mathbf{F}\mathbf{e}_1 = \mathbf{0}$ and $\mathbf{F}^T\mathbf{e}_2 = \mathbf{0}$
- $\mathbf{F}$ is rank 2



Estimating **F**

- If we don't know $\mathbf{K}_1$, $\mathbf{K}_2$, $\mathbf{R}$, or $\mathbf{t}$, can we estimate **F** for two images?

- Yes, given enough correspondences

Estimating F – 8-point algorithm

- The fundamental matrix F is defined by

$$\mathbf{x}'^{\mathrm{T}}\mathbf{F}\mathbf{x} = 0$$

for any pair of matches x and x' in two images.

- Let x=$(u,v,1)^{\mathrm{T}}$ and x'=$(u',v',1)^{\mathrm{T}}$,

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

each match gives a linear equation

$$uu' f_{11} + vu' f_{12} + u' f_{13} + uv' f_{21} + vv' f_{22} + v' f_{23} + uf_{31} + vf_{32} + f_{33} = 0$$

8-point algorithm

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n' & v_nu_n' & u_n' & u_nv_n' & v_nv_n' & v_n' & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

- In reality, instead of solving $\mathbf{Af} = 0$, we seek $\mathbf{f}$ to minimize $\|\mathbf{Af}\|$, least eigenvector of $\mathbf{A}^{\mathrm{T}}\mathbf{A}$.

8-point algorithm – Problem?

- **F** should have rank 2
- To enforce that **F** is of rank 2, F is replaced by F' that minimizes $\|\mathbf{F}-\mathbf{F}'\|$ subject to the rank constraint.

- This is achieved by SVD. Let $\mathbf{F}=\mathbf{U}\Sigma\mathbf{V}^{\mathrm{T}}$, where

$$\Sigma=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix},\ \text{let}\quad \Sigma'=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & 0\end{bmatrix}$$

then $\mathbf{F}'=\mathbf{U}\Sigma'\mathbf{V}^{\mathrm{T}}$ is the solution.

8-point algorithm

- Pros: it is linear, easy to implement and fast
- Cons: susceptible to noise

- Normalized 8-point algorithm: Hartley

What about more than two views?

- The geometry of three views is described by a 3 x 3 x 3 tensor called the *trifocal tensor*
- The geometry of four views is described by a 3 x 3 x 3 x 3 tensor called the *quadrifocal tensor*
- After this it starts to get complicated…
  - Structure from motion

Stereo reconstruction pipeline

- Steps
  - Calibrate cameras
  - Rectify images
  - Compute correspondence (and hence disparity)
  - Estimate depth

Correspondence algorithms

Algorithms may be classified into two types:

1. Dense: compute a correspondence at every pixel
2. Sparse: compute correspondences only for features

Example image pair – parallel cameras

First image

Second image

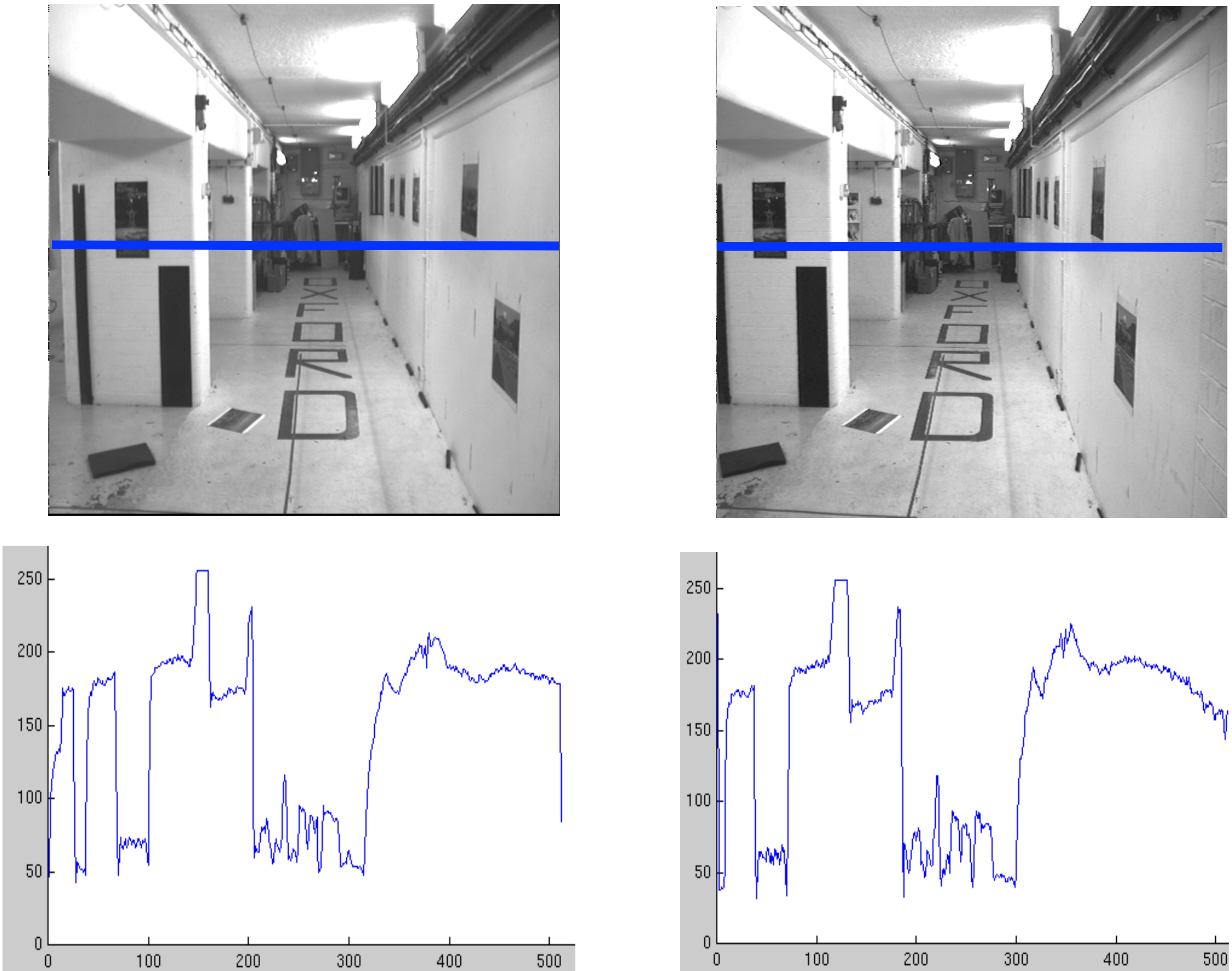
Dense correspondence algorithm

Search problem (geometric constraint): for each point in left image, corresponding point in right image lies on the epipolar line (1D ambiguity)

Disambiguating assumption (photometric constraint): the intensity neighborhood of corresponding points are similar across images

Measure similarity of neighborhood intensity by cross-correlation

Intensity profiles



- Clear correspondence, but also noise and ambiguity

Normalized Cross Correlation

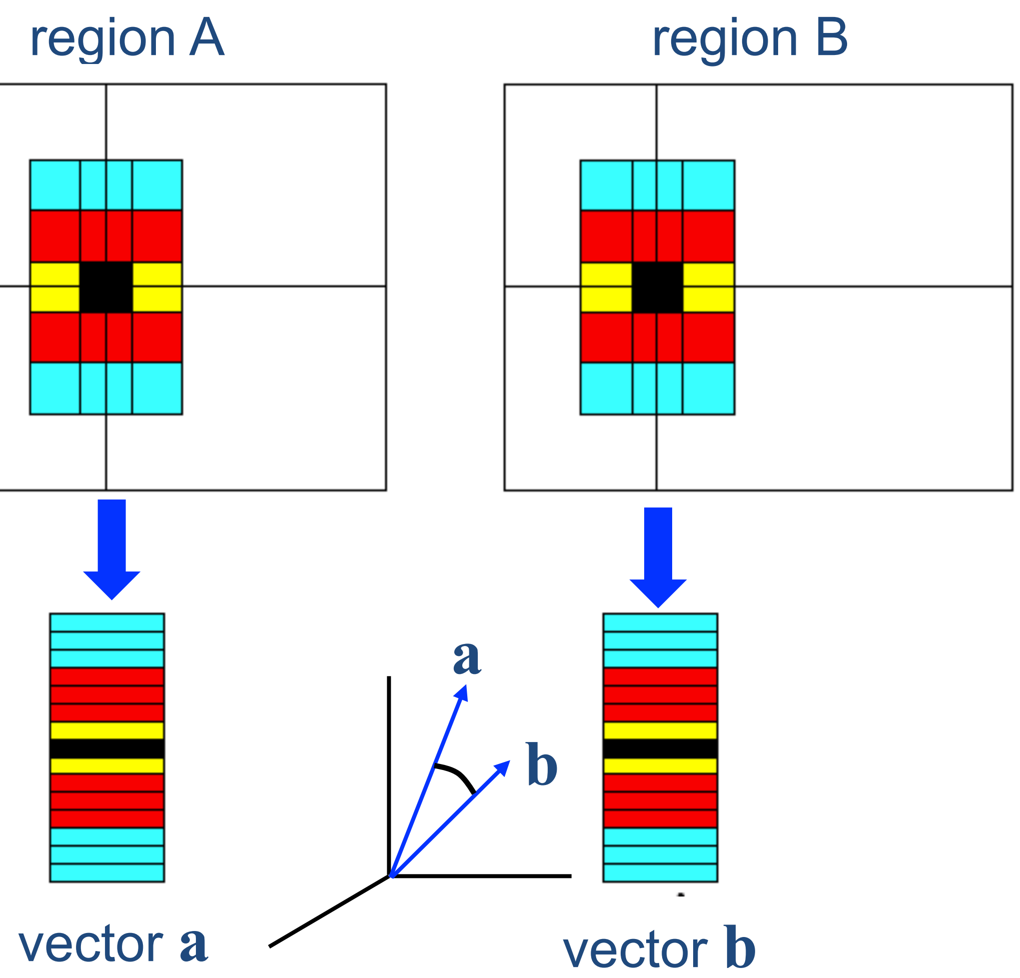
$$\text{NCC} = \frac{\sum_i \sum_j A(i,j)B(i,j)}{\sqrt{\sum_i \sum_j A(i,j)^2}\sqrt{\sum_i \sum_j B(i,j)^2}}$$

regions as vectors

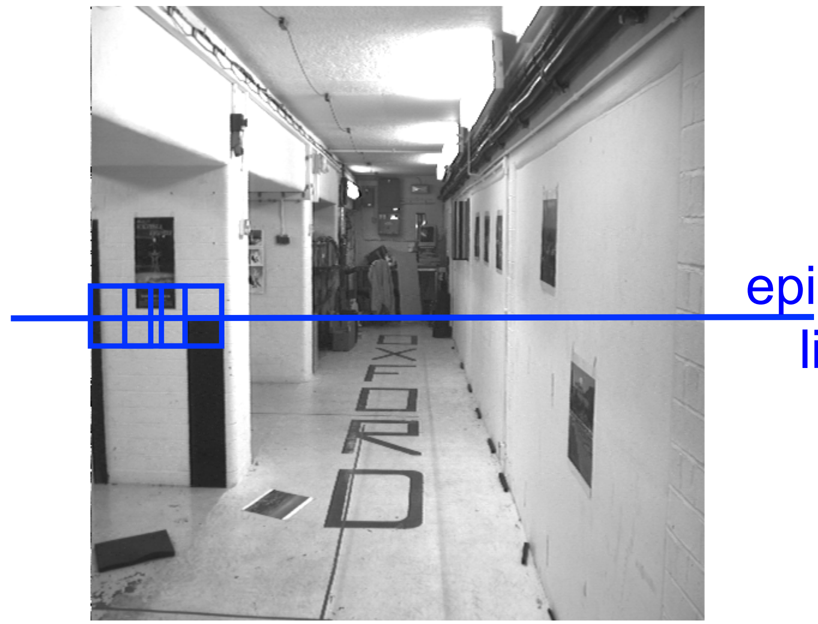
$$\mathtt{A} \rightarrow \mathbf{a}, \;\; \mathtt{B} \rightarrow \mathbf{b}$$

$$\text{NCC} = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

$$-1 \leq \text{NCC} \leq 1$$



Cross-correlation of neighborhood



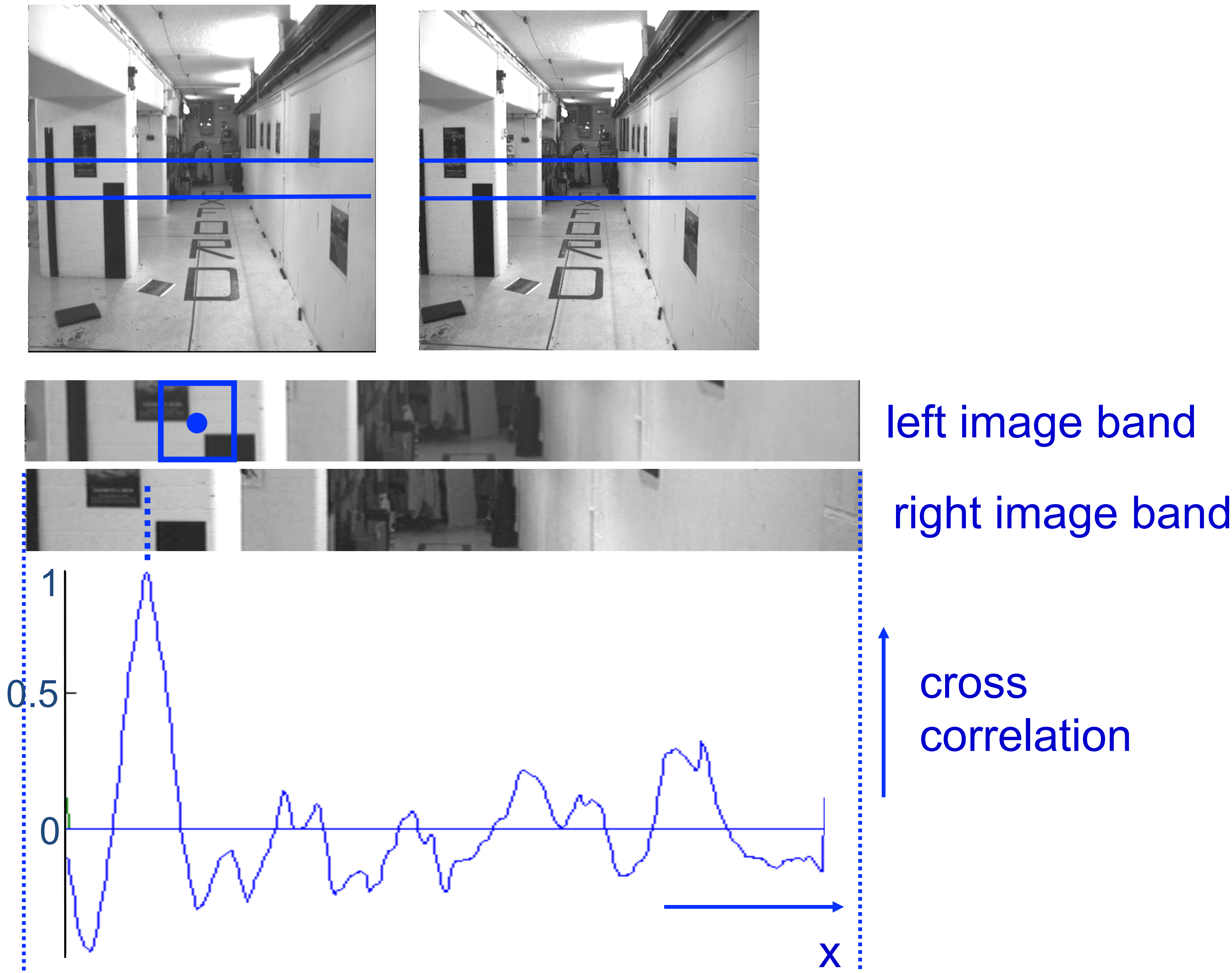
regions $\mathtt{A}, \mathtt{B}$, write as vectors $\mathbf{a}, \mathbf{b}$

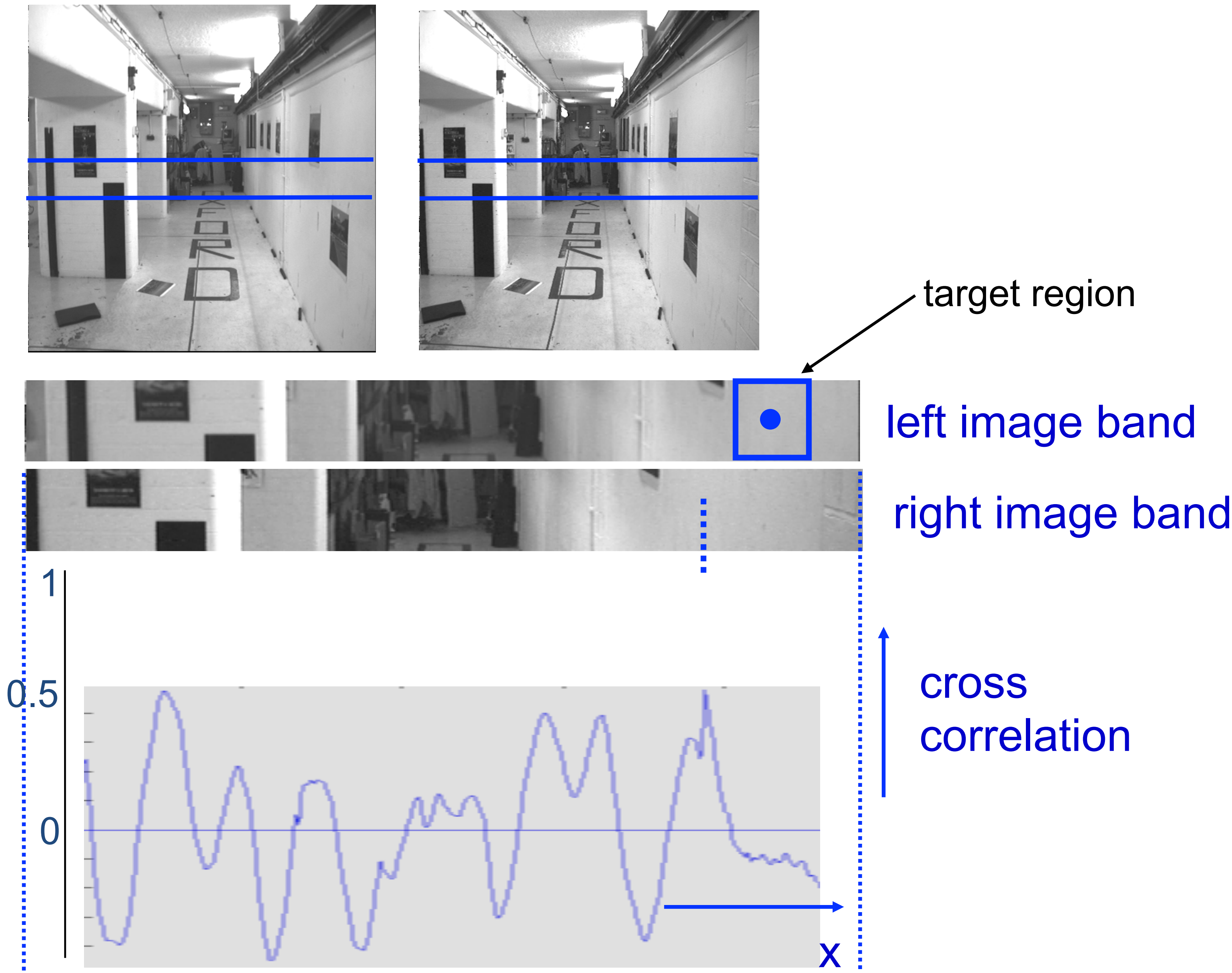
translate so that mean is zero

$$\mathtt{a} \rightarrow \mathtt{a} - \langle \mathbf{a} \rangle, \quad \mathtt{b} \rightarrow \mathtt{b} - \langle \mathbf{b} \rangle$$

$$\text{cross correlation} = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Invariant to $I \rightarrow \alpha I + \beta$

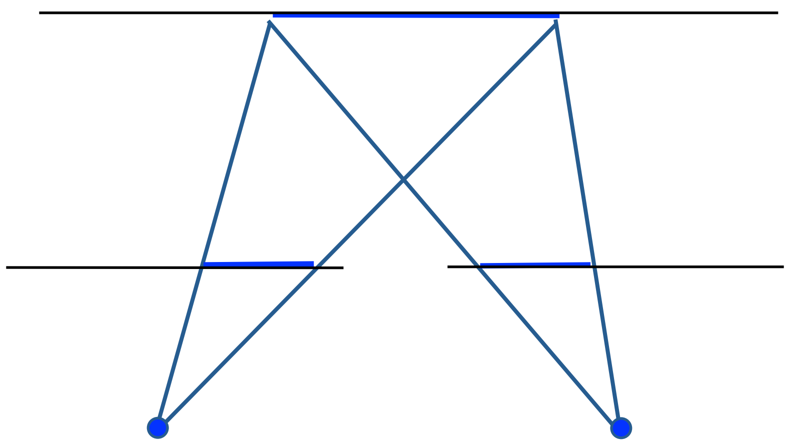

left image band
right image band
1
0.5
0
cross
correlation
x


target region
left image band
right image band
1
0.5
0
cross
correlation
x

Why is cross-correlation such a poor measure?

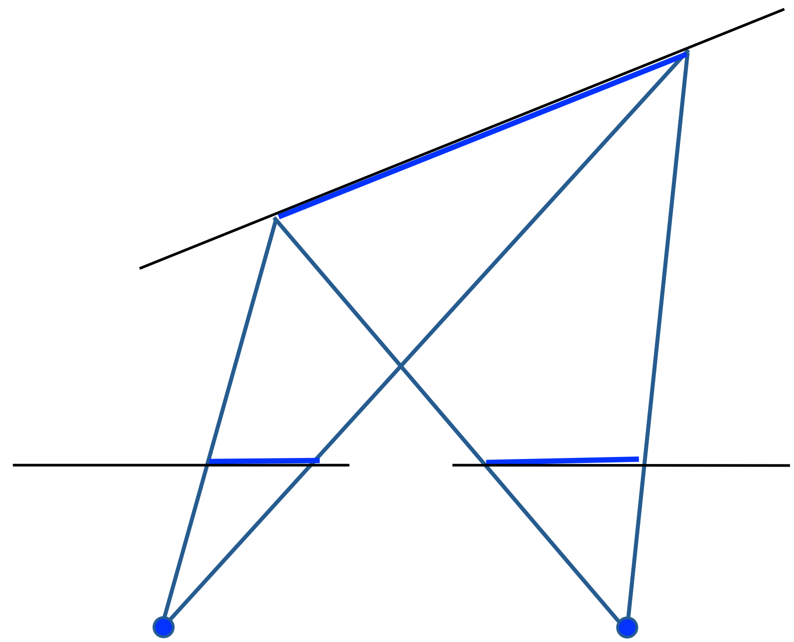
1. The neighborhood region does not have a “distinctive” spatial intensity distribution

2. Foreshortening effects



fronto-parallel surface

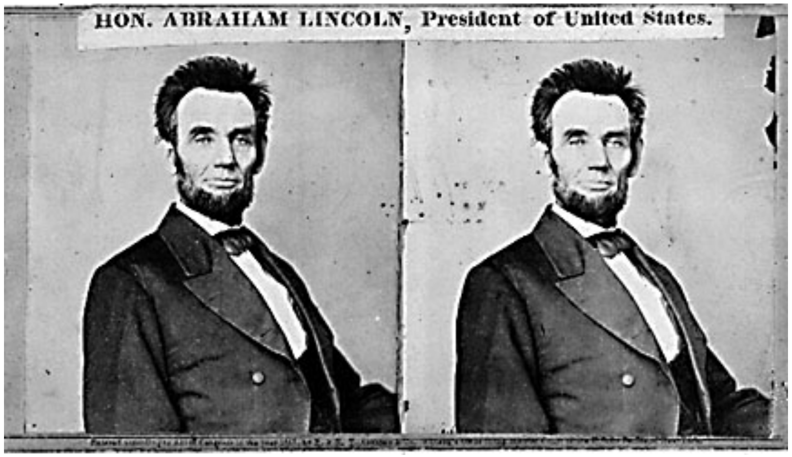
imaged length the same



slanting surface

imaged lengths differ

Limitations of similarity constraint



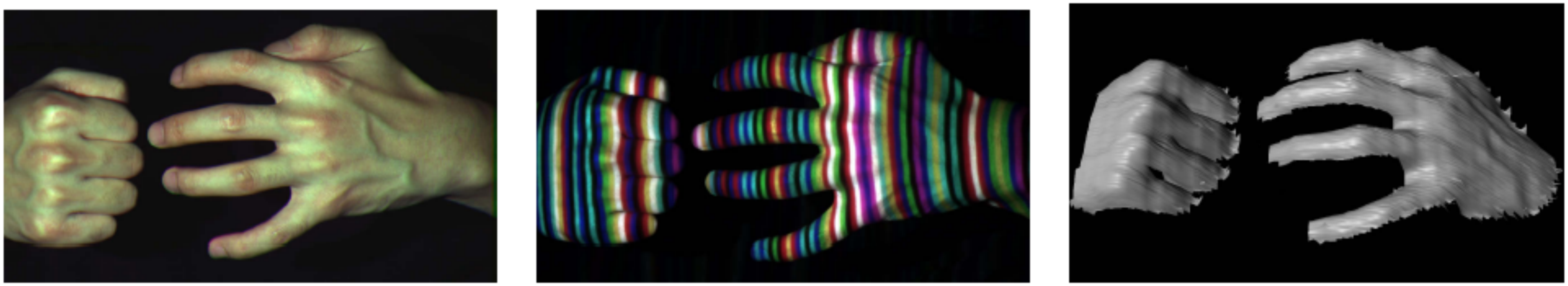
Textureless surfaces

Occlusions, repetition

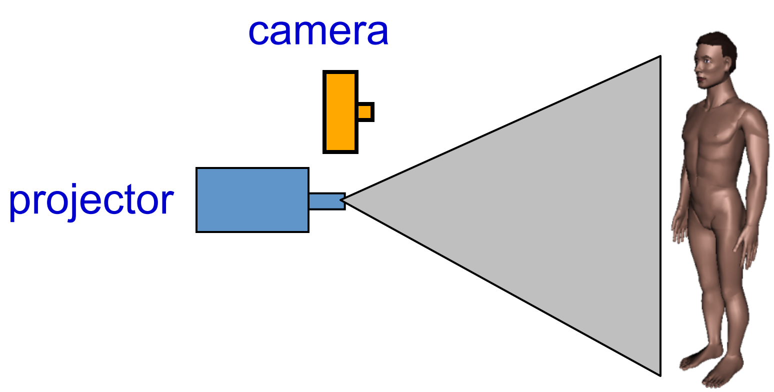
Non-Lambertian surfaces, specularities

Other approaches to obtaining 3D structure

Active stereo with structured light

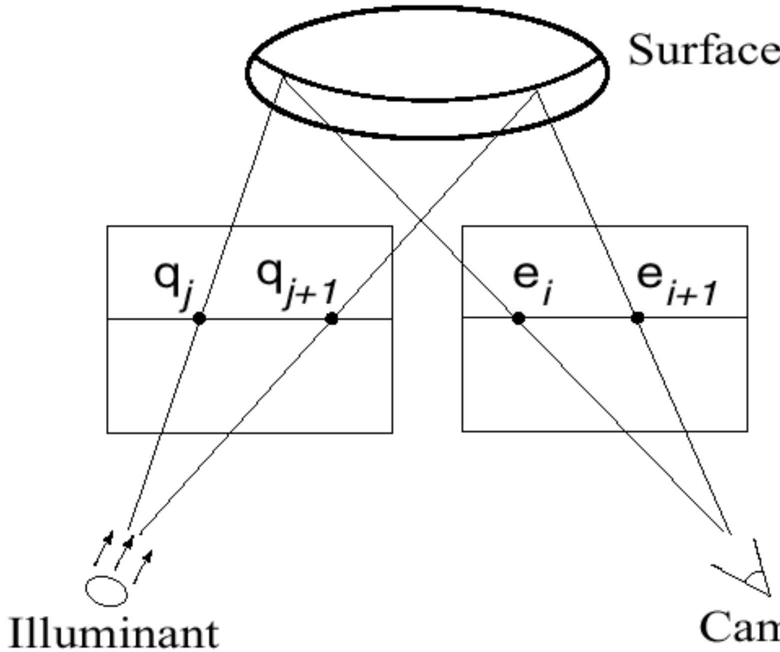


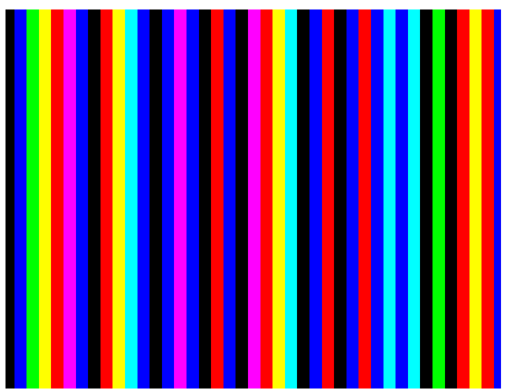
- Project "structured" light patterns onto the object
  - simplifies the correspondence problem
  - Allows us to use only one camera

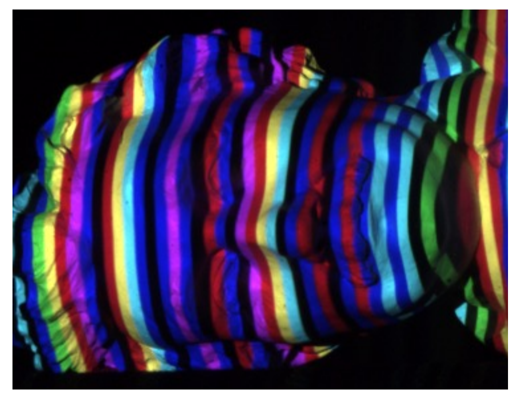


L. Zhang, B. Curless, and S. M. Seitz.
[Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming.]
3DPVT 2002

Active stereo with structured light







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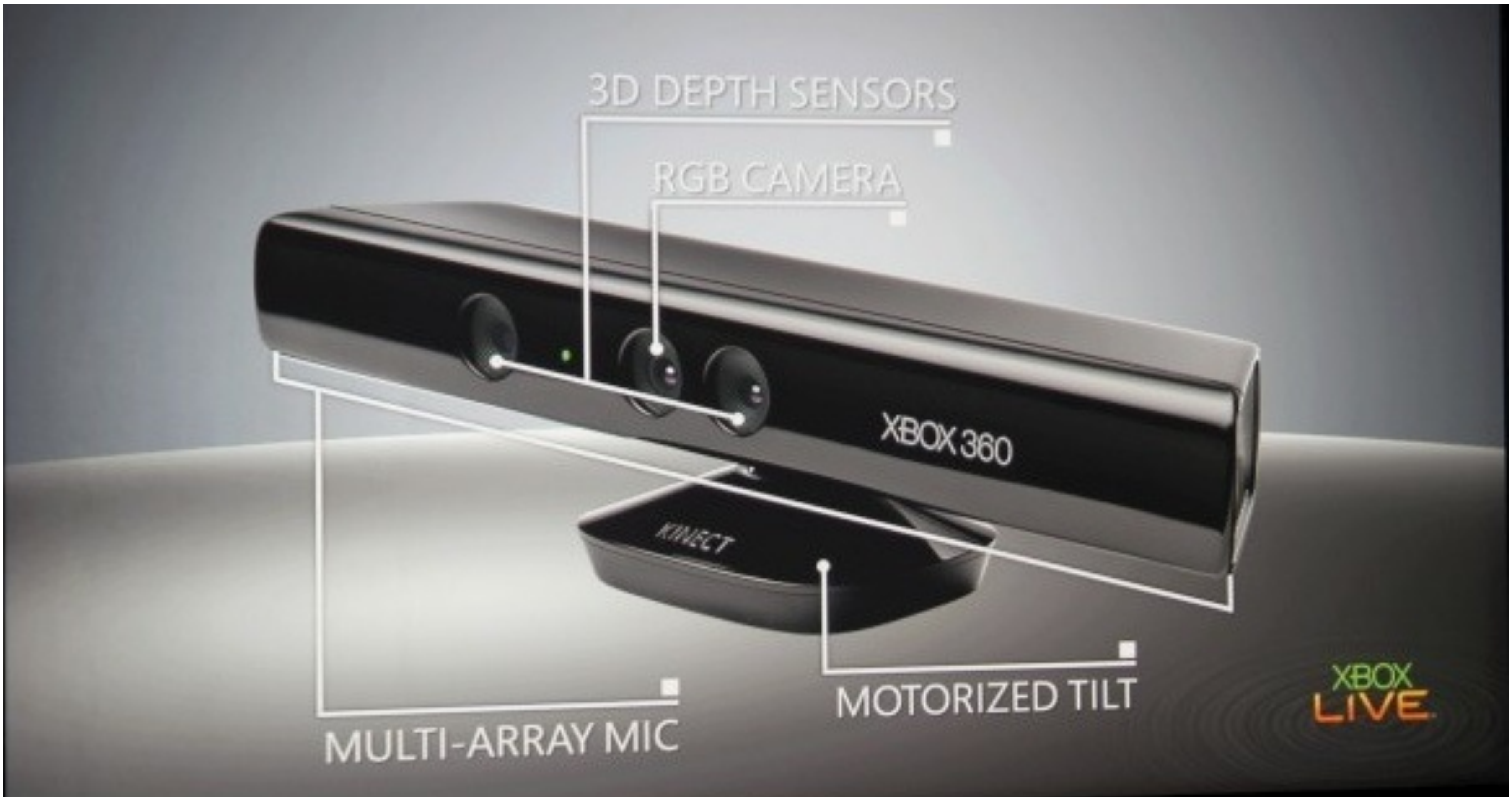
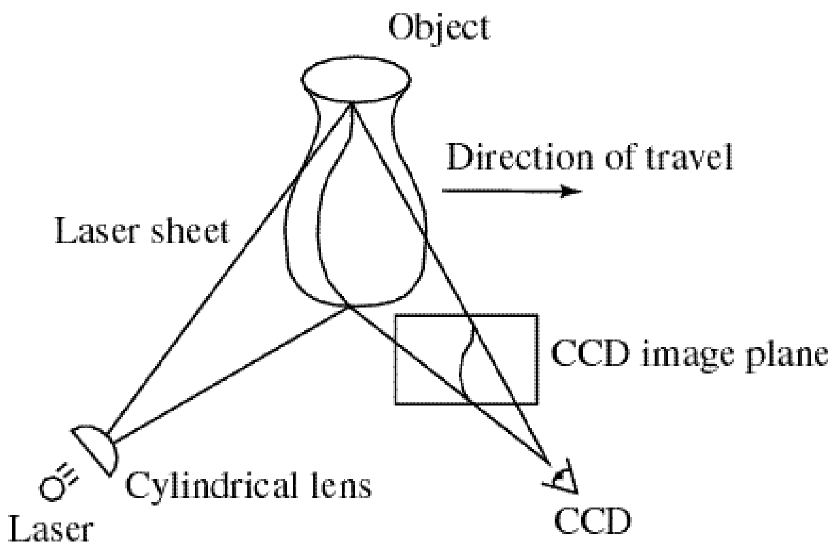
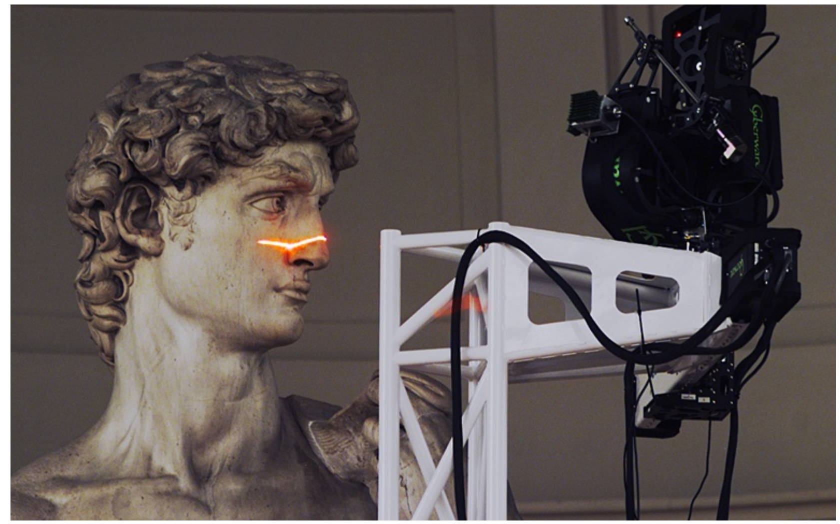
Microsoft Kinect



Laser scanning





Digital Michelangelo Project
http://graphics.stanford.edu/projects/mich/

- Optical triangulation
  - Project a single stripe of laser light
  - Scan it across the surface of the object
  - This is a very precise version of structured light scanning

Source: S. Seitz

Laser scanned models

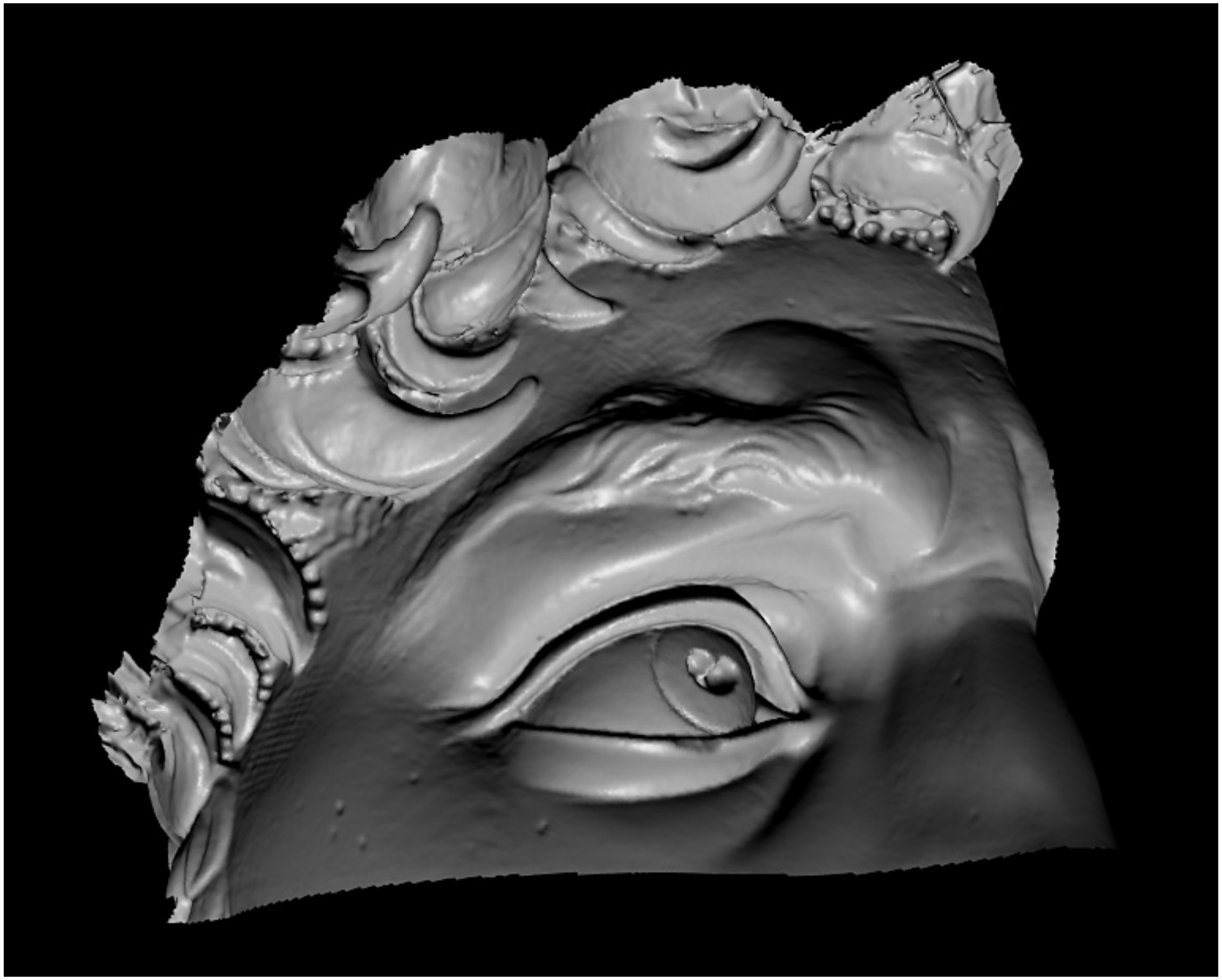
The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

Laser scanned models

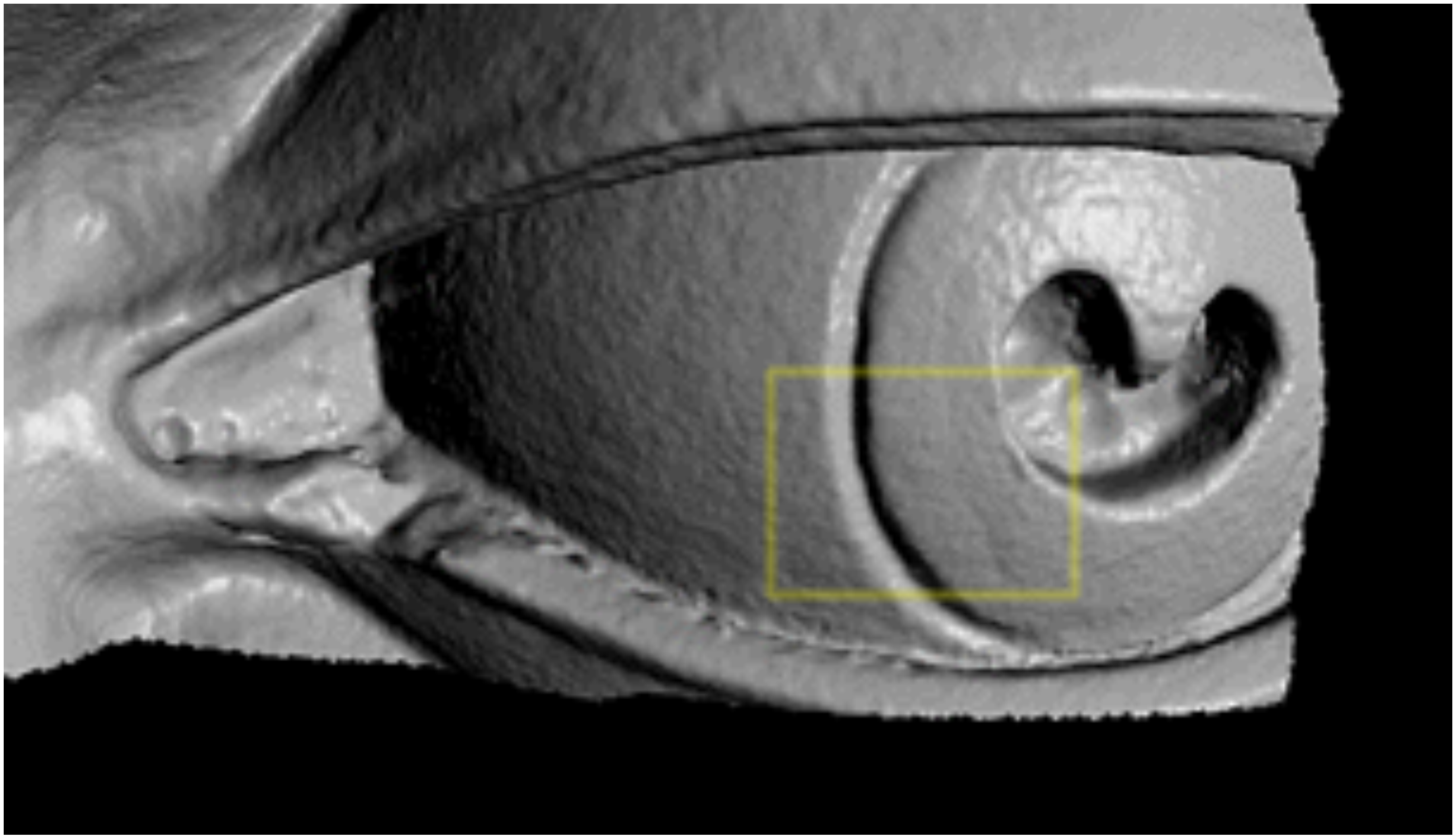
The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz



The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz



The Digital Michelangelo Project, Levoy et al.

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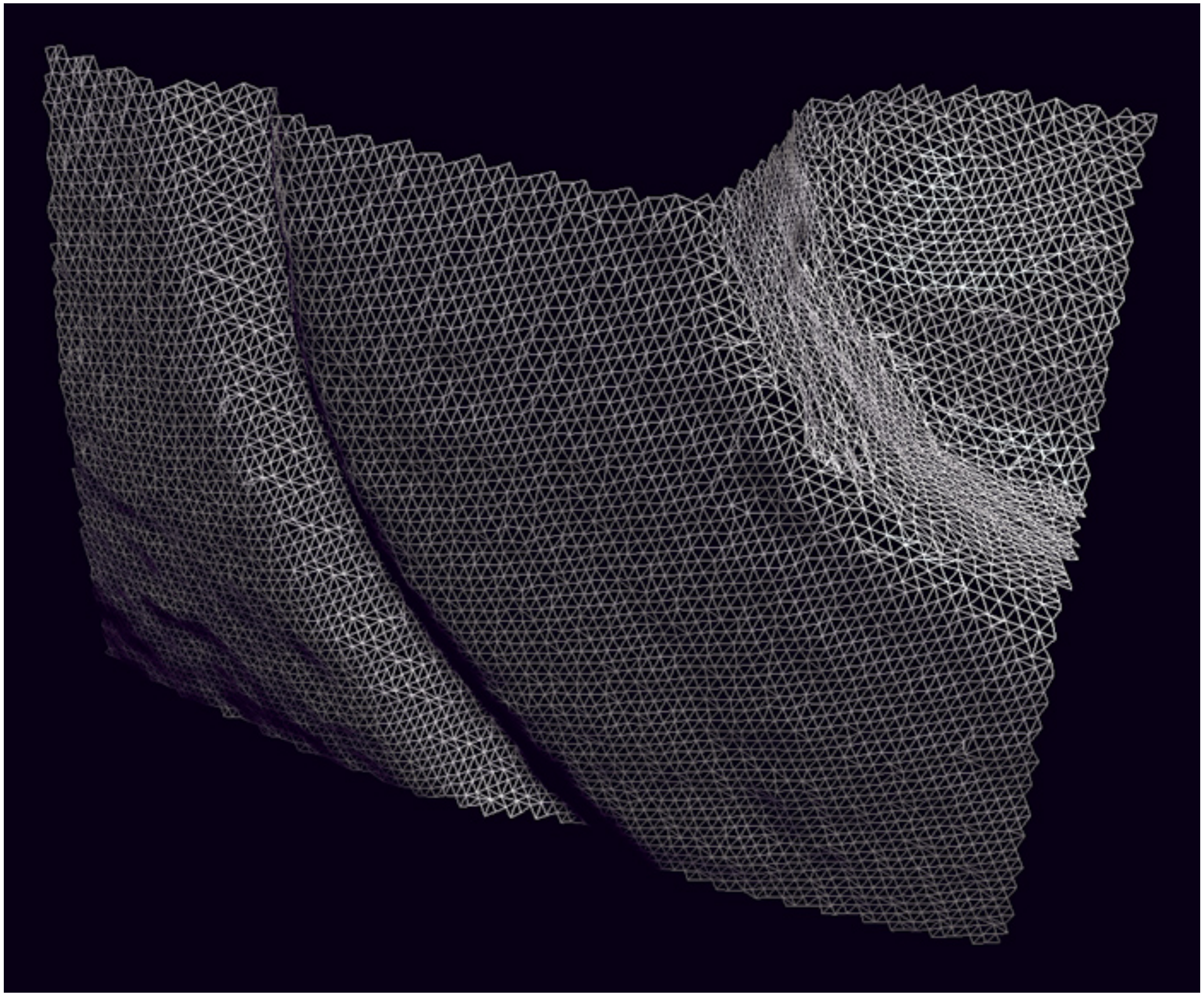

The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

Aligning range images

- A single range scan is not sufficient to describe a complex surface
- Need techniques to register multiple range images

  - … which brings us to *multi-view stereo*

Quiz