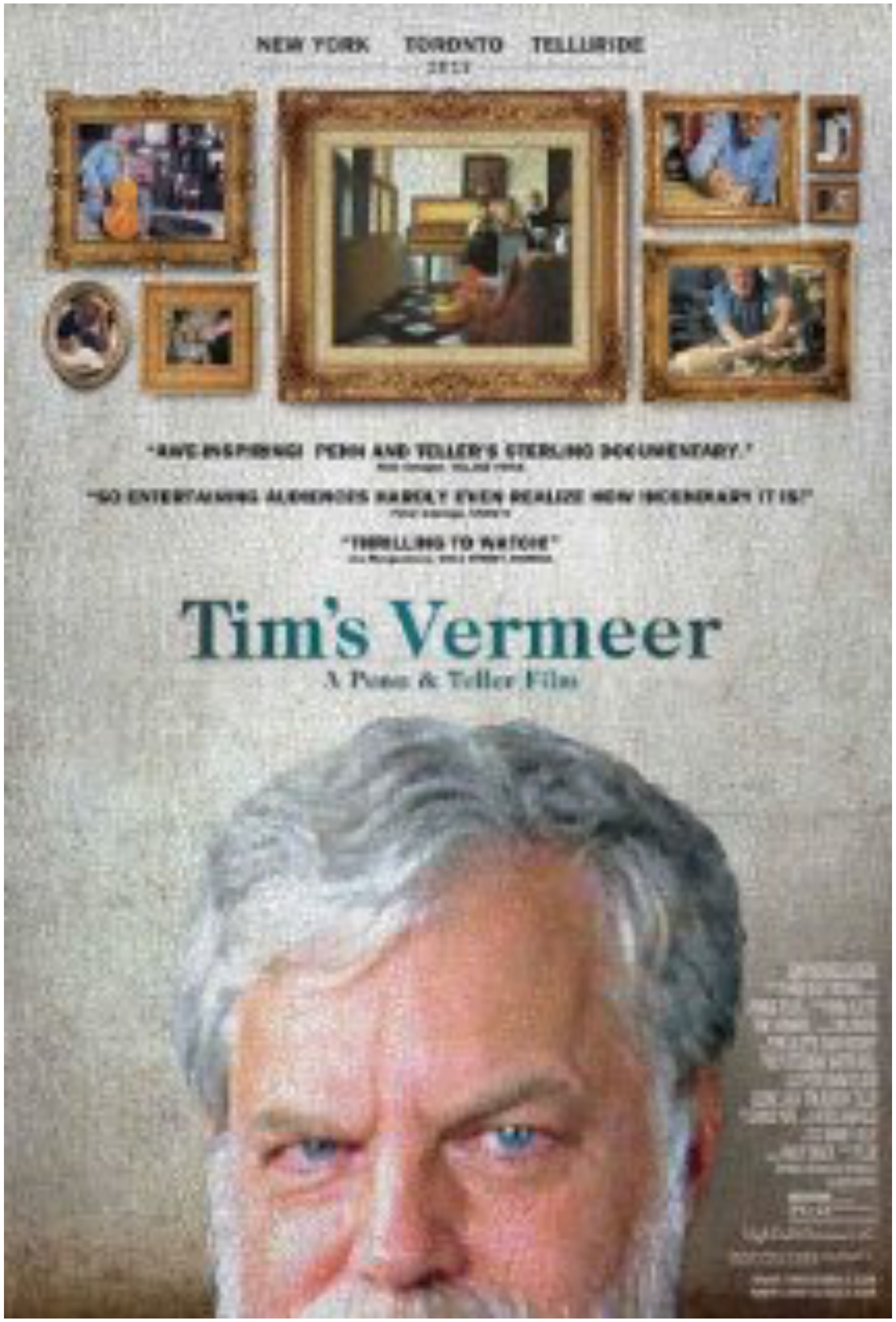
CS4670/5670: Computer Vision

Kavita Bala

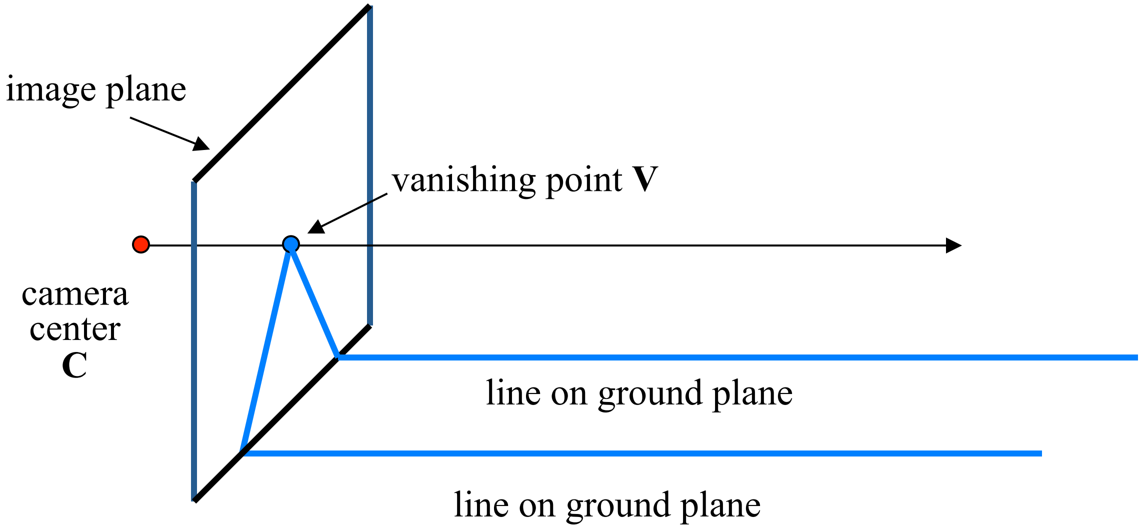
Lec 19: Single-view modeling 2


NEW YORK TORONTO TELLURIDE
2013
"AWE-INSPIRING! PENN AND TELLER'S STERLING DOCUMENTARY."
"SO ENTERTAINING AUDIENCES HARDLY EVEN REALIZE HOW INCENDIARY IT IS!"
"THRILLING TO WATCH"
Tim's Vermeer
A Penn & Teller Film

Today

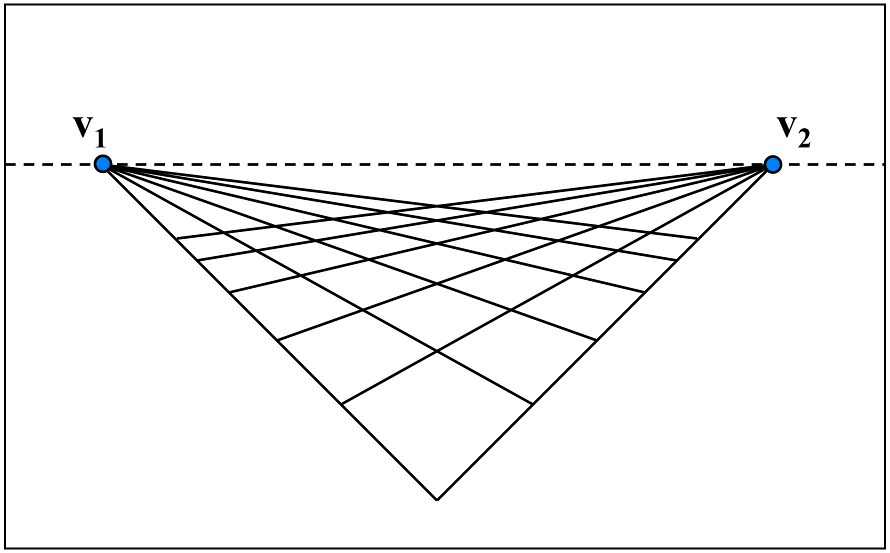
- Vanishing points in images are useful
  - Recover size
  - Camera calibration

Vanishing points



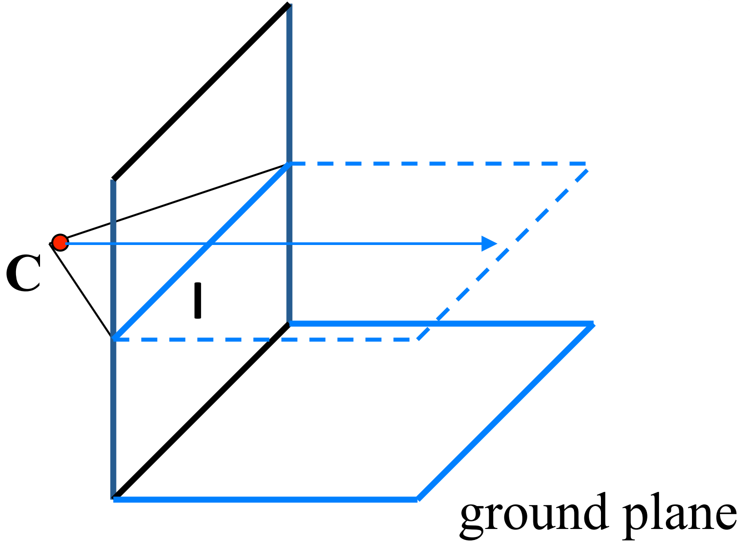
- Properties
  - Any two parallel lines (in 3D) have the same vanishing point **v**
  - The ray from **C** through **v** is parallel to the lines
  - An image may have more than one vanishing point
    - in fact, every image point is a potential vanishing point

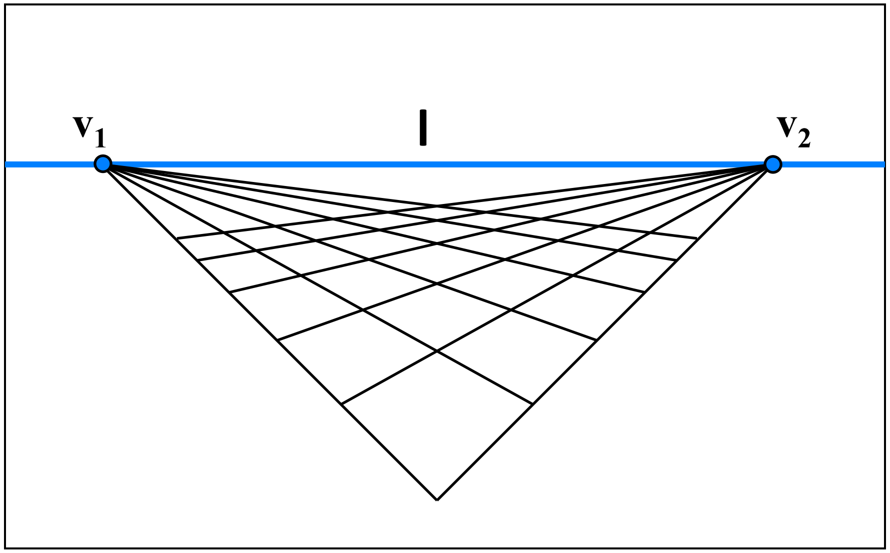
Vanishing lines



- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
  - Note that different planes (can) define different vanishing lines

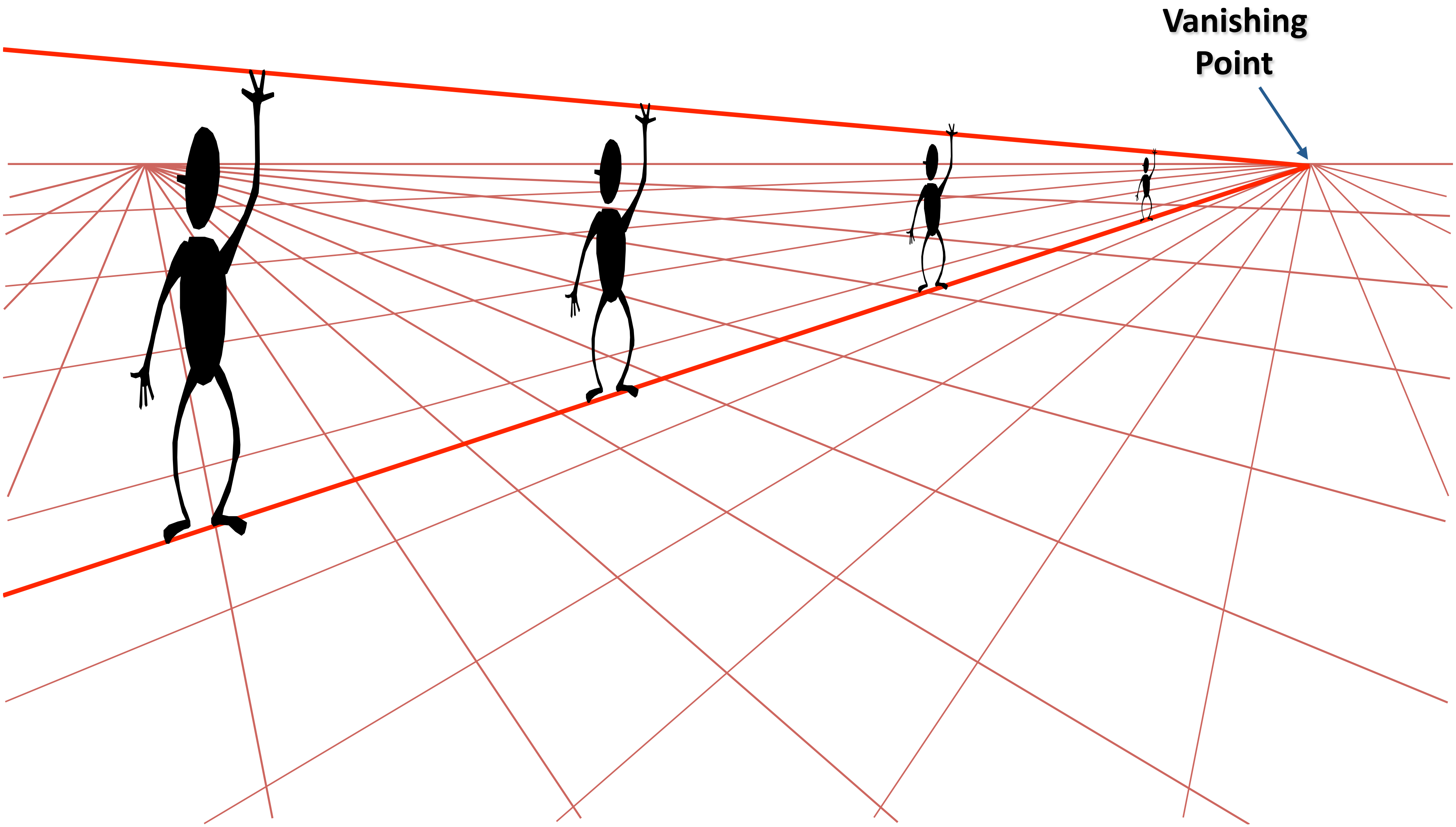
Computing vanishing lines





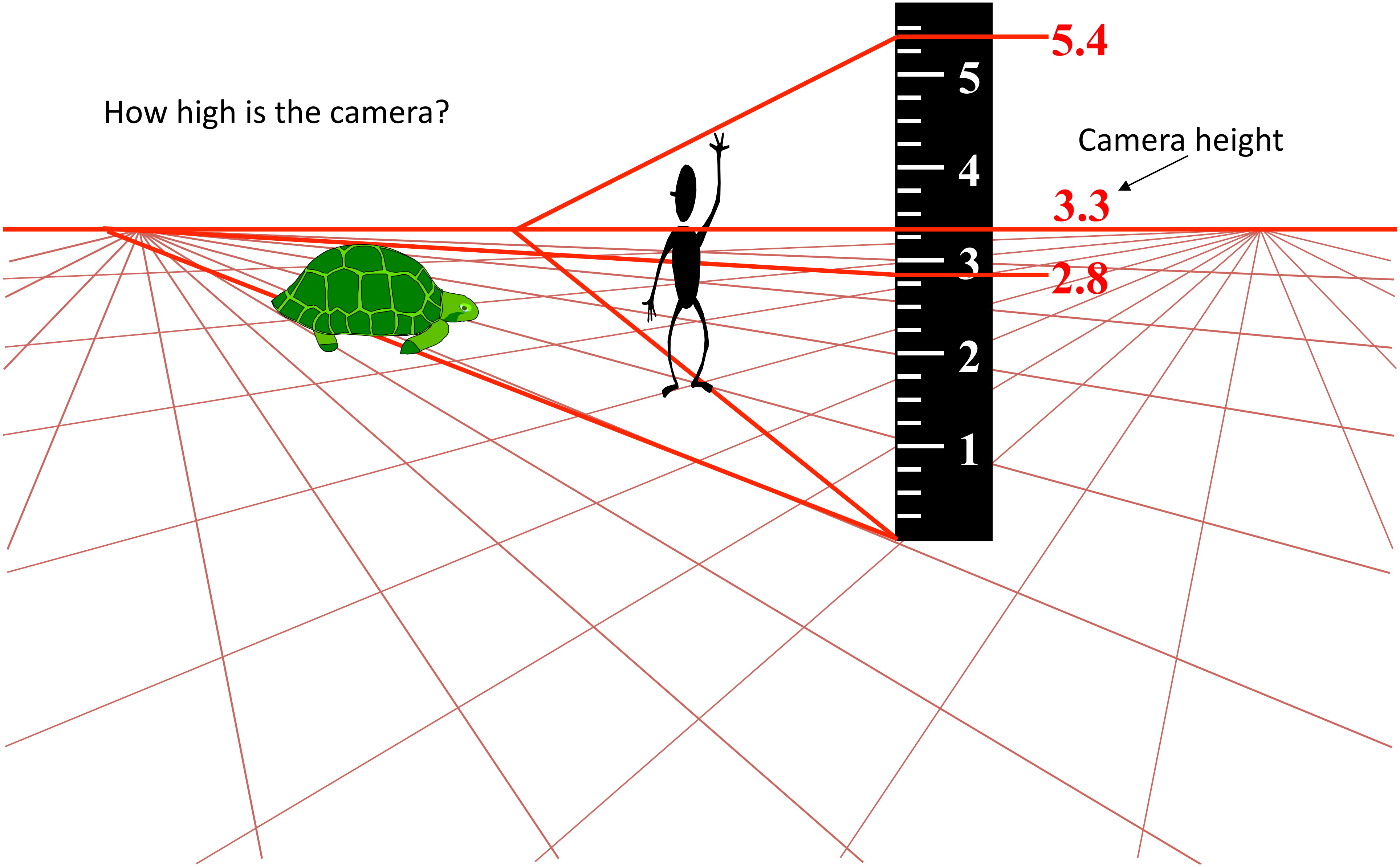
- Properties
  - **l** is intersection of horizontal plane through **C** with image plane
  - Compute **l** from two sets of parallel lines on ground plane
  - All points at same height as **C** project to **l**
    - points higher than C project above l
  - Provides way of comparing height of objects in the scene

Comparing heights

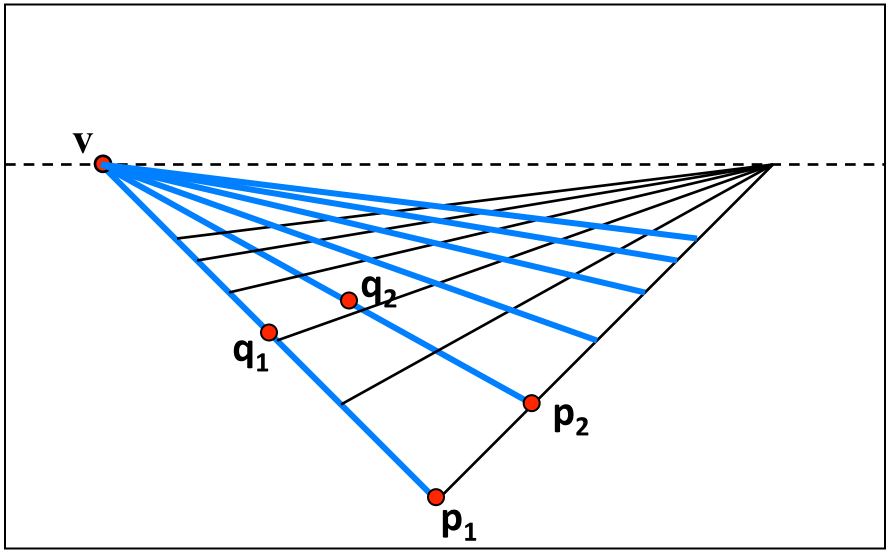


Measuring height

How high is the camera?



Computing vanishing points (from lines)



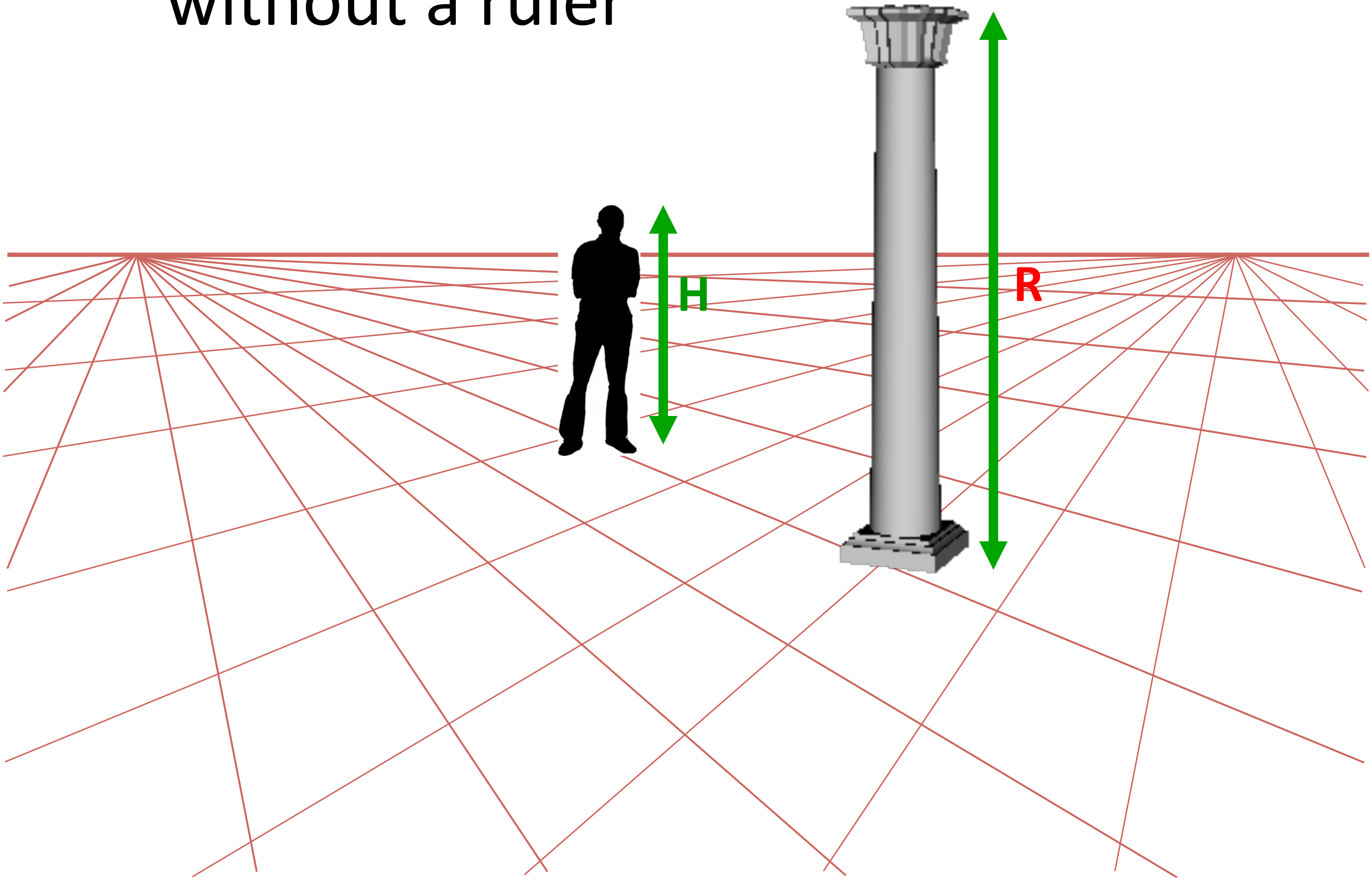
- Intersect p_1q_1 with p_2q_2

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

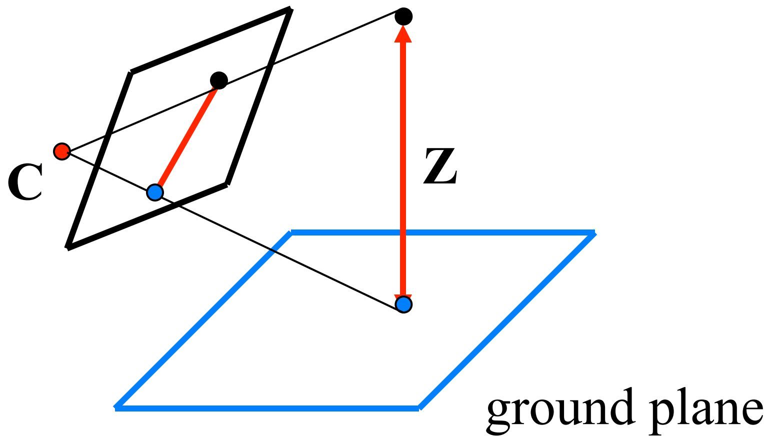
Least squares version

- Better to use more than two lines and compute the “closest” point of intersection
- See notes by Bob Collins for one good way of doing this:
  - http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt

Measuring height without a ruler



Measuring height without a ruler



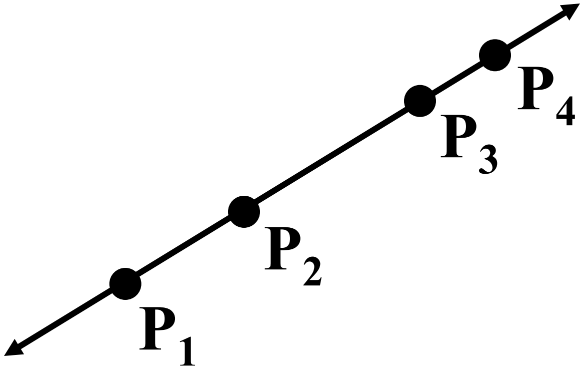
Compute Z from image measurements

- Actually get a scaled version of z

The cross ratio

- A Projective Invariant
  - Something that does not change under projective transformations (including perspective projection)

The *cross-ratio* of 4 collinear points



$$\frac{\|\mathbf{P}_3-\mathbf{P}_1\|\,\|\mathbf{P}_4-\mathbf{P}_2\|}{\|\mathbf{P}_3-\mathbf{P}_2\|\,\|\mathbf{P}_4-\mathbf{P}_1\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

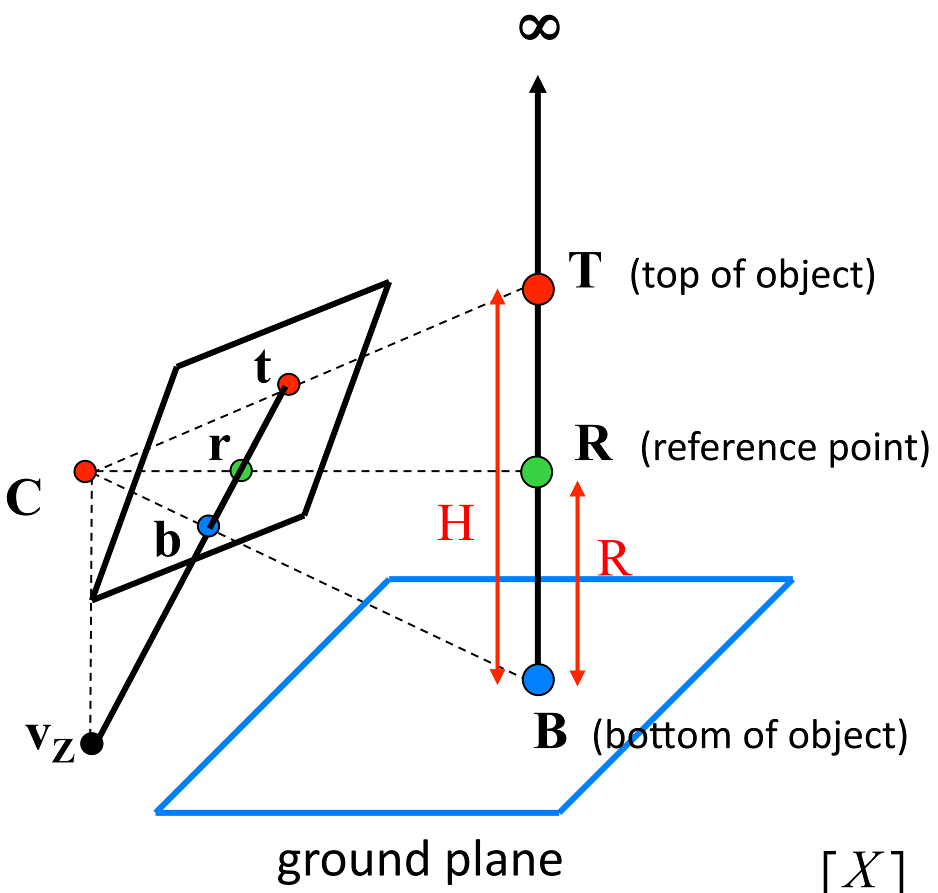
Can permute the point ordering

$$\frac{\|\mathbf{P}_1-\mathbf{P}_3\|\,\|\mathbf{P}_4-\mathbf{P}_2\|}{\|\mathbf{P}_1-\mathbf{P}_2\|\,\|\mathbf{P}_4-\mathbf{P}_3\|}$$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



$$\frac{\|\mathbf{T}-\mathbf{B}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{R}-\mathbf{B}\|\,\|\infty-\mathbf{T}\|}=\frac{H}{R}$$

scene cross ratio

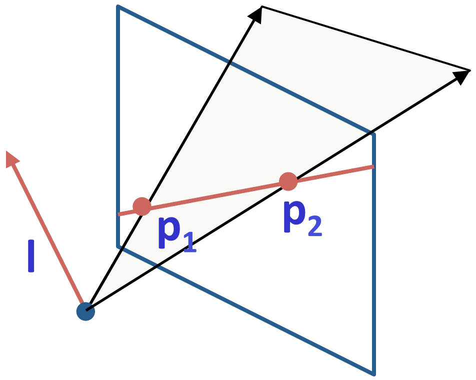
$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

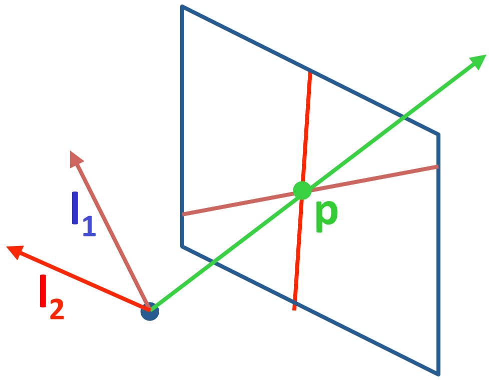
image cross ratio

scene points represented as $\mathbf{P}=\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$ image points as $\mathbf{p}=\begin{bmatrix}x\\y\\1\end{bmatrix}$

Point and line duality

- A line **l** is a homogeneous 3-vector





What is the line **l** spanned by rays $\mathbf{p}_1$ and $\mathbf{p}_2$?

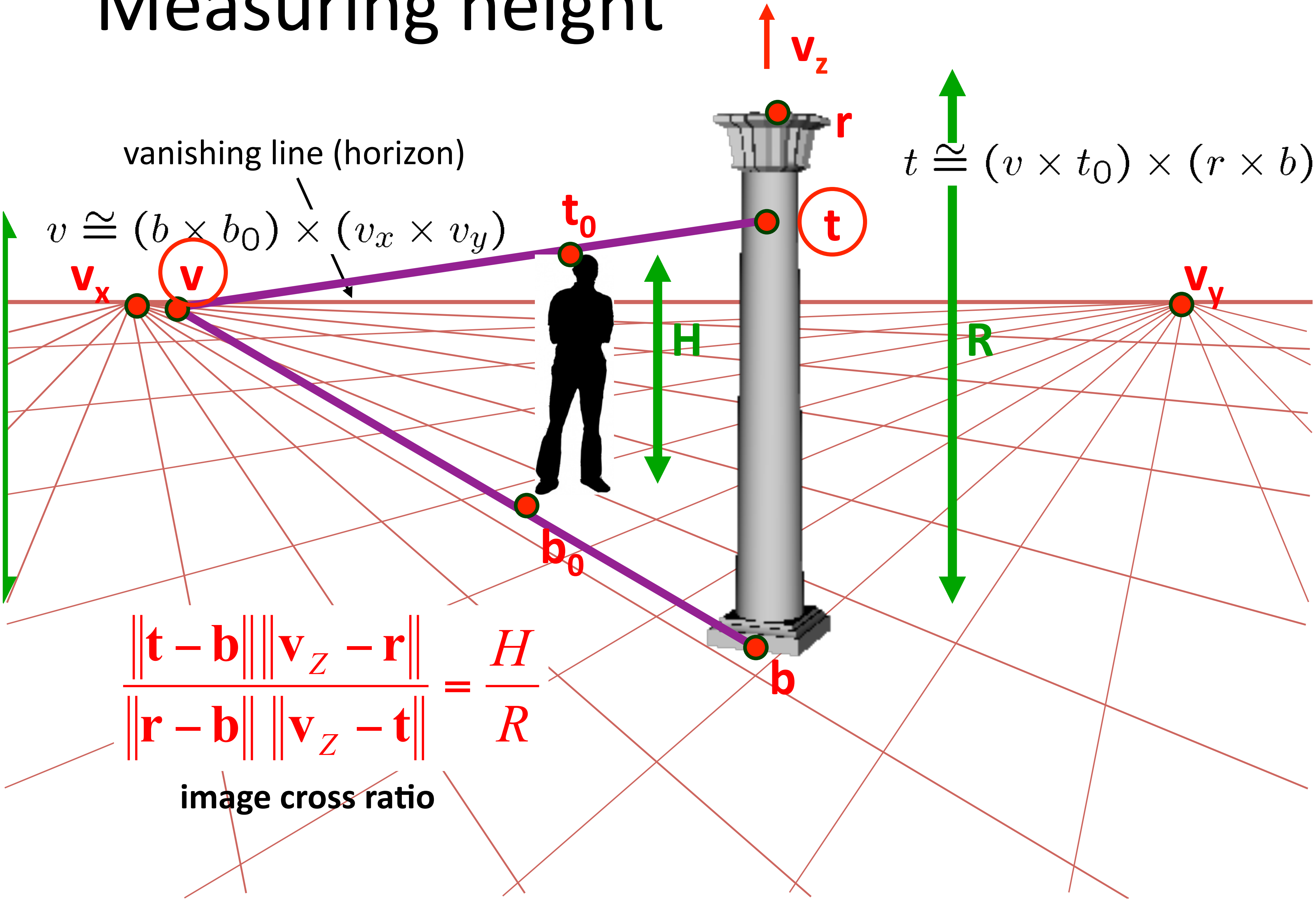
- **l** is $\perp$ to $\mathbf{p}_1$ and $\mathbf{p}_2$ $\Rightarrow$ $\mathbf{l} = \mathbf{p}_1 \times \mathbf{p}_2$
- **l** can be interpreted as a *plane normal*

What is the intersection of two lines $\mathbf{l}_1$ and $\mathbf{l}_2$?

- **p** is $\perp$ to $\mathbf{l}_1$ and $\mathbf{l}_2$ $\Rightarrow$ $\mathbf{p} = \mathbf{l}_1 \times \mathbf{l}_2$

Points and lines are *dual* in projective space

Measuring height



vanishing line (horizon)

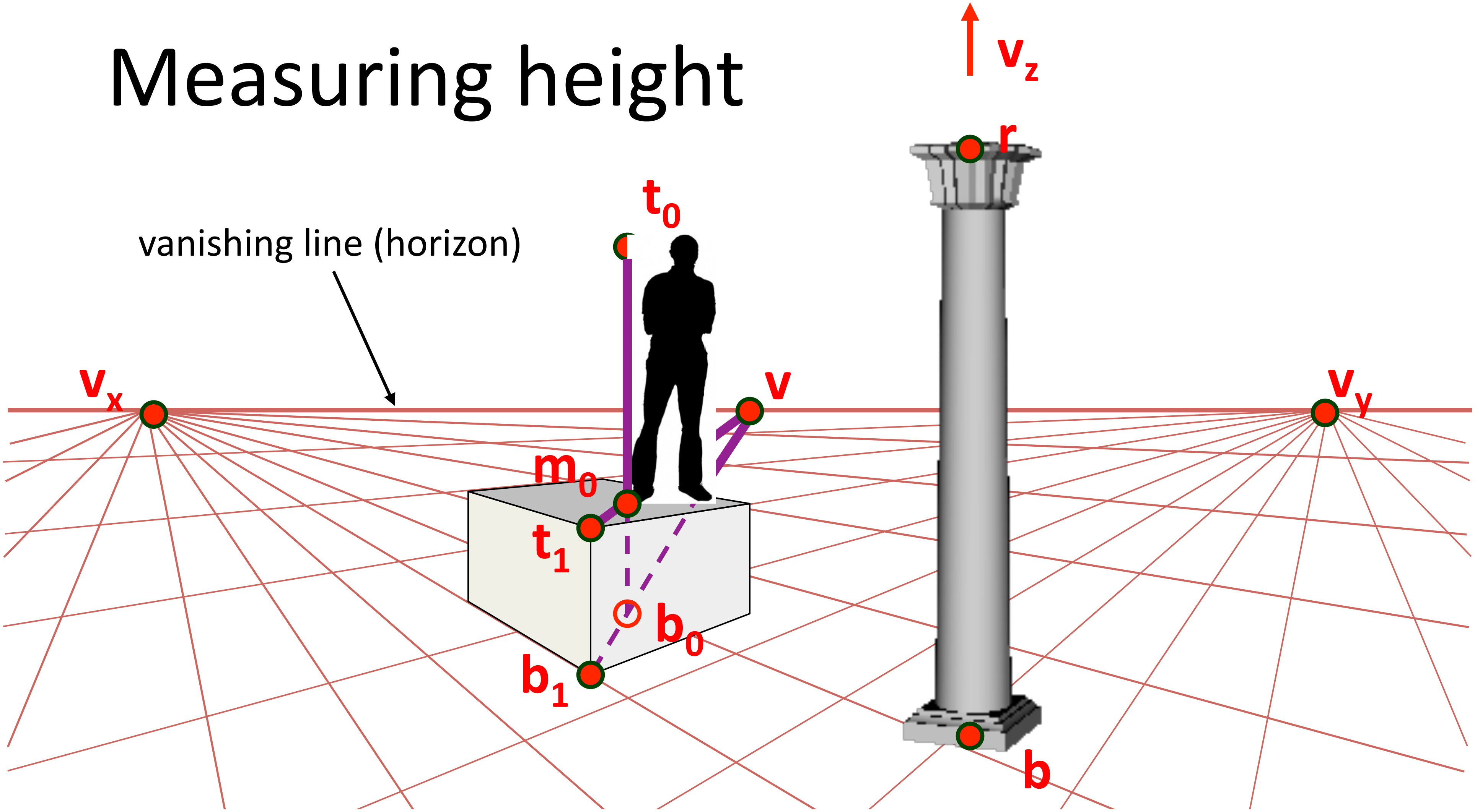
$$v \cong (b \times b_0) \times (v_x \times v_y)$$

$$t \cong (v \times t_0) \times (r \times b)$$

$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

Measuring height



What if the point on the ground plane $\mathbf{b_0}$ is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find $\mathbf{b_0}$ as shown above

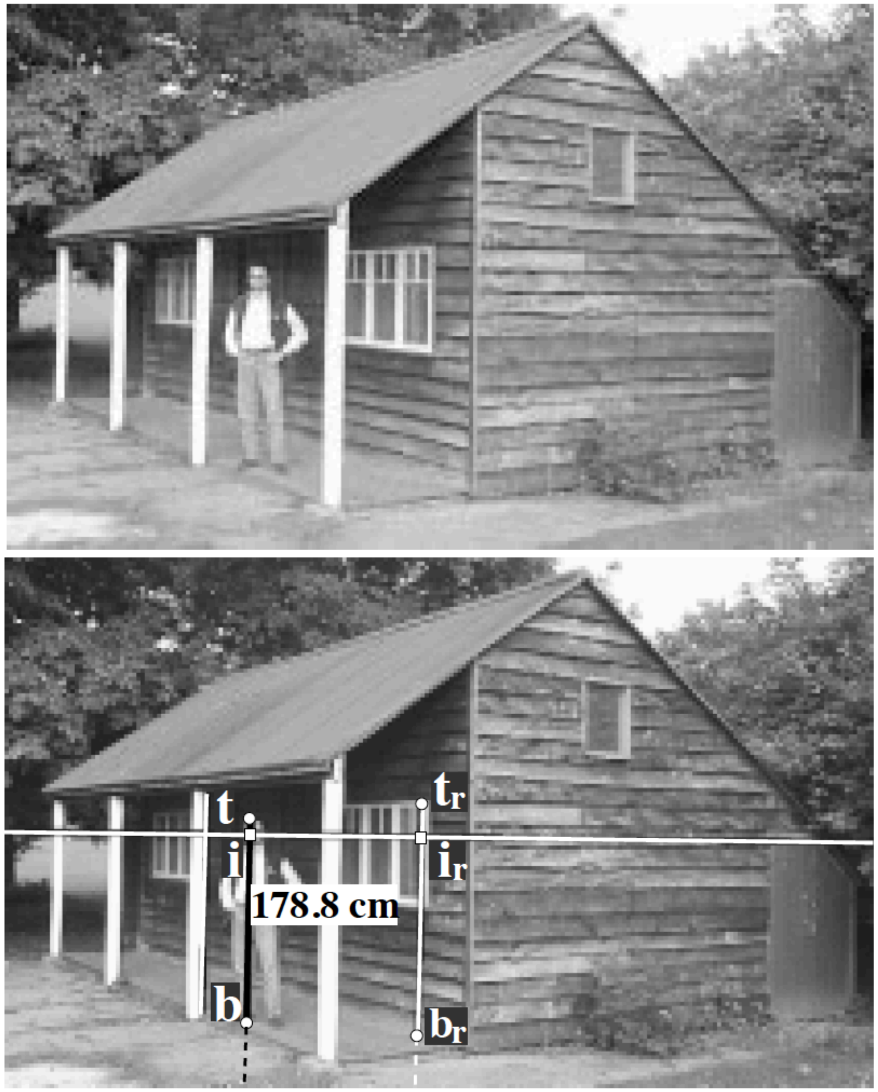


Figure 3: Measuring the height of a person: (top) original image; (bottom) the height of the person is computed from the image as 178.8cm (the true height is 180cm, but note that the person is leaning down a bit on his right foot). The vanishing line is shown in white and the reference height is the segment $(\mathbf{t}_r, \mathbf{b}_r)$. The vertical vanishing point is not shown since it lies well below the image. $\mathbf{t}$ is the top of the head and $\mathbf{b}$ is the base of the feet of the person while $\mathbf{i}$ is the intersection with the vanishing line.

3D Modeling from a photograph

St. Jerome in his Study, H. Steenwick

3D Modeling from a photograph

3D Modeling from a photograph

Flagellation, Piero della Francesca

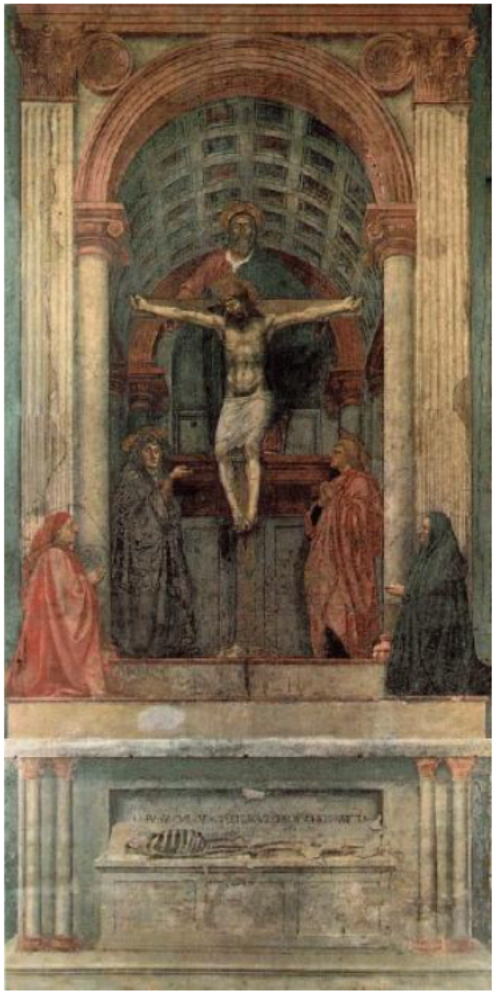
3D Modeling from a photograph

video by Antonio Criminisi

3D Modeling from a photograph

The following example shows the reconstruction of a chapel depicted in one of the earliest and most famous Renaissance frescoes: **La Trinita' (The Trinity)** (1427) by Masaccio (1401-1428).

original fresco



***La Trinita'* (1427)**
by Masaccio

images of the reconstructed 3D model

Some Related Techniques

- Image-Based Modeling and Photo Editing
  - Mok et al., SIGGRAPH 2001

- Single View Modeling of Free-Form Scenes
  - Zhang et al., CVPR 2001

- Tour Into The Picture
  - Anjyo et al., SIGGRAPH 1997

Camera calibration

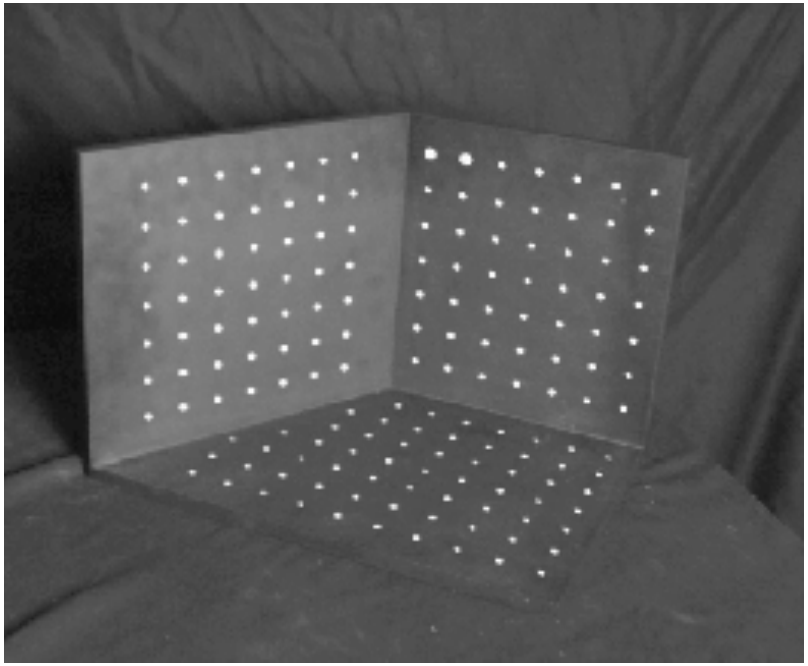
- Goal: estimate the camera parameters
  - Version 1: solve for projection matrix

$$\mathbf{x} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$

- Version 2: solve for camera parameters separately
  - intrinsics (focal length, principle point, pixel size)
  - extrinsics (rotation angles, translation)
  - radial distortion

Calibration using a reference object

- Place a known object in the scene
  - identify correspondence between image 2D and scene 3D
  - compute mapping from scene to image

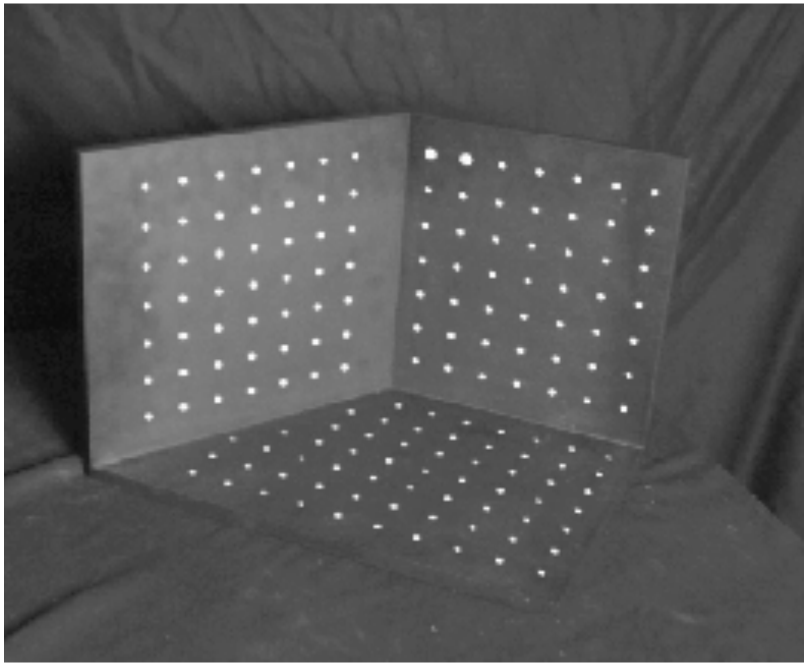


Issues

- must know geometry very accurately
- must know 3D->2D correspondence

Estimating the projection matrix

- Place a known object in the scene
  - identify correspondence between image and scene
  - compute mapping from scene to image



$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Direct linear calibration

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

$$\begin{aligned} u_i &= \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}} \\ v_i &= \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}} \end{aligned}$$

$$\begin{aligned} u_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}) &= m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03} \\ v_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}) &= m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13} \end{aligned}$$

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_iX_i & -u_iY_i & -u_iZ_i & -u_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_iX_i & -v_iY_i & -v_iZ_i & -v_i \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \\ m_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Direct linear calibration

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Can solve for m_{ij} by linear least squares

- use eigenvector trick that we used for homographies. A x = 0

$$\underset{2n \times 9}{\mathbf{A}} \qquad \underset{9}{\mathbf{h}} \qquad \underset{2n}{\mathbf{0}}$$

Defines a least squares problem: minimize $\|\mathbf{Ah} - \mathbf{0}\|^2$

- Since **h** is only defined up to scale, solve for unit vector **ĥ**
- Solution: **ĥ** = eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points

Direct linear calibration

- Advantage:
  - Very simple to formulate and solve
- Disadvantages:
  - Doesn’t tell you the camera parameters
  - Doesn’t model radial distortion
  - Hard to impose constraints (e.g., known *f*)
  - Doesn’t minimize the right error function

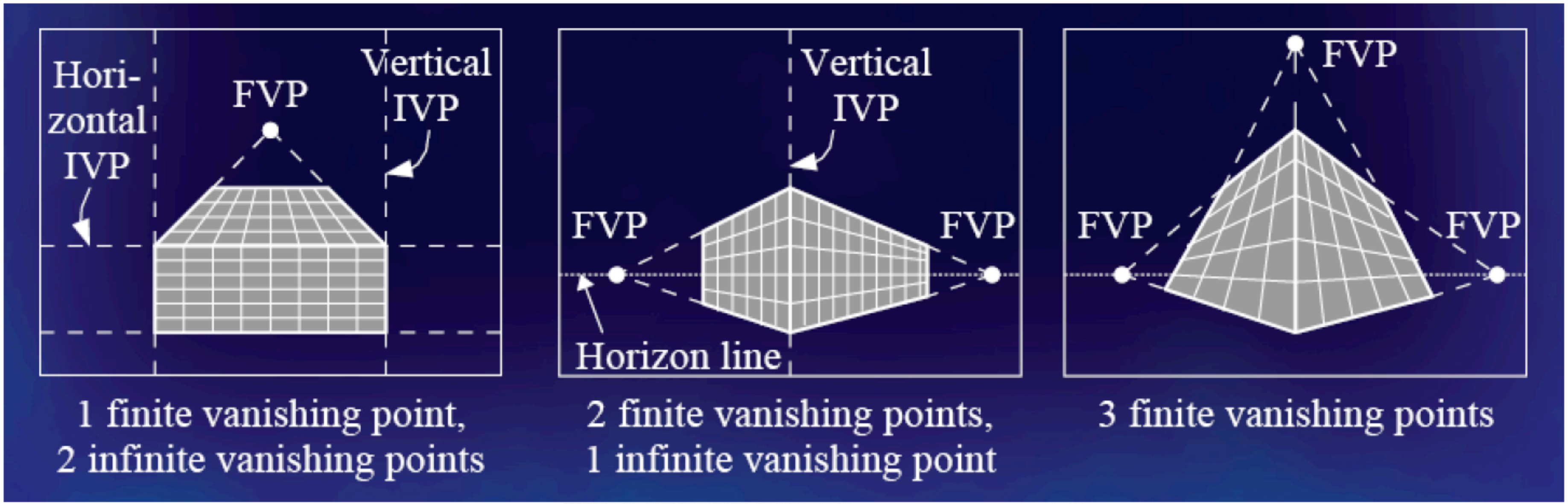
Nonlinear *methods* are preferred

- Define error function E between projected 3D points and image positions: nonlinear function of intrinsics, extrinsics, radial distortion
- Minimize E using nonlinear optimization techniques

Summary

- Known correspondences
  - (ui, vi) and (Xi, Yi, Zi)
- Compute mij solving system of linear equations
  - May use this to initialize non linear error minimization problem to recover more accurate mij

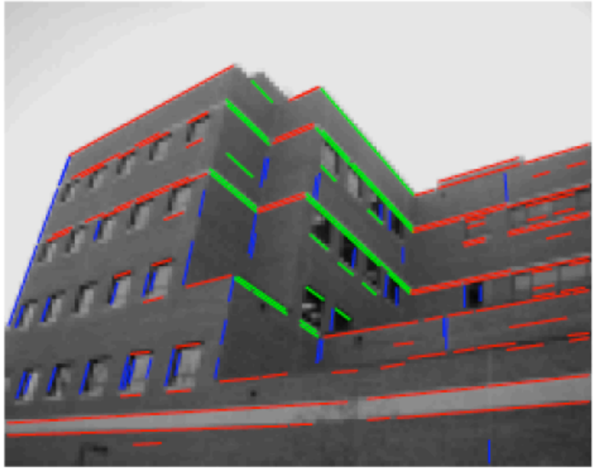
Calibration from vanishing points



1 finite vanishing point,
2 infinite vanishing points

2 finite vanishing points,
1 infinite vanishing point

3 finite vanishing points



- From vanishing points corresponding to 3 orthogonal directions of world

$$e_i = [1,0,0]^T,\ e_j = [0,1,0]^T,\ e_k = [0 \quad 0 \quad 1]^T$$

$$\mathbf{v}_i = KRe_i, \mathbf{v}_j = KRe_j, \mathbf{v}_k = KRe_k.$$

$$e_i^T e_j = 0$$

$$\mathbf{v}_i^T K^{-T} R R^T K^{-1} \mathbf{v}_j = \mathbf{v}_i^T K^{-T} K^{-1} \mathbf{v}_j = 0$$

$$K = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} K^{-1} = \begin{bmatrix} 1/f & 0 & -u_0/f \\ 0 & 1/f & -v_0/f \\ 0 & 0 & 1 \end{bmatrix}$$

$$v_i^T K^{-T} K^{-1} v_j = 0$$
$$v_j^T K^{-T} K^{-1} v_k = 0$$

$$v_i^T K^{-T} K^{-1} v_k = 0$$

- 3 finite vanishing points: get f, u0, v0

Rotation from vanishing points

- R_{1c} 1st column vector of Rotation matrix

$$R = \begin{bmatrix} R_{1c} & R_{2c} & R_{3c} \end{bmatrix}$$

$$\lambda v_i = KRe_i \qquad e_i = [1, 0, 0]^T$$

$$R_{1c} = \lambda K^{-1} v_i$$

- λ from $||R_{1c}||_2 = 1$

Vanishing points and projection matrix

$$\mathbf{\Pi} = \begin{array}{cccc} \left[\begin{array}{cccc} \boxed{\begin{array}{c} * \\ * \\ * \end{array}} & \boxed{\begin{array}{c} * \\ * \\ * \end{array}} & \boxed{\begin{array}{c} * \\ * \\ * \end{array}} & \boxed{\begin{array}{c} * \\ * \\ * \end{array}} \end{array}\right] \\ \boldsymbol{\pi}_1 \quad \boldsymbol{\pi}_2 \quad \boldsymbol{\pi}_3 \quad \boldsymbol{\pi}_4 \end{array} = \begin{bmatrix} \boldsymbol{\pi}_1 & \boldsymbol{\pi}_2 & \boldsymbol{\pi}_3 & \boldsymbol{\pi}_4 \end{bmatrix}$$

- Projection of x axis $= \mathbf{v}_X$ (X vanishing point)
- similarly, $\boldsymbol{\pi}_2 = \mathbf{v}_Y$, $\boldsymbol{\pi}_3 = \mathbf{v}_Z$
- $\boldsymbol{\pi}_4 = \mathbf{\Pi}\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$ = projection of world origin

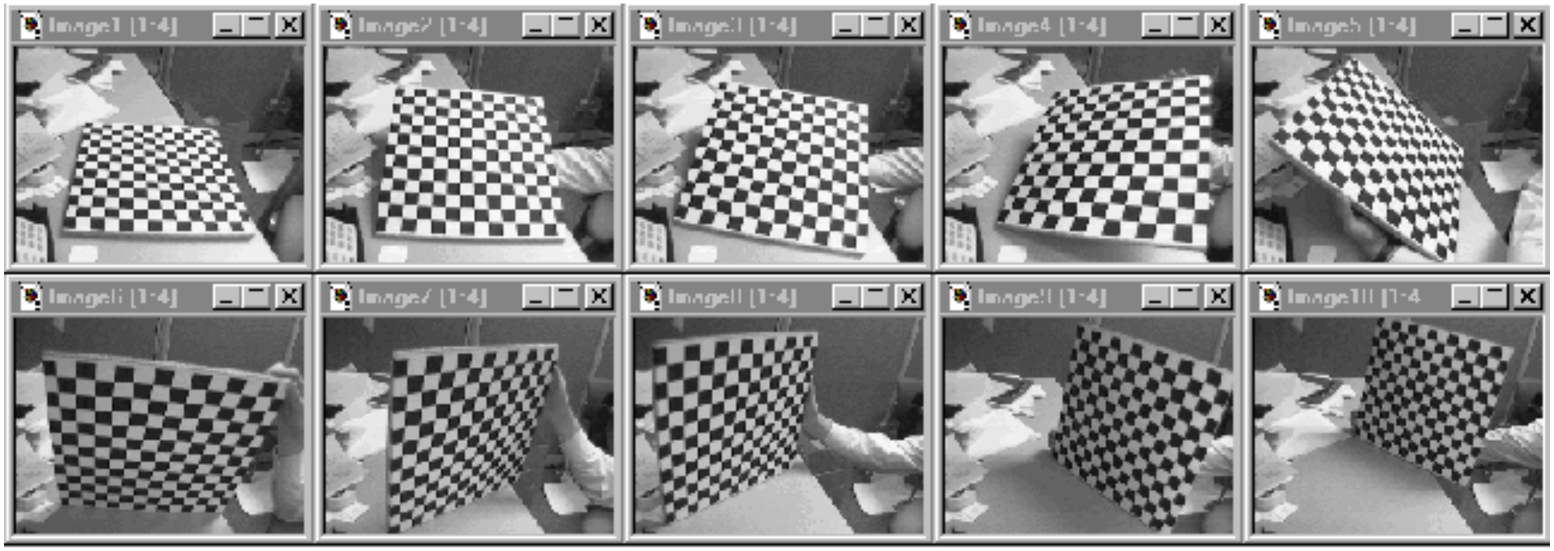
$$\mathbf{\Pi} = \begin{bmatrix} \mathbf{v}_X & \mathbf{v}_Y & \mathbf{v}_Z & \mathbf{o} \end{bmatrix}$$

Not So Fast! We only know **v**'s up to a scale factor

$$\mathbf{\Pi} = \begin{bmatrix} a\,\mathbf{v}_X & b\mathbf{v}_Y & c\mathbf{v}_Z & \mathbf{o} \end{bmatrix}$$

- Can fully specify by providing 3 reference points

Alternative: multi-plane calibration



Images courtesy Jean-Yves Bouguet, Intel Corp.

Advantage

- Only requires a plane
- Don’t have to know positions/orientations
- Good code available online! (including in OpenCV)
  - Matlab version by Jean-Yves Bouget: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
  - Zhengyou Zhang’s web site: http://research.microsoft.com/~zhang/Calib/

Next time

Stereo