CS4670/5670: Computer Vision

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Lec 18: Single-view modeling

Projective geometry

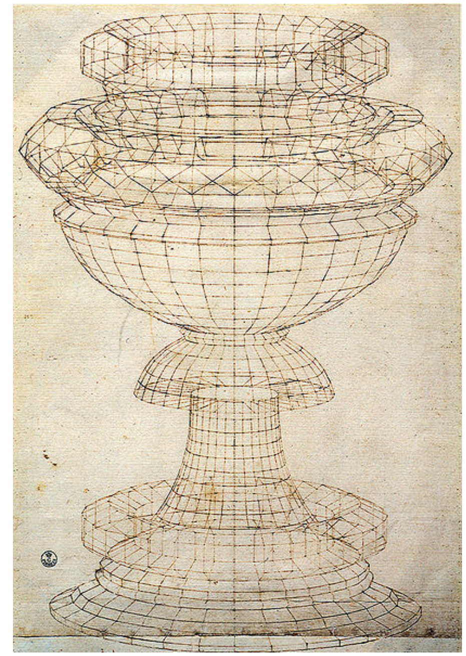
Ames Room

- Readings
  - Mundy, J.L. and Zisserman, A., Geometric Invariance in Computer Vision, Appendix: Projective Geometry for Machine Vision, MIT Press, Cambridge, MA, 1992, **(read 23.1 - 23.5, 23.10)**
    - available online: http://www.cs.cmu.edu/~ph/869/papers/zisser-mundy.pdf

The idea for this Appendix arose from our perception of a frustrating situation faced by vision researchers. For example, one is interested in some aspect of the theory of perspective image formation such as the epipolar line. The interested party goes to the library to check out a book on projective geometry filled with hope that the necessary mathematical machinery will be directly at hand. These expectations are quickly dashed. Upon opening the book, the expectant reader finds the presentation dominated by endless observations about harmonic relations and a few chapters which explore the minutiae of Pappus' theorem. Finally, as a last cruel twist of irony, the book ends in triumph with a rather exhilarating discourse on the conic pencil. All of the material is presented in the form of theorems defined on points, lines and conics without the use of coordinates, except perhaps for a quick pause to define barycentric coordinates just to taunt the reader. Dejected, the vision researcher throws the book aside and contents himself with some calculations using homogeneous coordinates and transformations which are covered briefly in Duda and Hart [93] or perhaps from a book on graphics [113].

Projective geometry—what's it good for?

- Uses of projective geometry
  - Drawing
  - Measurements
  - Mathematics for projection
  - Undistorting images
  - Camera pose estimation
  - Object recognition



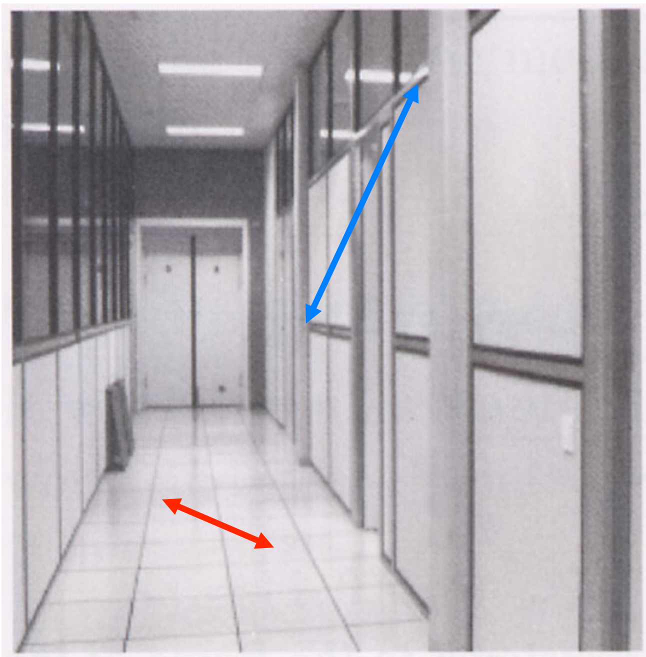
Paolo Uccello

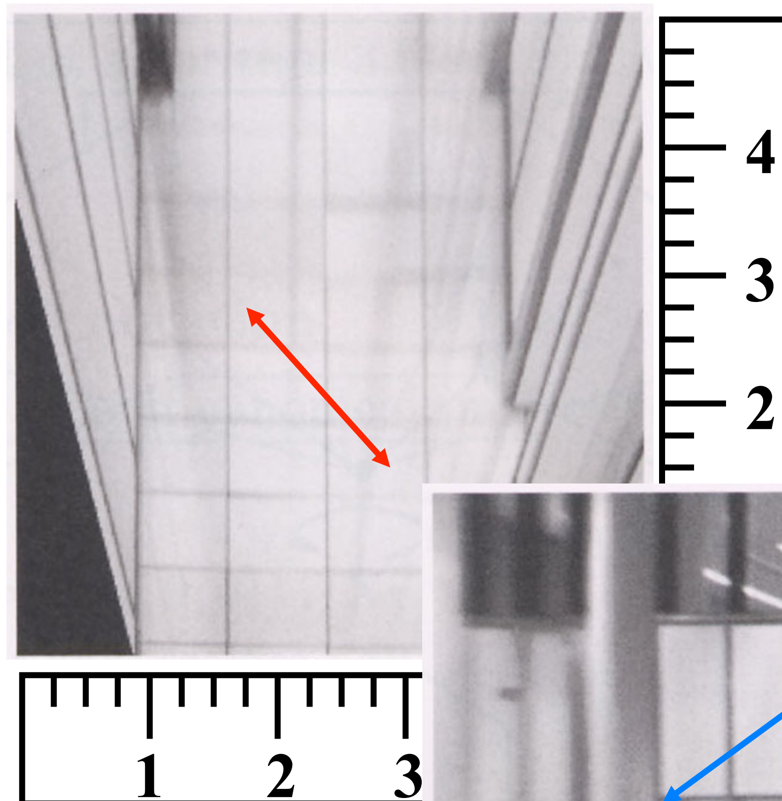
Applications of projective geometry

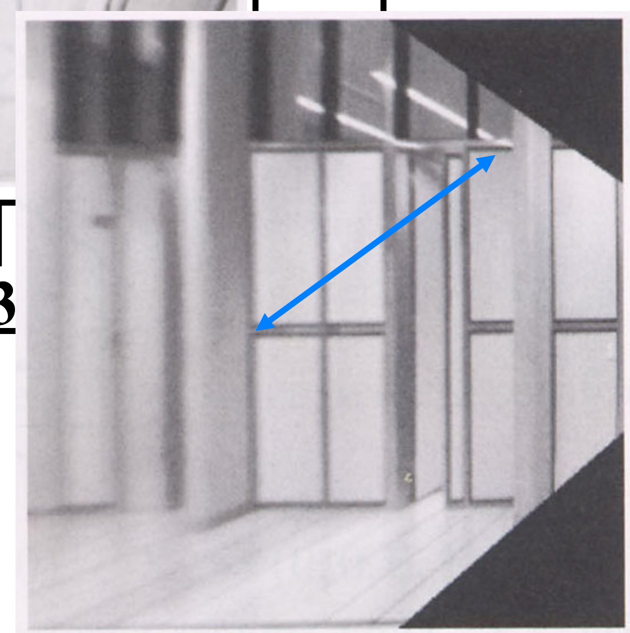
Vermeer's *Music Lesson*

Reconstructions by Criminisi et al.

Measurements on planes







Approach: unwarp then measure

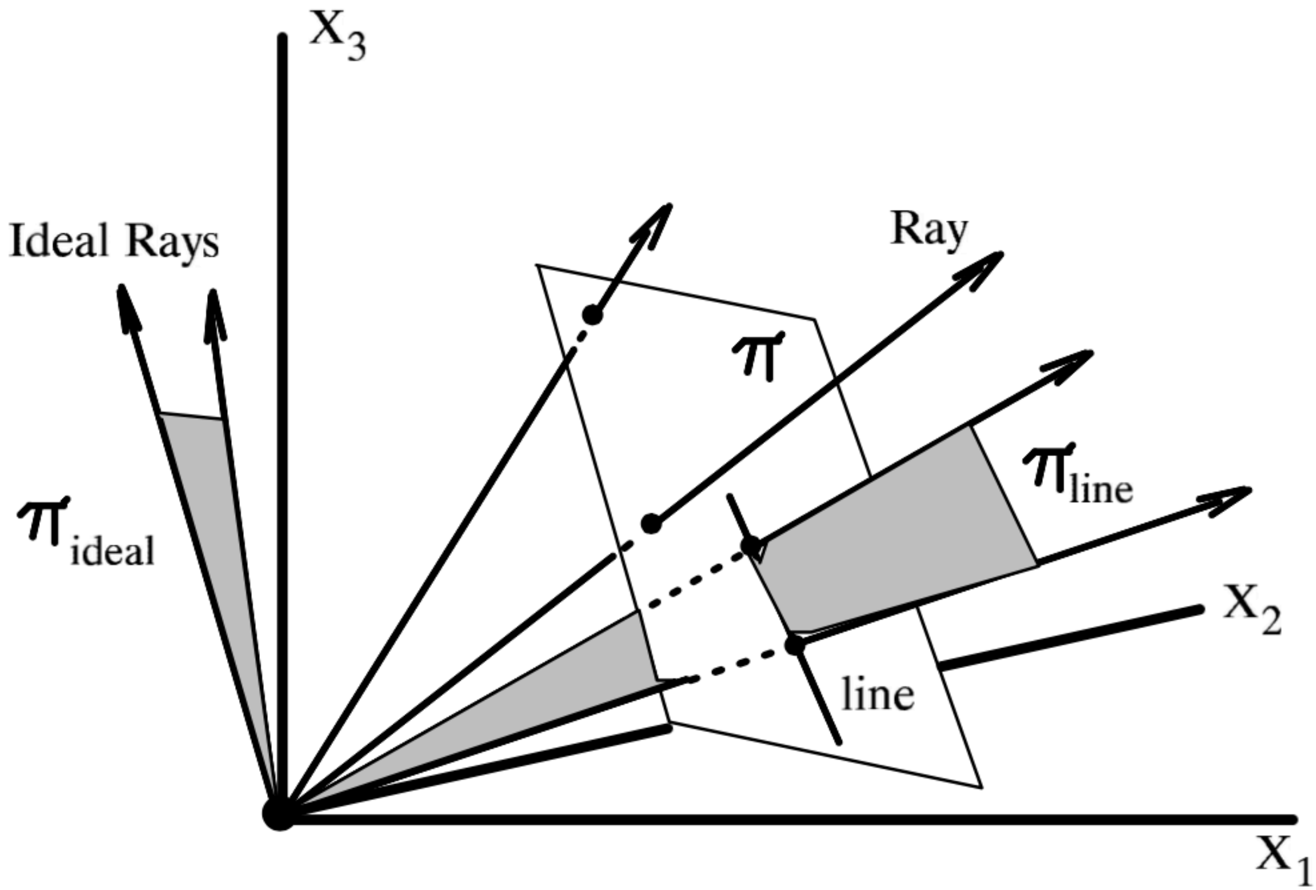
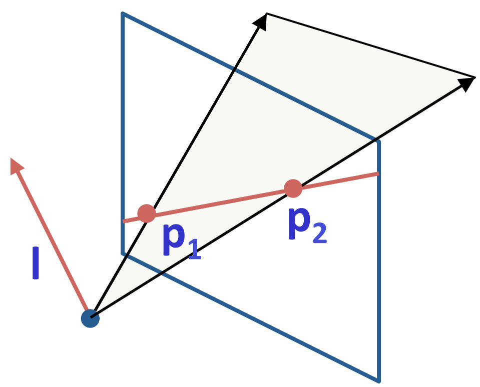
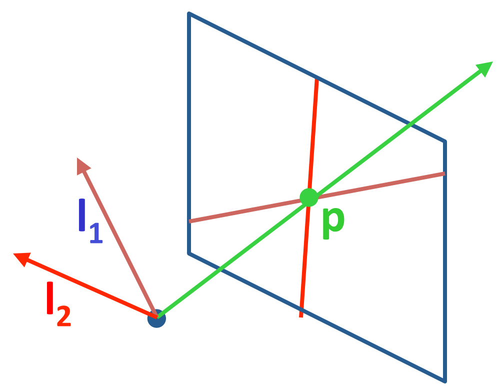


Figure 23.11
A model for the projective plane can be constructed by rays in 3D space. The rays correspond to points in the projective plane. Two rays through the origin define a unique plane though the origin. Any plane through the origin corresponds to a projective line.

Point and line duality

- A line **l** is a homogeneous 3-vector





What is the line **l** spanned by rays $\mathbf{p_1}$ and $\mathbf{p_2}$?

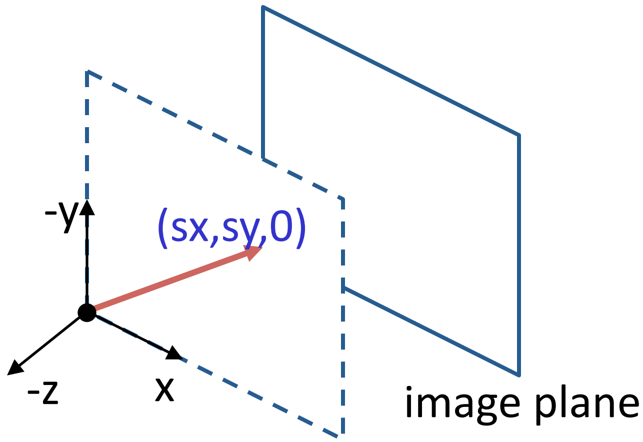
- **l** is $\perp$ to $\mathbf{p_1}$ and $\mathbf{p_2}$ $\Rightarrow$ $\mathbf{l} = \mathbf{p_1} \times \mathbf{p_2}$
- **l** can be interpreted as a *plane normal*

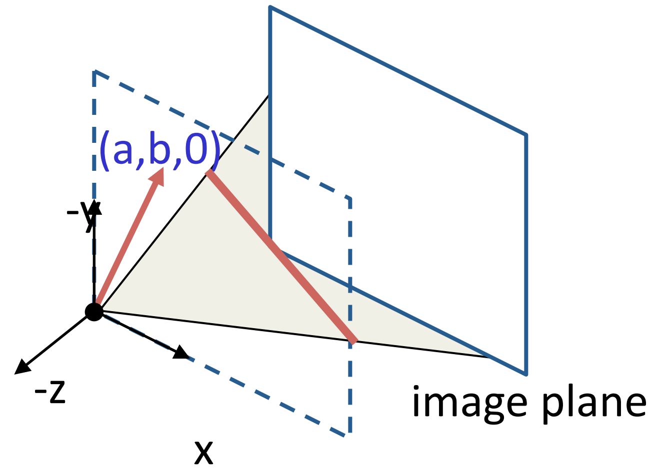
What is the intersection of two lines $\mathbf{l_1}$ and $\mathbf{l_2}$?

- **p** is $\perp$ to $\mathbf{l_1}$ and $\mathbf{l_2}$ $\Rightarrow$ $\mathbf{p} = \mathbf{l_1} \times \mathbf{l_2}$

Points and lines are *dual* in projective space

Ideal points and lines





- Ideal point ("point at infinity")
  - p ≅ (x, y, 0)

Ideal line

- l ≅ (a, b, 0) – parallel to image plane
- Corresponds to a line in the image (finite coordinates)
  - goes through image origin (*principal point*)

3D projective geometry

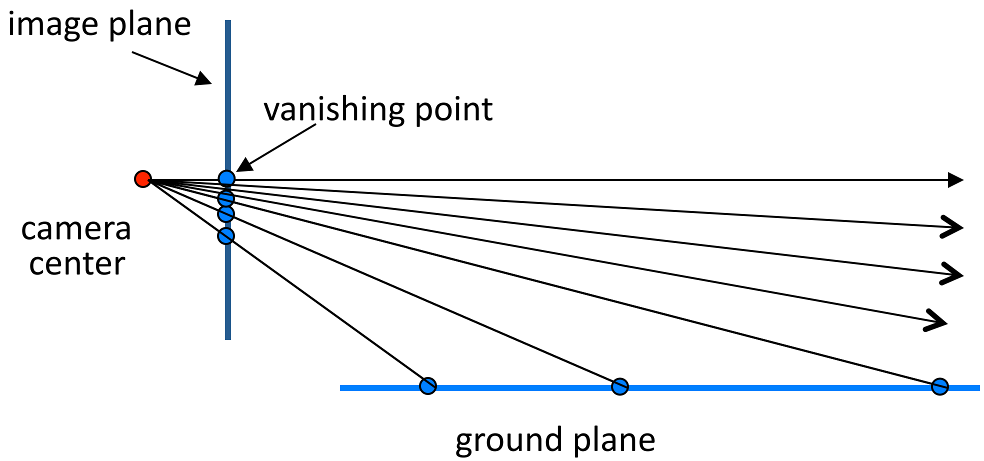
- These concepts generalize naturally to 3D
  - Homogeneous coordinates
    - Projective 3D points have four coords: **P** = (X,Y,Z,W)
  - Duality
    - A plane **N** is also represented by a 4-vector
    - Points and planes are dual in 3D: **N P**=0
    - Three points define a plane, three planes define a point

3D to 2D: perspective projection

Projection:

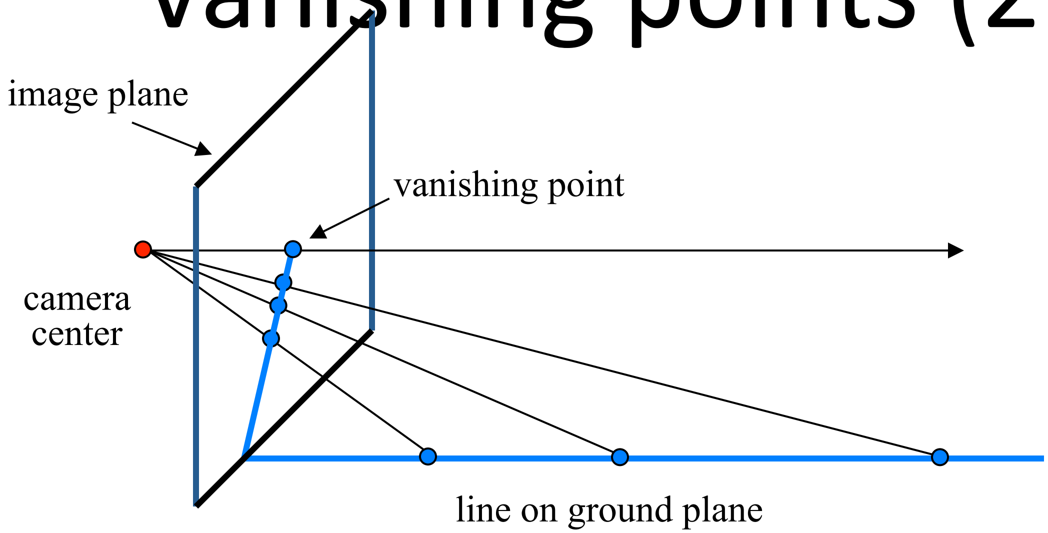
$$\mathbf{p} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi P}$$

Vanishing points (1D)

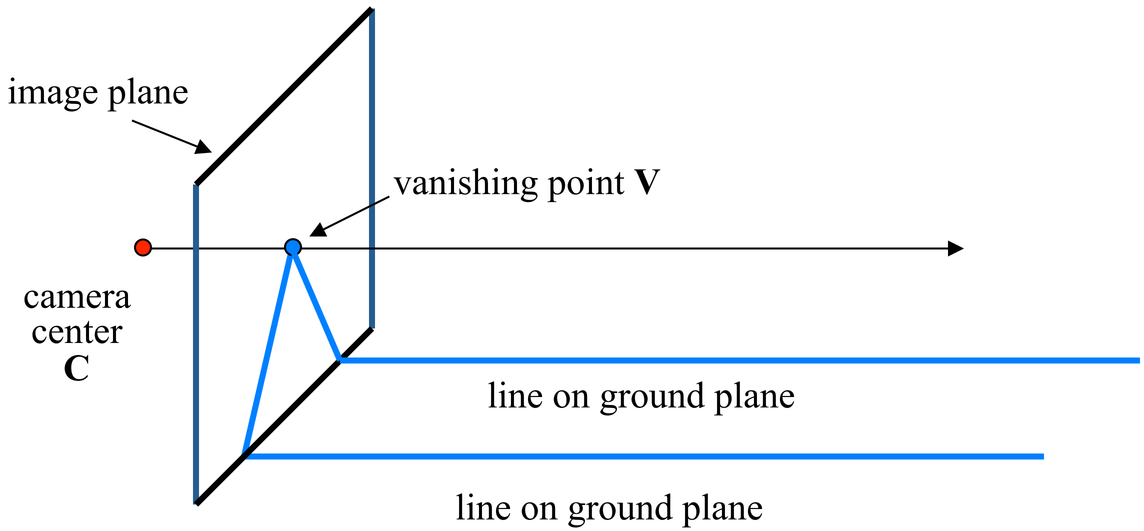


- Vanishing point
  - projection of a point at infinity
  - can often (but not always) project to a finite point in the image

Vanishing points (2D)

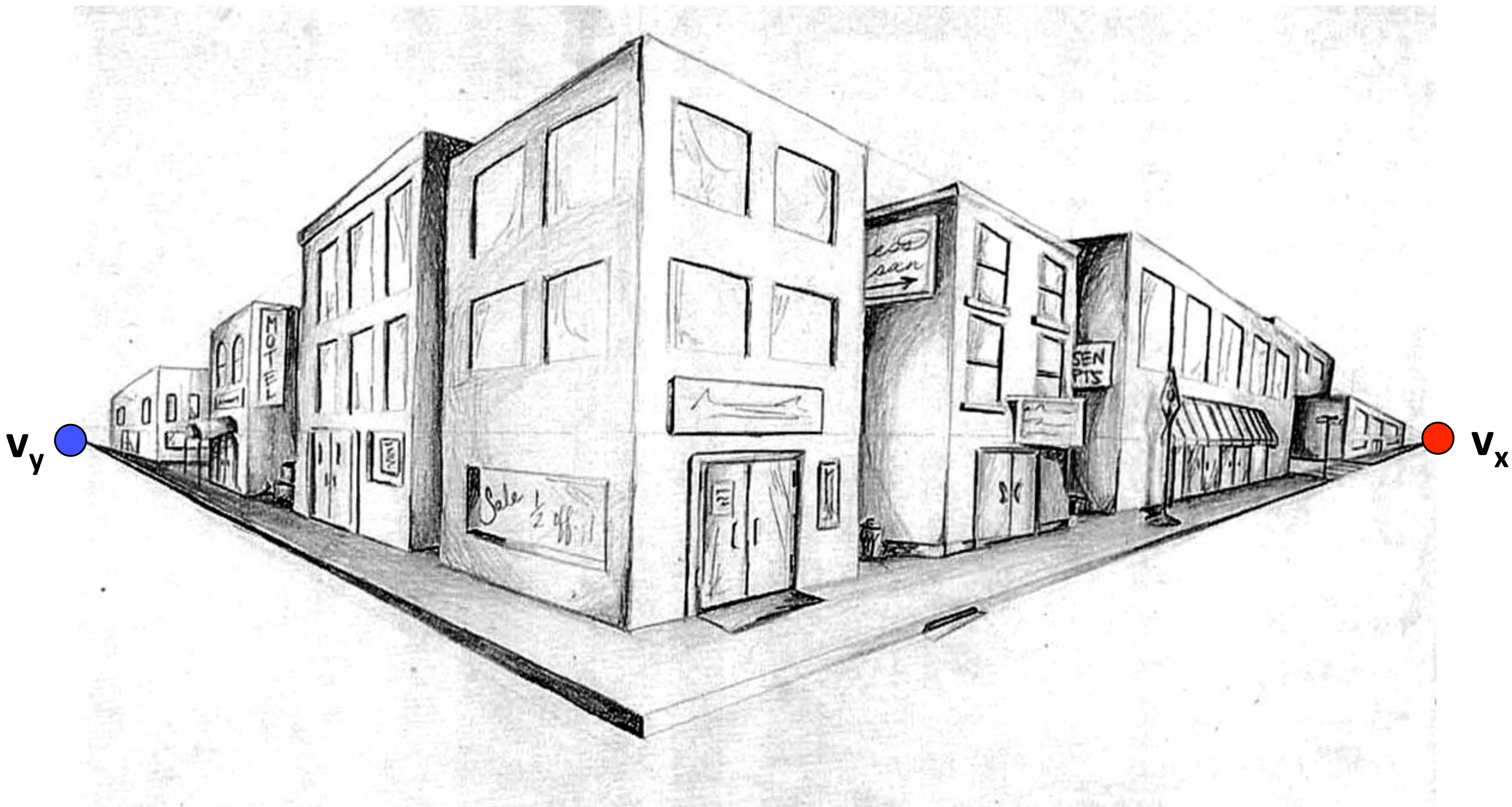


Vanishing points

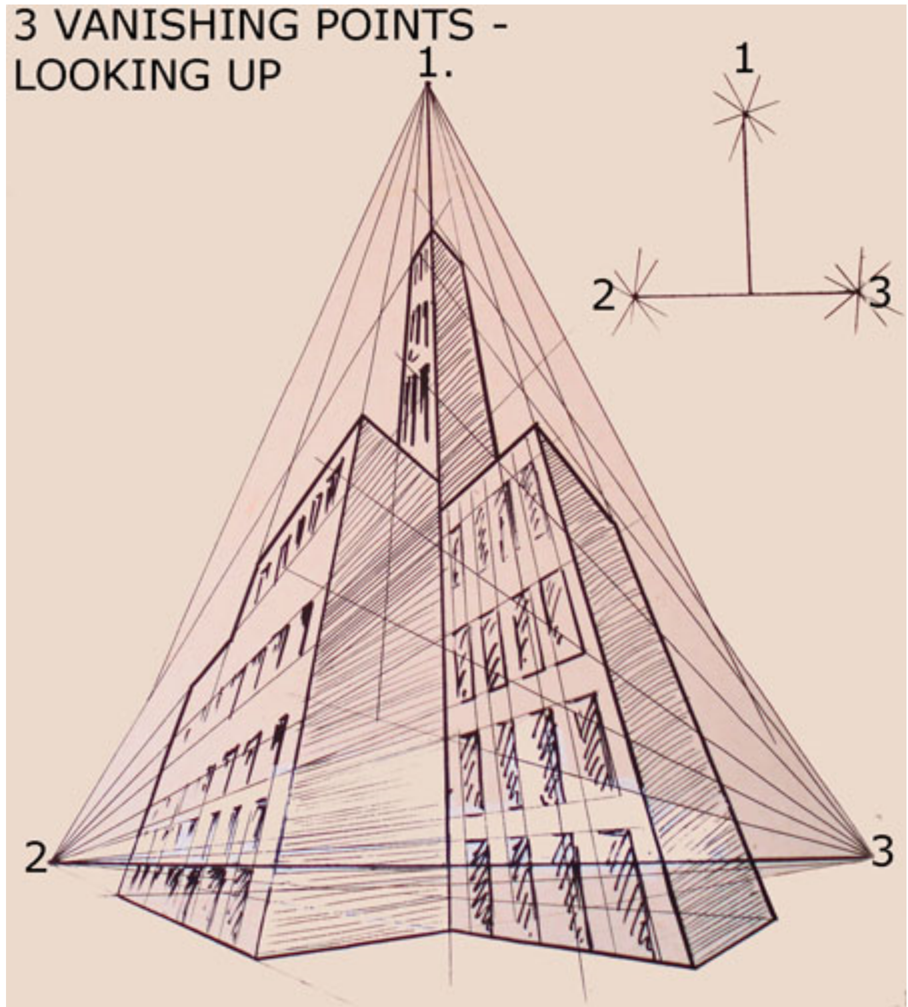


- Properties
  - Any two parallel lines (in 3D) have the same vanishing point **v**
  - The ray from **C** through **v** is parallel to the lines
  - An image may have more than one vanishing point
    - in fact, every image point is a potential vanishing point

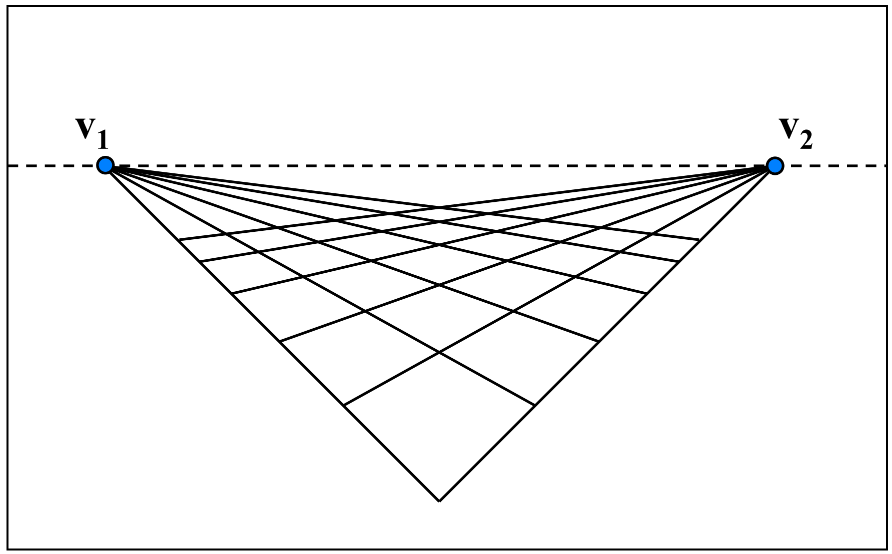
Two point perspective



Three point perspective

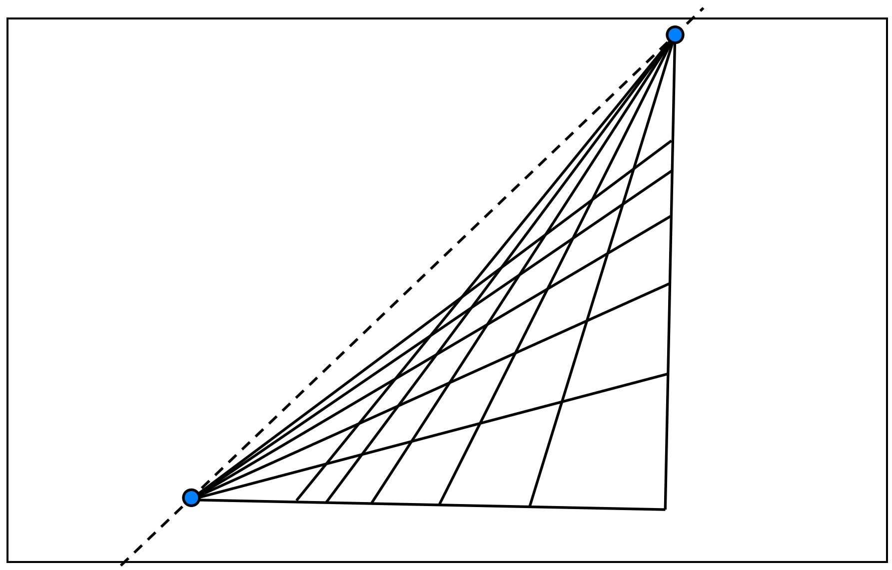


Vanishing lines



- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
  - Note that different planes (can) define different vanishing lines

Vanishing lines



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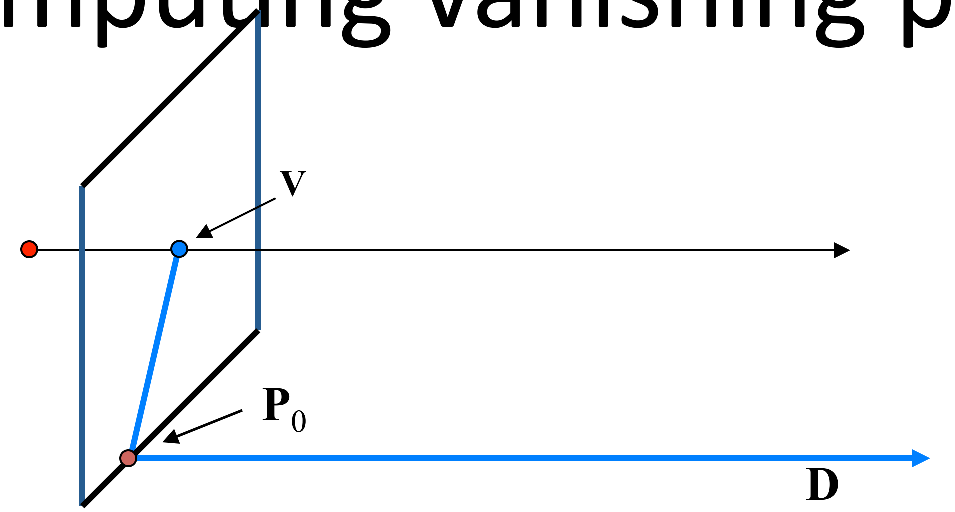
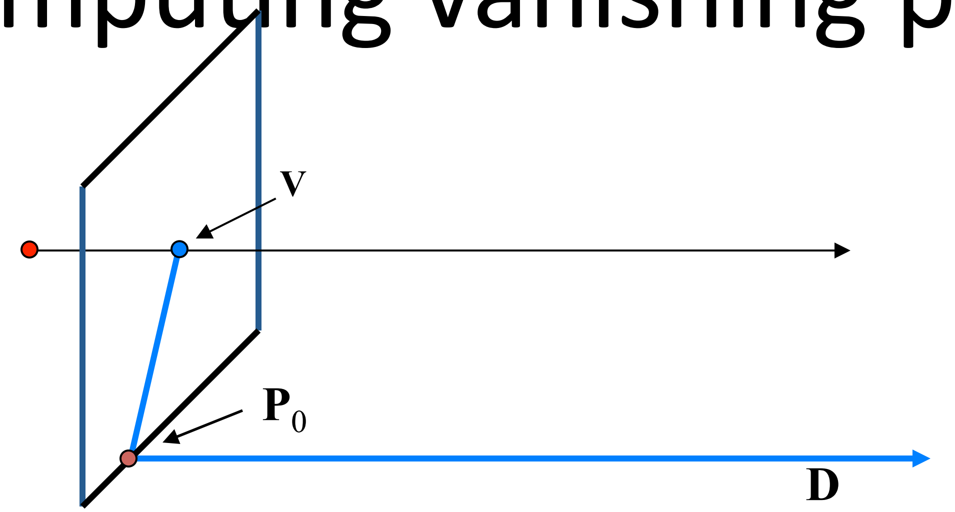
Computing vanishing points



$$\mathbf{P} = \mathbf{P}_0 + t\mathbf{D}$$

Computing vanishing points

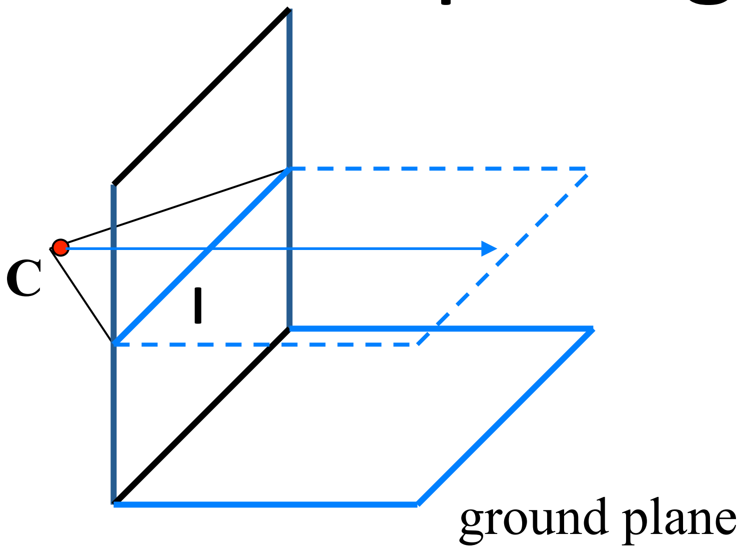


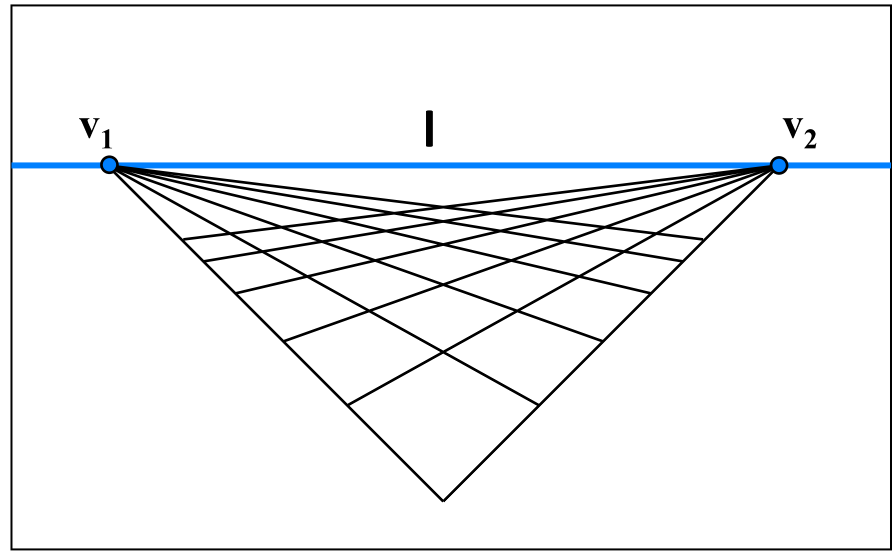
$$\mathbf{P} = \mathbf{P}_0 + t\mathbf{D}$$

$$\mathbf{P}_t = \begin{bmatrix} P_X + tD_X \\ P_Y + tD_Y \\ P_Z + tD_Z \\ 1 \end{bmatrix} \cong \begin{bmatrix} P_X / t + D_X \\ P_Y / t + D_Y \\ P_Z / t + D_Z \\ 1/t \end{bmatrix}$$

- Properties $\mathbf{v} = \mathbf{\Pi P}_\infty$
  - $\mathbf{P}_\infty$ is a point at *infinity*, **v** is its projection
  - Depends only on line *direction*
  - Parallel lines $\mathbf{P}_0 + t\mathbf{D}$, $\mathbf{P}_1 + t\mathbf{D}$ intersect at $\mathbf{P}_\infty$

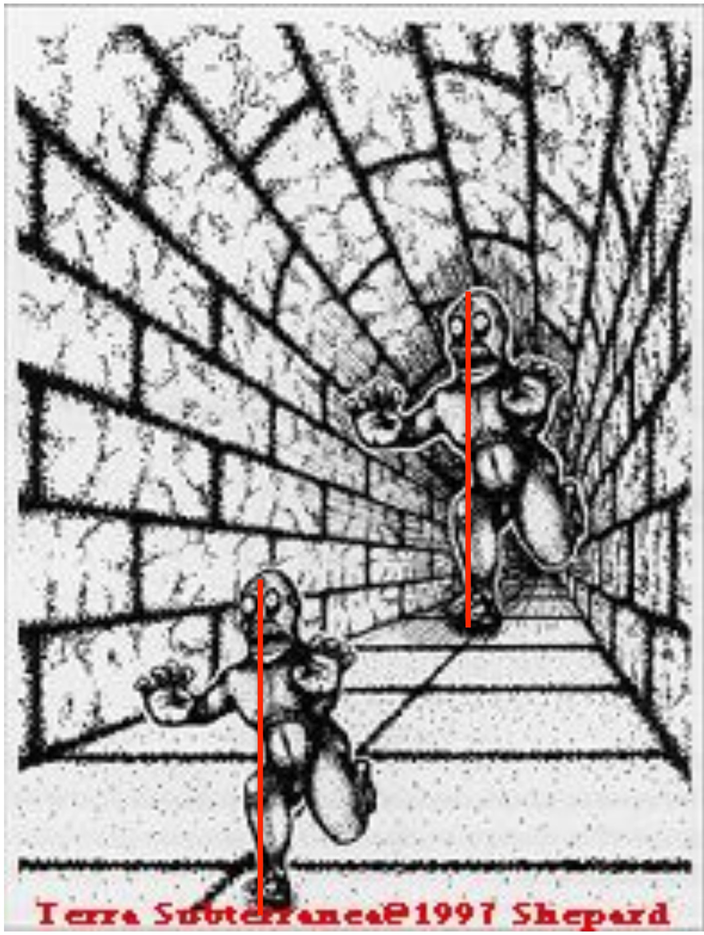
Computing vanishing lines

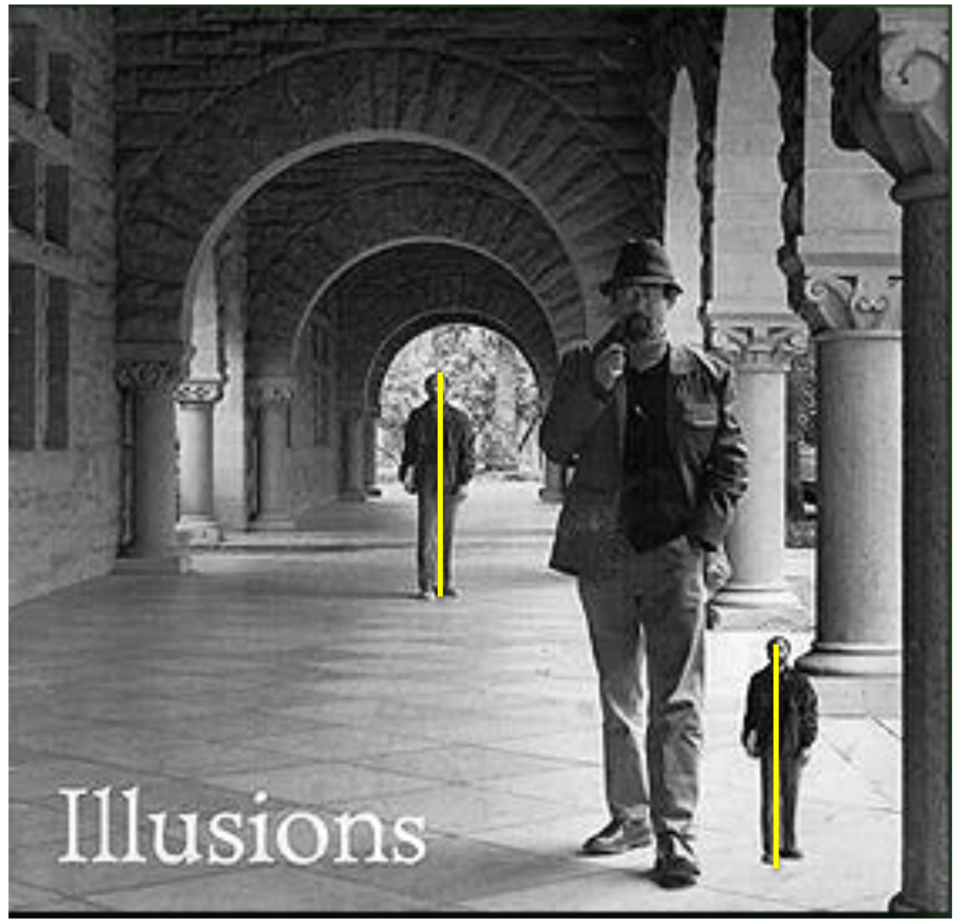




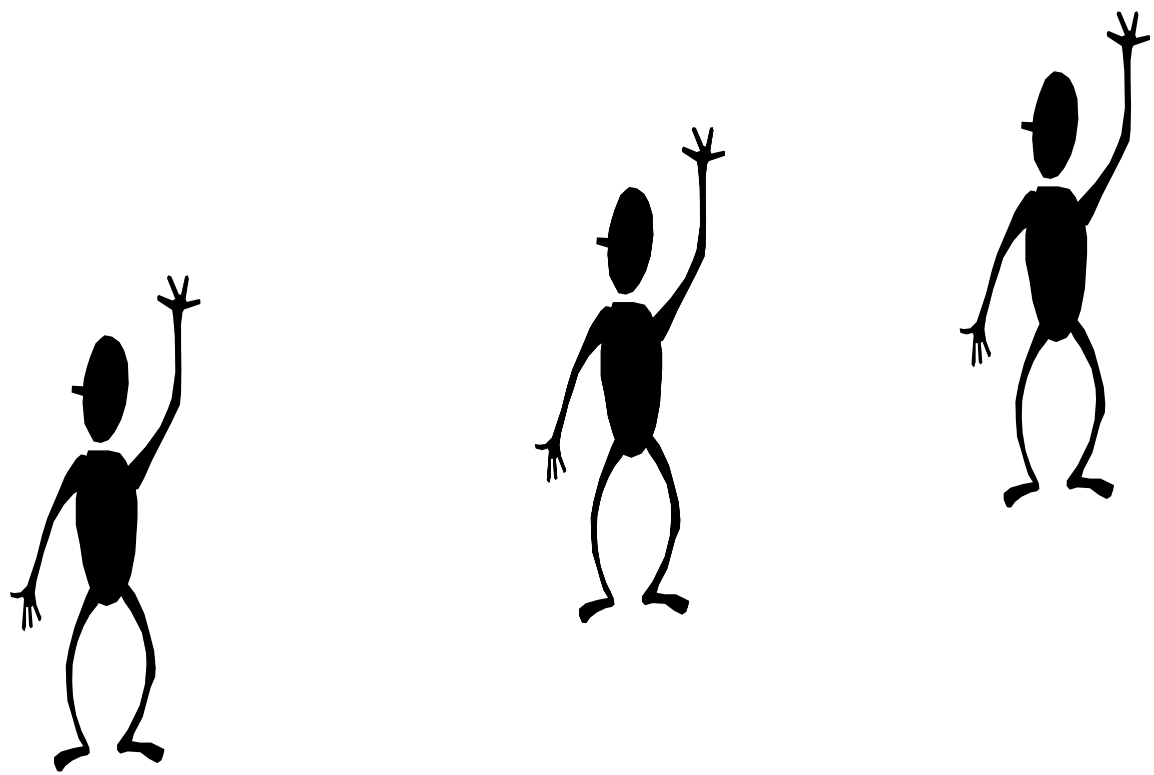
- Properties
  - **l** is intersection of horizontal plane through **C** with image plane
  - Compute **l** from two sets of parallel lines on ground plane
  - All points at same height as **C** project to **l**
    - points higher than C project above l
  - Provides way of comparing height of objects in the scene

Fun with vanishing points

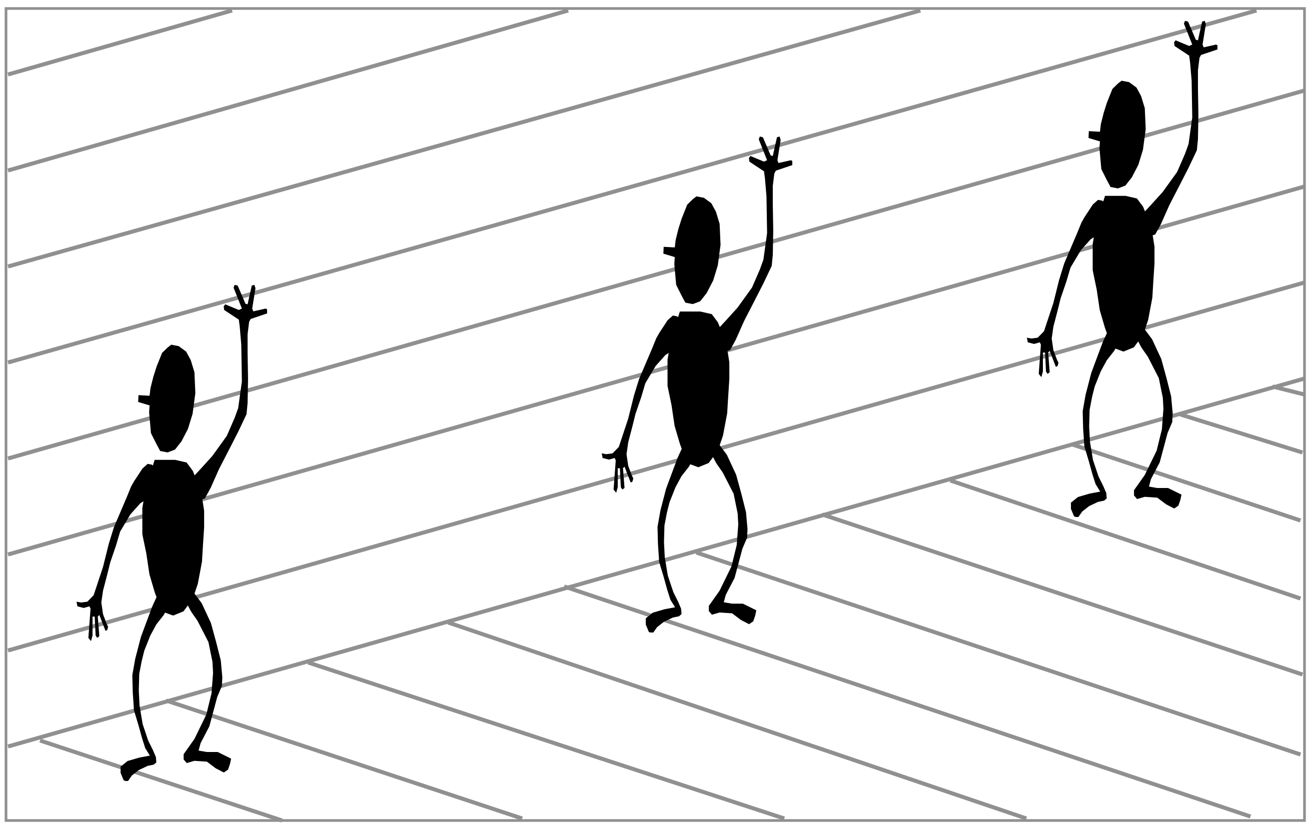




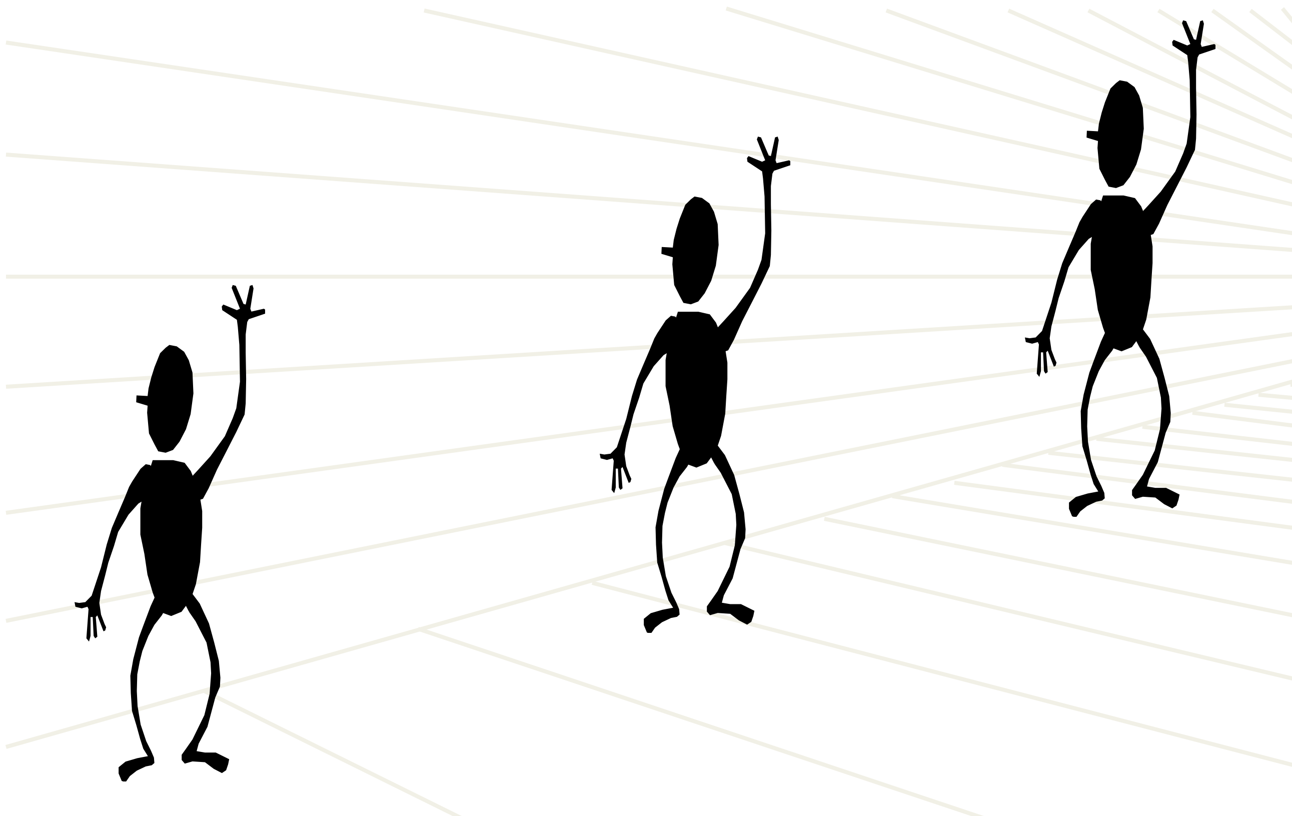
Perspective cues



Perspective cues

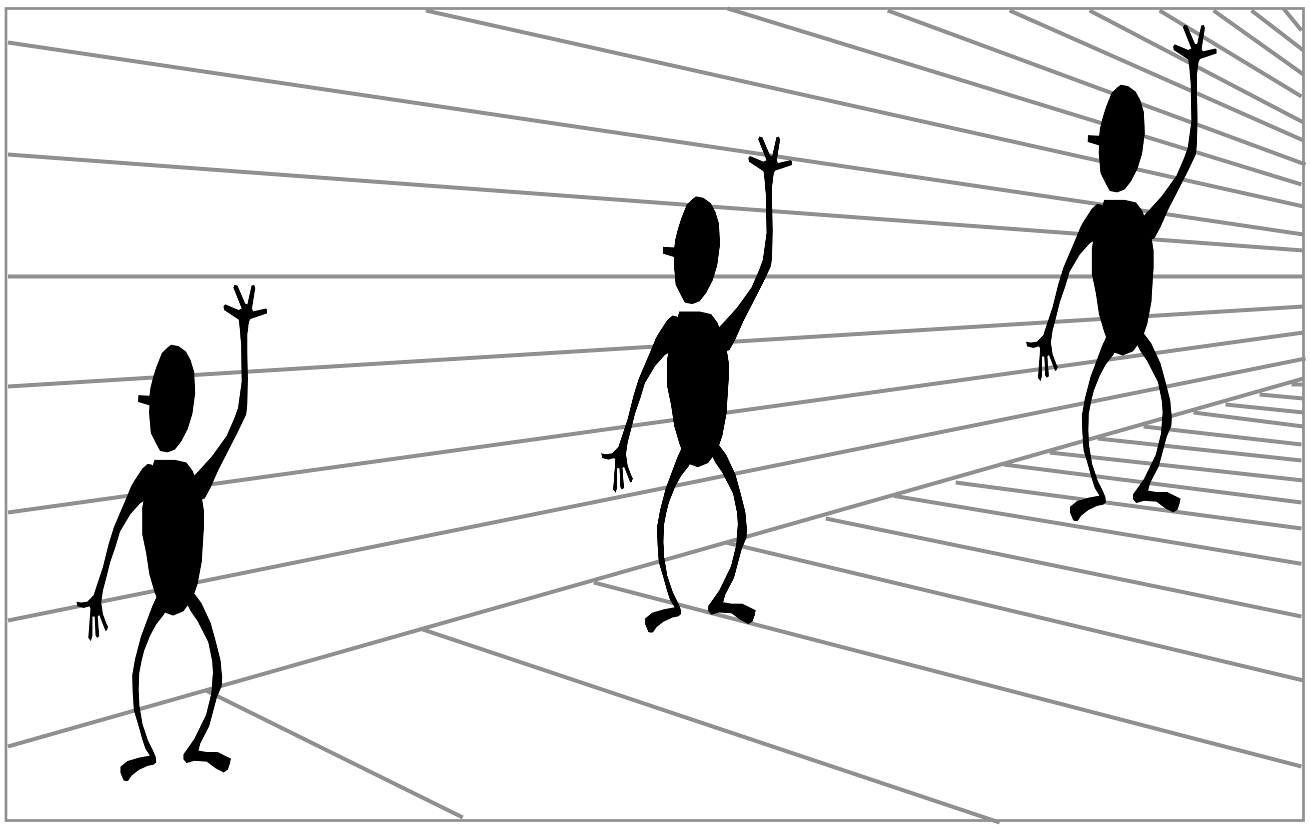


Perspective cues

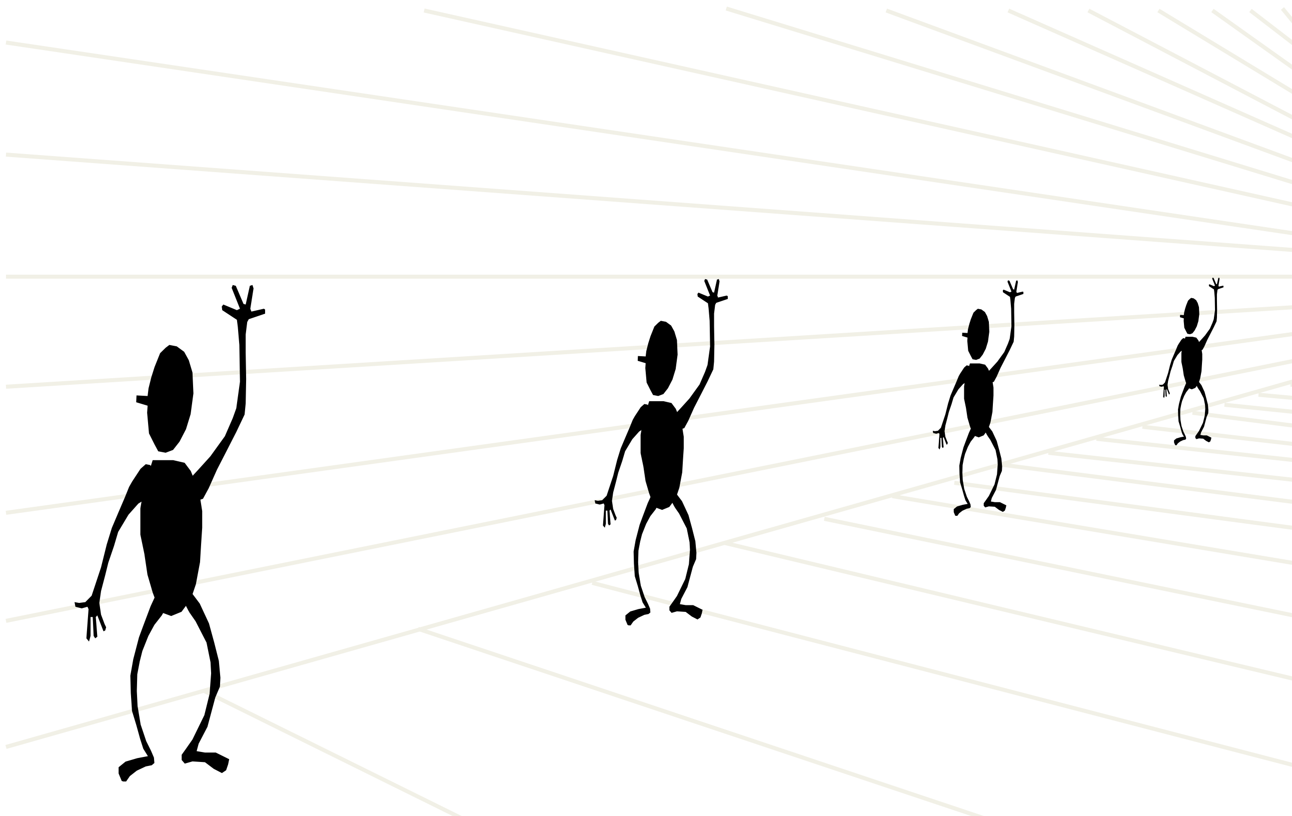


Slide by Steve Seitz

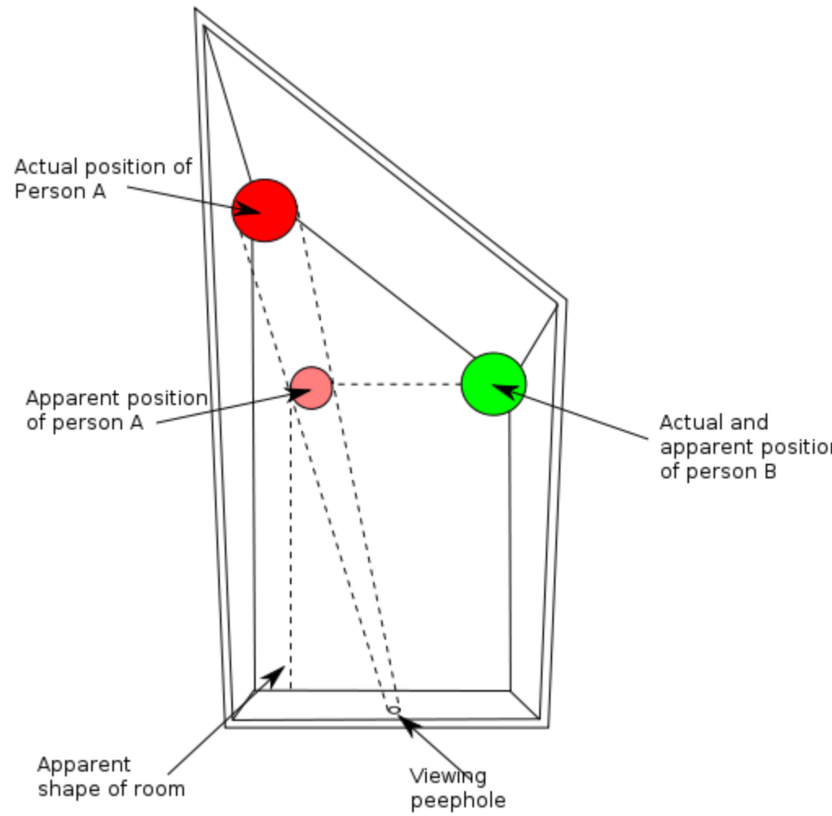
Perspective cues



Perspective cues



Ames Room


Actual position of
Person A
Apparent position
of person A
Actual and
apparent position
of person B
Apparent
shape of room
Viewing
peephole